

22. Electric Fields

Hadi Sobhani



Question: How does charged particle 2 push on charged particle 1 when they have no contact?



q_1



q_2

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q_1



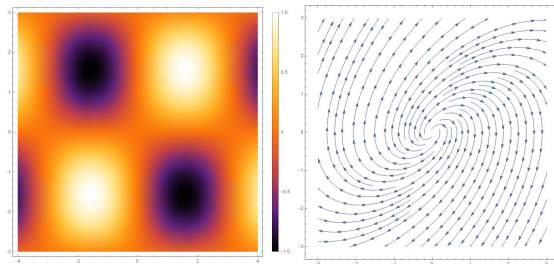
q_2

Solution: The electric field! Particle 2 sets up an electric field at all points in the surrounding space, even if the space is a vacuum. Thus, particle 2 pushes on particle 1 not by touching it.

Fields

There two types of field: and vector field.

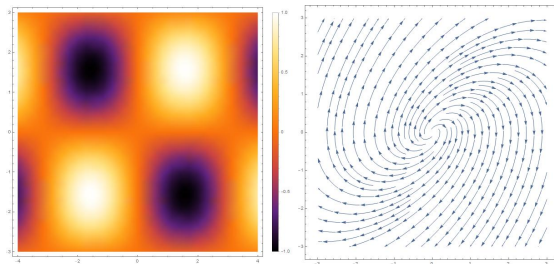
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Fields

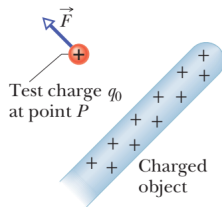
There two types of field: and vector field.

- a scalar field (the left one): it gives a scalar quantity at each point. temperature field, pressure field.
- a vector field (the right one): it gives a vector quantity at each point. electric field, magnetic field.



Electric field

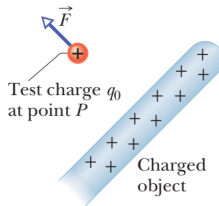
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- We place a particle with a small positive charge q_0 , called a test charge. We then measure the electrostatic force $\vec{F}$ that acts on the test charge. The electric field at that point is then

$$\vec{E} = \frac{\vec{F}}{q}.$$

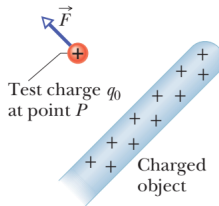


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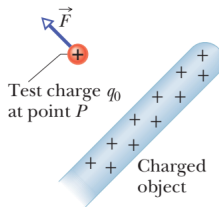


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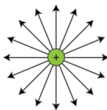
- Electric field is an abstract property set up by a charged object.
- the SI unit for the electric field is the newton per coulomb (N/C).



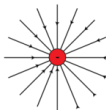
Electric Field Lines

Michael Faraday envisioned lines, now called **electric field lines**, in the space around any given charged particle or object.

Electric Fields of Individual Charged Particles (Point Charges):

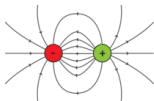


Electric field lines of a positive point charge

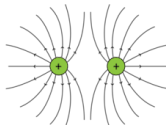


Electric field lines of a negative point charge

Interacting Electric Fields of Two Charged Particles:



Positively and Negatively Charged Particles

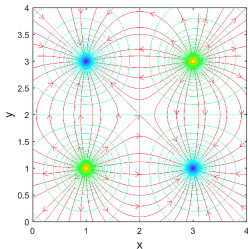


Two Positively Charged Particles

Electric field

The rules for drawing electric fields lines are these:

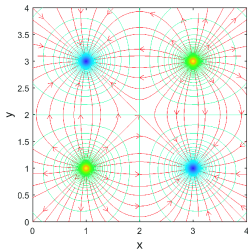
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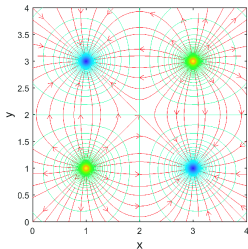
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2. In a plane perpendicular to the field lines, the relative density of the lines represents the relative magnitude of the field there, with greater density for greater magnitude.



