
LINEAR CONTROL SYSTEMS

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Lecture 9

Controller design in the frequency domain

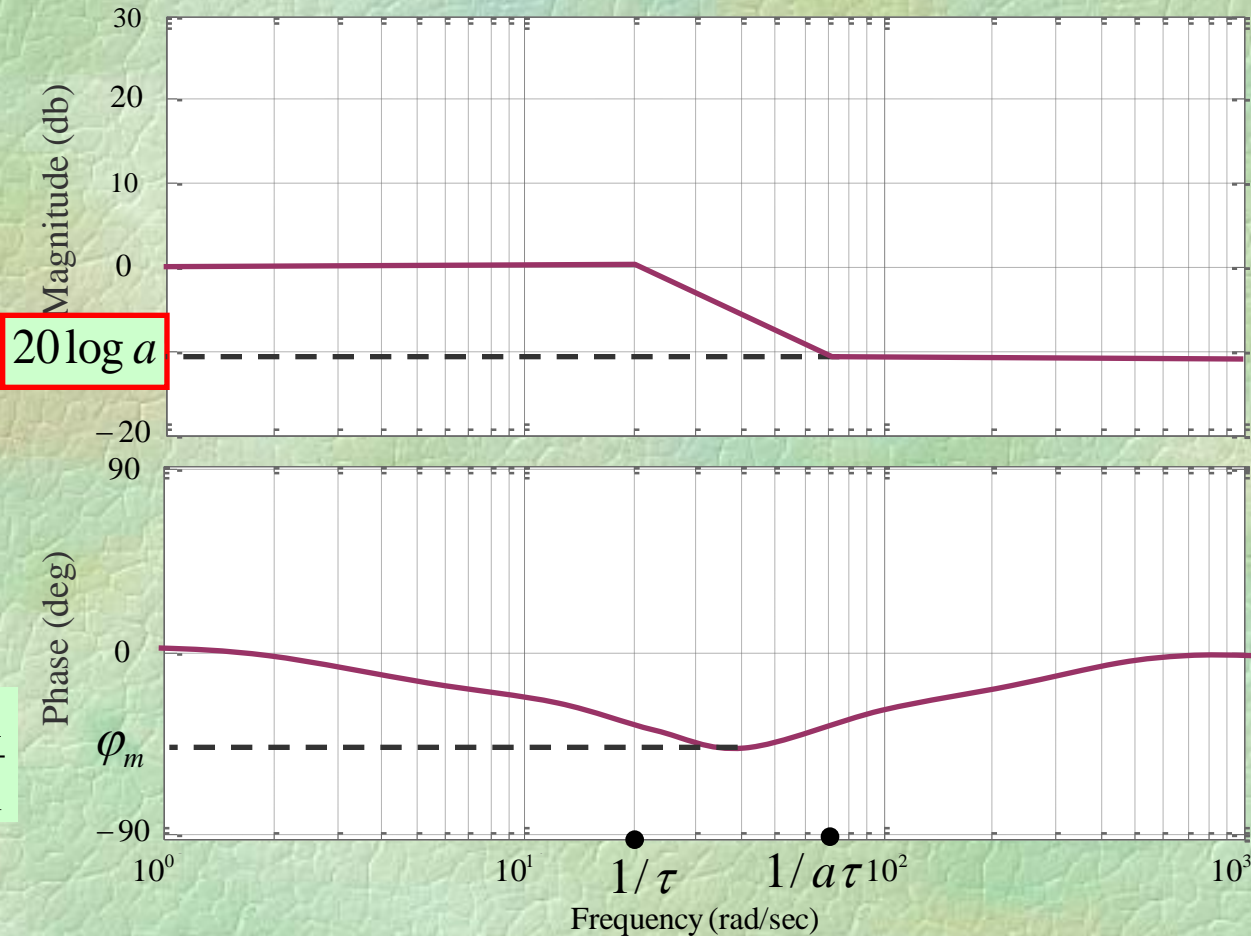
Topics to be covered include:

- ❖ Phase-lag controller design.
- ❖ Phase-lead controller design

A phase-lag controller

$$G(s) = \frac{a\tau s + 1}{\tau s + 1}$$

Let $a < 1$



$$\sin \varphi_m = \frac{a-1}{a+1}$$

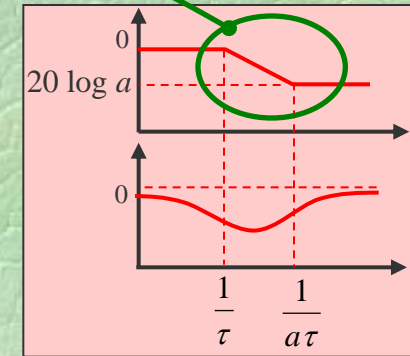
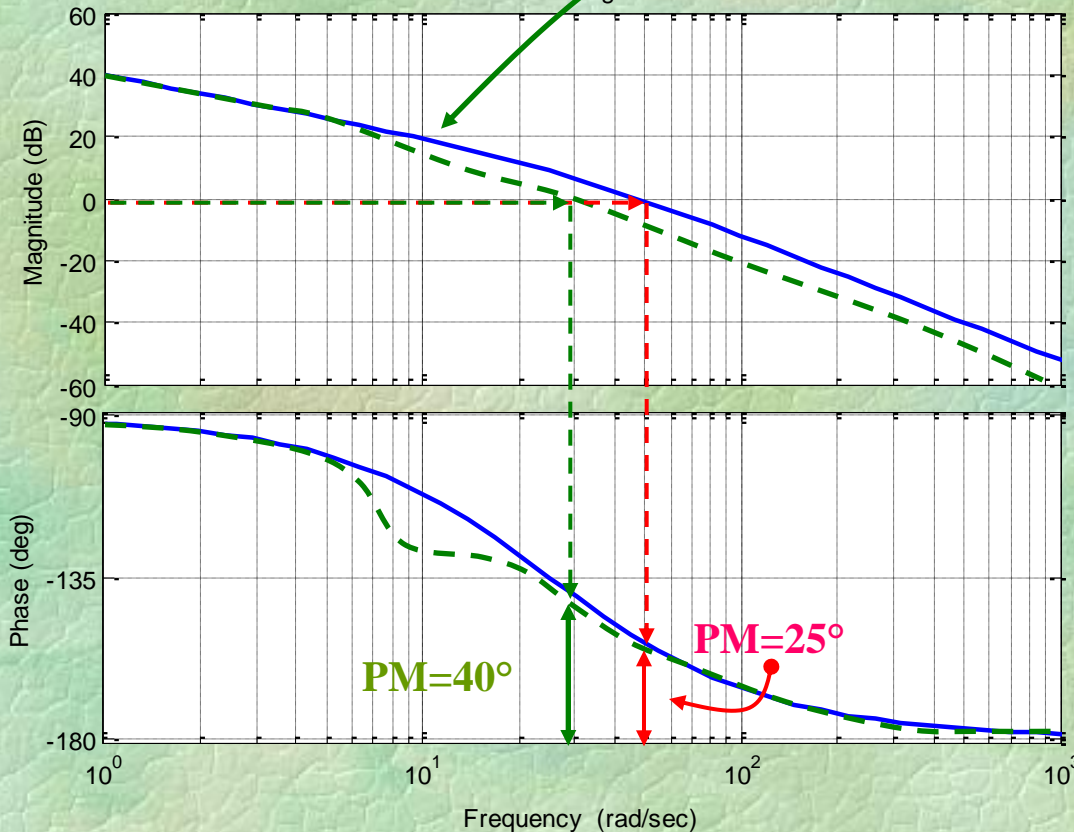
Design fundamental of a lag controller

Analysis

Design

Consider a minimum phase system.

Bode Diagram



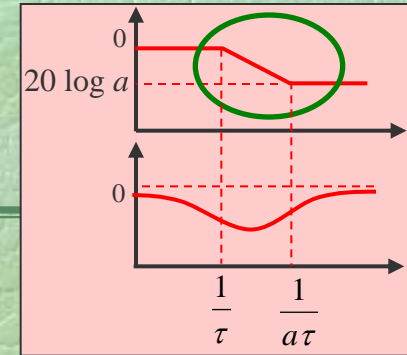
How can the lag controller help us?

Is it different from a Proportional controller?

What is the effect of a lag controller on **BW**? Speed of system?

What is the effect of a lag controller on noise effect?

Design procedure of a phase-lag controller in the frequency domain



Step 1: Consider $G_c(s) = k \frac{a\tau s + 1}{\tau s + 1}$ with $a < 1$ as a phase-lag controller.

Note: If the plant has another gain k , let $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$

Step 2: Try to fix k according to the performance request, otherwise let $k=1$ ✓

Step 3: Sketch the Bode plot of the system (with the fixed k) without controller.

Step 4: According to desired PM (GM) choose the new gain crossover frequency (Phase crossover frequency). (reduce it a little).?

Step 5: Find the required gain by lag controller and **derive the parameter a** .

$$20\log(a) = \dots \Rightarrow a = \checkmark$$

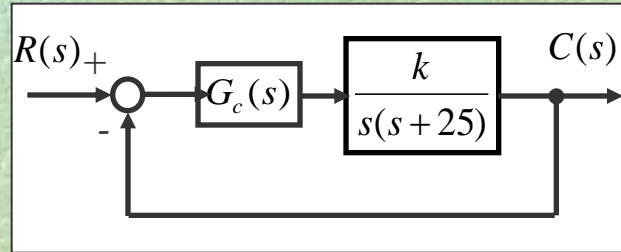
Step 6: Put the right corner of the controller sufficiently far from crossover frequency.

$$\frac{1}{a\tau} = \frac{\omega_{cross}^{new}}{10} \Rightarrow \tau = \checkmark$$

Step 7: Check the designed controller.

Phase-lag controller design in the frequency domain

Example 1: Design a lag controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.



Step 1: Consider $G_c(s) = k \frac{a\tau s + 1}{\tau s + 1}$ with $a < 1$ as a phase-lag controller.

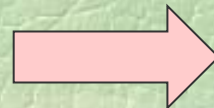
Note: If the plant has another gain k , let $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$



$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} \quad a < 1$$

Step 2: Try to fix k according to the performance request, otherwise let $k=1$

$$k_v = \lim_{s \rightarrow 0} s G_c(s) \frac{k}{s(s+25)} = 100$$



$$k = 2500$$

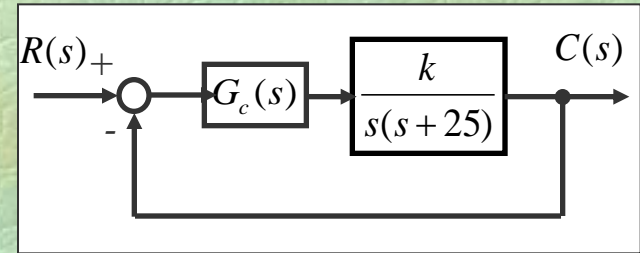
Phase-lag controller design in the frequency domain

Step 3: Sketch the Bode plot of the system (with the fixed k) without controller.

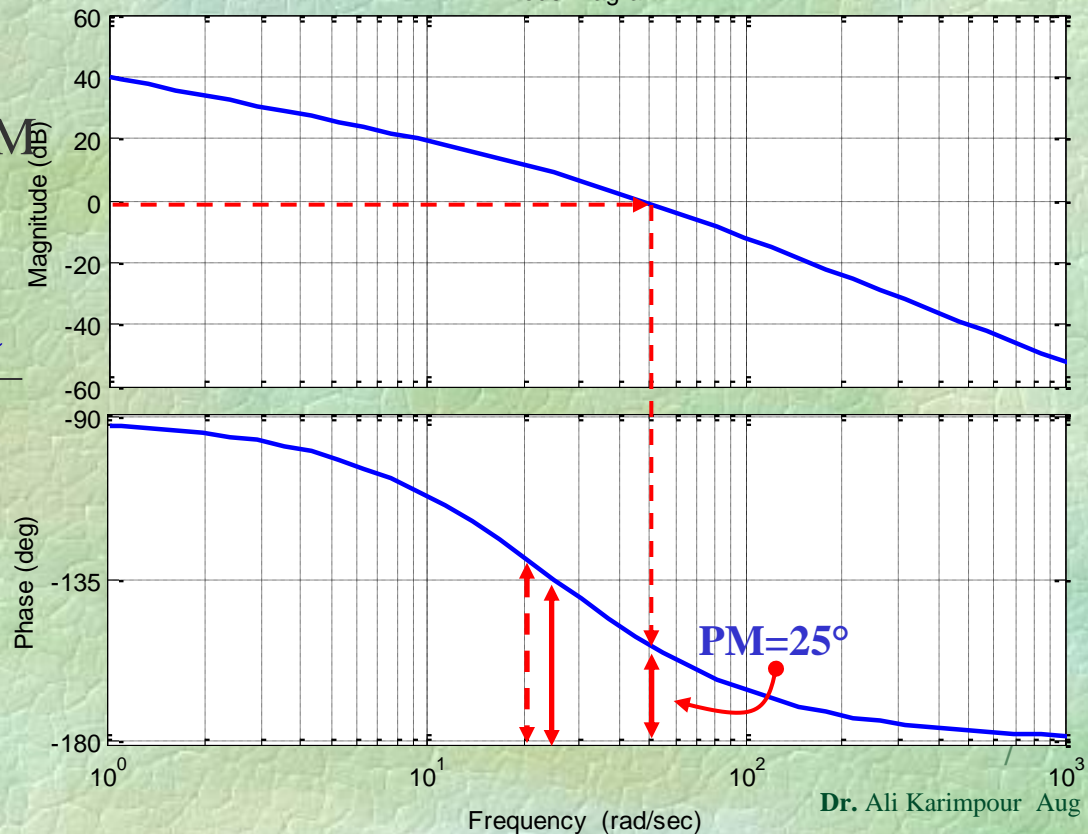
$$G(s) = \frac{2500}{s(s+25)} = \frac{100}{s(s/25+1)}$$

Step 4: According to desired PM (GM) choose the new gain crossover frequency (Phase crossover frequency). (reduce it a little).

$$\cancel{\omega_{cross}^{new} = 25} \quad \omega_{cross}^{new} = 20$$

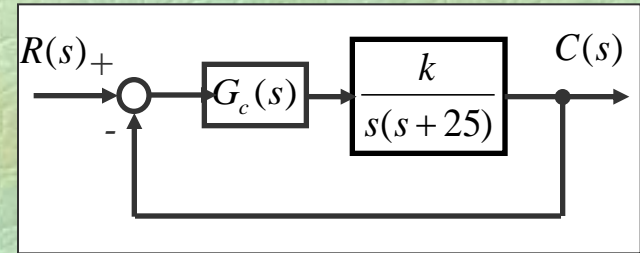


Bode Diagram



Phase-lag controller design in the frequency domain

Step 5: Find the required gain by lag controller and **derive the parameter a**.



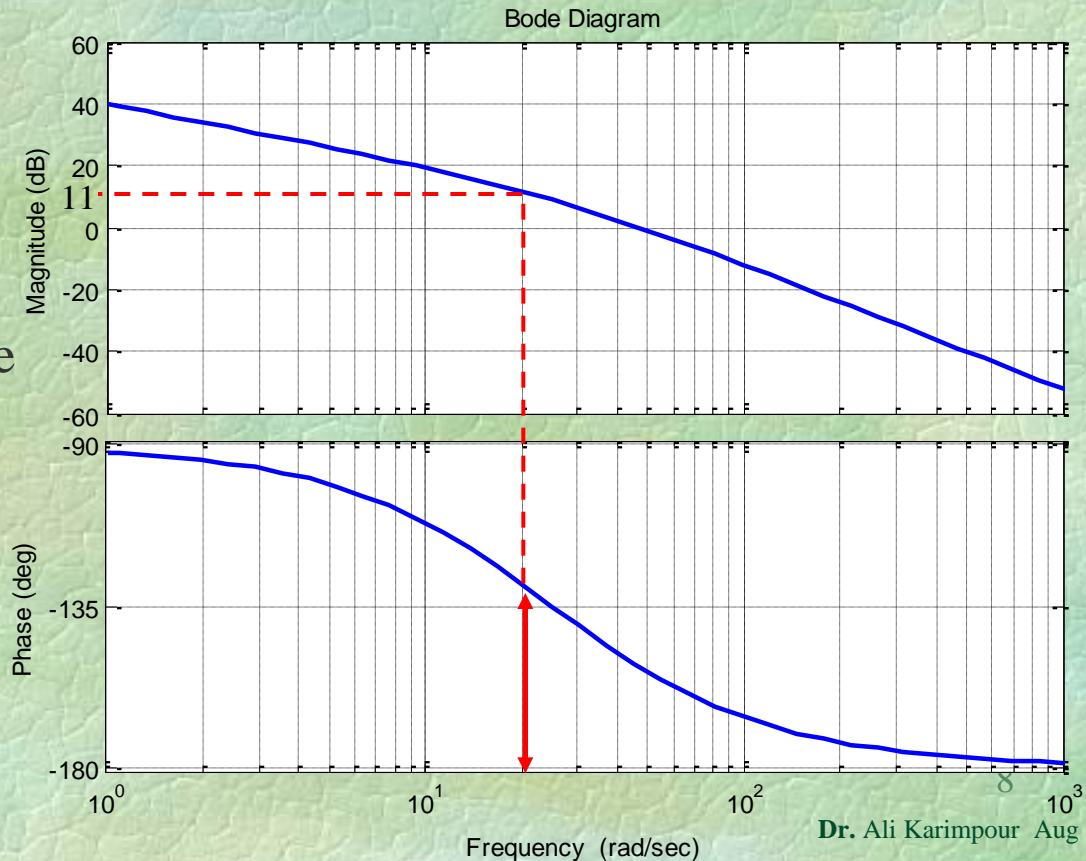
$$20\log(a) = -11$$

$$a = 10^{-\frac{11}{20}} = 0.28$$

Step 6: Put the right corner of the controller sufficiently far from crossover frequency.

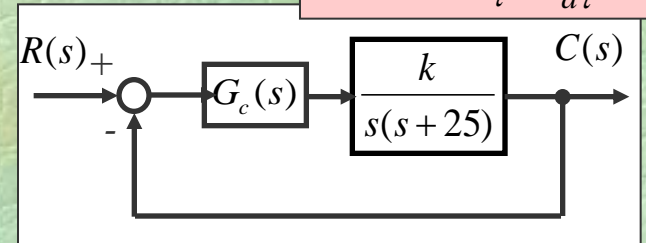
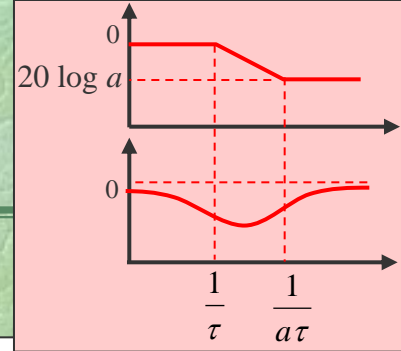
$$\frac{1}{a\tau} = \frac{\omega_{cross}^{new}}{10} = 2 \quad \tau = 1.79$$

$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{0.5s + 1}{1.79s + 1}$$



Phase-lag controller design in the frequency domain

Step 7: Check the designed controller.



