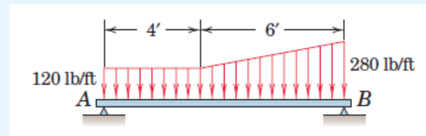
بسمه تعالی

فصل پنجم- بخش سوم: تیرها

مثالهای تکمیلی:

SAMPLE PROBLEM 5/11

Determine the equivalent concentrated load(s) and external reactions for the simply supported beam which is subjected to the distributed load shown.



Solution. The area associated with the load distribution is divided into the rectangular and triangular areas shown. The concentrated-load values are determined by computing the areas, and these loads are located at the centroids of the respective areas. (1)

Once the concentrated loads are determined, they are placed on the free-body diagram of the beam along with the external reactions at A and B. Using principles of equilibrium, we have

$[\Sigma M_A = 0]$ $\qquad 1200(5) + 480(8) - R_B(10) = 0$

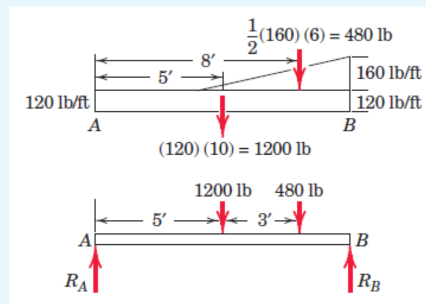
$R_B = 984$ lb $\qquad$ *Ans.*

$[\Sigma M_B = 0]$ $\qquad R_A(10) - 1200(5) - 480(2) = 0$

$R_A = 696$ lb $\qquad$ *Ans.*

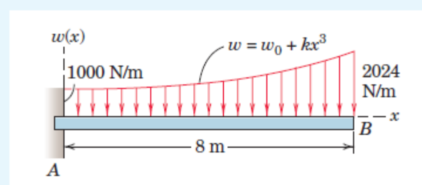
Helpful Hint

(1) Note that it is usually unnecessary to reduce a given distributed load to a *single* concentrated load.



SAMPLE PROBLEM 5/12

Determine the reaction at the support A of the loaded cantilever beam.



Solution. The constants in the load distribution are found to be $w_0 = 1000$ N/m and $k = 2$ N/m^4. The load R is then (1)

$$R = \int w\, dx = \int_0^8 (1000 + 2x^3)\, dx = \left(1000x + \frac{x^4}{2}\right)\Big|_0^8 = 10\ 050 \text{ N}$$

(2) The x-coordinate of the centroid of the area is found by

$$\bar{x} = \frac{\int xw\, dx}{R} = \frac{1}{10\ 050}\int_0^8 x(1000 + 2x^3)\, dx$$

$$= \frac{1}{10\ 050}\left(500x^2 + \tfrac{2}{5}x^5\right)\Big|_0^8 = 4.49 \text{ m}$$

From the free-body diagram of the beam, we have

$[\Sigma M_A = 0]$ $\qquad M_A - (10\ 050)(4.49) = 0$

$M_A = 45\ 100$ N·m $\qquad$ *Ans.*

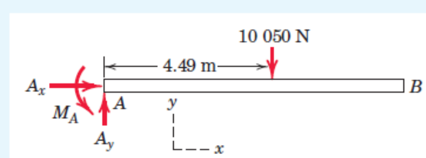
$[\Sigma F_y = 0]$ $\qquad A_y = 10\ 050$ N $\qquad$ *Ans.*

Note that $A_x = 0$ by inspection.