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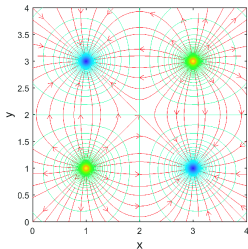
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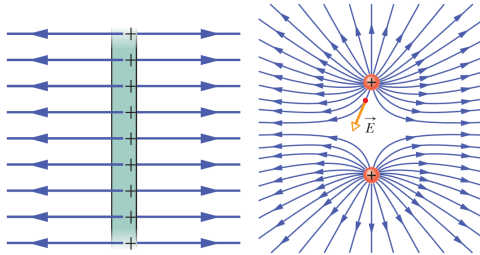
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Let's watch a programming.

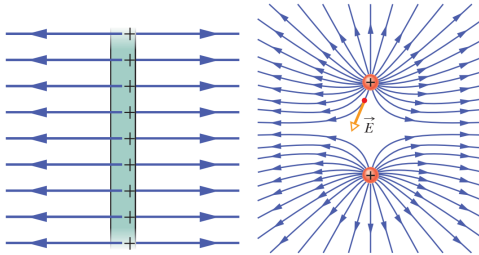
According to the field lines we have

- *uniform electric field*, meaning that the electric field has the same magnitude and direction at every point within the field (the left one).



According to the field lines we have

- *uniform electric field*, meaning that the electric field has the same magnitude and direction at every point within the field (the left one).
- *a nonuniform field*, where there is variation from point to point (the right one).



The Electric Field Due to a Point Charge

To find the electric field due to a charged particle, we place a positive test charge at any point near the particle, at distance r . From Coulomb's law we have

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{q_0} \left(\frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \hat{r} \right) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

The direction of $\vec{E}$ matches that of the force on the positive test charge: directly away from the point charge if q is positive and directly toward it if q is negative.

Magnitude value of the field is

$$E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}.$$

Addition of fields

Forces obey the principle of superposition, so we just add the forces as vectors:

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots \vec{F}_{0n}.$$

Multiplying both sides by $1/q_0$ gives

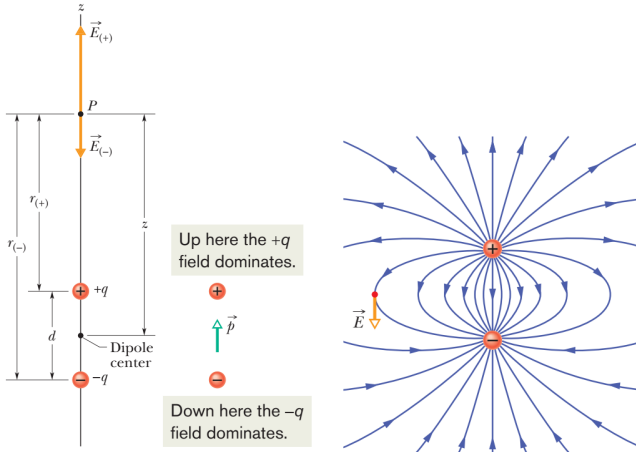
$$\begin{aligned}\vec{E} &= \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots \frac{\vec{F}_{0n}}{q_0}, \\ \vec{E}_0 &= \vec{E}_{01} + \vec{E}_{02} + \cdots \vec{E}_{0n}.\end{aligned}$$

This tells us that electric fields also obey the principle of superposition.

If you want the net electric field at a given point due to several particles, find the electric field due to each particle, and then sum the fields as vectors.

The Electric Field Due to an Electric Dipole

An electric dipole consists of two particles with charges of equal magnitude q but opposite signs, separated by a small distance d .



