



Sharif University of Technology
School of Mechanical Engineering

Instructor:

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Automatic Control

Chapter 9:

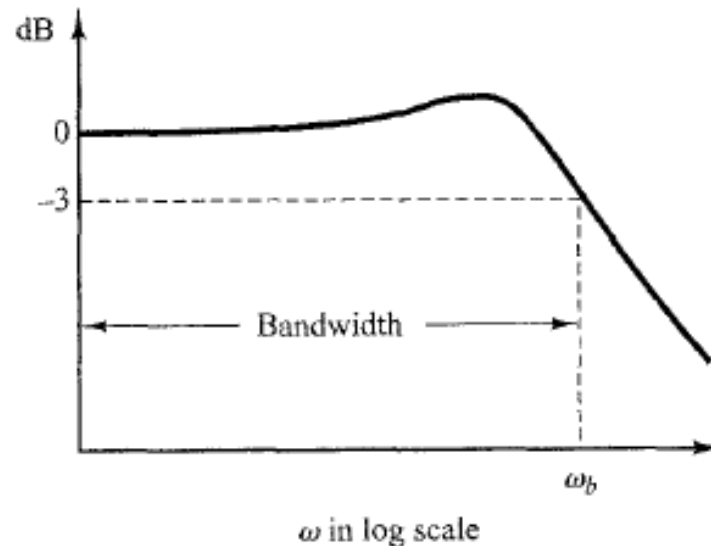
Control Systems
Design by
Frequency
Response Method

- Chapter 1: Introduction to Control Systems and Laplace Transformation
- Chapter 2: Mathematical Modeling of Control Systems
 - Part 1: Introduction to Control Systems
 - Transfer Function
 - Block Diagram and Closed Loop Control Systems
 - Simplification of Block Diagrams
 - Control Theory (Clip)
 - Part 2: Mason's Rule for Simplification of Block Diagrams
- Chapter 3: Modeling of Mechanical, Electrical and Fluid Systems

- Chapter 4: Modeling of Pneumatic, Hydraulic and Thermal Systems
- Chapter 5: Transient and Steady-State Response Analysis
- Chapter 6: Control Systems Analysis by Root-Locus Method
- Chapter 7: Control Systems Design by Root-Locus Method
- Chapter 8: Control Systems Analysis by Frequency Response Method
- Chapter 9: Control Systems Design by Frequency Response Method
- Chapter 10: PID Controller Design by Ziegler-Nichols Method

*ملزومات پاسخ فرکانسی مدار باز:

- ضریب بهره در فرکانس های پایین و نزدیک به ω_g باید بزرگتر باشد.
- شیب منحنی در نزدیکی ω_g باید -20db/dec باشد.
- در مکانهای بالا ضریب باید با سرعت زیادی کاهش یابد.



دیاگرام بود سیستم مدار باز

Lead Compensation

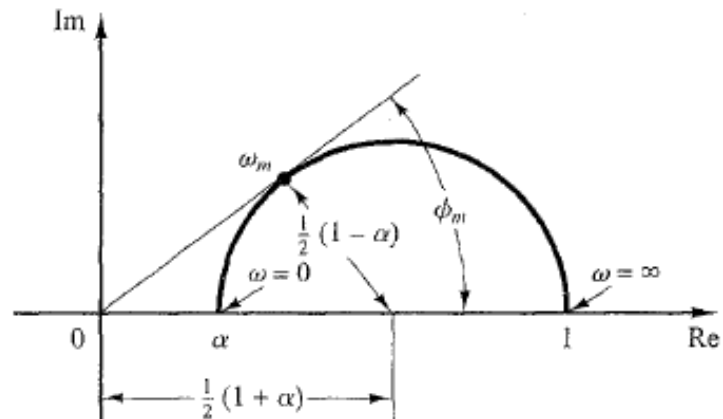
$$G_c(s) = k_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = k_c \alpha \frac{Ts + 1}{Ts\alpha + 1} = kG'_{c(s)}$$

