بسمه تعالی

کار مجازی

Equilibrium and stability of this articulated crane as a function of its position can be determined using the methods of work and energy, which are explained in this chapter.

Ref. Engineering mechanics, Statics, Thirteenth edition, R. C. Hibbler, 2013.

مثالهای تکمیلی

EXAMPLE 11.1

Determine the angle θ for equilibrium of the two-member linkage shown in Fig. 11–6*a*. Each member has a mass of 10 kg.

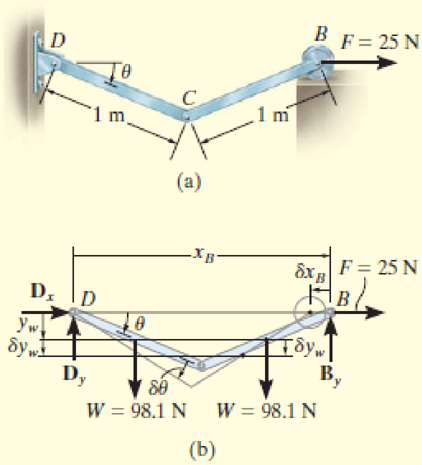


Fig. 11–6

SOLUTION

Free-Body Diagram. The system has only one degree of freedom since the location of both links can be specified by the single coordinate, $(q =)\ \theta$. As shown on the free-body diagram in Fig. 11–6*b*, when θ has a *positive* (clockwise) virtual rotation $\delta\theta$, only the force **F** and the two 98.1-N weights do work. (The reactive forces $\mathbf{D}_x$ and $\mathbf{D}_y$ are fixed, and $\mathbf{B}_y$ does not displace along its line of action.)

Virtual Displacements. If the origin of coordinates is established at the *fixed* pin support D, then the position of **F** and **W** can be specified by the *position coordinates* x_B and y_w. In order to determine the work, note that, as required, these coordinates are parallel to the lines of action of their associated forces. Expressing these position coordinates in terms of θ and taking the derivatives yields

$$x_B = 2(1 \cos\theta) \text{ m} \qquad \delta x_B = -2 \sin\theta\, \delta\theta \text{ m} \qquad (1)$$

$$y_w = \tfrac{1}{2}(1 \sin\theta) \text{ m} \qquad \delta y_w = 0.5 \cos\theta\, \delta\theta \text{ m} \qquad (2)$$

It is seen by the *signs* of these equations, and indicated in Fig. 11–6*b*, that an *increase* in θ (i.e., $\delta\theta$) causes a *decrease* in x_B and an *increase* in y_w.

Virtual-Work Equation. If the virtual displacements δx_B and δy_w were *both positive*, then the forces **W** and **F** would do positive work since the forces and their corresponding displacements would have the same sense. Hence, the virtual-work equation for the displacement $\delta\theta$ is

$$\delta U = 0; \qquad W\,\delta y_w + W\,\delta y_w + F\,\delta x_B = 0 \qquad (3)$$

Substituting Eqs. 1 and 2 into Eq. 3 in order to relate the virtual displacements to the common virtual displacement $\delta\theta$ yields

$$98.1(0.5 \cos\theta\, \delta\theta) + 98.1(0.5 \cos\theta\, \delta\theta) + 25(-2 \sin\theta\, \delta\theta) = 0$$

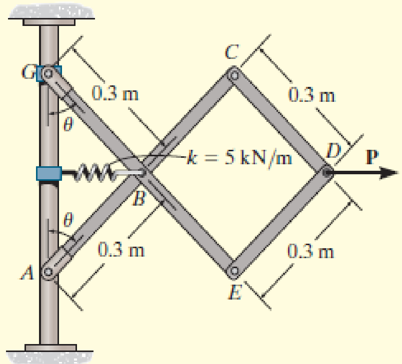
Notice that the "negative work" done by **F** (force in the opposite sense to displacement) has actually been *accounted for* in the above equation by the "negative sign" of Eq. 1. Factoring out the *common displacement* $\delta\theta$ and solving for θ, noting that $\delta\theta \neq 0$, yields

$$(98.1 \cos\theta - 50 \sin\theta)\, \delta\theta = 0$$

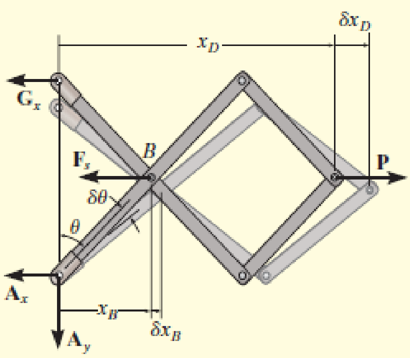
$$\theta = \tan^{-1}\frac{98.1}{50} = 63.0^\circ \qquad \textit{Ans.}$$

NOTE: If this problem had been solved using the equations of equilibrium, it would be necessary to dismember the links and apply three scalar equations to *each* link. The principle of virtual work, by means of calculus, has eliminated this task so that the answer is obtained directly.