Magnitude of Electric Field Due to an Electric Dipole I

$$\begin{aligned}E &= E_{(+)} - E_{(-)}, \\&= \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(+)}^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(-)}^2}, \\&= \frac{q}{4\pi\epsilon_0 \left(z - \frac{1}{2}d\right)^2} - \frac{q}{4\pi\epsilon_0 \left(z + \frac{1}{2}d\right)^2}, \\&= \frac{q}{4\pi\epsilon_0 z^2} \left(\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2} \right),\end{aligned}$$

After forming a common denominator and multiplying its terms, we come to

$$\begin{aligned}E &= \frac{q}{4\pi\epsilon_0 z^2} \frac{2d/z}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}, \\&= \frac{q}{2\pi\epsilon_0 z^3} \frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}.\end{aligned}$$

Magnitude of Electric Field Due to an Electric Dipole II

For the large distance, $z \gg d$, we have $d/2z \ll 1$. Thus we obtain

$$E = \frac{1}{2\pi\epsilon_0} \frac{qd}{z^3} = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3},$$

where $p = qd$ is known as the **electric dipole moment**. This equation shows that

- we can never find q and d separately; instead, we can find only their product.
- Although this equation holds only for distant points along the dipole axis, it turns out that E for a dipole varies as $\frac{1}{r^3}$ for all distant points, regardless of whether they lie on the dipole axis; here r is the distance between the point in question and the dipole center.
- Thus the electric field of a dipole ($\propto 1/z^3$) decreases more rapidly with distance than does the electric field of a single charge ($\propto 1/r^2$).

The electric field due to an extended object at a point

- To find the electric field of an extended object at a point, we first consider the electric field set up by a charge element dq in the object, where the element is small enough for us to apply the equation for a particle. Then we sum, via integration, components of the electric fields $d\vec{E}$ from all the charge elements.

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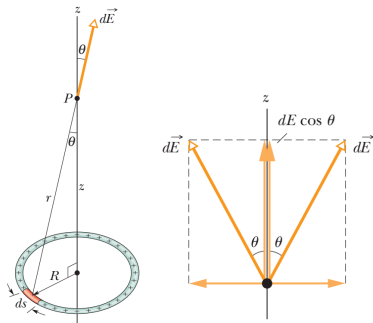
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- Because the individual electric fields $d\vec{E}$ have different magnitudes and point in different directions, we first see if symmetry allows us to cancel out any of the components of the fields, to simplify the integration.
- For a uniform distribution of charge, find the linear charge density $\lambda = dq/ds$ for charge along a line, the surface charge density $\sigma = dq/dA$ for charge on a surface, and the volume charge density $\rho = dq/dV$ for charge in a volume

The Electric Field Due to a Line of Charge I

The differential electric field $d\vec{E}$ at P , at distance r from the element is obtained as

$$dq = \lambda ds,$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{z^2 + R^2},$$



The Electric Field Due to a Line of Charge II

The components perpendicular to the z axis cancel; the parallel components add. Thus we have

$$\begin{aligned}\cos \theta &= \frac{z}{r} = \frac{z}{\sqrt{z^2 + R^2}} \\ dE \cos \theta &= \frac{1}{4\pi \epsilon_0} \frac{\lambda}{(z^2 + R^2)} \frac{z}{(z^2 + R^2)^{1/2}} \\ dE \cos \theta &= \frac{1}{4\pi \epsilon_0} \frac{\lambda z}{(z^2 + R^2)^{3/2}}\end{aligned}$$

Because we must sum a huge number of these components, each small, we set up an integral that moves along the ring, from element to element, from a starting point (call it $s = 0$) through the full circumference ($s = 2\pi R$). We find

$$\begin{aligned}E &= \int dE \cos \theta = \frac{1}{4\pi \epsilon_0} \frac{\lambda z}{(z^2 + R^2)^{3/2}} \int_0^{2\pi R} ds, \\ E &= \frac{1}{4\pi \epsilon_0} \frac{\lambda z(2\pi R)}{(z^2 + R^2)^{3/2}} = \frac{qz}{4\pi \epsilon_0 (z^2 + R^2)^{3/2}},\end{aligned}$$

where $q = \lambda(2\pi R)$ is the total charge.