$$G'_c(j\omega) = \frac{Tj\omega + 1}{Tj\omega\alpha + 1} \rightarrow M'_{c(\omega)} = \frac{\sqrt{\omega^2 T^2 + 1}}{\sqrt{\alpha^2 \omega^2 T^2 + 1}} \rightarrow \begin{cases} M'_{c(0)} = 1 \\ M'_{c(\infty)} = \frac{1}{\alpha} \end{cases}$$

$$\phi'_c(\omega) = \phi_c(\omega) = \tan^{-1} \omega T - \tan^{-1} \alpha \omega T \rightarrow \begin{cases} \phi'_c(0) = 0 \\ \phi'_c(\infty) = 90 - 90 = 0 \end{cases}$$

*Polar plot of a lead compensator

$\frac{\alpha(j\omega T + 1)}{j\omega T + 1}$, Where $0 < \alpha < 1$.



Lead Compensation

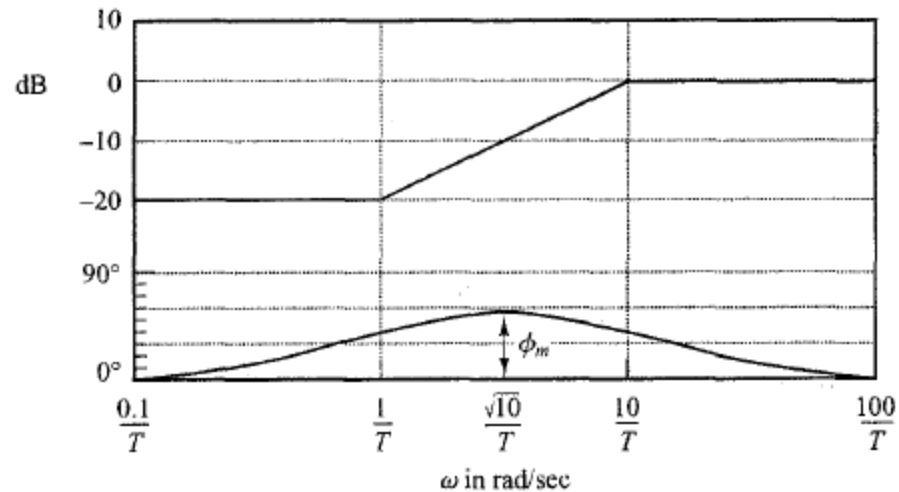
$$\sin \phi_m = \frac{1-\alpha}{2\alpha} = \frac{1-\alpha}{1+\alpha}$$

$$M'_c(\omega_m) = |G'_c(\omega_m)| = \sqrt{\left(\frac{1+\alpha}{2\alpha}\right)^2 - \left(\frac{1-\alpha}{2\alpha}\right)^2} = \frac{1}{\sqrt{\alpha}} = \frac{\sqrt{\omega^2 T^2 + 1}}{\sqrt{\alpha^2 \omega^2 T^2 + 1}}$$

$$\Rightarrow \omega_m = \frac{1}{T\sqrt{\alpha}}$$

*Bode diagram of a lead compensator

$\frac{\alpha(j\omega T + 1)}{j\omega T + 1}$, Where $\alpha = 0.1$



*روش طراحی:

۱. از روی میزان بهبود دقت K را بیابید .
به طور مثال برای سروسیستم نوع ۱ از روی k_v .

$$\bar{k}_v = \lim_{s \rightarrow 0} sG(s)G_c(s) = k_v G_c(0) = k_v k$$

$$\frac{k_v}{k} = K$$

۲. دیاگرام بود (Bode) را برای سیستم رسم کنید.

$$G_c(s)G(s) = G'_c(s) \cdot \underbrace{k \cdot G(s)}_{G_1(s)} = G'_c(s)G_1(s)$$

$$M_1(\omega) = |G_1(j\omega)| = k |G(j\omega)|$$

$$M_1(\omega_{gc}) = 1 \rightarrow \gamma_1 = 180 + \phi_1(\omega_{gc})$$

۳. مقدار زاویه ϕ مناسب برای اینکه به مقدار کرانه فاز کتلوب برسیم بیابید .