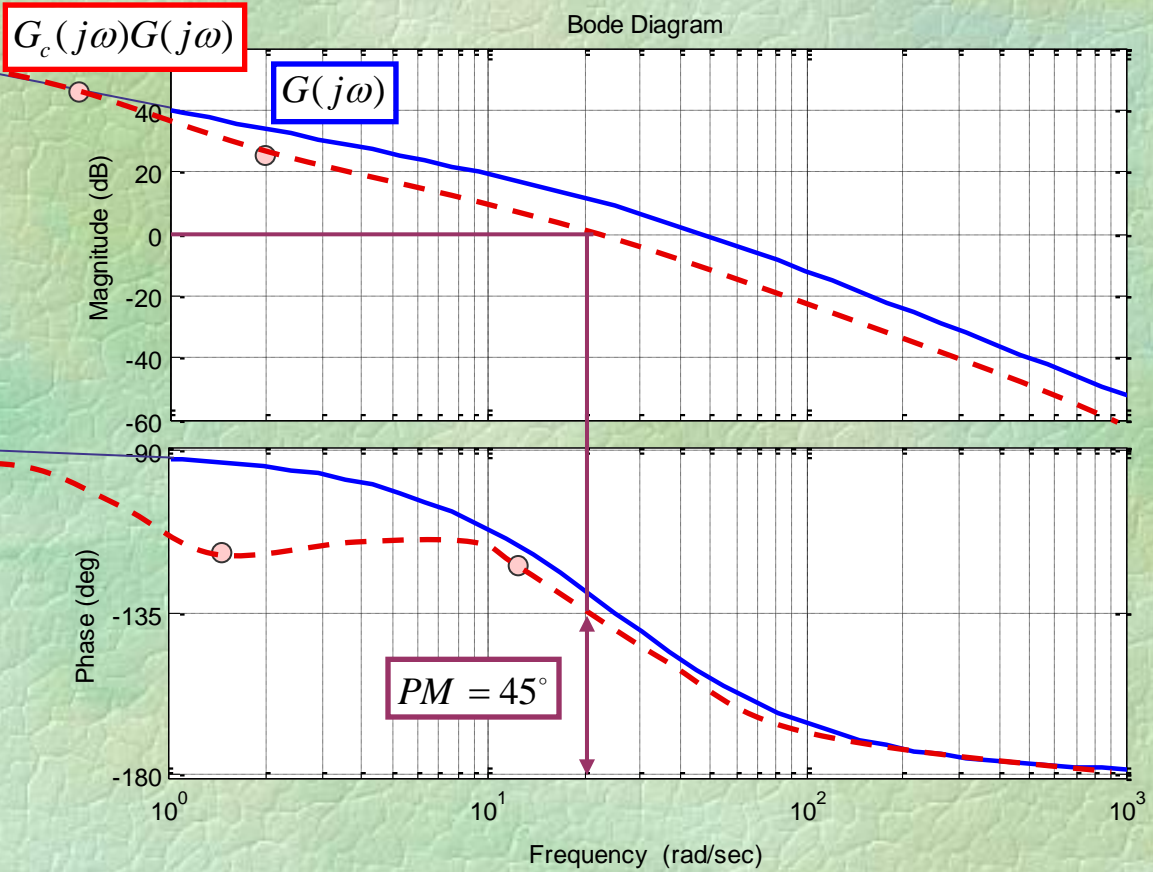
$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{1.79 \times 0.28s + 1}{1.79s + 1}$$

$$1/\tau = 0.55, 1/a\tau = 2$$

$$20 \log a = -11 \text{ dB}$$

$$1/\sqrt{a\tau} = 1.05$$

$$\sin \varphi_m = \frac{0.28 - 1}{0.28 + 1} \Rightarrow \varphi_m = -34^\circ$$



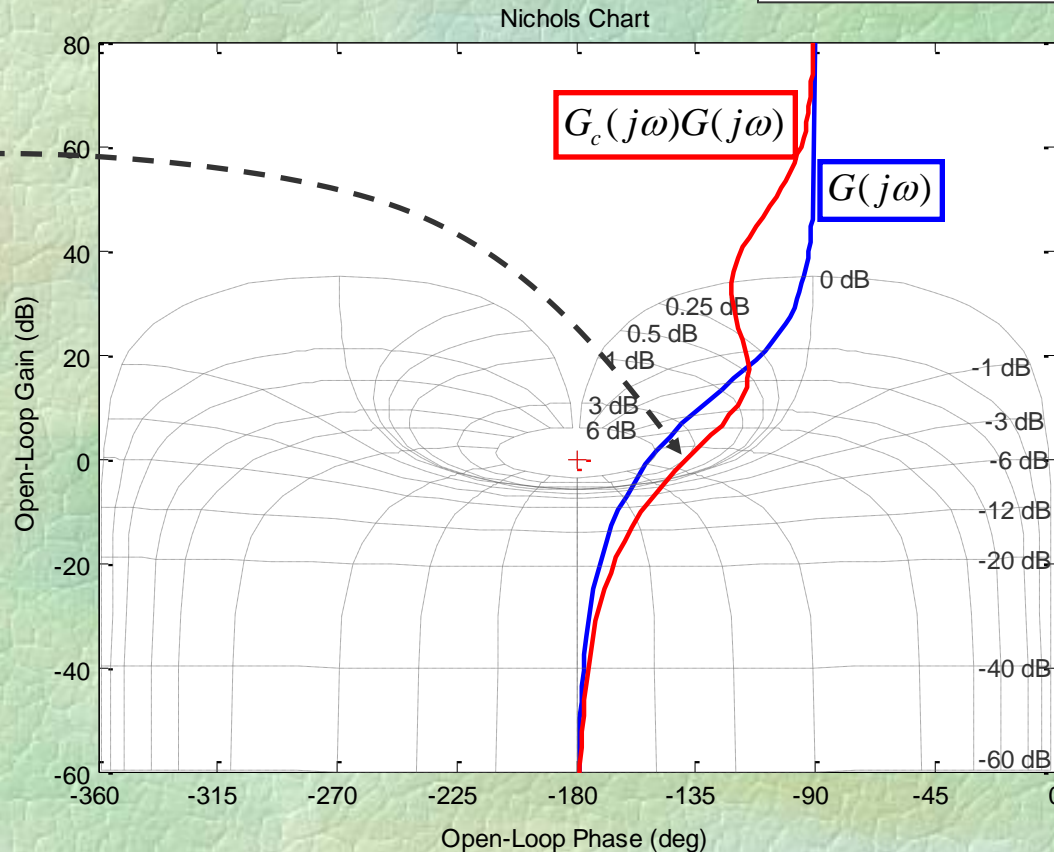
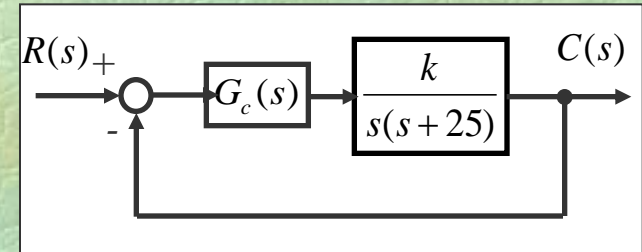
Phase-lag controller design in the frequency domain

Finding M_p

Without controller $M_p = 6 \text{ db}$

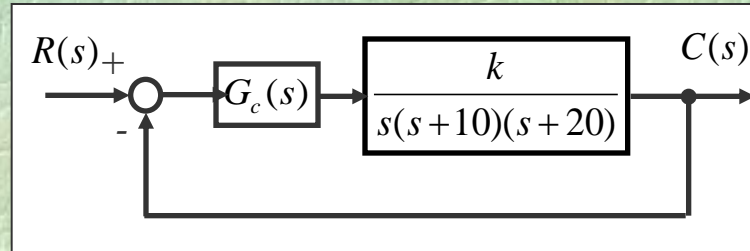
After applying
controller

$M_p = 3 \text{ db}$



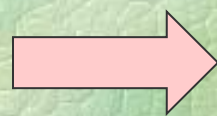
Phase-lag controller design in the frequency domain

Example 2: Design a lag controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



Step 1: Consider $G_c(s) = k \frac{a\tau s + 1}{\tau s + 1}$ with $a < 1$ as a phase-lag controller.

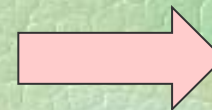
Note: If the plant has another gain k , let $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$



$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} \quad a < 1$$

Step 2: Try to fix k according to the performance request, otherwise let $k=1$

$$k_v = \lim_{s \rightarrow 0} s G_c(s) \frac{k}{s(s+10)(s+20)} = 100$$



$$k = 20000$$

Phase-lag controller design in the frequency domain

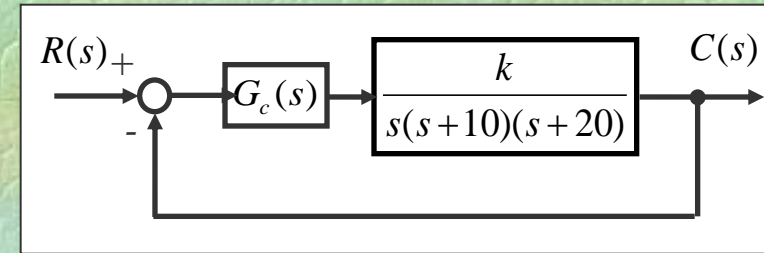
Step 3: Sketch the Bode plot of the system (with the fixed k) without controller.

$$G(s) = \frac{20000}{s(s+10)(s+20)} = \frac{100}{s(s/10+1)(s/20+1)}$$

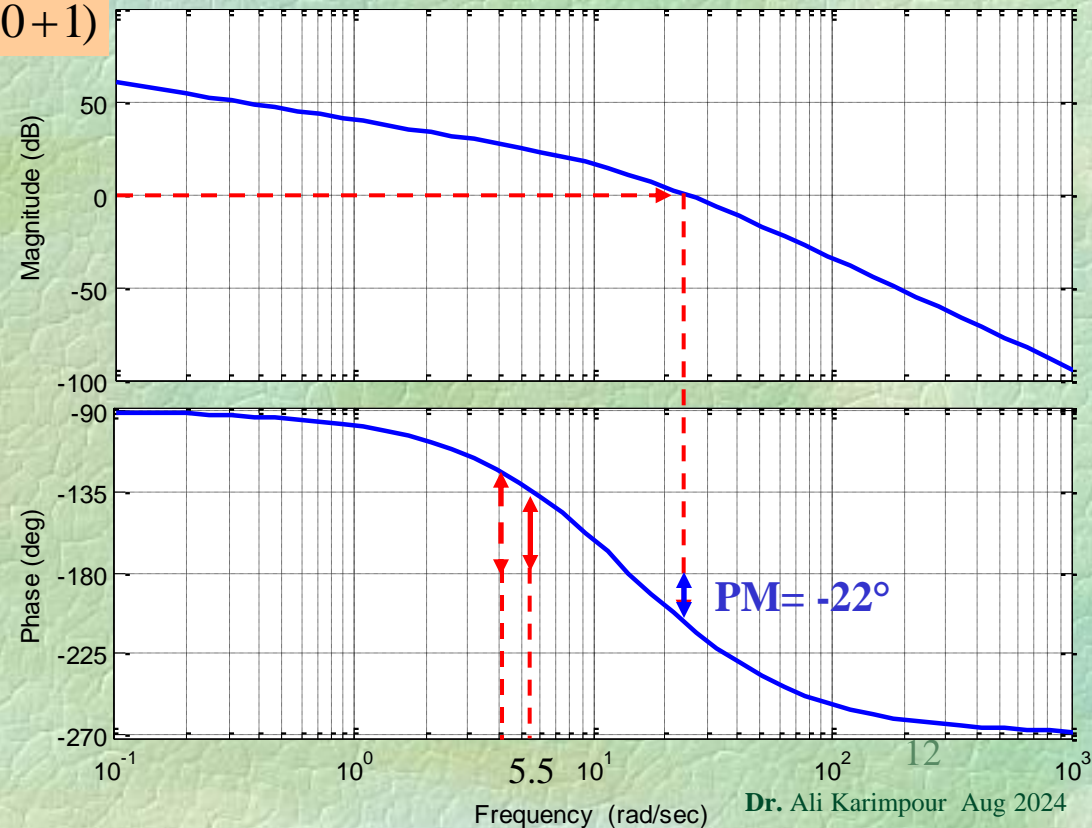
Step 4: According to desired PM (GM) choose the new gain crossover frequency (Phase crossover frequency). (reduce it a little).

~~$$\omega_{cross}^{new} = 5.5$$~~

$$\omega_{cross}^{new} = 4$$



Bode Diagram



Phase-lag controller design in the frequency domain

Step 5: Find the required gain by lag controller and **derive the parameter a**.

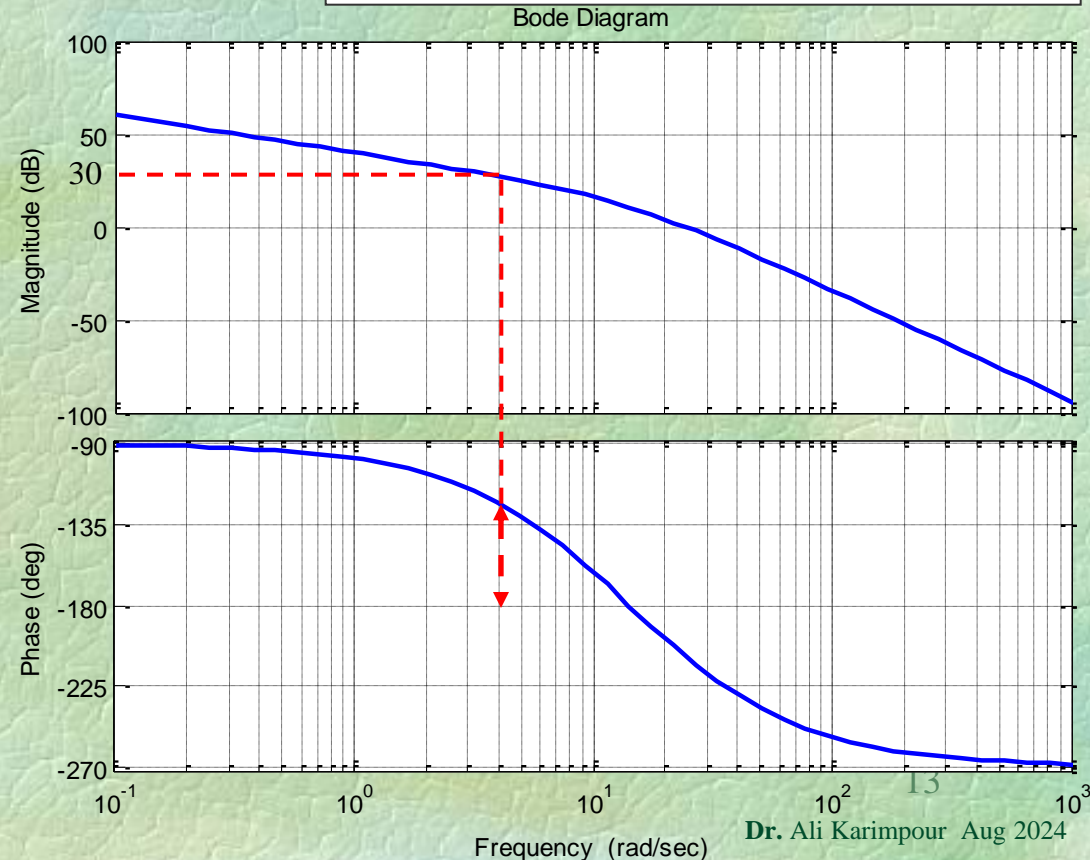
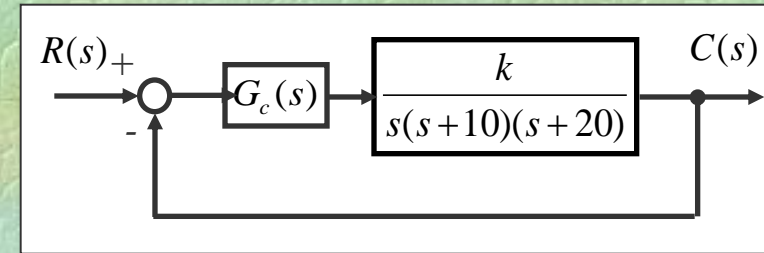
$$20\log(a) = -30$$

$$a = 10^{-\frac{30}{20}} = 0.0316$$

Step 6: Put the right corner of the controller sufficiently far from crossover frequency.