The Electric Field Due to a Line of Charge III

Consider a point on the central axis that is so far away that $z \gg R$. In this case we have

$$E = \frac{qz}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}} = \frac{qz}{4\pi\epsilon_0 \left[z^2 \left(1 + \cancel{(R/z)^2} \right) \right]^{3/2}} \approx \frac{qz}{4\pi\epsilon_0 z^3},$$
$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{z^2}.$$

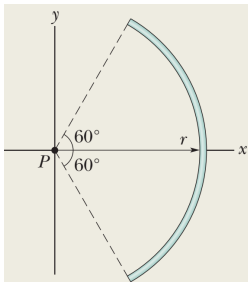
For a point at the center of the ring, we have

$$E = \frac{qz}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}} \xrightarrow{z=0} E = 0,$$

because if we were to place a test charge at the center of the ring, there would be no net electrostatic force acting on it; the force due to any element of the ring would be canceled by the force due to the element on the opposite side of the ring.

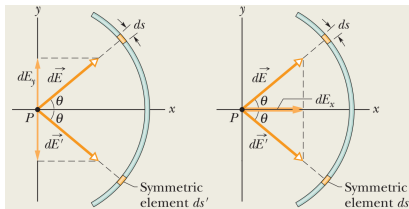
Example 1 I

Figure 22-13a shows a plastic rod with a uniform charge $-Q$. It is bent in a 120° circular arc of radius r and symmetrically placed across an x axis with the origin at the center of curvature P of the rod. In terms of Q and r , what is the electric field due to the rod at point P ?



Example 1 II

Solution:



$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{R^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{R^2}.$$

Due to the symmetry existing in the problem, to find the electric field set up by the rod, we need sum (via integration) only the x components of the differential electric fields set up by all the differential elements of the rod. So we can write the x component dE_x as

$$dE_x = dE \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{\lambda}{r^2} \cos \theta ds.$$

Replacing ds , using the relation $ds = R d\theta$, where $d\theta$ is the angle at P that includes arc length ds , we can integrate dE_x over the angle made by the rod at P , from $\theta = -60^\circ$ to $\theta = 60^\circ$; that will give us the field magnitude at P :

$$\begin{aligned}
 E &= \int dE_x = \int_{-60^\circ}^{60^\circ} \frac{1}{4\pi\epsilon_0} \frac{\lambda}{R^2} \cos\theta R d\theta, \\
 &= \frac{\lambda}{4\pi\epsilon_0 R} \int_{-60^\circ}^{60^\circ} \cos\theta d\theta = \frac{\lambda}{4\pi\epsilon_0 R} \left[\sin\theta \right]_{-60^\circ}^{60^\circ}, \\
 &= \frac{\lambda}{4\pi\epsilon_0 R} [\sin(60^\circ) - \sin(-60^\circ)] = \frac{\sqrt{3}\lambda}{4\pi\epsilon_0 R} = \frac{1.73\lambda}{4\pi\epsilon_0 R}.
 \end{aligned}$$

To evaluate λ , we note that the full rod subtends an angle of 120° and so is one-third of a full circle. Its arc length is then $2\pi R/3$, and its linear charge density must be

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi R/3} = \frac{0.477Q}{R}.$$

By this value of λ we may recast the electric field magnitude as

$$E = \frac{(1.73)(0.477Q)}{4\pi\epsilon_0 R^2} = \frac{0.83Q}{4\pi\epsilon_0 R^2}.$$

The direction of $\vec{E}$ is toward the rod, along the axis of symmetry of the charge distribution. We can write $\vec{E}$ in unit-vector notation as

$$\vec{E} = \frac{0.83Q}{4\pi\epsilon_0 R^2} \hat{i}.$$

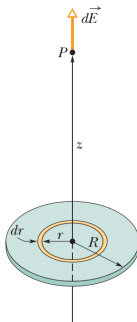
The Electric Field Due to a Charged Disk I