Helpful Hints

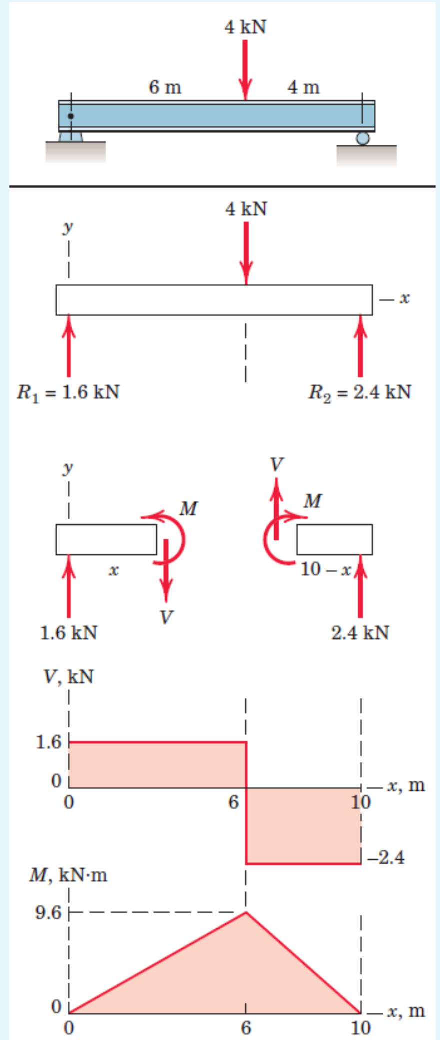
(1) Use caution with the units of the constants w_0 and k.

(2) The student should recognize that the calculation of R and its location $\bar{x}$ is simply an application of centroids as treated in Art. 5/3.



SAMPLE PROBLEM 5/13

Determine the shear and moment distributions produced in the simple beam by the 4-kN concentrated load.



Solution. From the free-body diagram of the entire beam we find the support reactions, which are

$$R_1 = 1.6 \text{ kN} \qquad R_2 = 2.4 \text{ kN}$$

A section of the beam of length x is next isolated with its free-body diagram on which we show the shear V and the bending moment M in their positive directions. Equilibrium gives

$[\Sigma F_y = 0]$ $\qquad 1.6 - V = 0 \qquad V = 1.6 \text{ kN}$

$[\Sigma M_{R_1} = 0]$ $\qquad M - 1.6x = 0 \qquad M = 1.6x$

① These values of V and M apply to all sections of the beam to the left of the 4-kN load.

A section of the beam to the right of the 4-kN load is next isolated with its free-body diagram on which V and M are shown in their positive directions. Equilibrium requires

$[\Sigma F_y = 0]$ $\qquad V + 2.4 = 0 \qquad V = -2.4 \text{ kN}$

$[\Sigma M_{R_2} = 0]$ $\qquad -(2.4)(10 - x) + M = 0 \qquad M = 2.4(10 - x)$

These results apply only to sections of the beam to the right of the 4-kN load.

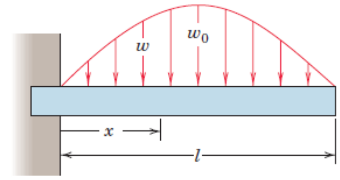
The values of V and M are plotted as shown. The maximum bending moment occurs where the shear changes direction. As we move in the positive x-direction starting with $x = 0$, we see that the moment M is merely the accumulated area under the shear diagram.

Helpful Hint

① We must be careful not to take our section at a concentrated load (such as $x = 6$ m) since the shear and moment relations involve discontinuities at such positions.

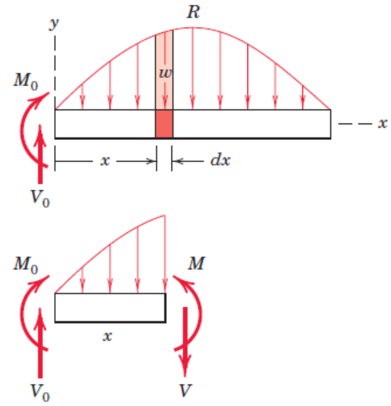
SAMPLE PROBLEM 5/14

The cantilever beam is subjected to the load intensity (force per unit length) which varies as $w = w_0 \sin(\pi x/l)$. Determine the shear force V and bending moment M as functions of the ratio x/l.