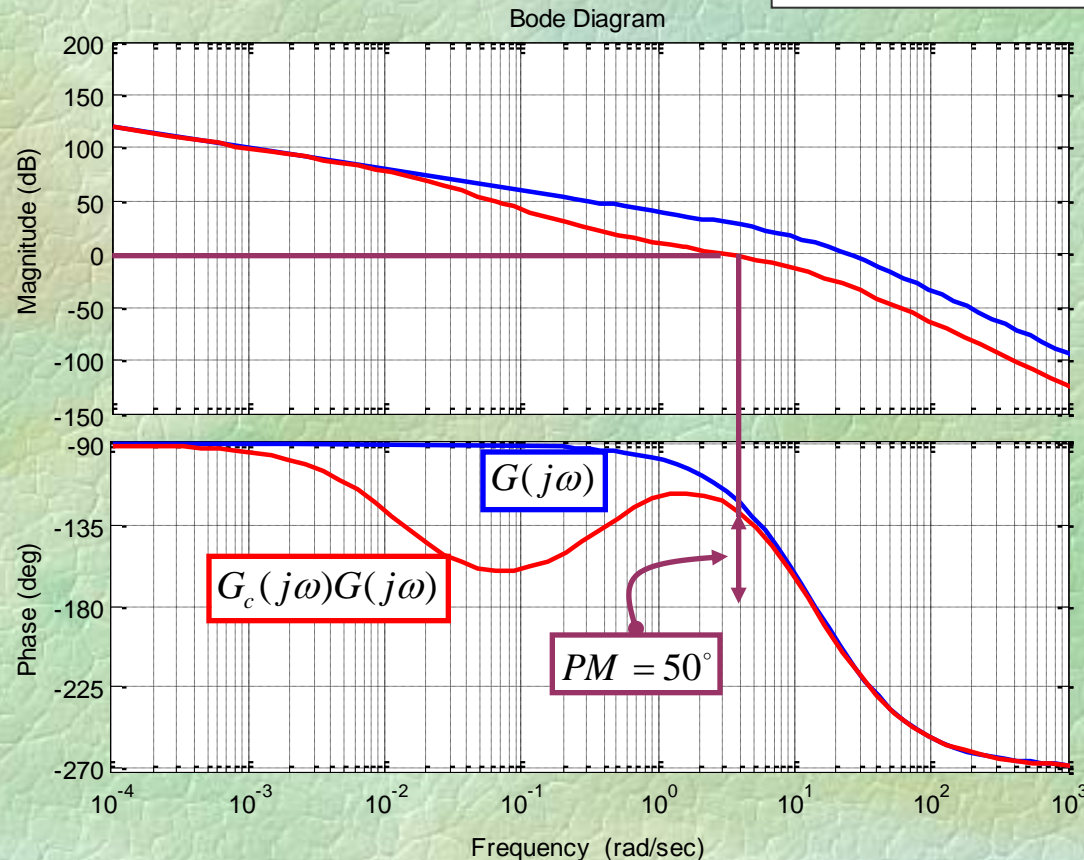
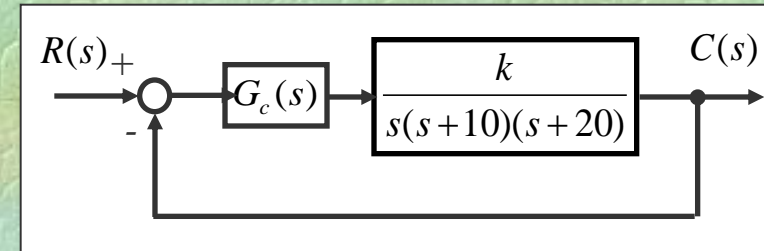
$$\frac{1}{a\tau} = \frac{\omega_{cross}^{new}}{10} = 0.4 \quad \tau = 79$$

$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{2.5s + 1}{79s + 1}$$



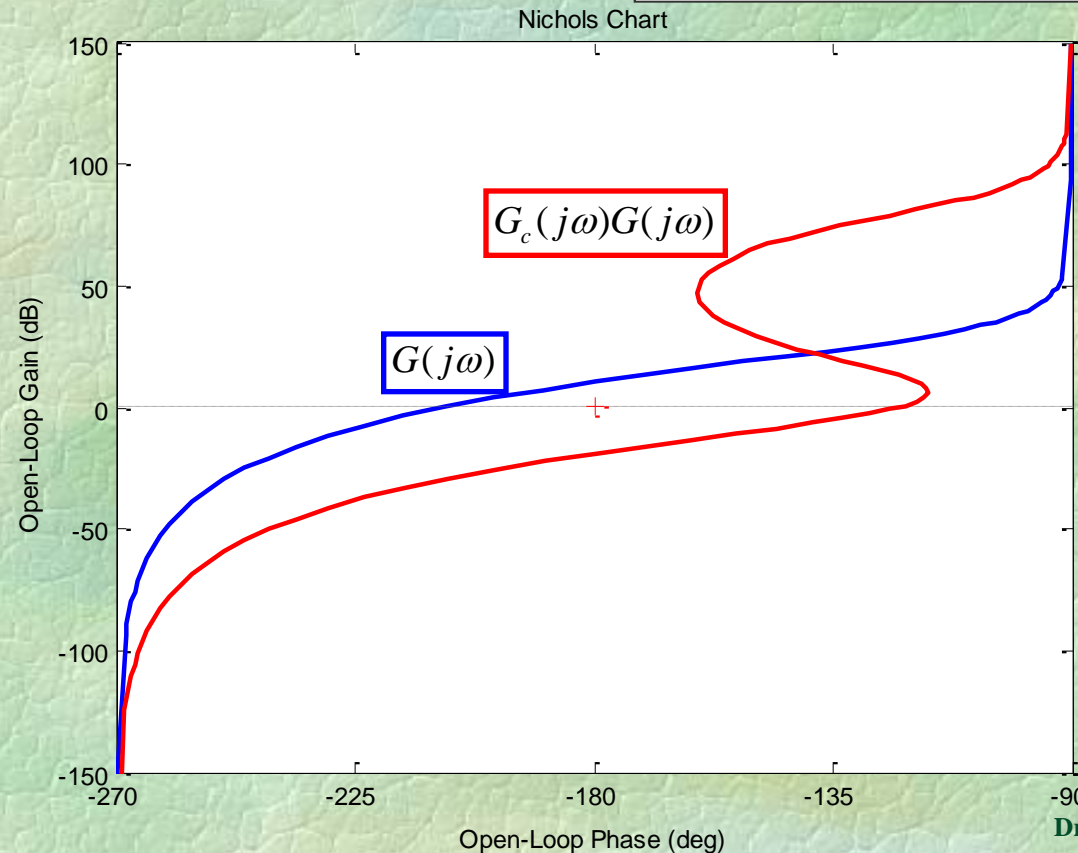
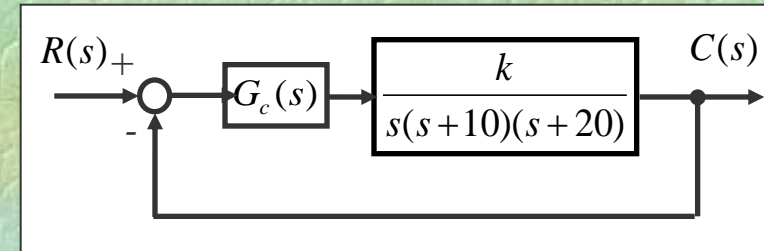
Phase-lag controller design in the frequency domain

Step 7: Check the designed controller.



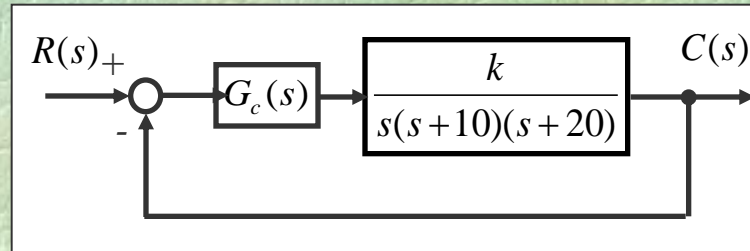
Phase-lag controller design in the frequency domain

Step 7: Check the designed controller.



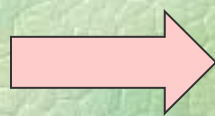
Phase-lag controller design in the frequency domain

Example 3: Design a lag controller for the following system such that the gain margin be 10 db and the ramp error constant be 100.



Step 1: Consider $G_c(s) = k \frac{a\tau s + 1}{\tau s + 1}$ with $a < 1$ as a phase-lag controller.

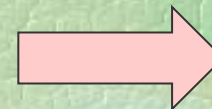
Note: If the plant has another gain k , let $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$



$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} \quad a < 1$$

Step 2: Try to fix k according to the performance request, otherwise let $k=1$

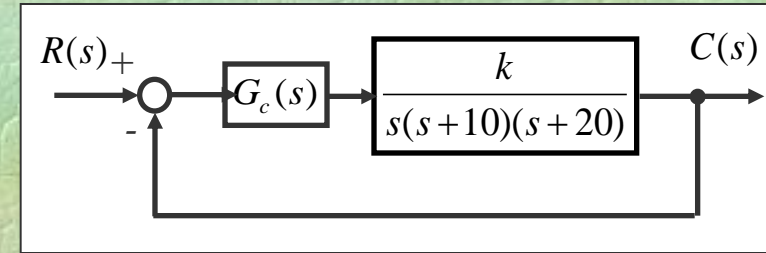
$$k_v = \lim_{s \rightarrow 0} s G_c(s) \frac{k}{s(s+10)(s+20)} = 100$$



$$k = 20000$$

Phase-lag controller design in the frequency domain

Step 3: Sketch the Bode plot of the system (with the fixed k) without controller.

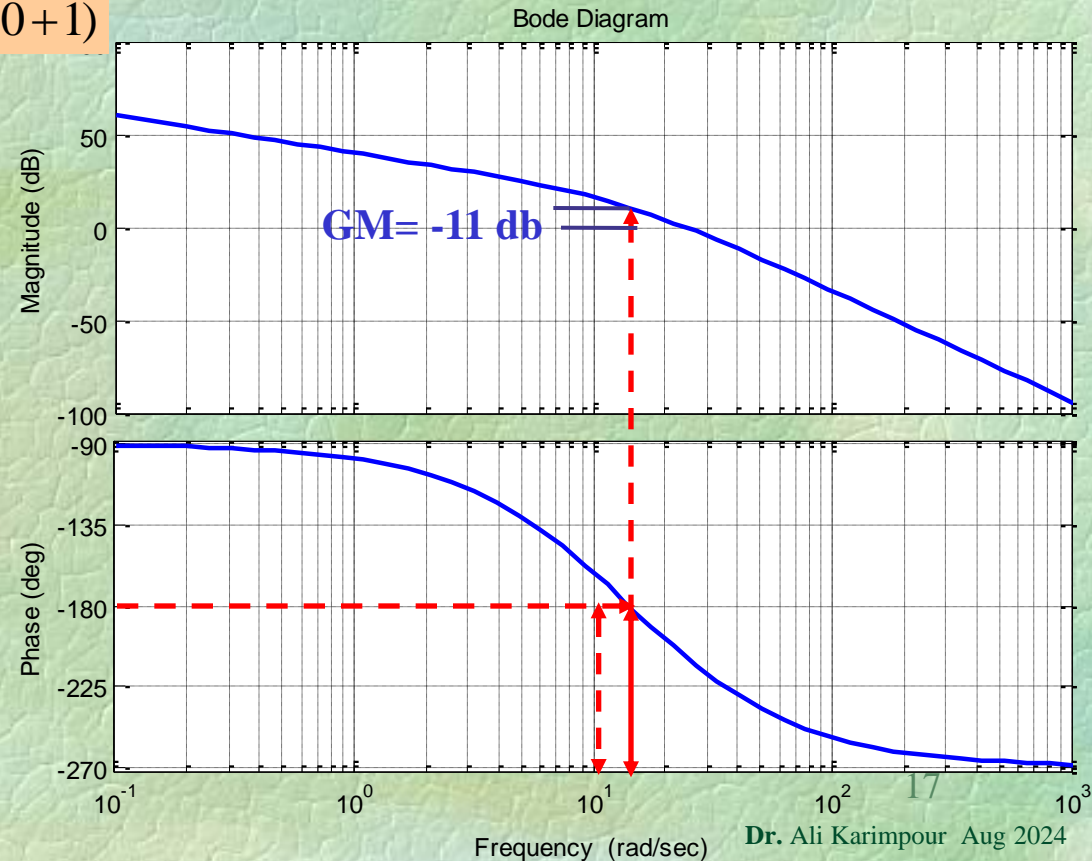


$$G(s) = \frac{20000}{s(s+10)(s+20)} = \frac{100}{s(s/10+1)(s/20+1)}$$

Step 4: According to desired PM (GM) choose the new gain crossover frequency (Phase crossover frequency). (reduce it a little).

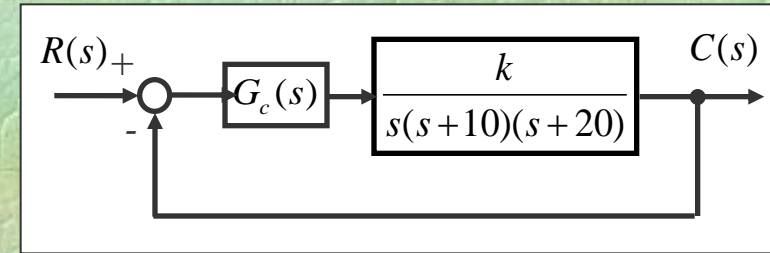
~~$$\omega_{cross}^{new} = 15$$~~

$$\omega_{cross}^{new} = 10$$



Phase-lag controller design in the frequency domain

Step 5: Find the required gain by lag controller and **derive the parameter a**.



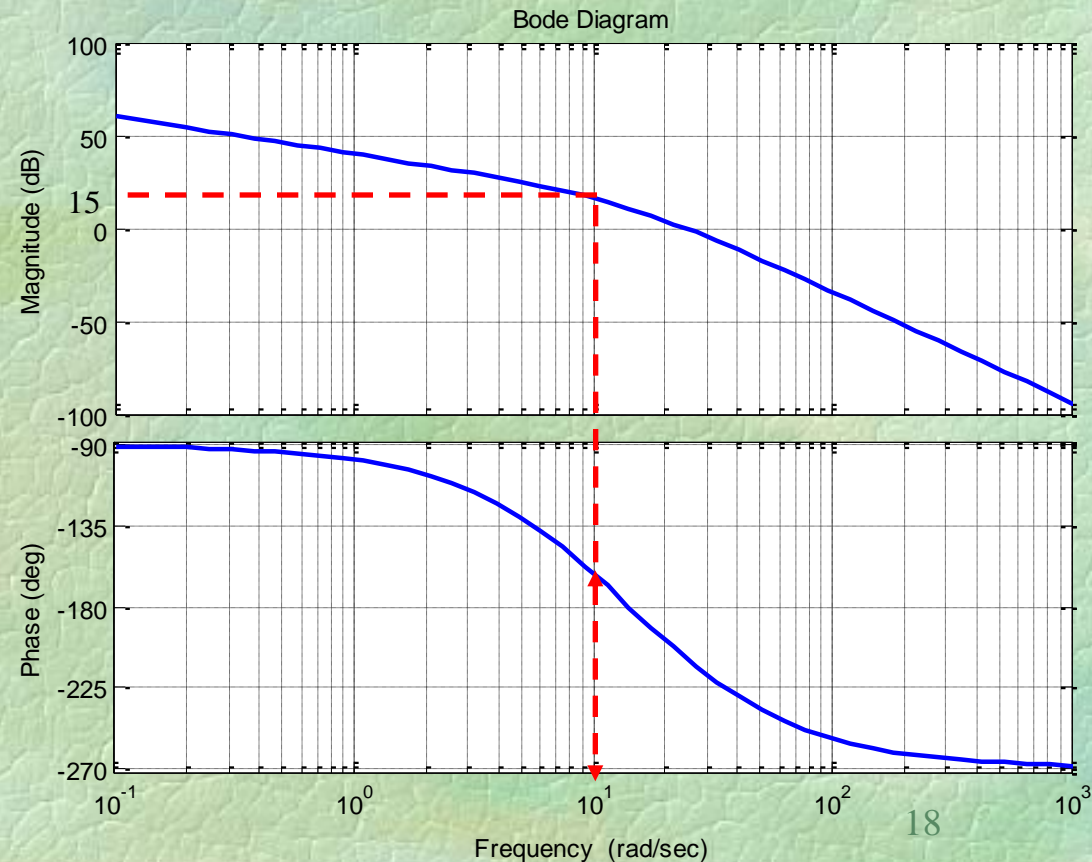
$$20\log(a) = -25$$

$$a = 10^{-\frac{25}{20}} = 0.056$$

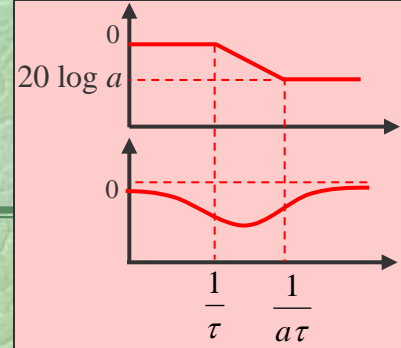
Step 6: Put the right corner of the controller sufficiently far from crossover frequency.

$$\frac{1}{a\tau} = \frac{10}{10} = 1 \quad \tau = 17.9$$

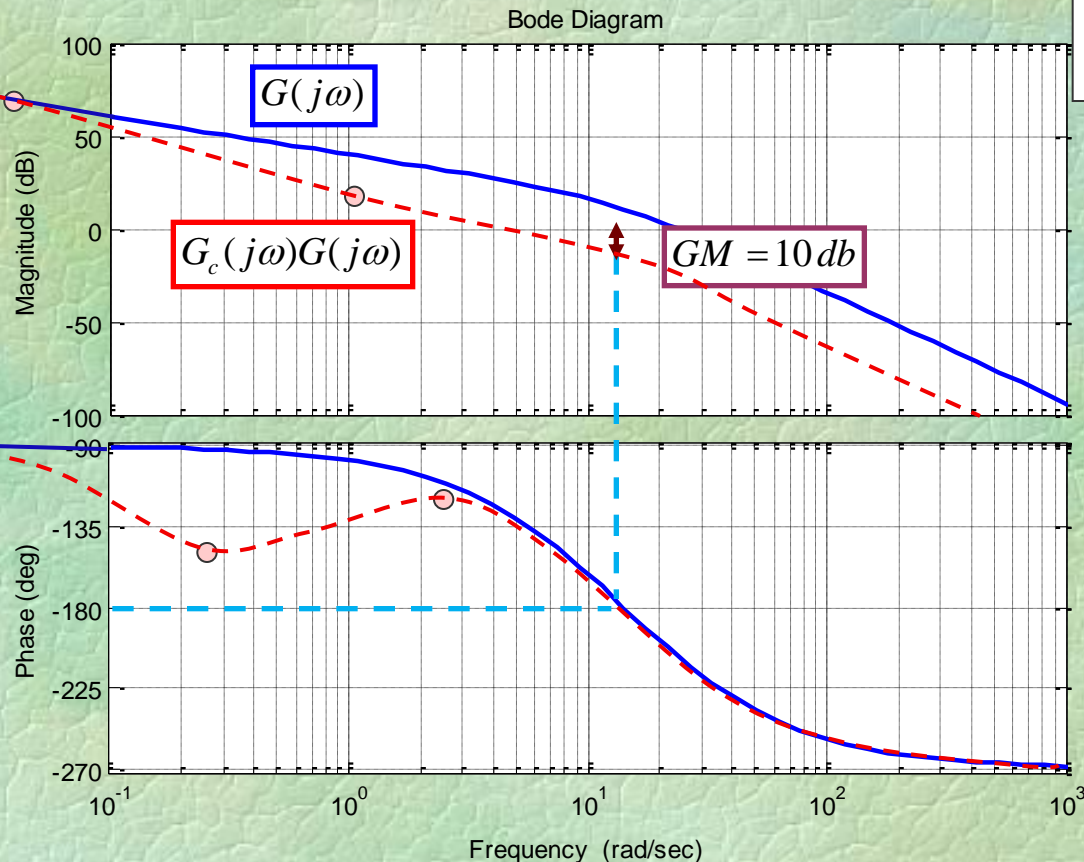
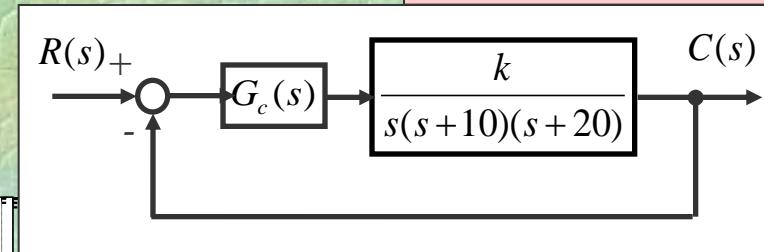
$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{s + 1}{17.9s + 1}$$



Phase-lag controller design in the frequency domain



Step 7: Check the designed controller.



$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{0.056 \times 17.9s + 1}{17.9s + 1}$$

$$1/\tau = 0.06, 1/a\tau = 1$$

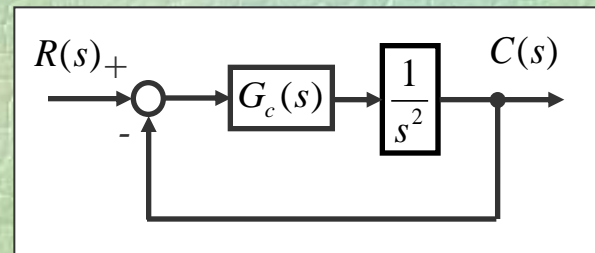
$$20 \log a = -25 \text{ db}$$

$$1/\sqrt{a\tau} = 0.25$$

$$\sin \varphi_m = \frac{0.056 - 1}{0.056 + 1} \Rightarrow \varphi_m = -63^\circ$$

Phase-lag controller design in the frequency domain

Example 4: Design a lag controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



It is not possible explain why?

