

بسمه تعالی

فصل پنجم- بخش دوم: مرکز هندسی اجسام مرکب

مثالهای تکمیلی:

EXAMPLE 9.9

Locate the centroid of the wire shown in Fig. 9-16a.

SOLUTION

Composite Parts. The wire is divided into three segments as shown in Fig. 9-16b.

Moment Arms. The location of the centroid for each segment is determined and indicated in the figure. In particular, the centroid of segment ① is determined either by integration or by using the table on the inside back cover.

Summations. For convenience, the calculations can be tabulated as follows:

Segment	L (mm)	\tilde{x} (mm)	\tilde{y} (mm)	\tilde{z} (mm)	$\tilde{x}L$ (mm ²)	$\tilde{y}L$ (mm ²)	$\tilde{z}L$ (mm ²)
1	$\pi(60) = 188.5$	60	-38.2	0	11 310	-7200	0
2	40	0	20	0	0	800	0
3	20	0	40	-10	0	800	-200
	$\Sigma L = 248.5$				$\Sigma \tilde{x}L = 11\,310$	$\Sigma \tilde{y}L = -5600$	$\Sigma \tilde{z}L = -200$

Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}L}{\Sigma L} = \frac{11\,310}{248.5} = 45.5 \text{ mm} \quad \text{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}L}{\Sigma L} = \frac{-5600}{248.5} = -22.5 \text{ mm} \quad \text{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z}L}{\Sigma L} = \frac{-200}{248.5} = -0.805 \text{ mm} \quad \text{Ans.}$$

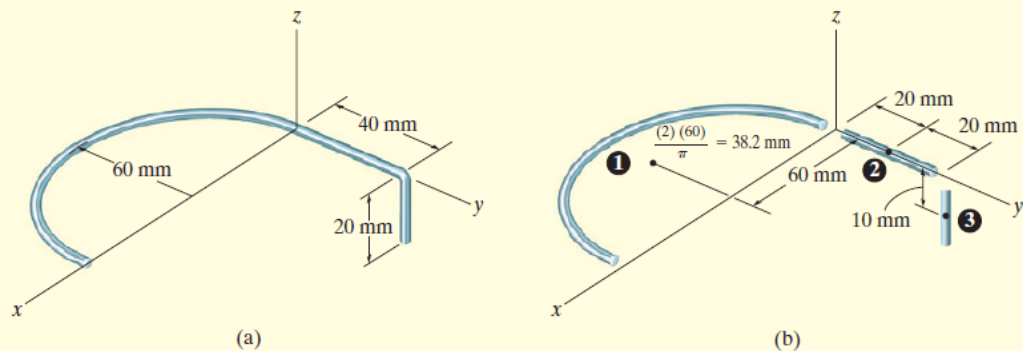


Fig. 9-16

SAMPLE PROBLEM 5/6

Locate the centroid of the shaded area.

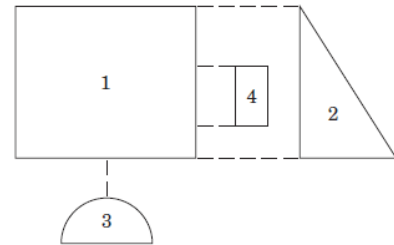
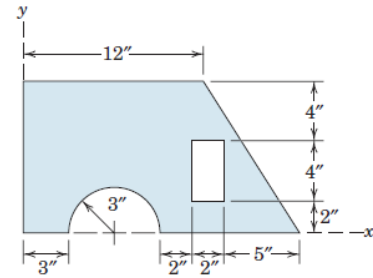
Solution. The composite area is divided into the four elementary shapes shown in the lower figure. The centroid locations of all these shapes may be obtained from Table D/3. Note that the areas of the “holes” (parts 3 and 4) are taken as negative in the following table:

PART	A in. ²	\bar{x} in.	\bar{y} in.	$\bar{x}A$ in. ³	$\bar{y}A$ in. ³
1	120	6	5	720	600
2	30	14	10/3	420	100
3	-14.14	6	1.273	-84.8	-18
4	-8	12	4	-96	-32
TOTALS	127.9			959	650

The area counterparts to Eqs. 5/7 are now applied and yield

$$\left[\bar{X} = \frac{\Sigma A \bar{x}}{\Sigma A} \right] \quad \bar{X} = \frac{959}{127.9} = 7.50 \text{ in.} \quad \text{Ans.}$$

$$\left[\bar{Y} = \frac{\Sigma A \bar{y}}{\Sigma A} \right] \quad \bar{Y} = \frac{650}{127.9} = 5.08 \text{ in.} \quad \text{Ans.}$$



SAMPLE PROBLEM 5/8

Locate the center of mass of the bracket-and-shaft combination. The vertical face is made from sheet metal which has a mass of 25 kg/m^2 . The material of the horizontal base has a mass of 40 kg/m^2 , and the steel shaft has a density of 7.83 Mg/m^3 .

Solution. The composite body may be considered to be composed of the five elements shown in the lower portion of the illustration. The triangular part will be taken as a negative mass. For the reference axes indicated it is clear by symmetry that the x -coordinate of the center of mass is zero.

