

24. Electric potential

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Electric Potential

If we release particle 1 at P , it begins to move and thus has kinetic energy. Energy cannot appear by magic, so from where does it come?



It comes from the electric potential energy U associated with the force between the two particles in the arrangement of Fig. 24-1.

We are going to define the electric potential (or potential for short) in terms of electric potential energy, so our first job is to figure out how to measure that potential energy.

Back in Chapter 8, we measured gravitational potential energy U of an object by

1. assigning $U = 0$ for a reference configuration (such as the object at table level) and
2. then calculating the work W the gravitational force does if the object is moved up or down from that level.

We then defined the potential energy as being

$$U = -W. \quad (24-1)$$

We want to find the potential energy U associated with a positive test charge q_0 located at point P in the electric field of a charged rod. First, we need a reference configuration for which $U = 0$. A reasonable choice is for the test charge to be infinitely far from the rod, because then there is no interaction with the rod.

The potential energy of the final configuration is then given by Eq. 24-1, where W is now the work done by the electric force.

Next, we define the electric potential V at P in terms of the work done by the electric force and the resulting potential energy:

$$V = \frac{-W_{\infty}}{q_0} = \frac{U}{q_0}, \quad \text{electric potential.} \quad (24-2)$$

That is, the electric potential is the amount of electric potential energy per unit charge when a positive test charge is brought in from infinity.

Repeating this procedure we find that an electric potential is set up at every point in the rod's electric field. In fact, every charged object sets up electric potential V at points throughout its electric field. If we happen to place a particle with, say, charge q at a point where we know the pre-existing V , we can immediately find the potential energy of the configuration:

$$(\text{electric potential energy}) = (\text{particle's charge}) \left(\frac{\text{electric potential}}{\text{unit charge}} \right),$$

$$U = qV, \quad (24-3)$$

where q can be positive or negative.

The SI unit for potential that follows from Eq. 24-2 is the joule per coulomb. This combination occurs so often that a special unit, the volt (abbreviated V), is used to represent it. Thus,

$$1 \text{ volt} = 1 \text{ joule per coulomb.}$$

With two unit conversions, we can now switch the unit for electric field from newtons per coulomb to a more conventional unit:

$$\begin{aligned} 1 \text{ N/C} &= \left(\frac{\text{N}}{\text{C}} \right) \left(\frac{1 \text{ V}}{1 \text{ J/C}} \right) \left(\frac{1 \text{ J}}{1 \text{ N.m}} \right) \\ &= 1 \text{ V/m.} \end{aligned}$$

If we move a particle with charge q from i to f , then, from Eq. 24-3, the potential energy of the system changes by

$$\Delta U = q\Delta V = q (V_f - V_i) . \quad (24-4)$$

The change can be positive or negative, depending on the signs of q and V . It can also be zero, if there is no change in potential from i to f (the points have the same value of potential). Because the electric force is conservative, the change in potential energy U between i and f is the same for all paths between those points (it is *path independent*).

Work by the Field.

We can relate the potential energy change U to the work W done by the electric force as the particle moves from i to f by applying the general relation for a conservative force:

$$W = -\Delta U. \quad (24-5)$$

Next, we can relate that work to the change in the potential by substituting from Eq. 24-4:

$$W = -\Delta U = -q\Delta V = -q(V_f - V_i). \quad (24-6)$$

Conservation of Energy.

If a charged particle moves through an electric field with no force acting on it other than the electric force due to the field, then the mechanical energy is conserved. Let's assume that we can assign the electric potential energy to the particle alone. Then we can write the conservation of mechanical energy of the particle that moves from point i to point f as

$$U_i + K_i = U_f + K_f, \quad (24-7)$$

$$\Delta K = -\Delta U. \quad (24-8)$$

Substituting Eq. 24-4, we find a very useful equation for the change in the particle's kinetic energy as a result of the particle moving through a potential difference:

$$\Delta K = -q\Delta V = -q(V_f - V_i). \quad (24-9)$$

Work by an Applied Force.

If some force in addition to the electric force acts on the particle, we say that the additional force is an *applied force* or *external force*, which is often attributed to an *external agent*. We account for that work W_{app} by modifying Eq. 24-7:

$$\begin{aligned}(\text{initial energy}) + (\text{work by applied force}) &= (\text{final energy}), \\ U_i + K_i + W_{app} &= U_f + K_f.\end{aligned}\tag{24-10}$$

Rearranging and substituting from Eq. 24-4, we can also write this as

$$\Delta K = -\Delta U + W_{app} = -q\Delta V + W_{app}.\tag{24-11}$$

In the special case where the particle is stationary before and after the move, the kinetic energy terms in Eqs. 24-10 and 24-11 are zero and we have

$$W_{app} = q\Delta V, \quad (\text{for } K_i = K_f).\tag{24-12}$$

In this special case, the work W_{app} involves the motion of the particle through the potential difference V and not a change in the particle's kinetic energy.