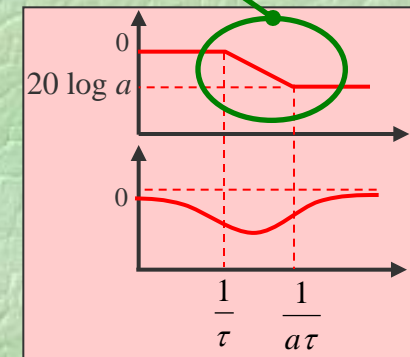
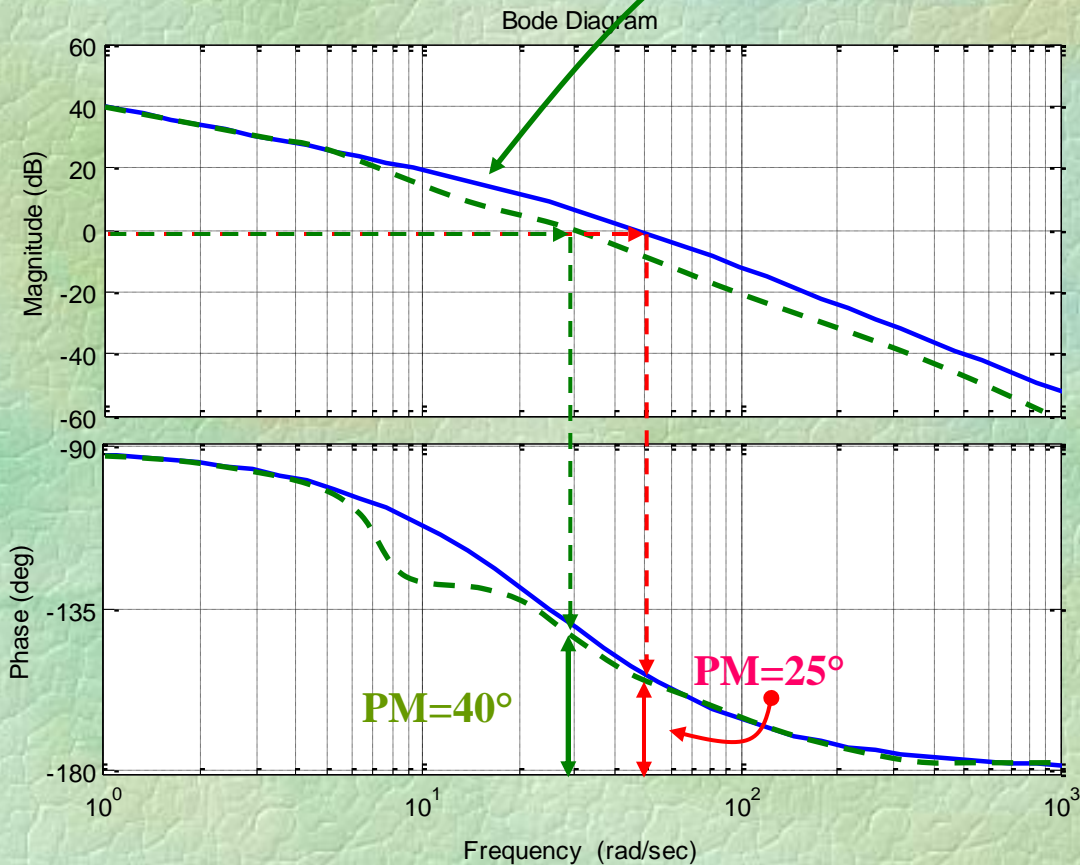
Controller design in the frequency domain

Topics to be covered include:

- ❖ Phase-lag controller design.
- ❖ **Phase-lead controller design**

Design fundamental of a lag controller (remember)

Consider a minimum phase system.



BW reduced.

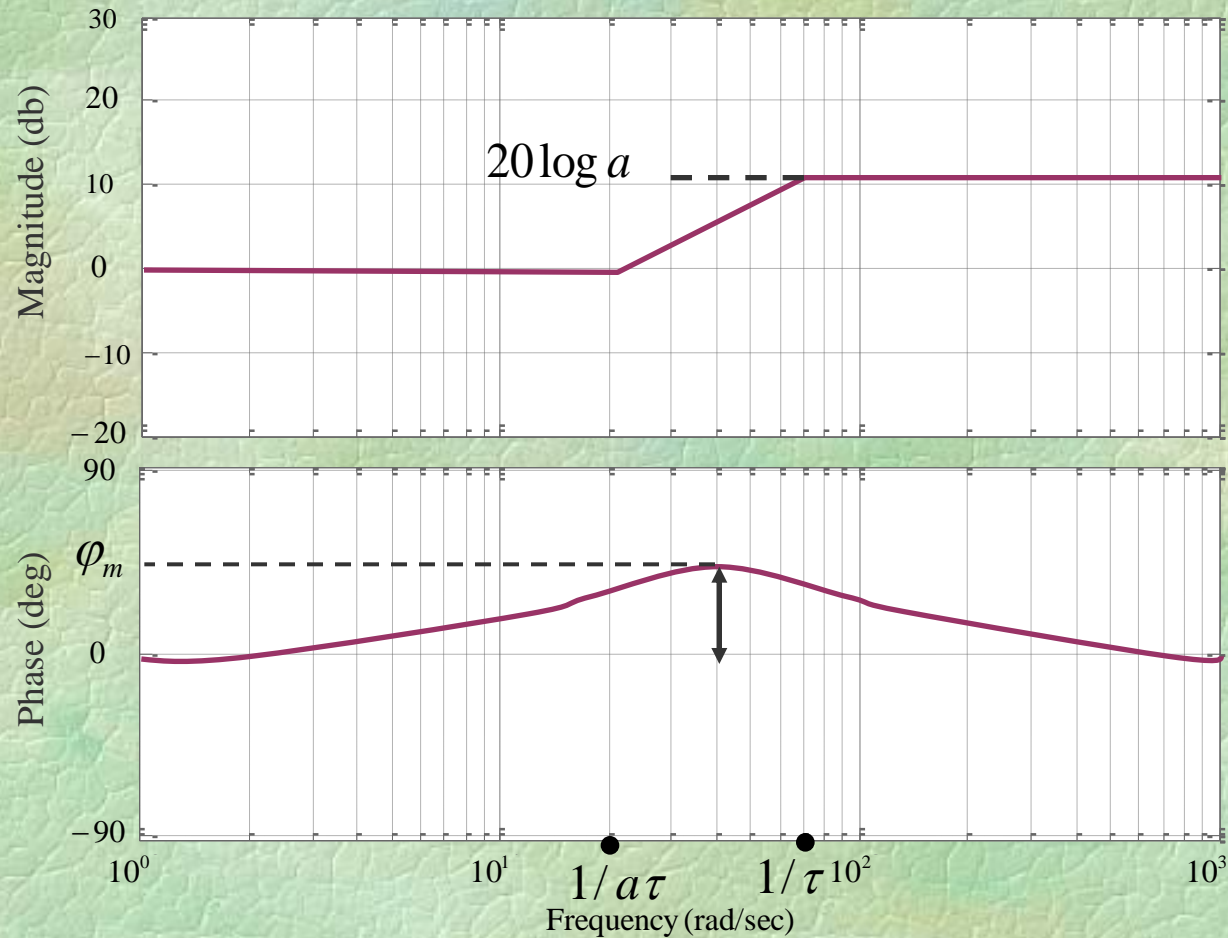
Speed of system reduced.

Effect of the noise is reduced.

A phase-lead controller

$$G(s) = \frac{a\tau s + 1}{\tau s + 1}$$

Let $a > 1$



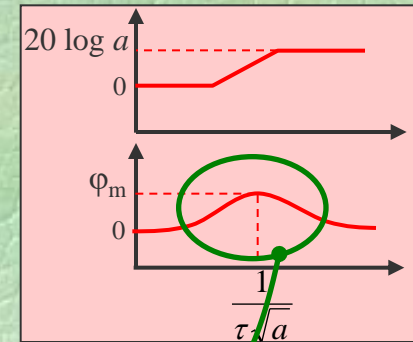
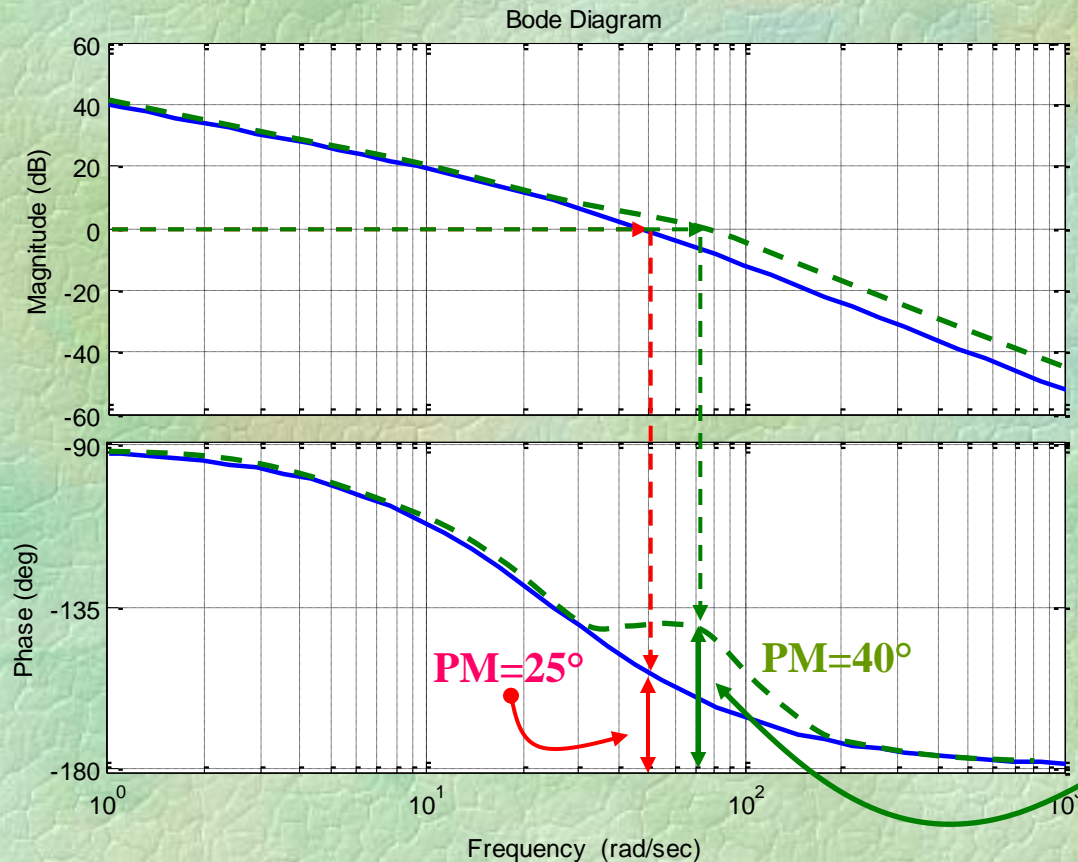
$$\sin \varphi_m = \frac{a-1}{a+1}$$

Design fundamental of the lead controller

Analysis

Design

Consider a minimum phase system.

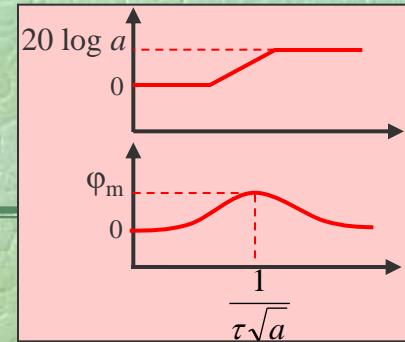


How can the lead controller help us?

What is the effect of a lead controller on **BW**? Speed of system?

What is the effect of a lead controller on noise effect?

Design procedure of the phase-lead controller in the frequency domain



Step 1: Consider $G_c(s) = k \frac{a\tau s + 1}{\tau s + 1}$ with $a > 1$ as a phase lead controller.

Note: If the plant has another gain k , let $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$

Step 2: Try to fix k according to the performance request, otherwise let $k=1$

Step 3: Sketch the Bode plot of the system (with the fixed k) without controller.

Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta \quad ? \quad \sin \varphi_m = \frac{a-1}{a+1} \quad \Rightarrow \quad a = \checkmark$$

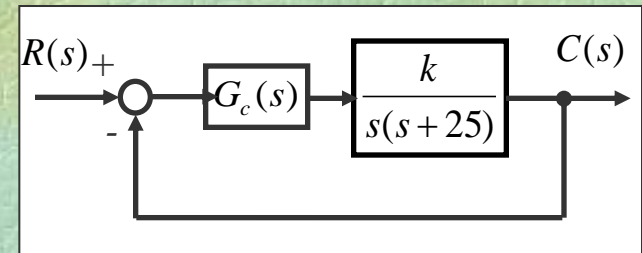
Step 5: Put the center of the controller in the new gain crossover frequency:

$$20 \log |G(j\omega)|_{\omega=\omega_c^{new}} + 10 \log(a) = 0 \quad \omega_c^{new} = \checkmark \quad \omega_c^{new} = \frac{1}{\tau\sqrt{a}} \quad \Rightarrow \quad \tau = \checkmark$$

Step 6: Check the controller.

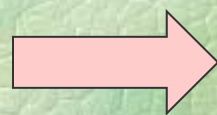
Phase-lead controller design in the frequency domain

Example 5: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100. Find the M_p of overall system.



Step 1: Consider $G_c(s) = k \frac{a\tau s + 1}{\tau s + 1}$ with $a > 1$ as a phase-lead controller.

Note: If the plant has another gain k , let $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$



$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} \quad a > 1$$

Step 2: Try to fix k according to the performance request, otherwise let $k=1$

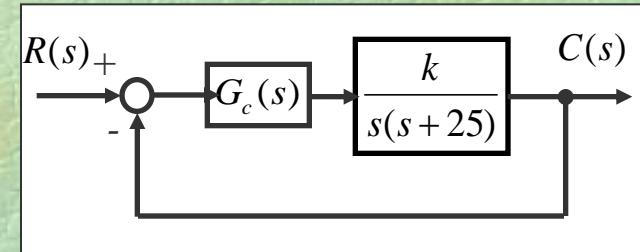
$$k_v = \lim_{s \rightarrow 0} s G_c(s) \frac{k}{s(s+25)} = 100$$



$$k = 2500$$

Phase-lead controller design in the frequency domain

Step 3: Sketch the Bode plot of the system (with the fixed k) without controller.



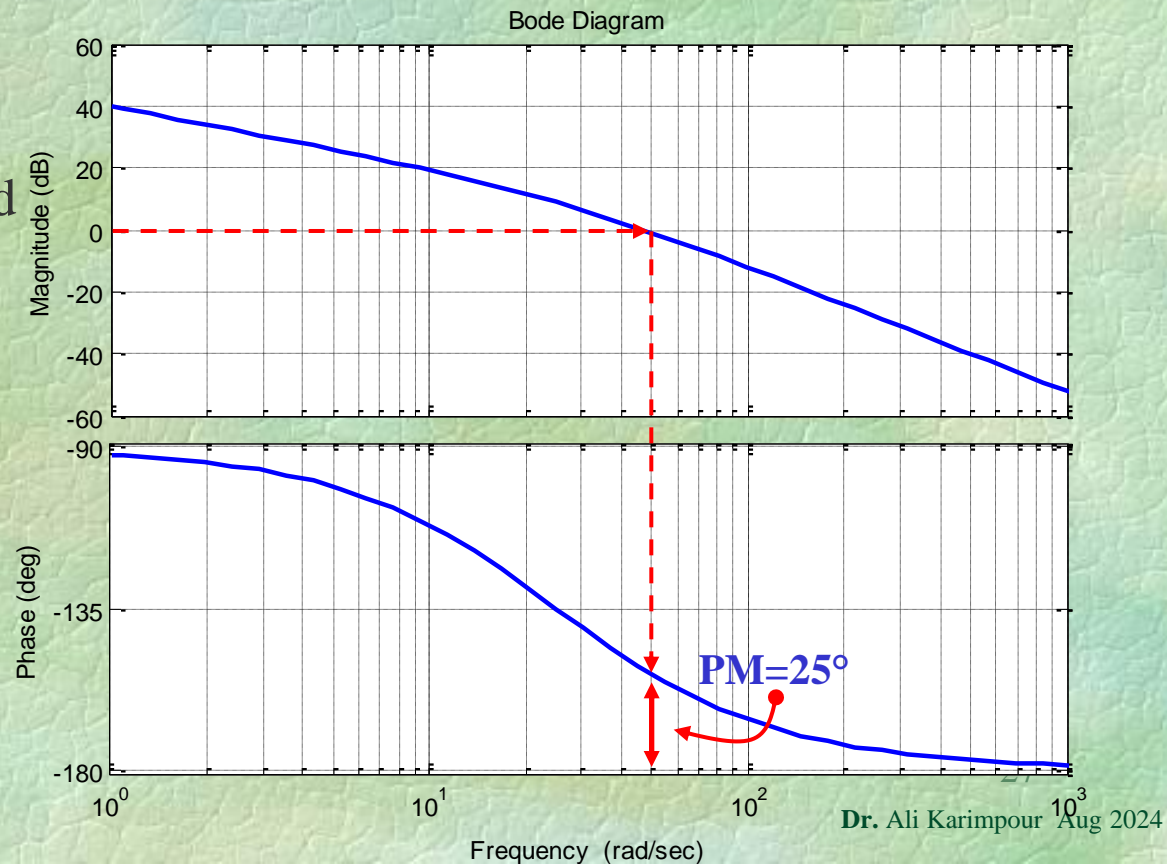
$$G(s) = \frac{2500}{s(s+25)} = \frac{100}{s(s/25+1)}$$

Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - 25^\circ + 5^\circ = 25^\circ$$

$$\sin 25 = \frac{a-1}{a+1} \Rightarrow a = 2.46$$



Phase-lead controller design in the frequency domain

Step 5: Put the center of the controller in the new gain crossover frequency:

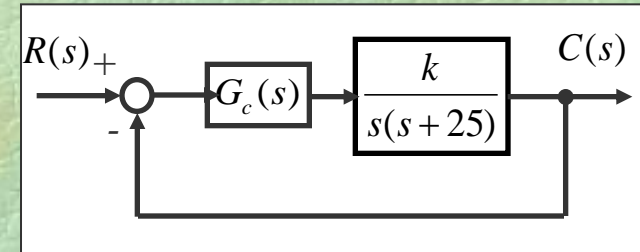
$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} = -10\log(a) = -3.91$$

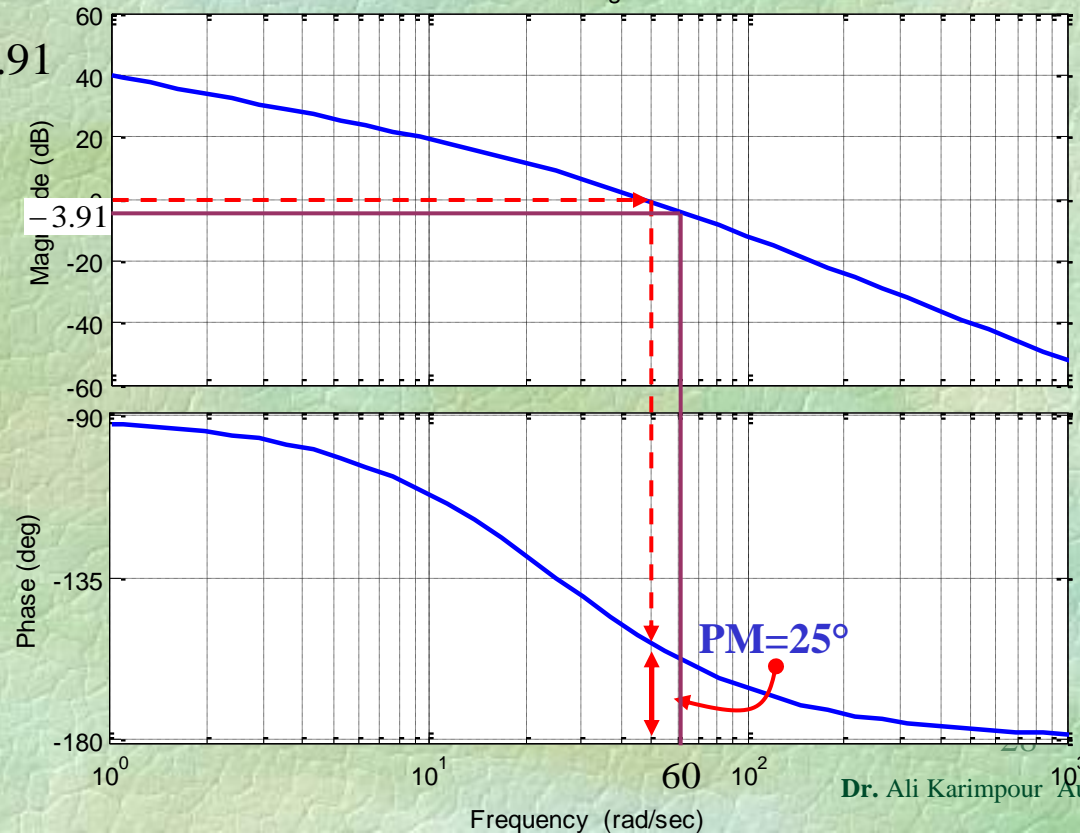
$$\omega_c^{new} = 60$$

$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}} \Rightarrow \tau = 0.0106$$

$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{0.0261s + 1}{0.0106s + 1}$$



Bode Diagram



Phase-lead controller design in the frequency domain

Step 6: Check the controller.