To find its small contribution dE to the electric field at point P , we rewrite Eq. 22-16 in terms of the ring's charge dq and radius r :

$$dE = \frac{dq z}{4\pi\epsilon_0(z^2 + r^2)^{3/2}}$$

where

$$dq = \sigma dA = \sigma(2\pi r dr)$$



The Electric Field Due to a Charged Disk II

After substituting this into Eq. 22-22 and simplifying slightly, we can sum all the dE contributions with

$$E = \int dE = \frac{\sigma z}{4\epsilon_0} \int_0^R (z^2 + r^2)^{-3/2} (2r) dr$$

To solve this integral, we cast it in the form

$$\int X^m dX = \frac{X^{m+1}}{m+1}.$$

Thus we have $m = -\frac{3}{2}$ and

$$X = z^2 + r^2 \Rightarrow dX = (2r)dr.$$

After integrating we have

$$E = \frac{\sigma z}{4\epsilon_0} \left[\frac{(z^2 + r^2)^{-1/2}}{-1/2} \right]_0^R.$$

Taking the limits and rearranging, we find

$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right).$$



The Electric Field Due to a Charged Disk III

As special cases both for $R \rightarrow \infty$ or $z \rightarrow 0$ we obtain

$$E = \frac{\sigma}{2\epsilon_0}.$$

This is the electric field produced by an infinite sheet of uniform charge located on one side of a nonconductor such as plastic.



A Point Charge in an Electric Field I

The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative, i.e

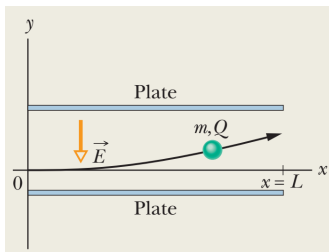
$$\vec{E} = \frac{\vec{F}}{q} \Rightarrow \vec{F} = q\vec{E}.$$

Let's watch the Millikan's experiment which determines the fundamental constant so-called the elementary charge, $1.60 \times 10^{-19}\text{C}$.



Example 2 I

Figure 22-19 shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes. An ink drop with a mass m of 1.3×10^{-10} kg and a negative charge of magnitude $Q = 1.5 \times 10^{-13}$ C enters the region between the plates, initially moving along the x axis with speed $v_x 18\text{ m/s}$. The length L of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field $\vec{E}$ is downward directed, is uniform, and has a magnitude of 1.4×10^6 N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)



Example 2 II

Solution: Applying Newton's second law $\vec{F} = m\vec{a}$ for components along the y axis, we find that

$$a_y = \frac{F}{m} = \frac{QE}{m}.$$

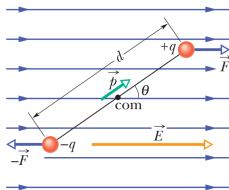
Let t represent the time required for the drop to pass through the region between the plates. During t the vertical and horizontal displacements of the drop are

$$y = \frac{1}{2}a_y t^2, \quad L = v_x t,$$

respectively. Eliminating t between these two equations and substituting for a_y , we find

$$\begin{aligned} y &= \frac{QEL^2}{2mv_x^2}, \\ &= \frac{(1.5 \times 10^{-13} C)(1.4 \times 10^4 N/C)(1.6 \times 10^{-2} m)^2}{(2)(1.3 \times 10^{-10} kg)(18 m/s)^2}, \\ &= 6.4 \times 10^{-4} m, \\ &= 0.64 mm. \end{aligned}$$