Electron-volts

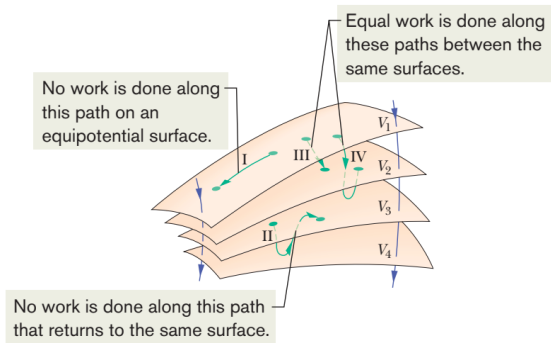
In atomic and subatomic physics, energy measures in the SI unit of joules often require awkward powers of ten. A more convenient (but nonSI unit) is the electron-volt (eV), which is defined to be equal to the work required to move a single elementary charge e (such as that of an electron or proton) through a potential difference ΔV of exactly one volt. From Eq. 24-6, we see that the magnitude of this work is $q\Delta V$. Thus,

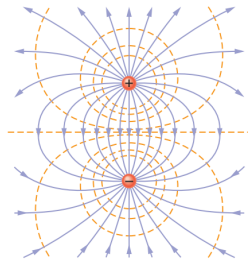
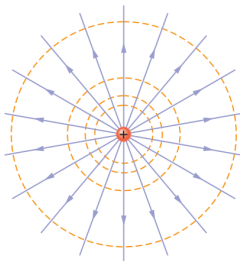
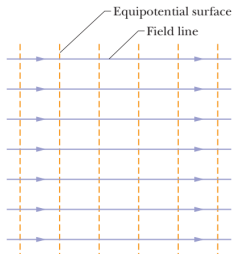
$$\begin{aligned}1 \text{ eV} &= e(1\text{V}), \\&= (1.602 \times 10^{-19}\text{C})(1\text{J/C}) = 1.602 \times 10^{-19}\text{J}.\end{aligned}\tag{24-14}$$

Equipotential Surfaces .

Adjacent points that have the same electric potential form an **equipotential surface**, which can be either an imaginary surface or a real, physical surface.

No net work W is done on a charged particle by an electric field when the particle moves between two points i and f on the same equipotential surface. This follows from Eq. 24-6, which tells us that W must be zero if $V_f = V_i$. Because of the path independence of work (and thus of potential energy and potential), $W = 0$ for any path connecting points i and f on a given equipotential surface regardless of whether that path lies entirely on that surface.





Calculating the Potential from the Field

We can calculate the potential difference between any two points i and f in an electric field if we know the electric field vector all along any path connecting those points. To make the calculation, we find the work done on a positive test charge by the field as the charge moves from i to f , and then use Eq. 24-6.

we know that the differential work dW done on a particle by a force $\vec{F}$ during a displacement $d\vec{s}$ is given by the dot product of the force and the displacement:

$$dW = \vec{F} \cdot d\vec{s}, \quad (24-15)$$

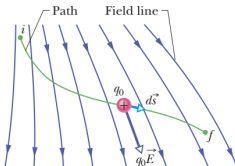
$$= q_0 \vec{E} \cdot d\vec{s}. \quad (1)$$

To find the total work W done on the particle by the field as the particle moves from point i to point f , we sum—via integration—the differential works done on the charge as it moves through all the displacements $d\vec{s}$ along the path:

$$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}. \quad (24-17)$$

If we substitute the total work W from Eq. 24-17 into Eq. 24-6, we find

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}. \quad (24-18)$$

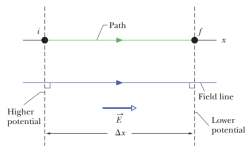


Thus, the potential difference $V_f - V_i$ between any two points i and f in an electric field is equal to the negative of the line integral (meaning the integral along a particular path) of from i to f . However, because the electric force is conservative, all paths (whether easy or difficult to use) yield the same result.

Equation 24-18 allows us to calculate the difference in potential between any two points in the field. If we set potential $V_i = 0$, then Eq. 24-18 becomes

$$V_f = V = - \int_i^f \vec{E} \cdot d\vec{s}. \quad (24-19)$$

Uniform field Let's apply Eq. 24-18 for a uniform field as shown in Fig. 24-7.



The angle between $\vec{E}$ and $d\vec{s}$ in Eq. 24-18 is zero, and the dot product gives us

$$\vec{E} \cdot d\vec{s} = E ds \cos 0 = E ds$$

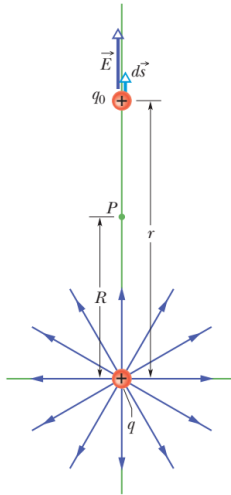
Because E is constant for a uniform field, Eq. 24-18 becomes

$$V_f - V_i = -E \int_i^f ds, \quad (24-20)$$

$$\Delta V = -E\Delta x. \quad (24-21)$$

This is the change in voltage ΔV between two equipotential lines in a uniform field of magnitude E , separated by distance Δx . If we move in the direction of the field by distance Δx , the potential decreases. In the opposite direction, it increases. I.e. **The electric field vector points from higher potential toward lower potential.**

Potential Due to a Charged Particle.



