

25. Capacitance

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Introduction

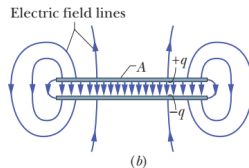
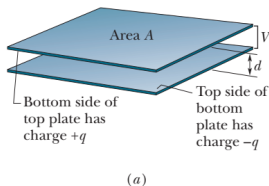


Figure 25-3a shows a less general but more conventional arrangement, called a parallel-plate capacitor, consisting of two parallel conducting plates of area A separated by a distance d .

The symbol we use to represent a capacitor



is based on the structure of a parallel-plate capacitor but is used for capacitors of all geometries. We assume for the time being that no material medium (such as glass or plastic) is present in the region between the plates.



When a capacitor is *charged*, its plates have charges of equal magnitudes but opposite signs: $+q$ and $-q$. However, we refer to the *charge of a capacitor* as being q , the absolute value of these charges on the plates. (Note that q is not the net charge on the capacitor, which is zero.)

The charge q and the potential difference V for a capacitor are proportional to each other; that is,

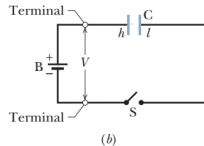
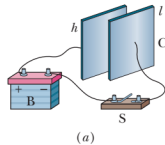
$$q = CV. \quad (25-1)$$

The proportionality constant C is called the **capacitance** of the capacitor.

The SI unit of capacitance that follows from Eq. 25-1 is the coulomb per volt. This unit occurs so often that it is given a special name, the *farad* (F):

$$1 \text{ farad} = 1 \text{ F} = 1 \text{ coulomb per volt} = 1 \text{ C/V}. \quad (25-2)$$

One way to charge a capacitor is to place it in an electric circuit with a battery. An *electric circuit* is a path through which charge can flow. A battery is a device that maintains a certain potential difference between its terminals (points at which charge can enter or leave the battery) by means of internal electrochemical reactions in which electric forces can move internal charge.



Calculating the Capacitance I

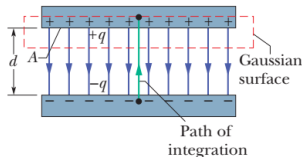
Our goal here is to calculate the capacitance of a capacitor once we know its geometry. Because we shall consider a number of different geometries, it seems wise to develop a general plan to simplify the work. In brief our plan is as follows:

1. Assume a charge q on the plates;
2. calculate the electric field between the plates in terms of this charge, using Gauss' law;
3. knowing E , calculate the potential difference V between the plates from Eq. 24-18;
4. calculate C from Eq. 25-1.

Note: The integration path starts on the negative plate and ends on the positive plate.

A Parallel-Plate Capacitor I

We use Gauss' law to relate q and E . Then we integrate the E to get the potential difference.



We draw a Gaussian surface that encloses just the charge q on the positive plate, as in Fig. 25-5. From Eq. 25-4 we can then write

$$q = \epsilon_0 EA \quad (25-7)$$

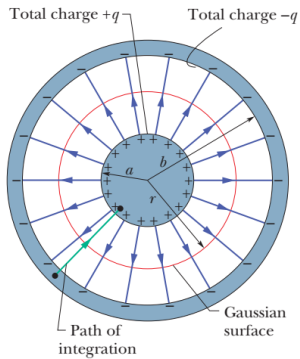
where A is the area of the plate. Equation 25-6 yields

$$V = \int_{-}^{+} E \, ds = E \int_0^d ds = Ed. \quad (25-8)$$

If we now substitute q from Eq. 25-7 and V from Eq. 25-8 into the relation $q = CV$ (Eq. 25-1), we find

$$C = \frac{\epsilon_0 A}{d} \quad (\text{parallel-plate capacitor}). \quad (25-9)$$

A Cylindrical Capacitor I



A Cylindrical Capacitor II

$$q = \epsilon_0 EA = \epsilon_0 E(2\pi rL),$$

$$E = \frac{q}{2\pi \epsilon_0 Lr}. \quad (25-12)$$

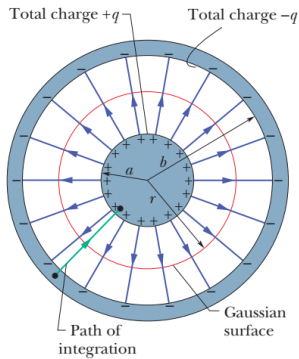
Substitution of this result into Eq. 25-6 yields

$$V = \int_{-}^{+} E \, ds = -\frac{q}{2\pi \epsilon_0 L} \int_b^a \frac{dr}{r} = \frac{q}{2\pi \epsilon_0 L} \ln\left(\frac{b}{a}\right), \quad (25-13)$$

where we have used the fact that here $ds = -dr$ (we integrated radially inward). From the relation $C = q/V$, we then have

$$C = 2\pi \epsilon_0 \frac{L}{\ln(b/a)}, \quad (\text{cylindrical capacitor}) \quad (25-14)$$