Solution. The free-body diagram of the entire beam is drawn first so that the shear force V_0 and bending moment M_0 which act at the supported end at $x = 0$ can be computed. By convention V_0 and M_0 are shown in their positive mathematical senses. A summation of vertical forces for equilibrium gives

$$[\Sigma F_y = 0] \qquad V_0 - \int_0^l w\,dx = 0 \qquad V_0 = \int_0^l w_0 \sin\frac{\pi x}{l}\,dx = \frac{2w_0 l}{\pi}$$



(1) A summation of moments about the left end at $x = 0$ for equilibrium gives

$$[\Sigma M = 0] \qquad -M_0 - \int_0^l x(w\,dx) = 0 \qquad M_0 = -\int_0^l w_0 x \sin\frac{\pi x}{l}\,dx$$

$$M_0 = \frac{-w_0 l^2}{\pi^2}\left[\sin\frac{\pi x}{l} - \frac{\pi x}{l}\cos\frac{\pi x}{l}\right]_0^l = -\frac{w_0 l^2}{\pi}$$

From a free-body diagram of an arbitrary section of length x, integration of Eq. 5/10 permits us to find the shear force internal to the beam. Thus,

$$\text{(2)} \quad [dV = -w\,dx] \qquad \int_{V_0}^{V} dV = -\int_0^x w_0 \sin\frac{\pi x}{l}\,dx$$

$$V - V_0 = \left[\frac{w_0 l}{\pi}\cos\frac{\pi x}{l}\right]_0^x \qquad V - \frac{2w_0 l}{\pi} = \frac{w_0 l}{\pi}\left(\cos\frac{\pi x}{l} - 1\right)$$

or in dimensionless form

$$\frac{V}{w_0 l} = \frac{1}{\pi}\left(1 + \cos\frac{\pi x}{l}\right) \qquad \textit{Ans.}$$

The bending moment is obtained by integration of Eq. 5/11, which gives

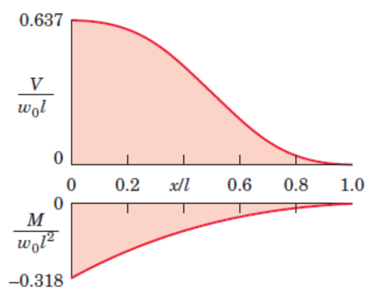
$$[dM = V\,dx] \qquad \int_{M_0}^{M} dM = \int_0^x \frac{w_0 l}{\pi}\left(1 + \cos\frac{\pi x}{l}\right)dx$$

$$M - M_0 = \frac{w_0 l}{\pi}\left[x + \frac{l}{\pi}\sin\frac{\pi x}{l}\right]_0^x$$

$$M = -\frac{w_0 l^2}{\pi} + \frac{w_0 l}{\pi}\left[x + \frac{l}{\pi}\sin\frac{\pi x}{l} - 0\right]$$

or in dimensionless form

$$\frac{M}{w_0 l^2} = \frac{1}{\pi}\left(\frac{x}{l} - 1 + \frac{1}{\pi}\sin\frac{\pi x}{l}\right) \qquad \textit{Ans.}$$



The variations of $V/w_0 l$ and $M/w_0 l^2$ with x/l are shown in the bottom figures. The negative values of $M/w_0 l^2$ indicate that physically the bending moment is in the direction opposite to that shown.