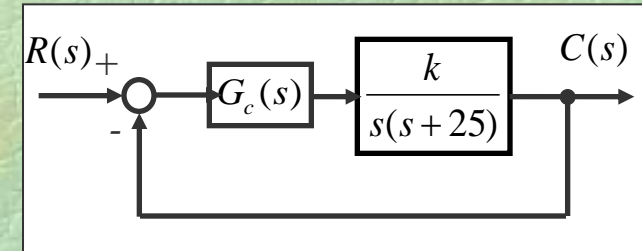
$$G_c(s) = \frac{0.0261s + 1}{0.0106s + 1}$$

$$1/\tau = 94.33, 1/a\tau = 38.3$$

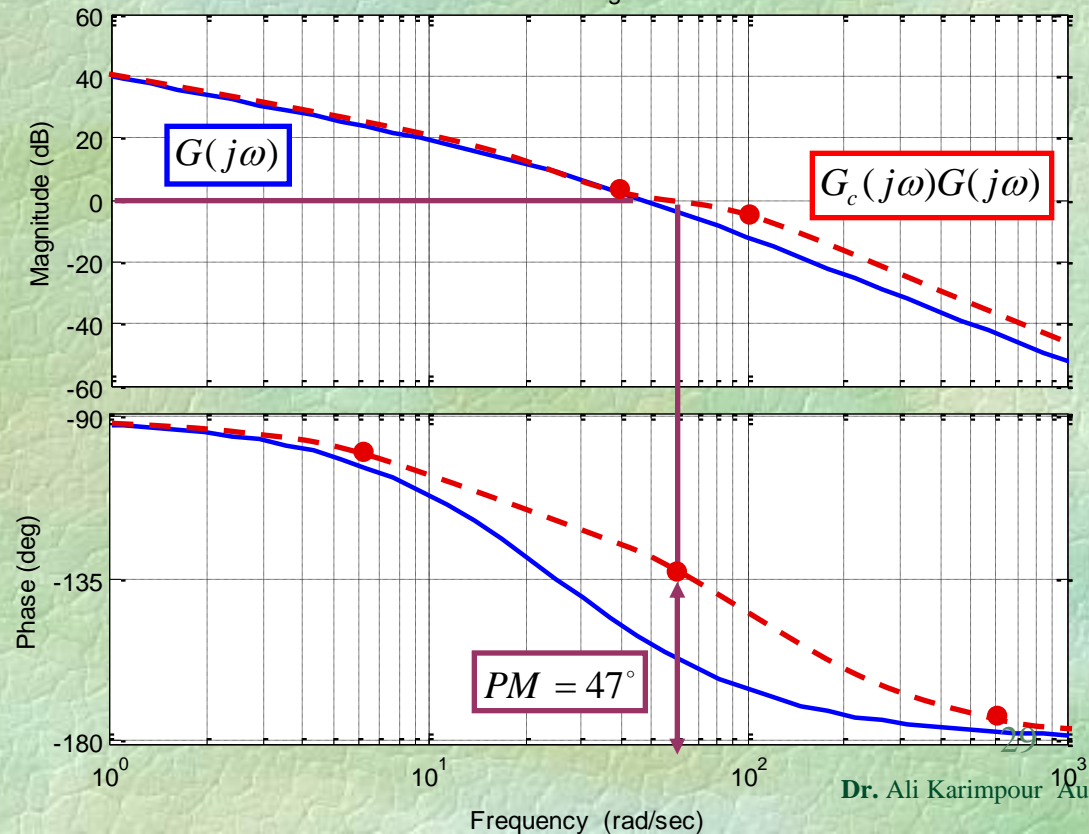
$$20 \log a = 7.8$$

$$1/\sqrt{a\tau} = 60$$

$$\varphi_m = 25^\circ$$



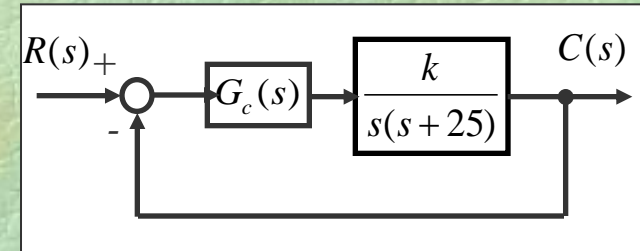
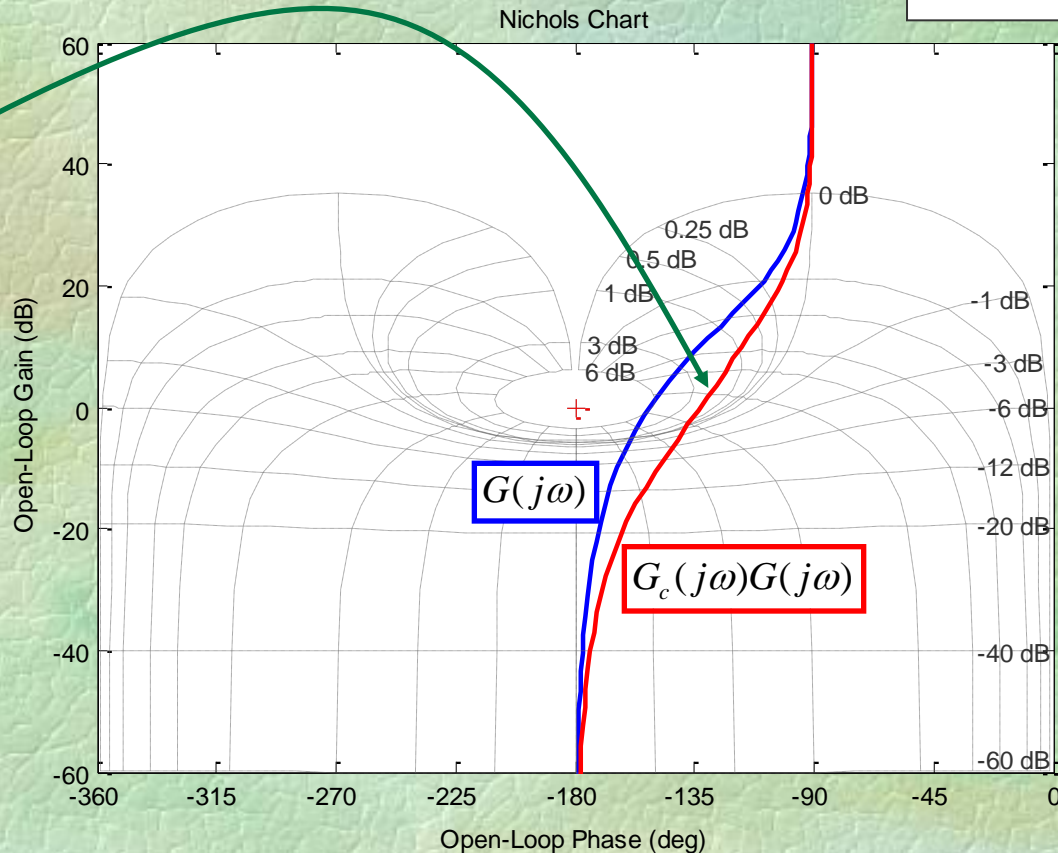
Bode Diagram



Phase-lead controller design in the frequency domain

Find M_p

$M_p = 2 \text{ db}$

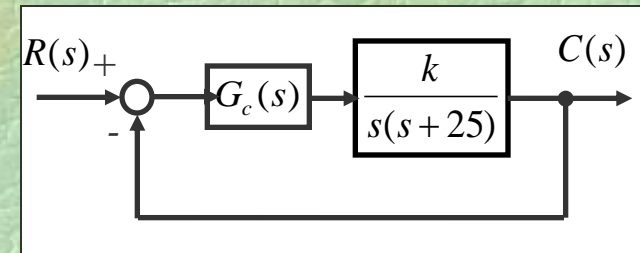


Phase-lead controller design in the frequency domain

Example 6: Find the step response of example 5 and compare it with lag design in example 1.

$$G_{clead}(s) = \frac{0.0261s + 1}{0.0106s + 1} \quad \text{Lead controller of example 5}$$

$$G_{clag}(s) = \frac{0.5s + 1}{1.79s + 1} \quad \text{Lag controller of example 1}$$



$$M_1(s) = \frac{G(s)}{1 + G(s)} = \frac{2500}{s^2 + 25s + 2500}$$

Closed loop transfer function without controller

$$M_2(s) = \frac{G_{clead}(s).G(s)}{1 + G_{clead}(s).G(s)} = \frac{6156s + 235800}{s^3 + 119.3s^2 + 8514s + 235800}$$

Closed loop transfer function with a phase-lead controller

$$M_3(s) = \frac{G_{clag}(s).G(s)}{1 + G_{clag}(s).G(s)} = \frac{698.3s + 1397}{s^3 + 25.6s^2 + 712.3s + 1397}$$

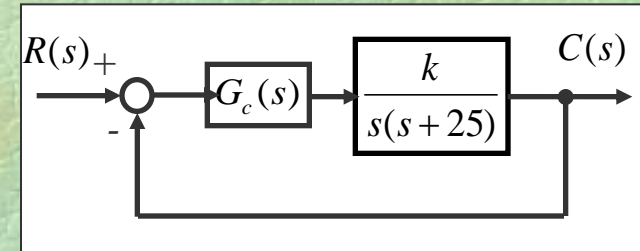
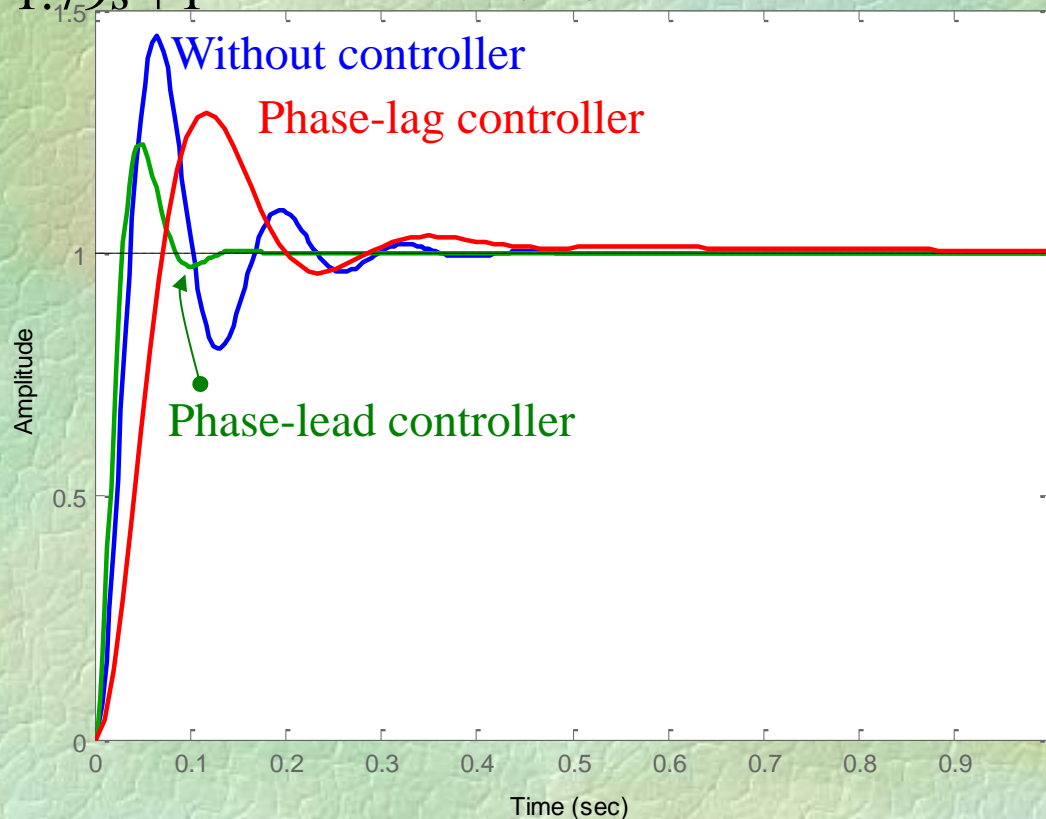
Closed loop transfer function with a phase-lag controller

Phase-lead controller design in the frequency domain

$$G_{clead}(s) = \frac{0.0261s + 1}{0.0106s + 1} \quad \text{Lead controller of example 5}$$

$$G_{clag}(s) = \frac{0.5s + 1}{1.79s + 1} \quad \text{Lag controller of example 1}$$

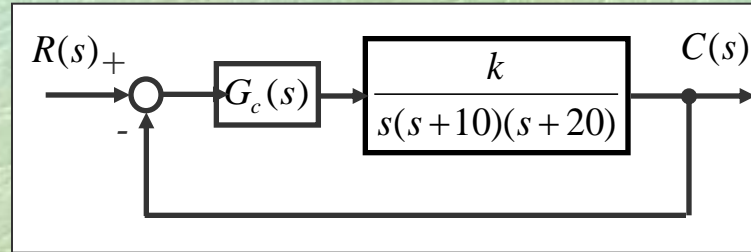
Step Response



Check the speed of different controllers and compare it ??

Phase-lead controller design in the frequency domain

Example 7: Design a lead controller for the following system such that the phase margin be 45° and the ramp error constant be 100.



Step 1: Consider $G_c(s) = k \frac{a\tau s + 1}{\tau s + 1}$ with $a > 1$ as a phase-lead controller.

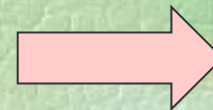
Note: If the plant has another gain k , let $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$

➔

$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} \quad a > 1$$

Step 2: Try to fix k according to performance request, otherwise let $k=1$

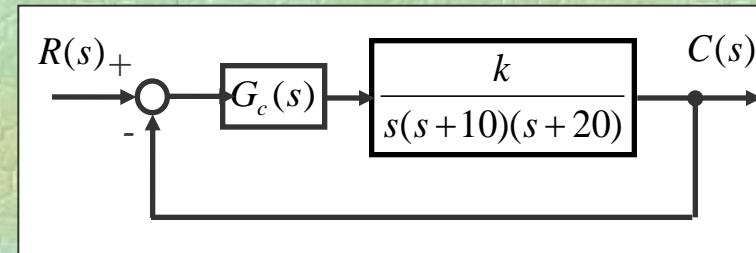
$$k_v = \lim_{s \rightarrow 0} s G_c(s) \frac{k}{s(s+10)(s+20)} = 100$$



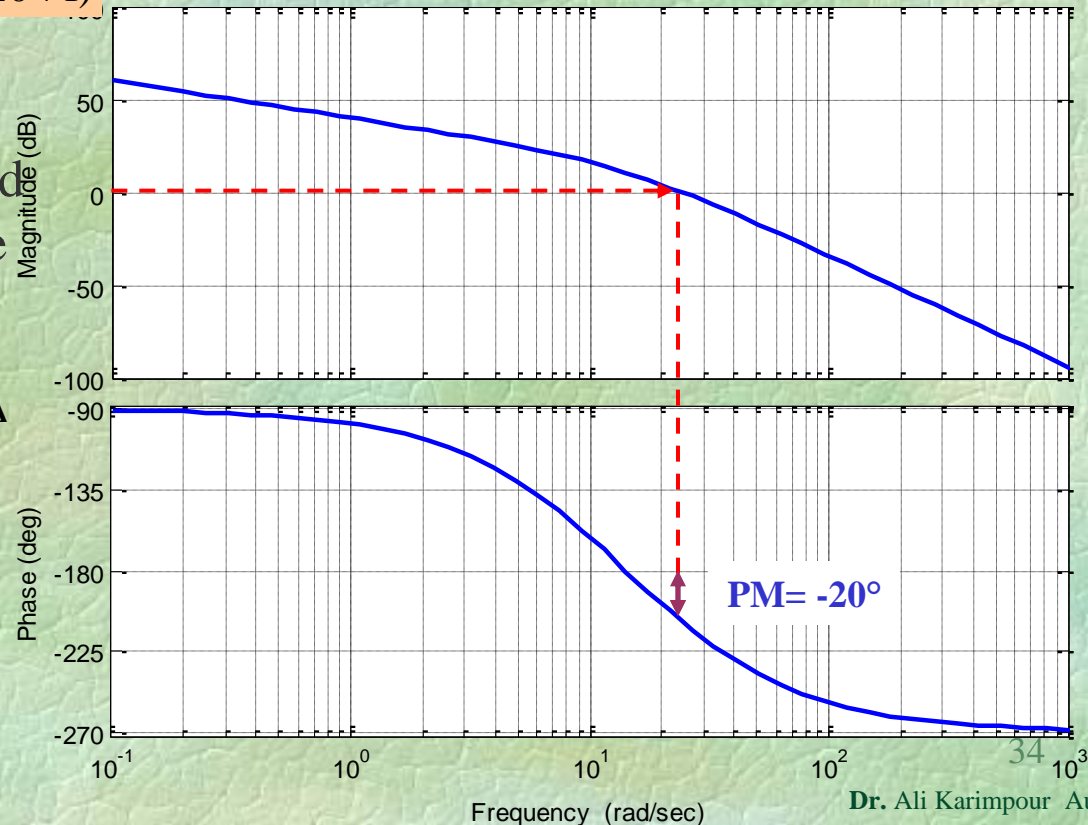
$$k = 20000$$

Phase-lead controller design in the frequency domain

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.