A Spherical Capacitor I



A Spherical Capacitor II

$$q = \epsilon_0 EA = \epsilon_0 E(4\pi r^2),$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}. \quad (25-15)$$

If we substitute this expression into Eq. 25-6, we find

$$V = \int_{-}^{+} E \, ds = -\frac{q}{4\pi\epsilon_0} \int_a^b \frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{q}{4\pi\epsilon_0} \frac{b-a}{ab}, \quad (25-17)$$

where again we have substituted $-dr$ for ds . If we now substitute Eq. 25-16 into Eq. 25-1 and solve for C , we find

$$C = 4\pi\epsilon_0 \frac{ab}{b-a}. \quad (25-17)$$

An Isolated Sphere I

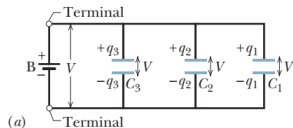
To find the capacitance of the conductor, we first rewrite Eq. 25-17 as

$$C = 4\pi \epsilon_0 \frac{a}{1 - a/b}.$$

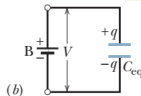
If we then let $b \rightarrow \infty$ and substitute R for a , we find

$$C = 4\pi \epsilon_0 R \quad (\text{isolated sphere}). \quad (25-18)$$

Capacitors in Parallel I



Parallel capacitors and their equivalent have the same V ("par- V ").



- When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.
- Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same total charge q and the same potential difference V as the actual capacitors.

Capacitors in Parallel II

To derive an expression for C_{eq} in Fig. 25-8b, we first use Eq. 25-1 to find the charge on each actual capacitor:

$$q_1 = C_1 V, \quad q_2 = C_2 V, \quad q_3 = C_3 V.$$

The total charge on the parallel combination of Fig. 25-8a is then

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

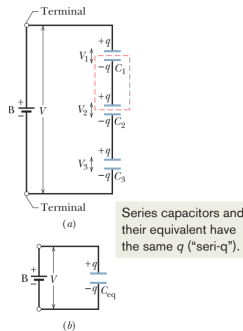
The equivalent capacitance, with the same total charge q and applied potential difference V as the combination, is then

$$C_{\text{eq}} = \frac{q}{V} = C_1 + C_2 + C_3,$$

a result that we can easily extend to any number n of capacitors, as

$$C_{\text{eq}} = \sum_{j=1}^n C_j \quad (n \text{ capacitors in parallel}). \quad (25-19)$$

Capacitors in Series I



- When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q . The sum of the potential differences across all the capacitors is equal to the applied potential difference V .
- Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same total potential difference V as the actual series capacitors.

Capacitors in Series II

To derive an expression for C_{eq} in Fig. 25-9b, we first use Eq. 25-1 to find the potential difference of each actual capacitor:

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad V_3 = \frac{q}{C_3},$$

The total potential difference V due to the battery is the sum

$$V = V_1 + V_2 + V_3 = q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right).$$

The equivalent capacitance is then

$$C_{\text{eq}} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

We can easily extend this to any number n of capacitors as

$$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^n \frac{1}{C_j} \quad (n \text{ capacitors in series}). \quad (25-20)$$

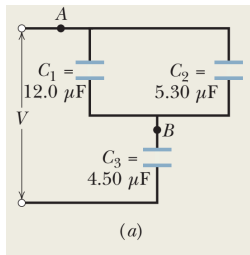


Sample Problem 25.02 I

- (a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10a, across which potential difference V is applied. Assume

$$C_1 = 12.0 \mu\text{F}, \quad C_2 = 5.30 \mu\text{F}, \quad C_3 = 4.50 \mu\text{F}.$$

- (b) The potential difference applied to the input terminals in Fig. 25-10a is $V = 12.5\text{V}$. What is the charge on C_1 ?



Sample Problem 25.02 II

(a)

$$C_{12} = C_1 + C_2 = 12.0\mu\text{F} + 5.30\mu\text{F} = 17.3\mu\text{F}.$$

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{17.3\mu\text{F}} + \frac{1}{4.50\mu\text{F}} = \frac{1}{0.280\mu\text{F}}\mu\text{F}.$$

(b)

$$q_{123} = C_{123}C = (3.57\mu\text{F})(12.5\text{V}) = 44.6\mu\text{C},$$

$$q_{12} = q_{123} = 44.6\mu\text{C}.$$

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\mu\text{C}}{17.3\mu\text{F}} = 2.58\text{V},$$

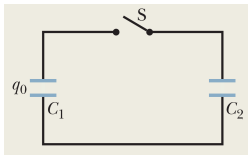
$$V_1 = V_{12} = 2.58\text{V},$$

$$q_1 = C_1 V_1 = (12.0\mu\text{F})(2.58\text{V}) = 31.0\mu\text{C}.$$



Sample Problem 25.03 I

Capacitor 1, with $C_1 = 3.55\text{mF}$, is charged to a potential difference $V_0 = 6.30\text{V}$, using a 6.30V battery. The battery is then removed, and the capacitor is connected as in Fig. 25-11 to an uncharged capacitor 2, with $C_2 = 8.95\text{mF}$. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.