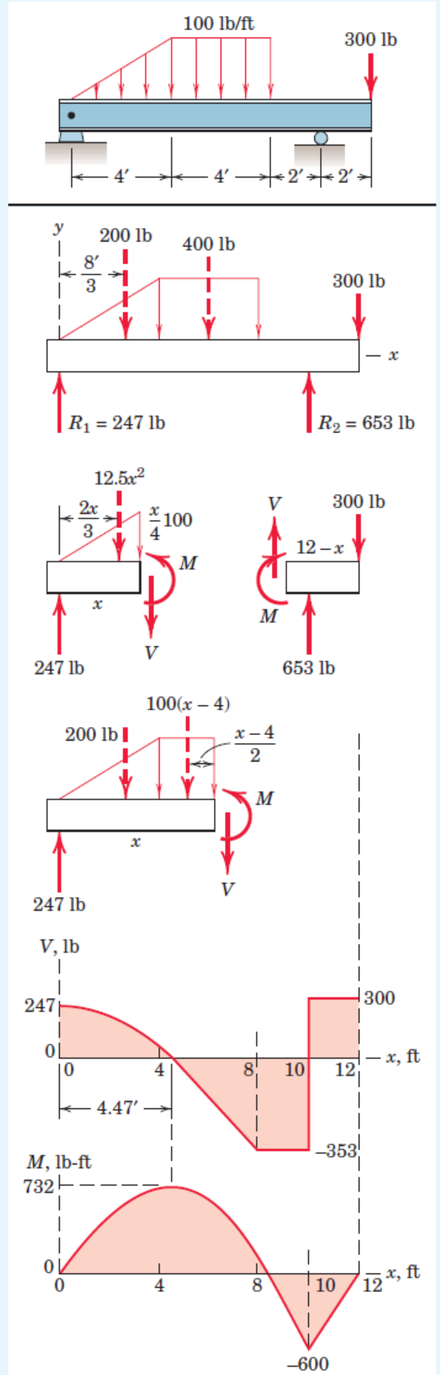
Helpful Hints

(1) In this case of symmetry it is clear that the resultant $R = V_0 = 2w_0 l/\pi$ of the load distribution acts at midspan, so that the moment requirement is simply $M_0 = -Rl/2 = -w_0 l^2/\pi$. The minus sign tells us that physically the bending moment at $x = 0$ is opposite to that represented on the free-body diagram.

(2) The free-body diagram serves to remind us that the integration limits for V as well as for x must be accounted for. We see that the expression for V is positive, so that the shear force is as represented on the free-body diagram.

SAMPLE PROBLEM 5/15

Draw the shear-force and bending-moment diagrams for the loaded beam and determine the maximum moment M and its location x from the left end.



Solution. The support reactions are most easily obtained by considering the resultants of the distributed loads as shown on the free-body diagram of the beam as a whole. The first interval of the beam is analyzed from the free-body diagram of the section for $0 < x < 4$ ft. A summation of vertical forces and a moment summation about the cut section yield

$$[\Sigma F_y = 0] \qquad V = 247 - 12.5x^2$$

$$[\Sigma M = 0] \qquad M + (12.5x^2)\frac{x}{3} - 247x = 0 \qquad M = 247x - 4.17x^3$$

These values of V and M hold for $0 < x < 4$ ft and are plotted for that interval in the shear and moment diagrams shown.

From the free-body diagram of the section for which $4 < x < 8$ ft, equilibrium in the vertical direction and a moment sum about the cut section give

$$[\Sigma F_y = 0] \qquad V + 100(x - 4) + 200 - 247 = 0 \qquad V = 447 - 100x$$

$$[\Sigma M = 0] \qquad M + 100(x - 4)\frac{x - 4}{2} + 200[x - \tfrac{2}{3}(4)] - 247x = 0$$

$$M = -267 + 447x - 50x^2$$

These values of V and M are plotted on the shear and moment diagrams for the interval $4 < x < 8$ ft.

The analysis of the remainder of the beam is continued from the free-body diagram of the portion of the beam to the right of a section in the next interval. It should be noted that V and M are represented in their positive directions. A vertical-force summation and a moment summation about the section yield

$$V = -353 \text{ lb} \qquad \text{and} \qquad M = 2930 - 353x$$

These values of V and M are plotted on the shear and moment diagrams for the interval $8 < x < 10$ ft.

The last interval may be analyzed by inspection. The shear is constant at +300 lb, and the moment follows a straight-line relation beginning with zero at the right end of the beam.

The maximum moment occurs at $x = 4.47$ ft, where the shear curve crosses the zero axis, and the magnitude of M is obtained for this value of x by substitution into the expression for M for the second interval. The maximum moment is

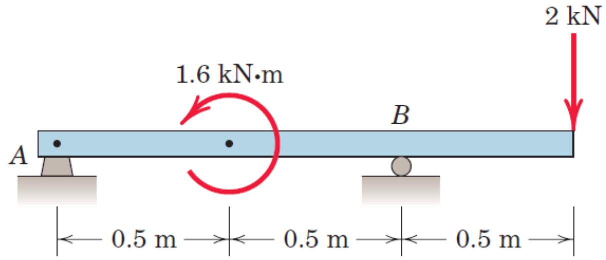
$$M = 732 \text{ lb-ft} \qquad \textit{Ans.}$$

As before, note that the change in moment M up to any section equals the area under the shear diagram up to that section. For instance, for $x < 4$ ft,

