Advanced Quantum Mechanics I, Fall 2023

Exam (0042)

Tuesday, July 06, 2024,1:30-3:30 pm

You have 120 minutes.

1- Prove that he eigenvalues of a Hermitian operator A are real and the eigenkets of the Hermitian operator A corresponding to different eigenvalues are orthogonal.

2- Consider a spin 1/2 system and let $|\alpha\rangle$ and $|\beta\rangle$ be $|S_z;+\rangle$ and $|S_x;+\rangle$, respectively. Write down explicitly the square matrix that corresponds to $|\alpha\rangle\langle\beta|$ in the usual (S_z diagonal) basis.

3- The Hamiltonian operator for a two-state system is given by

$$H = a(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|)$$

where a is a number with the dimension of energy. Find the energy eigenvalues and the corresponding energy eigenkets (as linear combinations of $|1\rangle$ and $|2\rangle$).

4- A certain observable in quantum mechanics has a 2×2 matrix representation as follows:

$$A = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$$

a) Find the normalized eigenvectors of this observable and the corresponding eigenvalues.
b) Find the unitary operator U which diagolizes A.

5- The Hamiltonian operator for a three-state system is given by

$$H = \varepsilon_\circ \begin{pmatrix} 1 & \circ & \circ \\ \circ & \circ & 1 \\ \circ & 1 & \circ \end{pmatrix}$$

Suppose that the system is in the state $\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Compute $\langle H^2 \rangle_\alpha$, the expectation value of H^2 taken with respect to state α.