Sample Problem 25.03 II

Solution:

$$\begin{aligned}q_0 &= C_1 V_0 = (3.55 \times 10^{-6} \text{F})(6.30 \text{V}) \\&= 22.365 \times 10^{-6} \text{C}.\end{aligned}$$

$$V_1 = V_2 \quad \text{equilibrium}$$

$$\frac{q_1}{C_1} = \frac{q_2}{C_2}.$$

The total charge after the transfer must be

$$q_0 = q_1 + q_2, \Rightarrow q_2 = q_0 - q_1.$$

We can now rewrite the second equilibrium equation as

$$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}$$

Solving this for q_1 and substituting given data, we find $q_1 = 6.35 \mu\text{C}$ and then $q_2 = 16.0 \mu\text{C}$.



Energy Stored in an Electric Field I

Suppose that, at a given instant, a charge q' has been transferred from one plate of a capacitor to the other. The potential difference V' between the plates at that instant will be q'/C . If an extra increment of charge dq' is then transferred, the increment of work required will be, from Eq. 24-6,

$$\begin{aligned}dW &= V' dq' = \frac{q'}{C} dq', \\ \Rightarrow W &= \int dW = \frac{1}{C} \int_0^q q' dq' = \frac{q^2}{2C}\end{aligned}$$

This work is stored as potential energy U in the capacitor, so that

$$U = \frac{q^2}{2C} = \frac{1}{2} CV^2. \quad (25-22)$$

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

Energy Density I

In a parallel-plate capacitor, neglecting fringing, the electric field has the same value at all points between the plates. Thus, the **energy density** u —that is, the potential energy per unit volume between the plates—should also be uniform. We can find u by dividing the total potential energy by the volume Ad of the space between the plates. Using Eq. 25-22, we obtain

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}. \quad (25-23)$$

Considering $C = \frac{\epsilon_0 A}{d}$, we have

$$u = \frac{1}{2} \epsilon_0 \left(\frac{V}{d} \right)^2. \quad (25-24)$$

However, from Eq. 24-42 ($E = -\Delta V = \Delta s$), V/d equals the electric field magnitude E ; so

$$u = \frac{1}{2} \epsilon_0 E^2. \quad (25-25)$$



Capacitor with a Dielectric I

- If the space between the plates of a capacitor is completely filled with a dielectric material, which is an insulating material such as mineral oil or plastic, the capacitance C in vacuum (or, effectively, in air) is multiplied by the material's dielectric constant κ , which is a number greater than 1.
- In a region that is completely filled by a dielectric, all electrostatic equations containing the permittivity constant ϵ_0 must be modified by replacing ϵ_0 with $\kappa \epsilon_0$. Thus, the magnitude of the electric field produced by a point charge inside a dielectric is given by this modified form of Eq. 23-15:

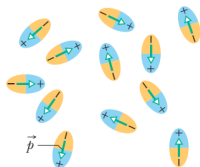
$$E = \frac{1}{4\pi\kappa\epsilon_0} \frac{q}{r^2} \quad (25-28)$$

Also, the expression for the electric field just outside an isolated conductor immersed in a dielectric (see Eq. 23-11) becomes

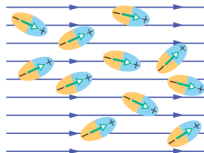
$$E = \frac{\sigma}{\kappa\epsilon_0}.$$

Because κ is always greater than unity, both these equations show that for a *fixed distribution of charges*, the effect of a dielectric is to weaken the electric field that would otherwise be present.

Dielectrics: An Atomic View I

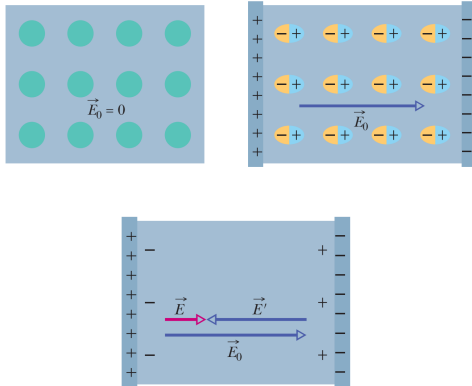


(a)



(b)

Dielectrics: An Atomic View II



Dielectrics and Gauss' Law I

We can write Gauss'law in the form

$$\epsilon_0 \kappa \oint \vec{E} \cdot d\vec{A} = q. \quad (25-36)$$

The End