Bode Diagram



Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (-20) + 5^\circ = 70^\circ$$

$$\sin 70 = \frac{a-1}{a+1} \Rightarrow a = 32$$

Phase-lead controller design in the frequency domain

Step 5: Put the center of controller in the new gain crossover frequency:

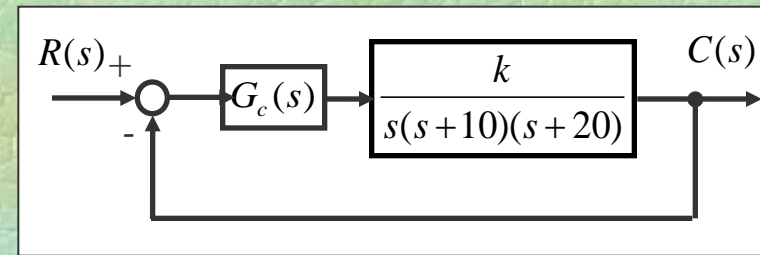
$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} = -10\log(a) = -15$$

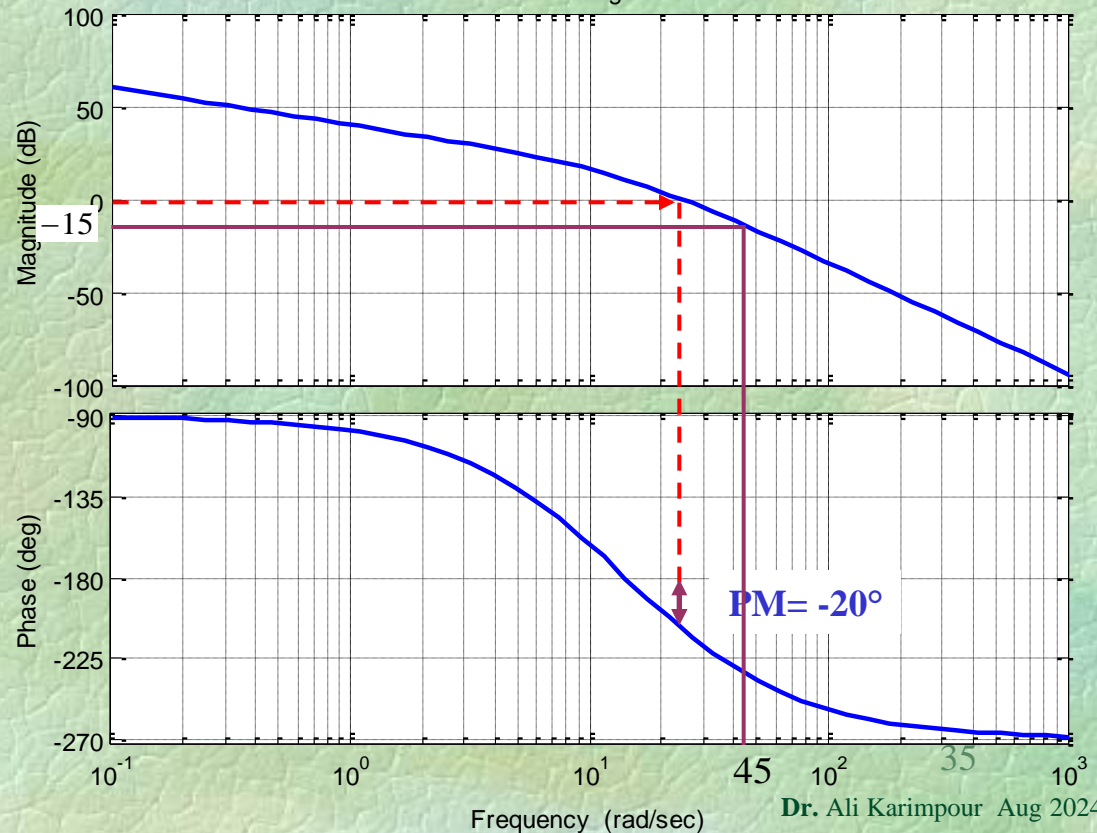
$$\omega_c^{new} = 45$$

$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}} \Rightarrow \tau = 0.0039$$

$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = \frac{0.1248s + 1}{0.0039s + 1}$$

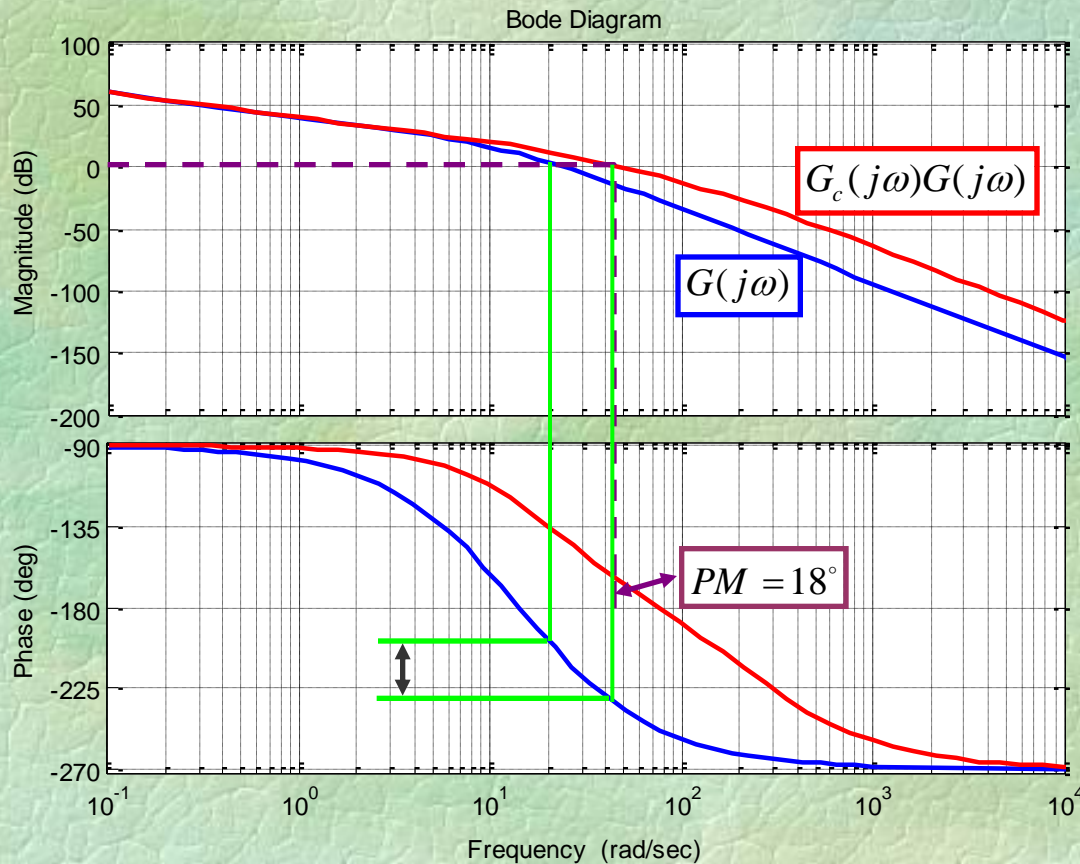
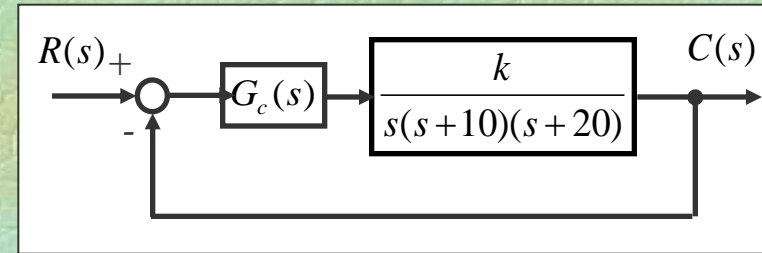


Bode Diagram



Phase-lead controller design in the frequency domain

Step 6: Check the controller.

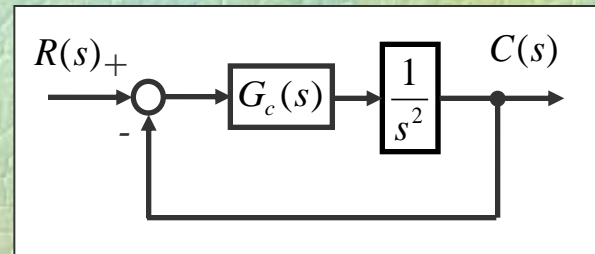


Note: Design is not possible!

Why?

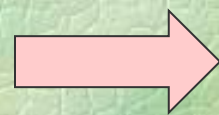
Phase-lead controller design in the frequency domain

Example 8: Design a lead controller for the following system such that the phase margin be 45° and the open loop bandwidth be 10 rad/sec



Step 1: Consider $G_c(s) = k \frac{a\tau s + 1}{\tau s + 1}$ with $a > 1$ as a phase-lag controller.

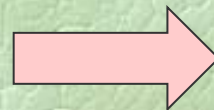
Note: If the plant has another gain k , let $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$



$$G_c(s) = k \frac{a\tau s + 1}{\tau s + 1} \quad a > 1$$

Step 2: Try to fix k according to performance request, otherwise let $k=1$

$$20 \log \left| \frac{k}{\omega^2} \right|_{\omega=5} = -3 \quad \text{Why?}$$



$$k = 17.7$$

Phase-lead controller design in the frequency domain

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

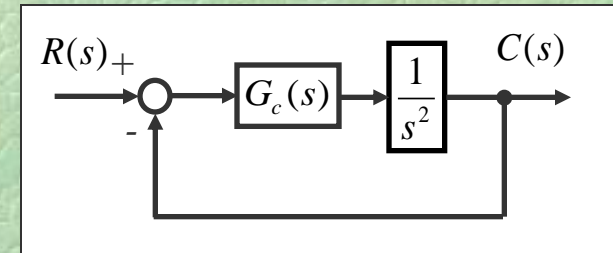
$$G(s) = \frac{17.7}{s^2}$$

Step 4: Find the system PM and if it is not sufficient choose the required phase by:

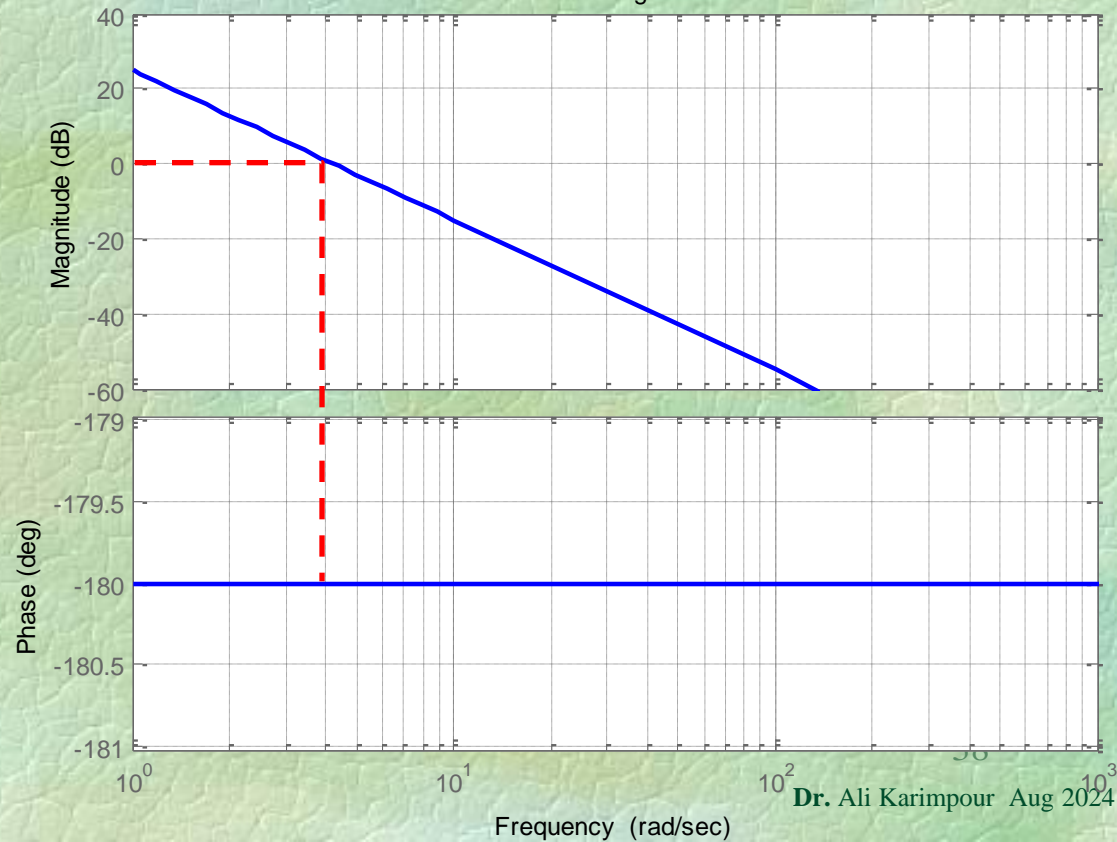
$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (0) + 0^\circ = 45^\circ$$

$$\sin 45^\circ = \frac{a-1}{a+1} \Rightarrow a = 5.8$$



Bode Diagram



Phase-lead controller design in the frequency domain

Step 5: Put the center of controller in the new gain crossover frequency:

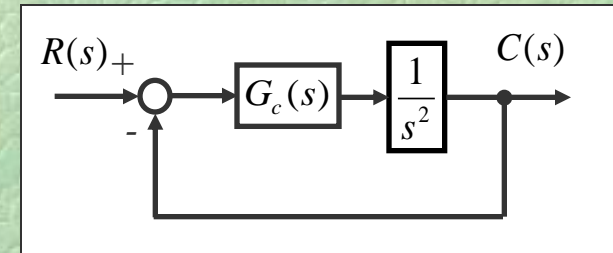
$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} = -10\log(a) = -7.63$$

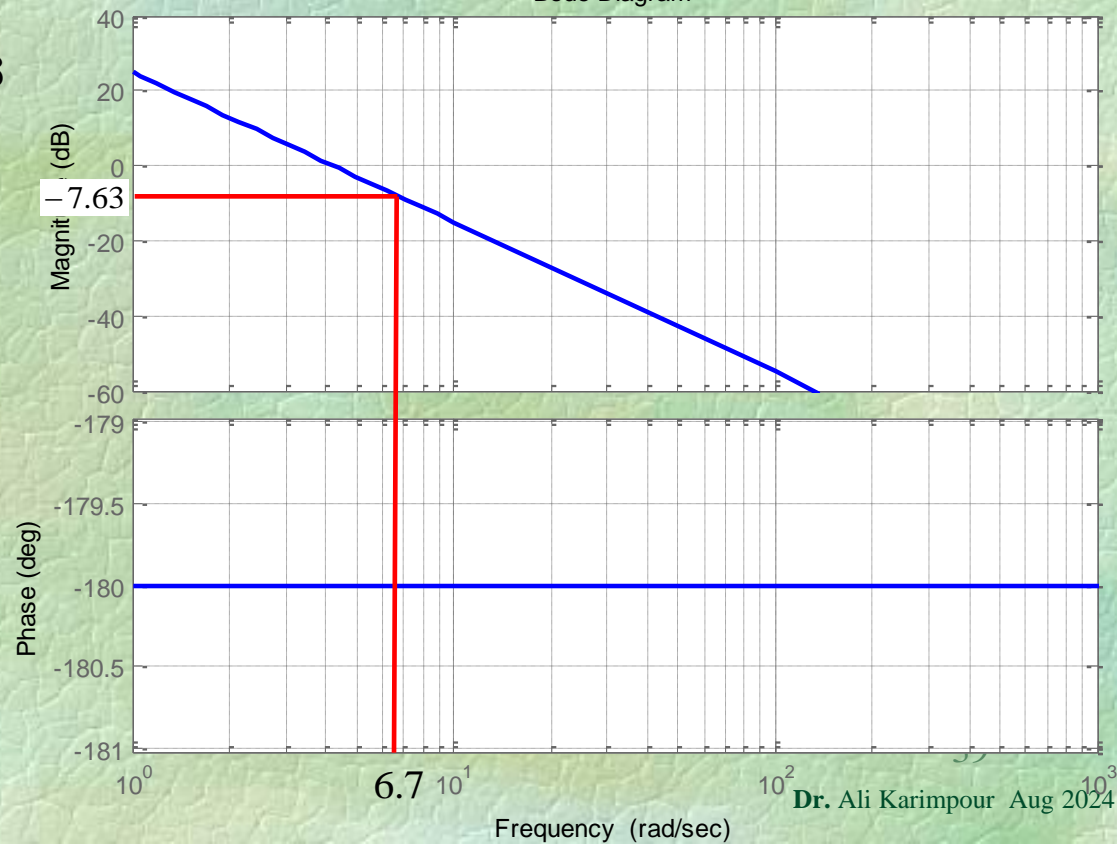
$$\omega_c^{new} = 6.7$$

$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}} \Rightarrow \tau = 0.062$$

$$G_c(s) = \frac{a\tau s + 1}{\tau s + 1} = 17.7 \frac{0.3596s + 1}{0.062s + 1}$$

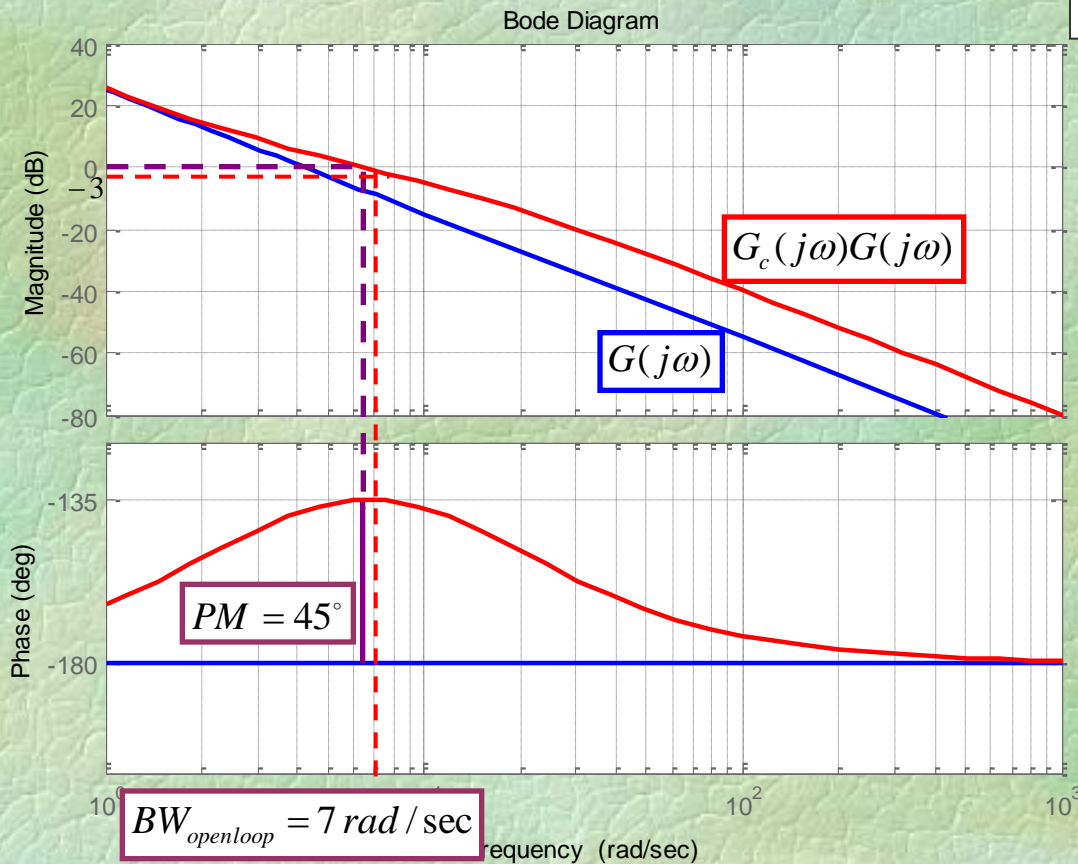
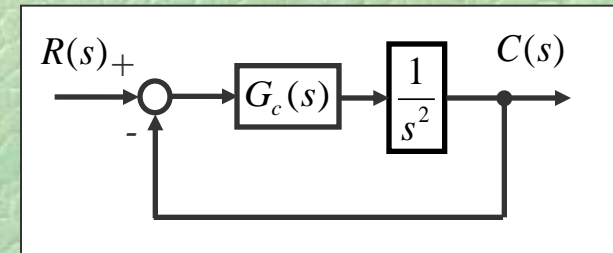


Bode Diagram



Phase-lead controller design in the frequency domain

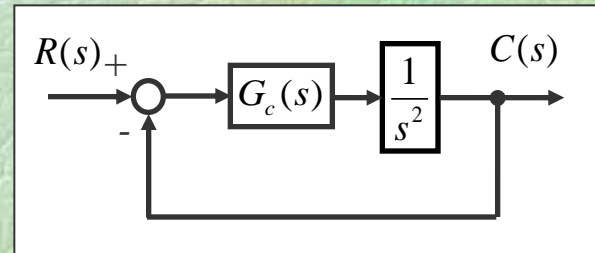
Step 6: Check the controller.