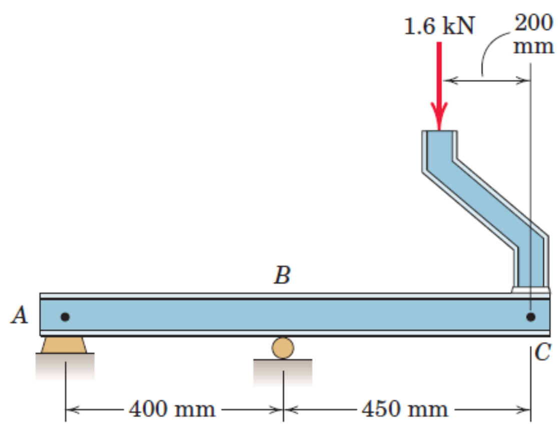
$$\left[\Delta M = \int V\,dx\right] \qquad M - 0 = \int_0^x (247 - 12.5x^2)\,dx$$

and, as above, $\qquad M = 247x - 4.17x^3$

تمرین: در تیرهای نشان داده شده در شکلهای زیر نمودار نیروی برشی و گشتاور خمشی را در طول تیر رسم کنید. بیشترین مقادیر این پارامترها و مکان آنها را تعیین نمایید.



شکل (۱)



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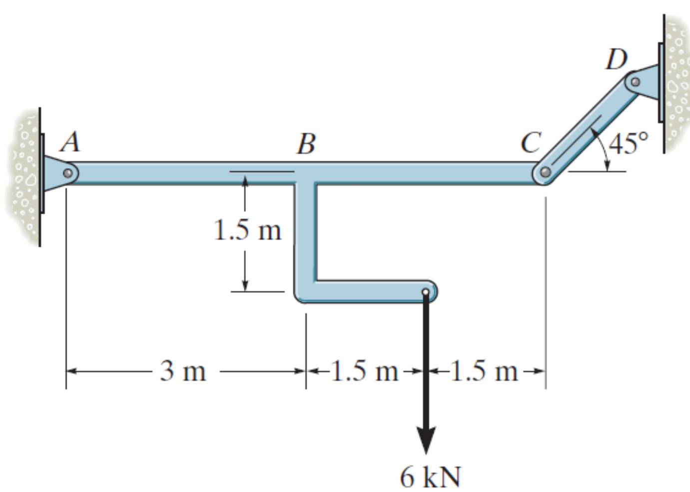

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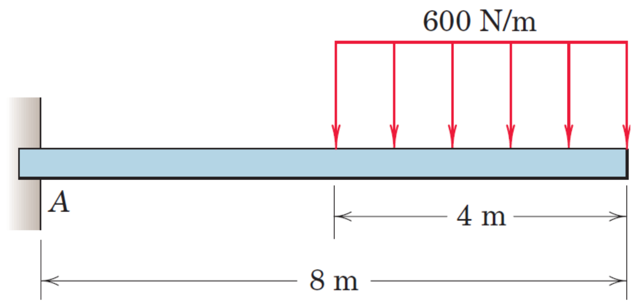
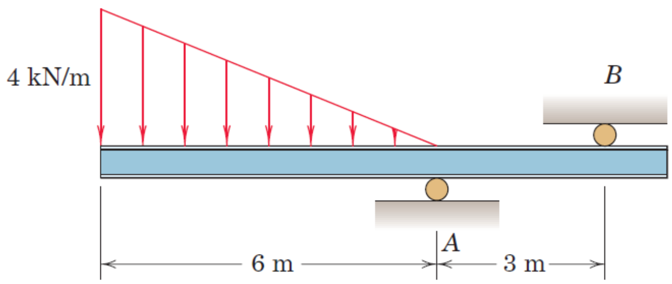
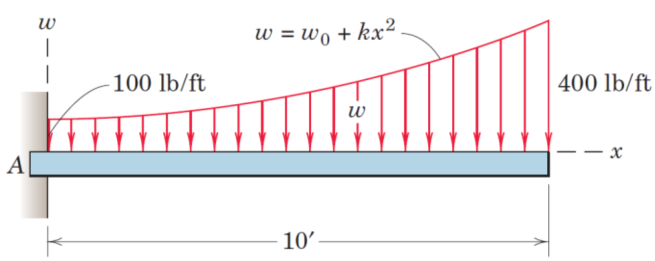


شکل (۴)



شکل (۵)



شکل (۶)

موفق باشید.