The mass m of each part is easily calculated and should need no further explanation. For Part 1 we have from Sample Problem 5/3

$$\bar{z} = \frac{4r}{3\pi} = \frac{4(50)}{3\pi} = 21.2 \text{ mm}$$

For Part 3 we see from Sample Problem 5/2 that the centroid of the triangular mass is one-third of its altitude above its base. Measurement from the coordinate axes becomes

$$\bar{z} = -[150 - 25 - \frac{1}{3}(75)] = -100 \text{ mm}$$

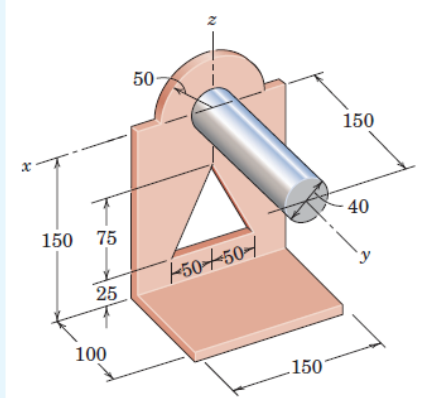
The y - and z -coordinates to the mass centers of the remaining parts should be evident by inspection. The terms involved in applying Eqs. 5/7 are best handled in the form of a table as follows:

PART	m kg	\bar{y} mm	\bar{z} mm	$m\bar{y}$ kg·m	$m\bar{z}$ kg·mm
1	0.098	0	21.2	0	2.08
2	0.562	0	-75.0	0	-42.19
3	-0.094	0	-100.0	0	9.38
4	0.600	50.0	-150.0	30.0	-90.00
5	1.476	75.0	0	110.7	0
TOTALS	2.642			140.7	-120.73

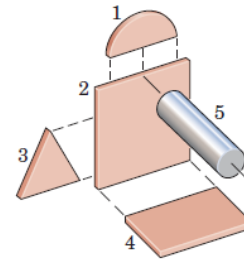
Equations 5/7 are now applied and the results are

$$\left[\bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} \right] \quad \bar{Y} = \frac{140.7}{2.642} = 53.3 \text{ mm} \quad \text{Ans.}$$

$$\left[\bar{Z} = \frac{\Sigma m\bar{z}}{\Sigma m} \right] \quad \bar{Z} = \frac{-120.73}{2.642} = -45.7 \text{ mm} \quad \text{Ans.}$$

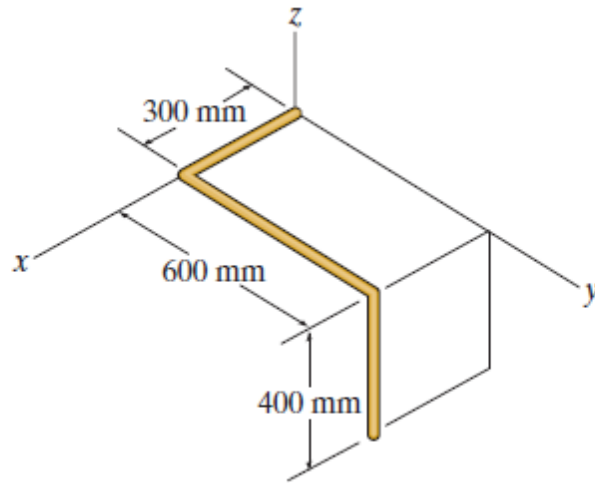


Dimensions in millimeters



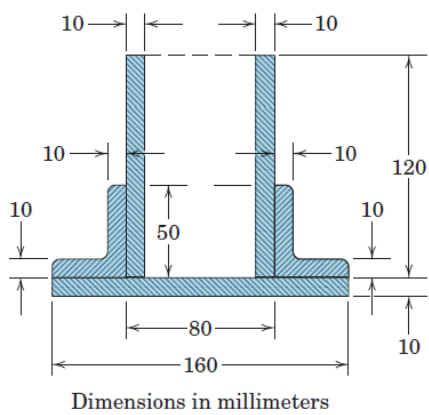
تمرینها:

تمرین ۱- مرکز هندسی سیم نشان داده شده در شکل ۱ را تعیین نمایید.

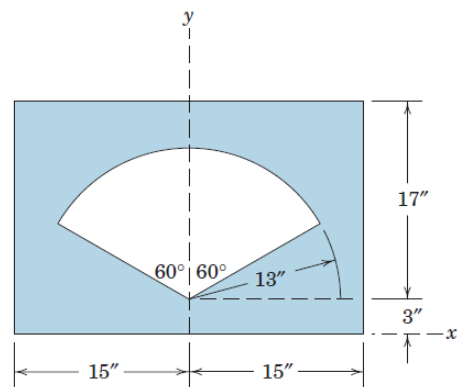


شکل (۱)

تمرین ۲- مرکز سطح سطوح نشان داده شده در شکل‌های ۲ و ۳ را تعیین نمایید.

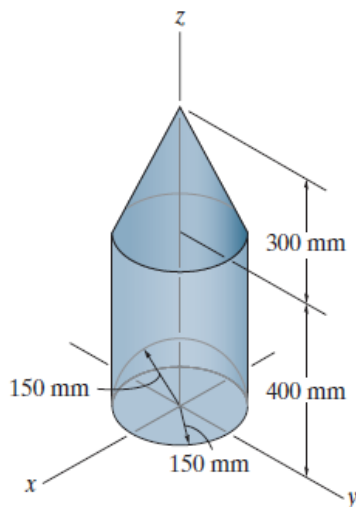


شکل (۳)



شکل (۲)

تمرین ۳- جسم نشان داده شده در شکل ۴ شامل یک استوانه و یک مخروط می‌باشد. در انتهای استوانه یک حفره نیم کره‌ای شکل ایجاد شده است. اگر چگالی استوانه و مخروط به ترتیب 2.7 Mg/m^3 و 7.8 Mg/m^3 باشد، مرکز جرم جسم مرکب را بیابید.



شکل (۴)

موفق باشید.