Controller is not ok

Try again

Phase-lead controller design in the frequency domain



Step 2: Try to fix k according to performance request, otherwise let $k=1$

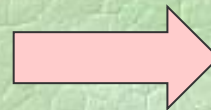
Open loop bandwidth is near to gain crossover frequency so:

$$20 \log \left| \frac{k}{\omega^2} \right|_{\omega=5} = -3$$



$$k = 17.7$$

$$20 \log \left| \frac{k}{\omega^2} \right|_{\omega=7} = -3$$



$$k = 35.7$$

Phase-lead controller design in the frequency domain

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

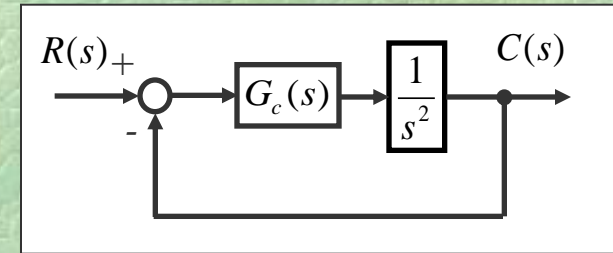
$$G(s) = \frac{35.7}{s^2}$$

Step 4: Find the system PM and if it is not sufficient choose the required phase by:

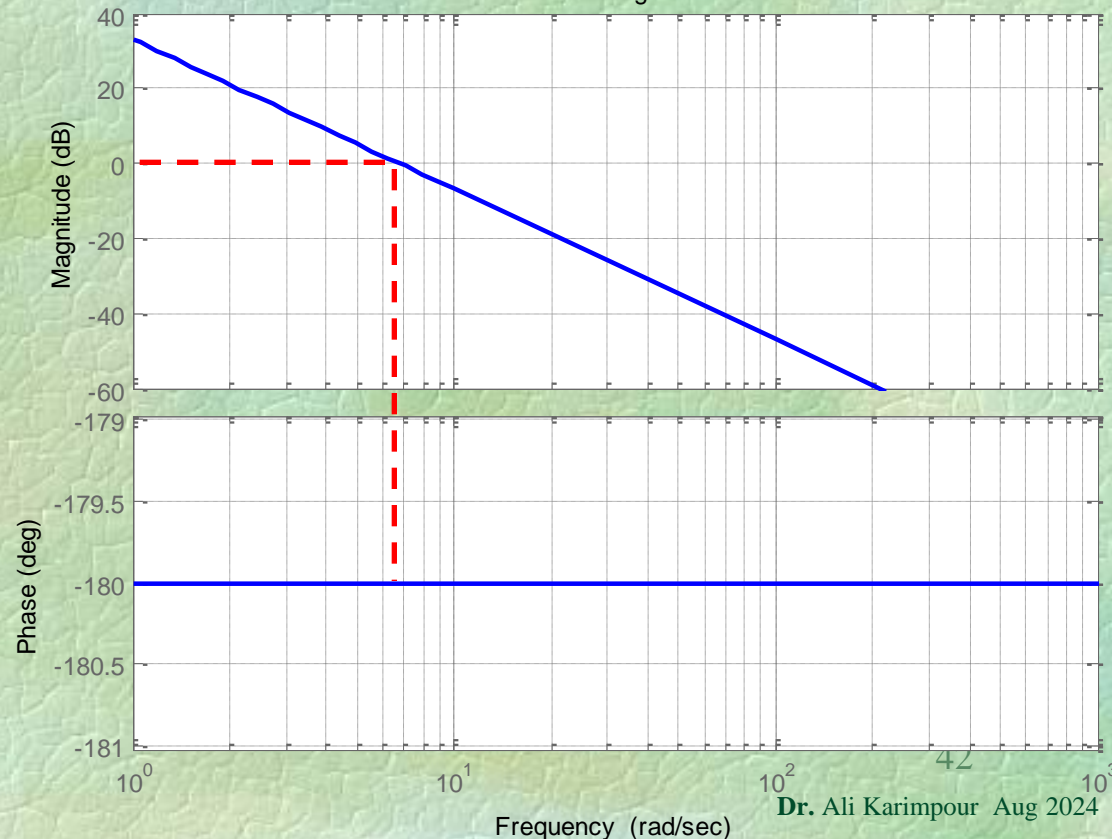
$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

$$\varphi_m = 45^\circ - (0) + 0^\circ = 45^\circ$$

$$\sin 45^\circ = \frac{a-1}{a+1} \Rightarrow a = 5.8$$



Bode Diagram



Phase-lead controller design in the frequency domain

Step 5: Put the center of controller in the new gain crossover frequency:

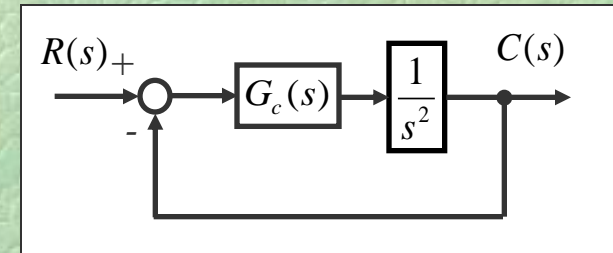
$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} = -10\log(a) = -7.63$$

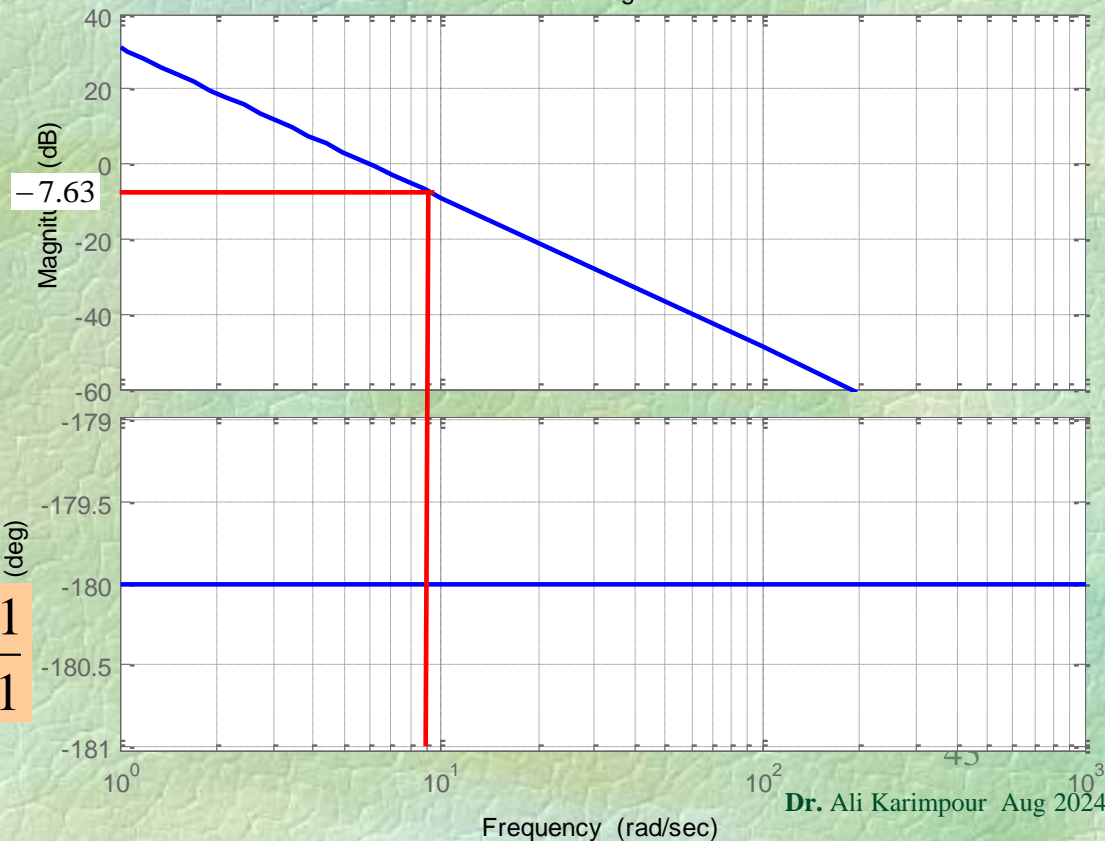
$$\omega_c^{new} = 9$$

$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}} \Rightarrow \tau = 0.0461$$

$$G_c(s) = k \frac{a\tau s + 1}{\tau s + 1} = 35.7 \frac{0.2674s + 1}{0.0461s + 1}$$

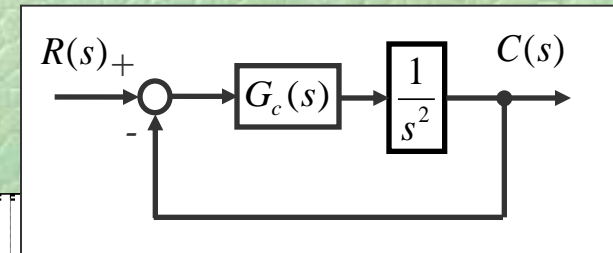
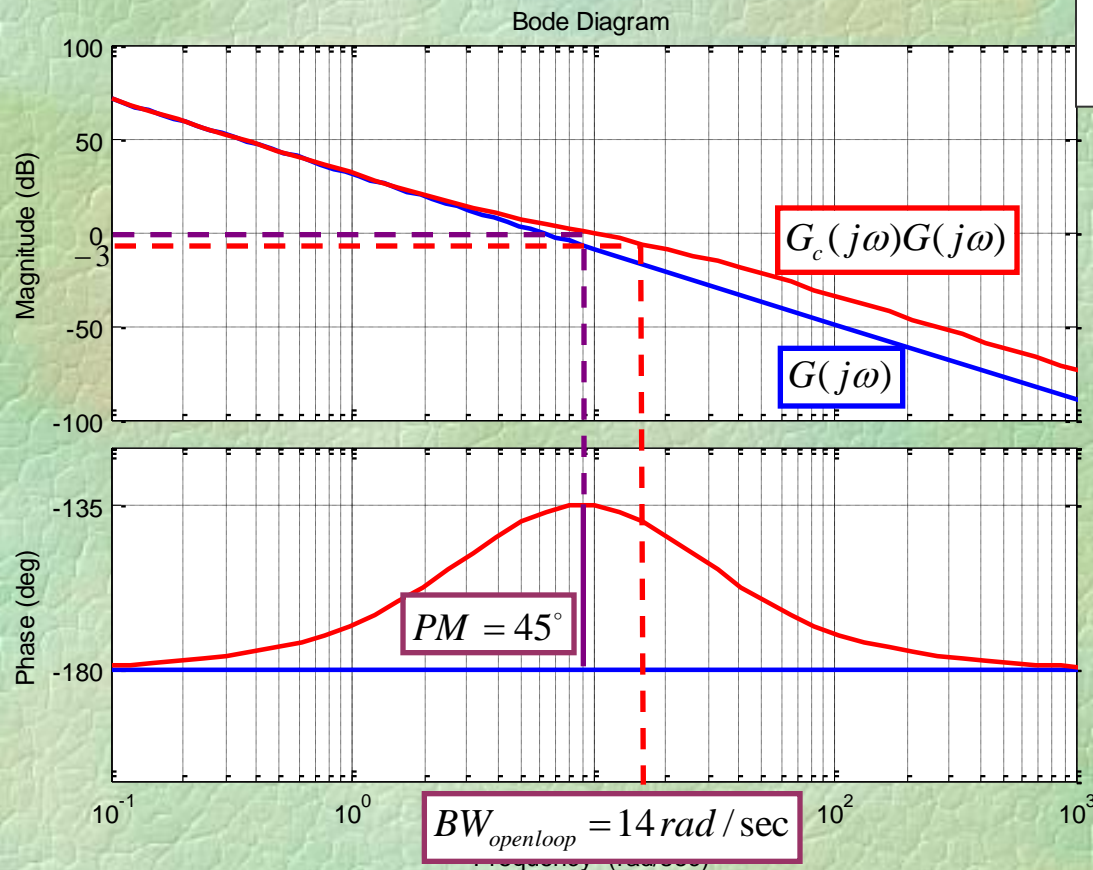


Bode Diagram



Phase-lead controller design in the frequency domain

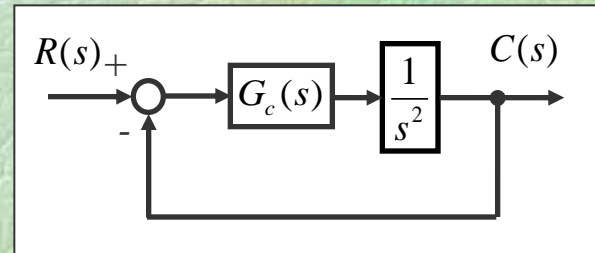
Step 6: Check the controller.



Controller is not ok

Try again

Phase-lead controller design in the frequency domain



Step 2: Try to fix k according to performance request, otherwise let $k=1$

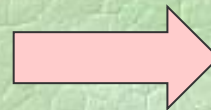
Open loop bandwidth is near to gain crossover frequency so:

$$20 \log \left| \frac{k}{\omega^2} \right|_{\omega=7} = -3$$



$$k = 35.7$$

$$20 \log \left| \frac{k}{\omega^2} \right|_{\omega=6} = -3$$



$$k = 25.5$$

Phase-lead controller design in the frequency domain

Step 3: Sketch the Bode plot of system (with the fixed k) without controller.

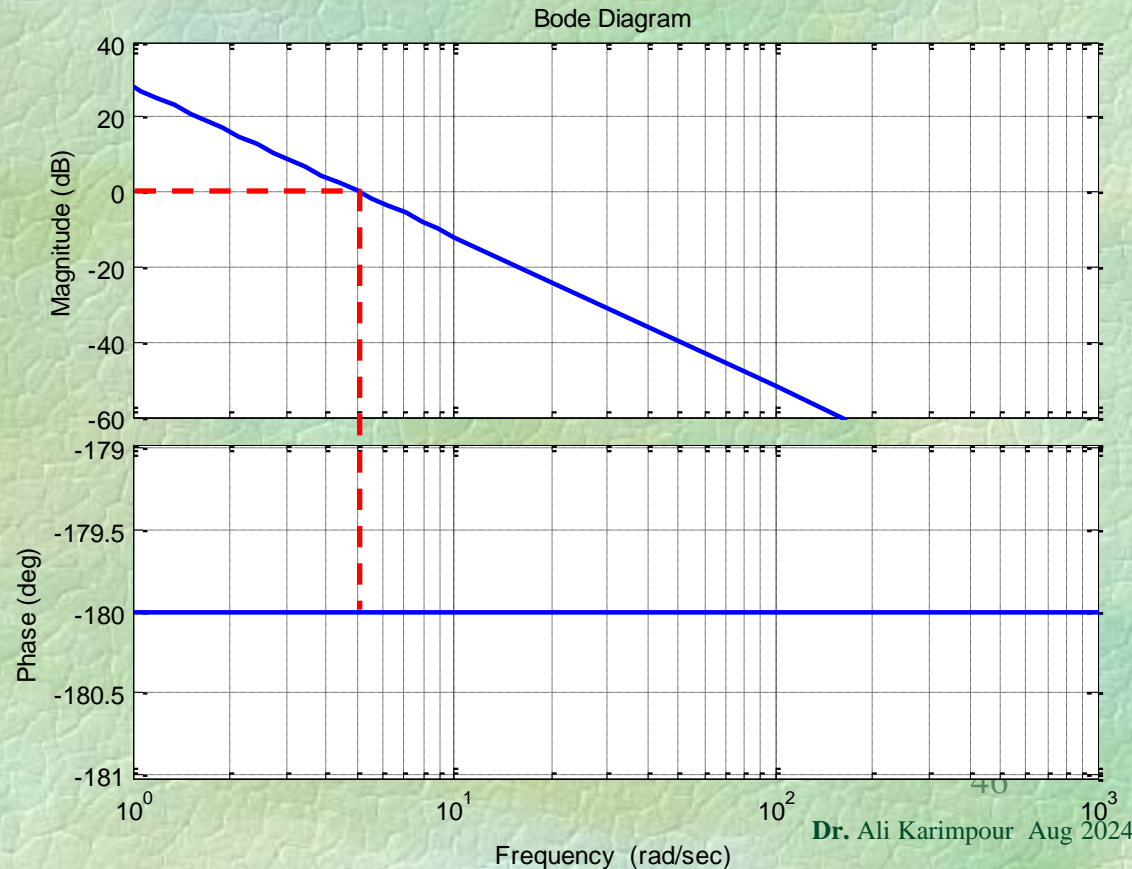
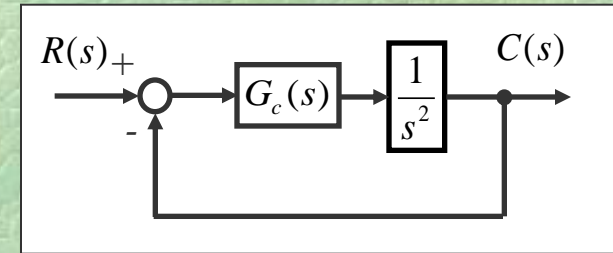
$$G(s) = \frac{25.5}{s^2}$$