A Dipole in an Electric Field



Electrostatic forces act on the charged ends of the dipole. Because the electric field is uniform, those forces act in opposite directions and with the same magnitude $F = qE$. Thus, **because the field is uniform, the net force on the dipole from the field is zero and the center of mass of the dipole does not move.** However, the forces on the charged ends do produce a net torque $\vec{\tau}$ on the dipole about its center of mass. The center of mass lies on the line connecting the charged ends, at some distance x from one end and thus a distance $d - x$ from the other end. From the torque definition, we can write the magnitude of the net torque $\vec{\tau}$ as

$$\tau = Fx \sin \theta + F(d - x) \sin \theta = Fd \sin \theta.$$

Recalling $p = qd$ and $F = qE$

$$\tau = Fd \sin \theta = (qE)(p/q) \sin \theta = pE \sin \theta.$$

We can generalize this equation to vector form as

$$\vec{\tau} = \vec{p} \times \vec{E} \quad (\text{torque on a dipole}).$$

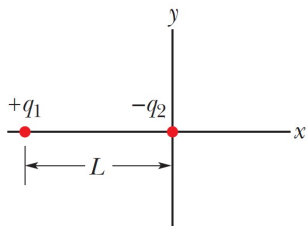


Potential Energy of an Electric Dipole

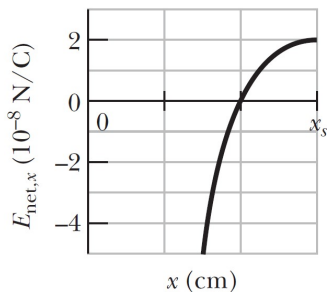
We then can find the potential energy U of the dipole at any other value of θ with $\Delta U = -W$ by calculating the work W done by the field on the dipole when the dipole is rotated. Thus we have

$$\begin{aligned} U &= -W = \int \tau d\theta = \int pE \sin \theta d\theta, \\ &= -pE \cos \theta = -\vec{p} \cdot \vec{E}. \end{aligned}$$

Further examples: 1



(a)



(b)

Figure 22-38 Problem 10.

Further examples: 2

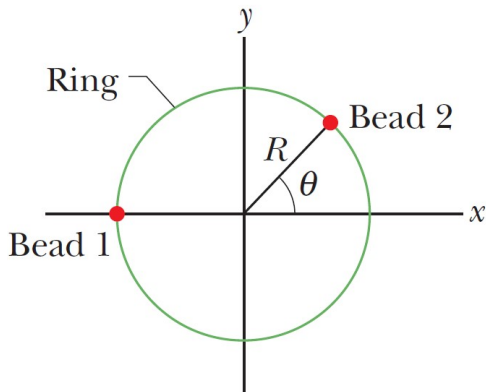


Figure 22-43 Problem 16.

Further examples: 3

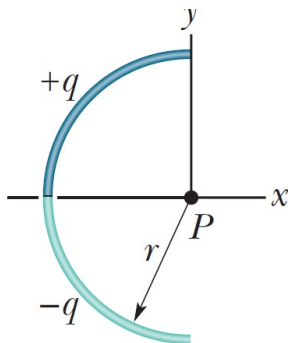


Figure 22-50
Problem 26.

Further examples: 4

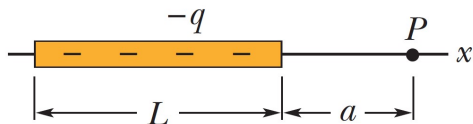


Figure 22-54 Problem 31.

Further examples: 5

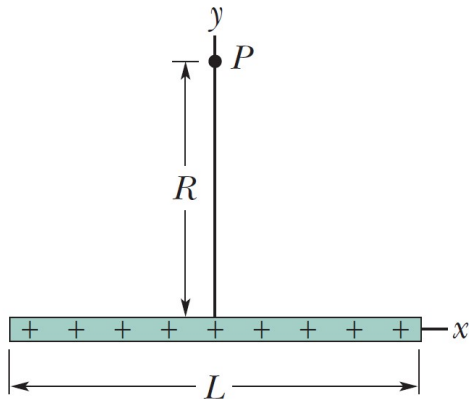


Figure 22-55 Problem 32.

The End