

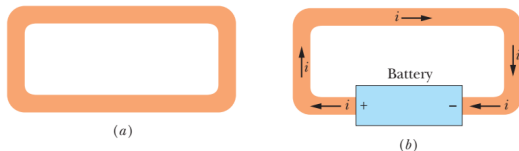
26. Current and Resistance

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Electric Current I

Although an electric current is a stream of moving charges, not all moving charges constitute an electric current. If there is to be an electric current through a given surface, there must be a net flow of charge through that surface.



$$i = \frac{dq}{dt} \quad (\text{definition of current}). \quad (26-1)$$

We can find the charge that passes through the plane in a time interval extending from 0 to t by integration

$$q = \int dq = \int_0^t i(t) dt \quad (26-2)$$

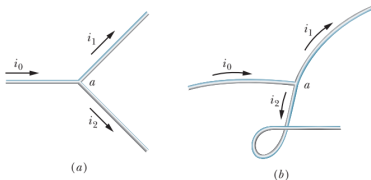
Electric Current II

The SI unit for current is the coulomb per second, or the ampere (A), which is an SI base unit:

$$1 \text{ ampere} = 1 \text{ A} = 1 \text{ coulomb per second} = 1 \text{ C/s.}$$

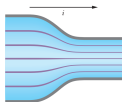
Current, as defined by Eq. 26-1, is a scalar because both charge and time in that equation are scalars. Because charge is conserved, the magnitudes of the currents in the branches must add to yield the magnitude of the current in the original conductor, so that

$$i_0 = i_1 + i_2. \quad (26-3)$$



A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

Current Density I



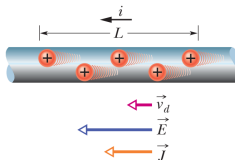
Sometimes we are interested in the current i in a particular conductor. At other times we take a localized view and study the flow of charge through a cross section of the conductor at a particular point. To describe this flow, we can use the **current density** $\vec{J}$ which has the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative. For each element of the cross section, the magnitude J is equal to the current per unit area through that element. We can write the amount of current through the element as $\vec{J} \cdot d\vec{A}$ where $d\vec{A}$ is the area vector of the element, perpendicular to the element. The total current through the surface is then

$$i = \int \vec{J} \cdot d\vec{A}. \quad (26-4)$$

$$\begin{aligned} i &= \int J dA = J \int dA = JA, \\ \Rightarrow J &= \frac{i}{A} \end{aligned} \quad (26-5)$$

Current Density II

When a conductor does not have a current through it, its conduction electrons move randomly, with no net motion in any direction. When the conductor does have a current through it, these electrons actually still move randomly, but now they tend to drift with a **drift speed** v_d in the direction opposite that of the applied electric field that causes the current.



The total charge of the carriers in the length L , each with charge e , is then

$$q = (nAL)e.$$

Because the carriers all move along the wire with speed v_d , this total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d}.$$

Current Density III

Equation 26-1 tells us that the current i is the time rate of transfer of charge across a cross section, so here we have

$$i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d.$$

Solving for v_d and recalling Eq. 26-5 ($J = i/A$), we obtain

$$v_d = \frac{i}{nAe} = \frac{J}{ne},$$

or, extended to vector form,

$$\vec{J} = (ne)\vec{v}_d. \quad (26-7)$$

Here the product ne , whose SI unit is the coulomb per cubic meter (C/m^3), is the *carrier charge density*.

If we apply the same potential difference between the ends of geometrically similar rods of copper and of glass, very different currents result. The characteristic of the conductor that enters here is its electrical resistance. We determine the resistance between any two points of a conductor by applying a potential difference V between those points and measuring the current i that results. The resistance R is then

$$R = \frac{V}{i}. \quad (26-8)$$

The SI unit for resistance ($\text{—}\Omega\text{—}$) that follows from Eq. 26-8 is the volt per ampere. This combination occurs so often that we give it a special name, the ohm (symbol Ω); that is,

$$1 \text{ ohm} = 1 \Omega = 1 \text{ volt per ampere} = 1 \text{ V/A}. \quad (26-9)$$

- resistivity:

$$\rho = \frac{E}{J}, \quad (\Omega \cdot \text{m}) \quad (26-10)$$

$$\vec{E} = \rho \vec{J}. \quad (26-10)$$

- conductivity:

$$\sigma = \frac{1}{\rho} \cdot (\Omega \cdot \text{m})^{-1} \quad (26-12)$$

$$\vec{J} = \sigma \vec{E}. \quad (26-13)$$

Resistance is a property of an object. Resistivity is a property of a material.

$$R = \rho \frac{L}{A}. \quad (26-16)$$

Variation of resistivity with respect to temperature

$$\rho - \rho_0 = \rho_0 \alpha (T[K] - T_0[K]). \quad (26-17)$$

Here T_0 is a selected reference temperature, ρ_0 is the resistivity at that temperature and $\alpha [K^{-1}]$ is a coefficient with certain value.

Ohm's law I

- Ohm's law is an assertion that the current through a device is always directly proportional to the potential difference applied to the device.
- A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.
- A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

Power in Electric Circuits I

The amount of charge dq that moves between those terminals in time interval dt is equal to $i dt$. This charge dq moves through a decrease in potential of magnitude V , and thus its electric potential energy decreases in magnitude by the amount

$$dU = dq V = i dt V, \quad (26-25)$$

The power P associated with that transfer is the rate of transfer dU/dt , which is given by Eq. 26-25 as

$$P = \frac{dU}{dt} = iV \quad (1)$$

The unit of power that follows from Eq. 26-26 is the volt-ampere ($V \cdot A$). We can write it as

$$1 V \cdot A = (1 J / C)(1 C / s) = 1 J / s = 1 W.$$

For a resistor or some other device with resistance R , we can combine Eqs. 26-8 ($R = V/i$) and 26-26 to obtain, for the rate of electrical energy dissipation due to a resistance, either

$$P = i^2 R. \quad (26-27)$$

$$P = \frac{V^2}{R}. \quad (26-28)$$



The End