EXAMPLE 11.2



(a)



(b)

Fig. 11–7

Determine the required force P in Fig. 11–7*a* needed to maintain equilibrium of the scissors linkage when $\theta = 60°$. The spring is unstretched when $\theta = 30°$. Neglect the mass of the links.

SOLUTION

Free-Body Diagram. Only $\mathbf{F}_s$ and $\mathbf{P}$ do work when θ undergoes a *positive* virtual displacement $\delta\theta$, Fig. 11–7*b*. For the arbitrary position θ, the spring is stretched $(0.3\text{ m})\sin\theta - (0.3\text{ m})\sin 30°$, so that

$$F_s = ks = 5000\text{ N/m}\,[(0.3\text{ m})\sin\theta - (0.3\text{ m})\sin 30°]$$
$$= (1500\sin\theta - 750)\text{ N}$$

Virtual Displacements. The position coordinates, x_B and x_D, measured from the *fixed point A*, are used to locate $\mathbf{F}_s$ and $\mathbf{P}$. These coordinates are parallel to the line of action of their corresponding forces. Expressing x_B and x_D in terms of the angle θ using trigonometry,

$$x_B = (0.3\text{ m})\sin\theta$$
$$x_D = 3[(0.3\text{ m})\sin\theta] = (0.9\text{ m})\sin\theta$$

Differentiating, we obtain the virtual displacements of points B and D.

$$\delta x_B = 0.3\cos\theta\,\delta\theta \qquad (1)$$
$$\delta x_D = 0.9\cos\theta\,\delta\theta \qquad (2)$$

Virtual-Work Equation. Force $\mathbf{P}$ does positive work since it acts in the positive sense of its virtual displacement. The spring force $\mathbf{F}_s$ does negative work since it acts opposite to its positive virtual displacement. Thus, the virtual-work equation becomes

$$\delta U = 0; \qquad -F_s\,\delta x_B + P\delta x_D = 0$$
$$-[1500\sin\theta - 750](0.3\cos\theta\,\delta\theta) + P(0.9\cos\theta\,\delta\theta) = 0$$
$$[0.9P + 225 - 450\sin\theta]\cos\theta\,\delta\theta = 0$$

Since $\cos\theta\,\delta\theta \neq 0$, then this equation requires

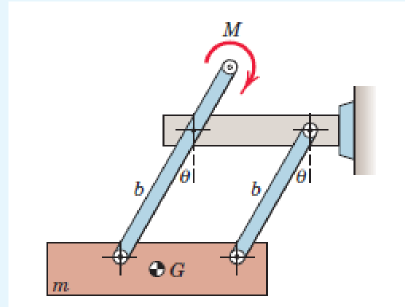
$$P = 500\sin\theta - 250$$

When $\theta = 60°$,

$$P = 500\sin 60° - 250 = 183\text{ N}$$ *Ans.*

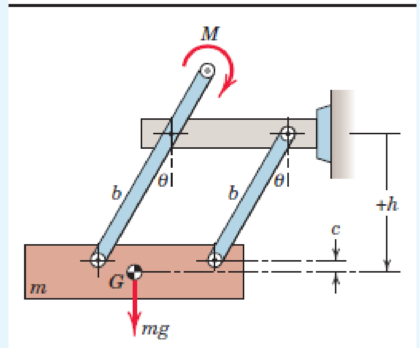
SAMPLE PROBLEM 7/2

The mass m is brought to an equilibrium position by the application of the couple M to the end of one of the two parallel links which are hinged as shown. The links have negligible mass, and all friction is assumed to be absent. Determine the expression for the equilibrium angle θ assumed by the links with the vertical for a given value of M. Consider the alternative of a solution by force and moment equilibrium.



Solution. The active-force diagram shows the weight mg acting through the center of mass G and the couple M applied to the end of the link. There are no other external active forces or moments which do work on the system during a change in the angle θ.