Step 4: Find the system PM and if it is not sufficient choose the required phase by:

$$\varphi_m = \text{Desired PM} - \text{Existed PM} + \Delta$$

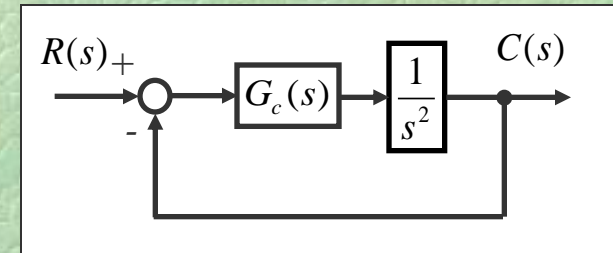
$$\varphi_m = 45^\circ - (0) + 0^\circ = 45^\circ$$

$$\sin 45^\circ = \frac{a-1}{a+1} \Rightarrow a = 5.8$$



Phase-lead controller design in the frequency domain

Step 5: Put the center of controller in the new gain crossover frequency:



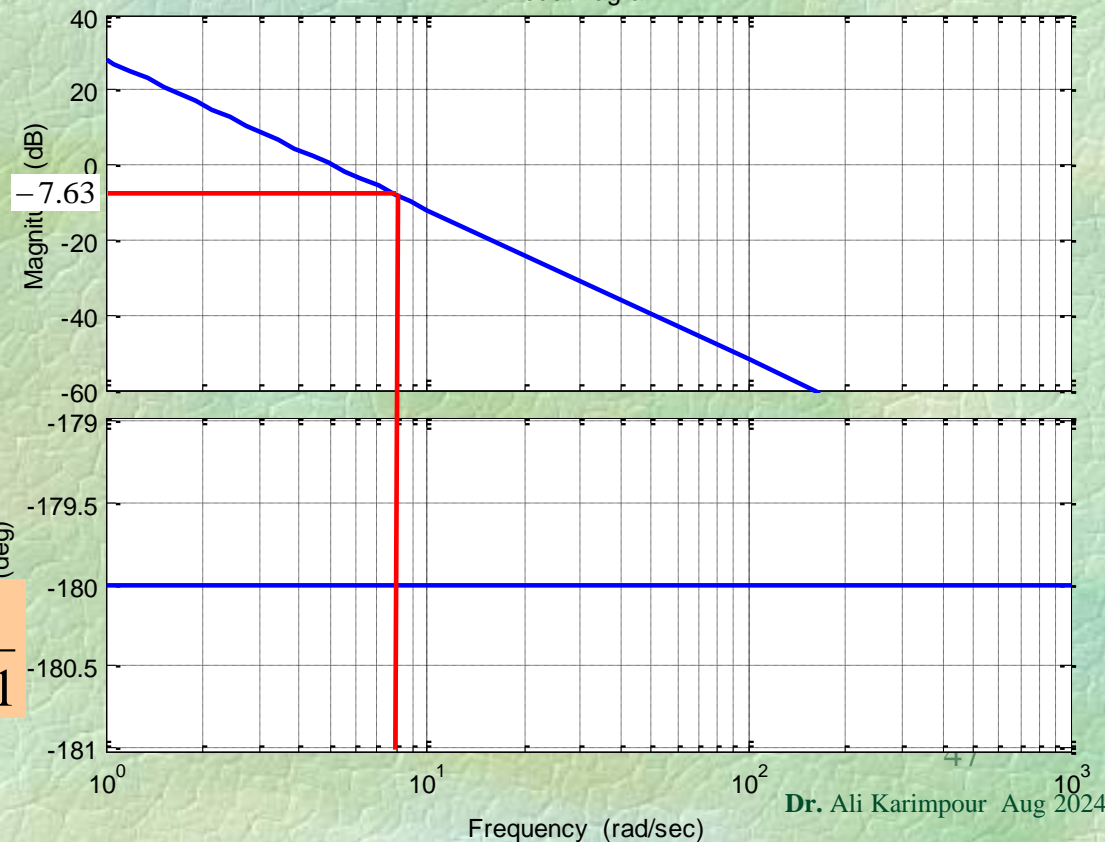
Bode Diagram

$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} + 10\log(a) = 0$$

$$20\log|G(j\omega)|_{\omega=\omega_c^{new}} = -10\log(a) = -7.63$$

$$\omega_c^{new} = 8$$

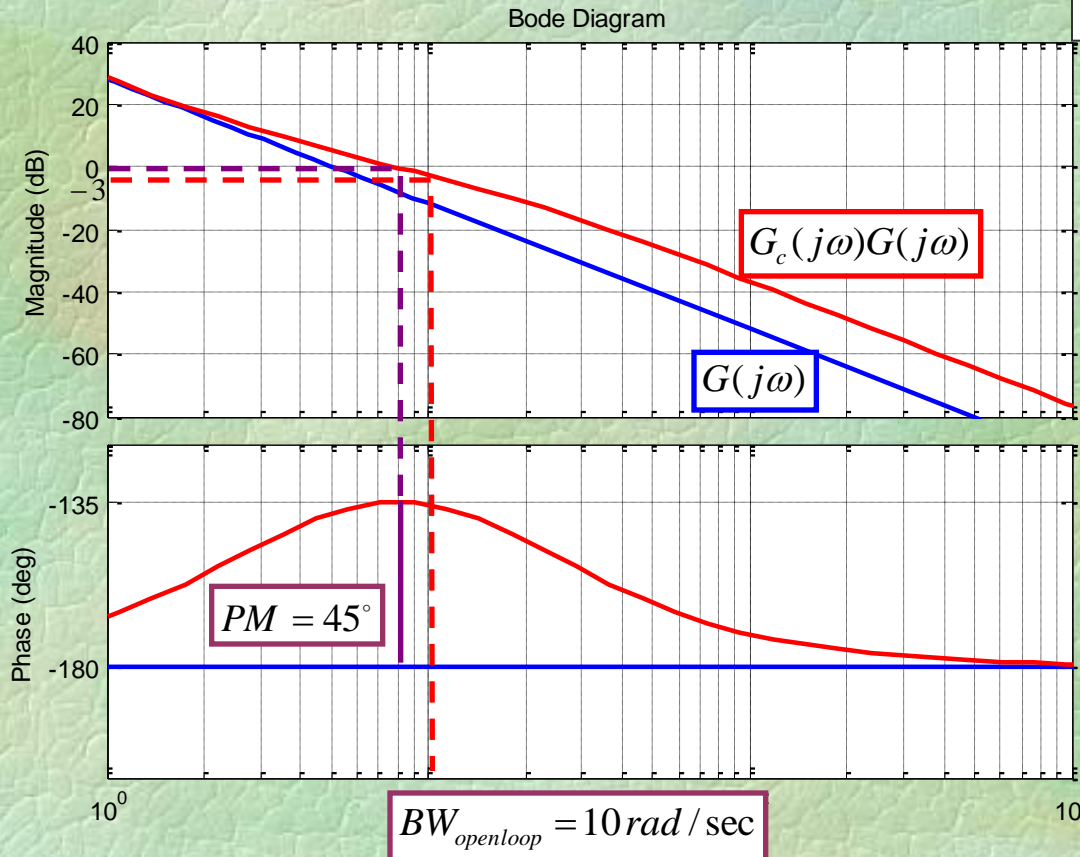
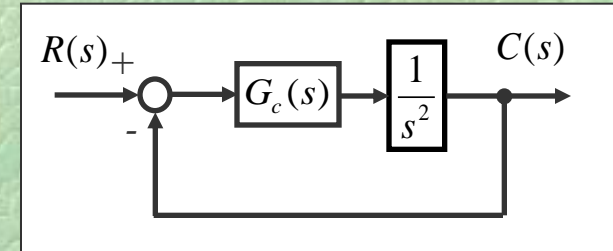
$$\omega_c^{new} = \frac{1}{\tau\sqrt{a}} \Rightarrow \tau = 0.0519$$



$$G_c(s) = k \frac{a\tau s + 1}{\tau s + 1} = 25.5 \frac{0.301s + 1}{0.0519s + 1}$$

Phase-lead controller design in the frequency domain

Step 6: Check the controller.



Controller is ok

It can be solved by a very better method without trial and error. Try to find it.

Exercises

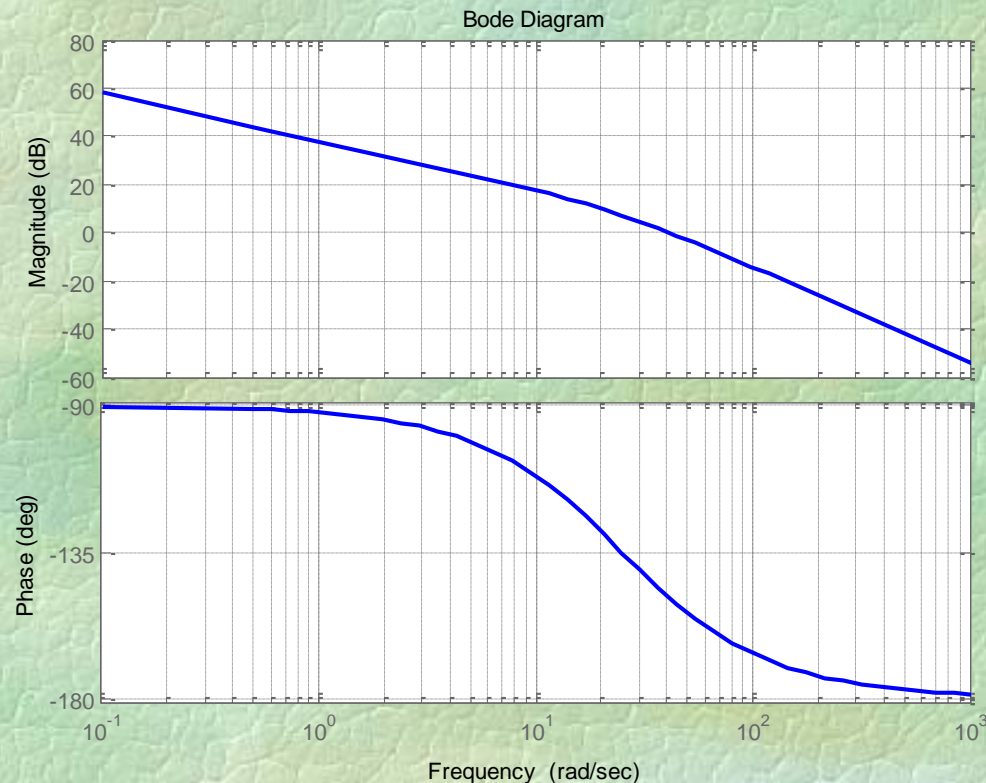
1- Following is the open loop transfer function of a system.

a) What is the velocity error constant. (answer 80)

b) Design lag controller such that $PM=45^\circ$. (answer $G_c(s)=(0.5s+1)/(1.56s+1)$)

c) Design lag controller such that $PM=45^\circ$ and the velocity error constant be 200.

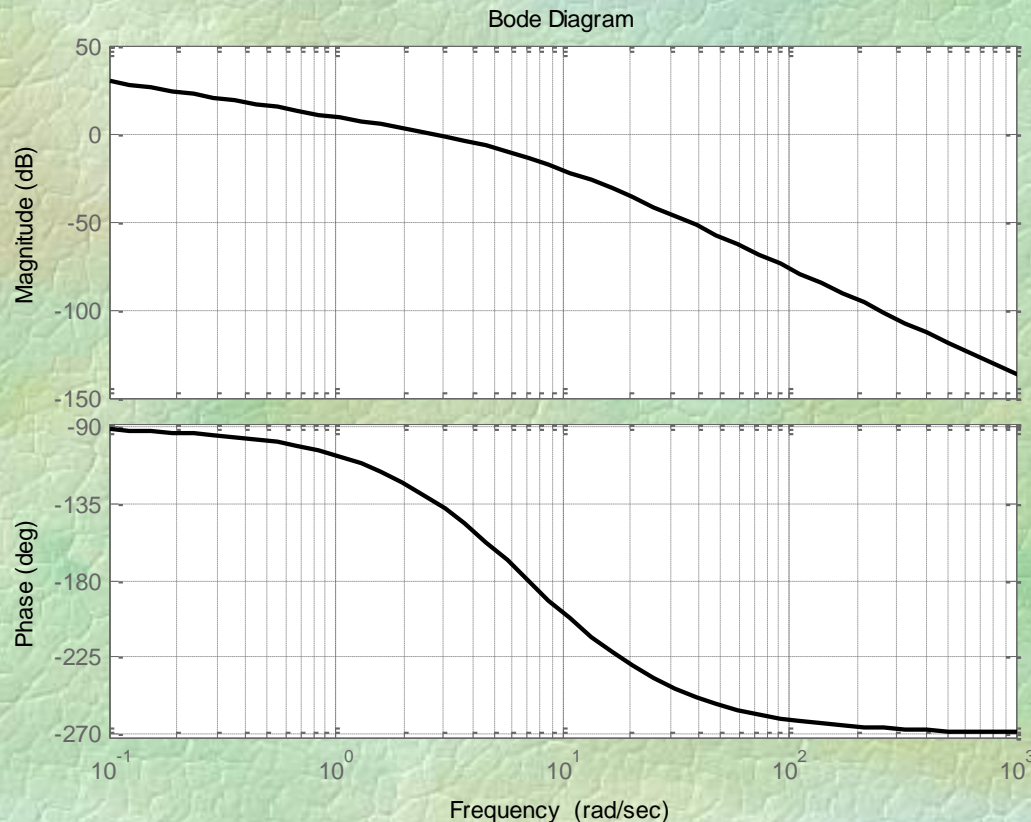
(answer $G_c(s)=(0.49s+1)/(3.75s+1)$)



Exercises

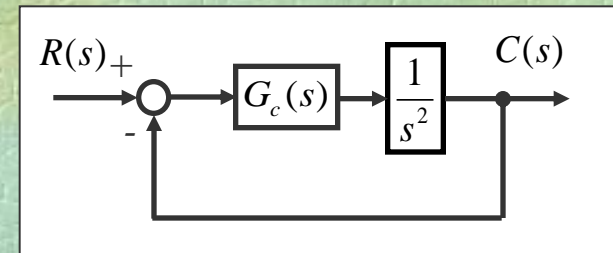
2- Following is the open loop transfer function of a system.

- Design a controller that the GM of system be 50 db.
- Design lag controller such that $PM=45^\circ$ and the velocity error constant be 30.

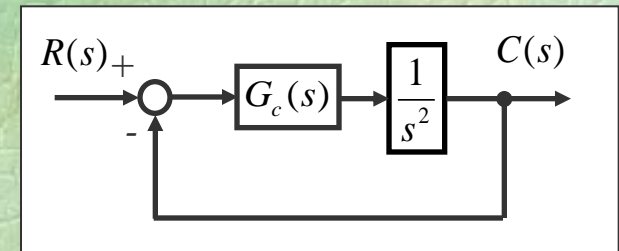


Exercises

3- Design a lead controller for the following system such that the phase margin be 40° and the open loop bandwidth be 23 rad/sec



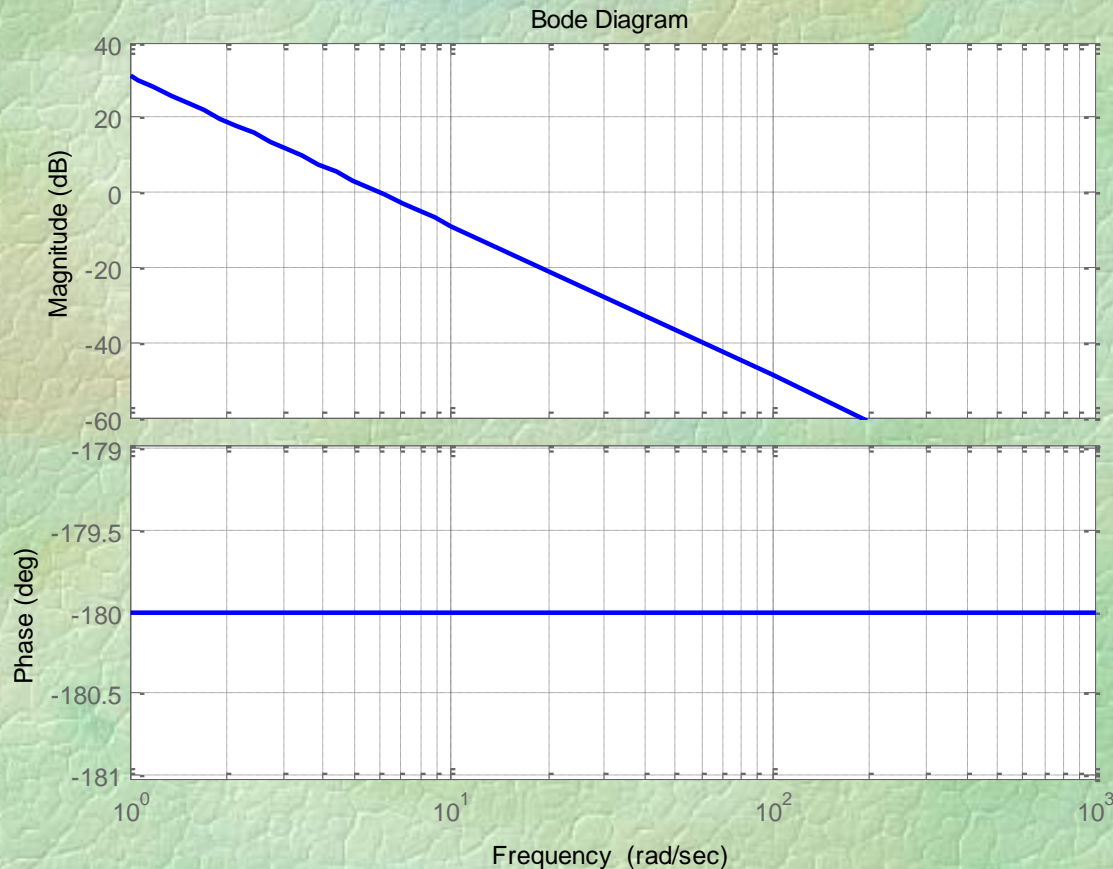
4- Design a lead controller for the following system such that the phase margin be 40° and the acceleration constant be 25.



$$\text{Answer : } G_c(s) = k \frac{0.301s + 1}{0.0519s + 1}$$

Exercises

5- Following is the open loop transfer function of a system. Design a lead controller such that $PM=45^\circ$. Draw the bode plot of compensated system.



Answer : $G_c(s) = k \frac{0.2674s + 1}{0.0461s + 1}$

Exercises

6- Following is the open loop transfer function of a system.

a) What is the velocity error constant. (answer 80)

b) Design a lead controller such that $PM=50^\circ$. (answer $G_c(s)=(0.0315s+1)/(0.0126s+1)$)

c) Design a lead controller such that $PM=50^\circ$ and the velocity error constant be 200. (answer $G_c(s)=2.5(0.0192s+1)/(0.0052s+1)$)