The electric field $\vec{E}$ in Fig. 24-9 is directed radially outward from the fixed particle. Thus, the differential displacement $d\vec{s}$ of the test particle along its path has the same direction as $\vec{E}$. That means that in Eq. 24-22, angle $\theta = 0$ and $\cos \theta = 1$. Because the path is radial, let us write ds as dr . Then, substituting the limits R and ∞ , we can write Eq. 24-18 as

$$V_f - V_i = - \int_R^\infty E \, dr. \quad (24-23)$$

Next, we set $V_f = 0$ (at ∞) and $V_i = V$ (at R). Then, for the magnitude of the electric field at the site of the test charge, we substitute from Eq. 22-3:

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}. \quad (24-24)$$

With these changes, Eq. 24-23 then gives us

$$\begin{aligned} 0 - V &= - \frac{q}{4\pi\epsilon_0} \int_R^\infty \frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} \right]_R^\infty, \\ &= - \frac{1}{4\pi\epsilon_0} \frac{q}{R} \end{aligned} \quad (2)$$

Solving for V and switching R to r , we then have

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}. \quad (24-26)$$

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.



Potential Due to a Group of Charged Particles.

We can find the net electric potential at a point due to a group of charged particles with the help of the superposition principle. Using Eq. 24-26 with the plus or minus sign of the charge included, we calculate separately the potential resulting from each charge at the given point. Then we sum the potentials. Thus, for n charges, the net potential is

$$V = \sum_{i=1}^n V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}. \quad (24-27)$$

The sum in Eq. 24-27 is an *algebraic sum*, not a vector sum like the sum that would be used to calculate the electric field resulting from a group of charged particles.

Potential Due to a Continuous Charge Distribution.

When a charge distribution q is continuous (as on a uniformly charged thin rod or disk), we cannot use the summation of Eq. 24-27 to find the potential V at a point P . Instead, we must choose a differential element of charge dq , determine the potential dV at P due to dq , and then integrate over the entire charge distribution

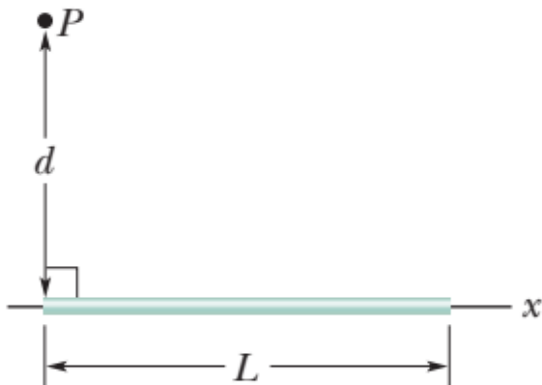
$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}. \quad (24-31)$$

Here r is the distance between P and dq . To find the total potential V at P , we integrate to sum the potentials due to all the charge elements:

$$V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}. \quad (24-32)$$

Line of Charge

In Fig. 24-15a, a thin nonconducting rod of length L has a positive charge of uniform linear density λ . Let us determine the electric potential V due to the rod at point P , a perpendicular distance d from the left end of the rod.



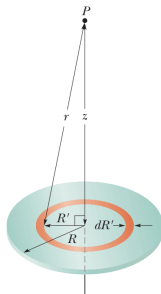
$$dq = \lambda dx, \quad (24-33)$$

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{(x^2 + d^2)^{1/2}}. \quad (24-34)$$

$$\begin{aligned} V &= \int dV = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{\lambda}{(x^2 + d^2)^{1/2}} dx \\ &= \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{(x^2 + d^2)^{1/2}}, \\ &= \frac{\lambda}{4\pi\epsilon_0} \left[\ln \left(x + (x^2 + d^2)^{1/2} \right) \right]_0^L, \\ &= \frac{\lambda}{4\pi\epsilon_0} \left[\ln \left(L + (L^2 + d^2)^{1/2} \right) - \ln d \right] \end{aligned}$$

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{L + (L^2 + d^2)^{1/2}}{d} \right]. \quad (24-35)$$

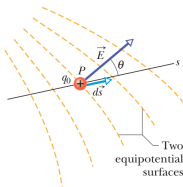
Charged Disk



$$dq = \sigma (2\pi R') (dR')$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{R' dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\epsilon_0} \left(\sqrt{z^2 + R^2} - z \right). \quad (24-37)$$

Calculating the Field from the Potential



$$dV = -\vec{E} \cdot d\vec{s}$$

$$\vec{E} = -\vec{\nabla}_s V,$$

$$\Rightarrow E = -\frac{\partial V}{\partial s}.$$

For Cartesian coordinates the gradient is

$$\vec{\nabla} = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \quad (3)$$

The End