The vertical position of the center of mass G is designated by the distance h below the fixed horizontal reference line and is $h = b \cos \theta + c$. The work done by mg during a movement δh in the direction of mg is



$$
\begin{aligned}
+mg\, \delta h &= mg\, \delta(b \cos \theta + c) \\
&= mg(-b \sin \theta\, \delta\theta + 0) \\
&= -mgb \sin \theta\, \delta\theta
\end{aligned}
$$

① The minus sign shows that the work is negative for a positive value of $\delta\theta$. The constant c drops out since its variation is zero.

With θ measured positive in the clockwise sense, $\delta\theta$ is also positive clockwise. Thus, the work done by the clockwise couple M is $+M\, \delta\theta$. Substitution into the virtual-work equation gives us

$[\delta U = 0]$ $$M\, \delta\theta + mg\, \delta h = 0$$

which yields

$$M\, \delta\theta = mgb \sin \theta\, \delta\theta$$

$$\theta = \sin^{-1} \frac{M}{mgb} \qquad \textit{Ans.}$$

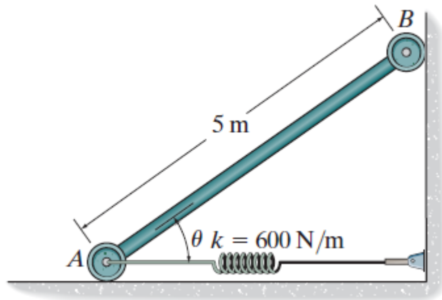
Inasmuch as $\sin \theta$ cannot exceed unity, we see that for equilibrium, M is limited to values that do not exceed mgb.

The advantage of the virtual-work solution for this problem is readily seen when we observe what would be involved with a solution by force and moment equilibrium. For the latter approach, it would be necessary for us to draw separate free-body diagrams of all of the three moving parts and account for all of the internal reactions at the pin connections. To carry out these steps, it would be necessary for us to include in the analysis the horizontal position of G with respect to the attachment points of the two links, even though reference to this position would finally drop out of the equations when they were solved. We conclude, then, that the virtual-work method in this problem deals directly with cause and effect and avoids reference to irrelevant quantities.

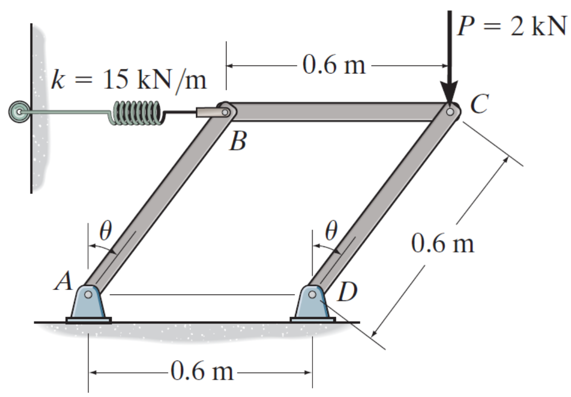
Helpful Hint

① Again, as in Sample Problem 7/1, we are consistent mathematically with our definition of work, and we see that the algebraic sign of the resulting expression agrees with the physical change.

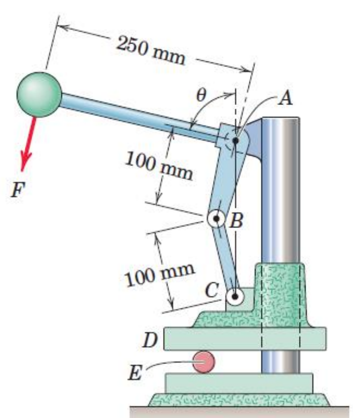
تمرین ۱- زاویه θ برای اینکه میله 50 kg در حال تعادل باشد را بیابید. در θ=60° فنر کشیدگی ندارد.



تمرین ۲- در اهرم بندی نشان داده شده وقتی θ=0° است فنر کشیدگی ندارد. اگر نیروی P به اهرم بندی وارد شود، زاویه θ تعادل رابیابید.



تمرین ۳- در مکانیزم پرس نشان داده شده میزان نیروی F برای اینکه نیروی 1000 N از سمت هر فک پرس به قطعه E وارد شود را بیابید. اصطکاک بین اجزاء ناچیز است.



موفق باشید.

تحویل: یک هفته بعد از بارگذاری