$$\phi_m = \gamma - \gamma_1 + 5$$

کرانه (حد) فاز مطلوب

چون در اثر افزودن lead محل ω_{gc} سمت راست شیفت پیدا می کند برای جبران کاهش زاویه اضافه می کنیم

۴. با استفاده از $\sin \phi_m = \frac{1-\alpha}{1+\alpha}$ مقدار α را بیابید.

$$\bar{M}(\omega) = |G_c(j\omega)| \cdot |G(j\omega)| = \left| \frac{j\omega T + 1}{j\alpha T + 1} \right| \cdot |G(j\omega)|$$

مقدار $\left| \frac{j\omega T + 1}{j\alpha T + 1} \right|$ در محل فرکانس مربوط به ϕ_m برابر است با $\frac{1}{\sqrt{\alpha}}$ پس در محل جدید $\bar{\omega}_g$ مقدار $|G_1(j\omega)|$ باشد $\sqrt{\alpha}$

$$M_1(\bar{\omega}_{gc}) = |G_1(j\bar{\omega}_{gc})| = \sqrt{\alpha}$$

$$\Rightarrow \omega_m = \bar{\omega}_{gc} \Rightarrow T = \frac{1}{\omega_m \sqrt{\alpha}}$$

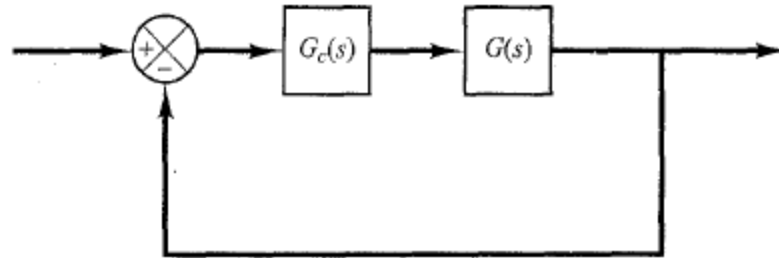
۵. ضرایب روبه رو را تعیین کنید.

$$\text{pole: } \omega = \frac{1}{\alpha T}$$

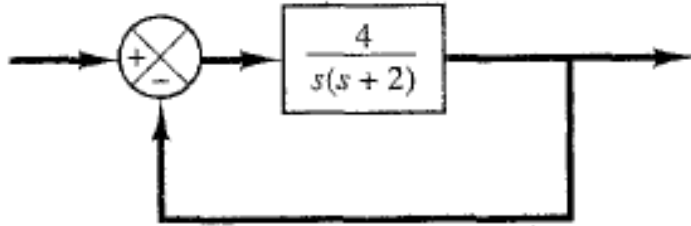
$$\text{zero: } \omega = \frac{1}{T}$$

۶. مقدار k_c را از رابطه زیر بدست آورید.

$$k = k_c \alpha \Rightarrow k_c = \frac{k}{\alpha}$$



*مثال :



$$G(s) = \frac{4}{s(s+2)}$$

$$\bar{k}_v = 20 \text{ sec}^{-1}, \bar{\gamma} \geq 50^\circ, \bar{k}_g \geq 10 \text{ dB}$$

*حل :

$$k_v = \lim_{s \rightarrow 0} sG(s) = 2 \Rightarrow k = \frac{\bar{k}_v}{k_v} = \frac{20}{2} = 10 = k_c \alpha$$

$$\omega_{gc} : M_1(\omega_{gc_1}) = 1 \rightarrow \frac{4 \times 10}{\omega_{gc_1} \sqrt{\omega_{gc_1}^2 + 4}} = 1$$

$$\phi_1(\omega_{gc_1}) = -90 - \tan^{-1} \frac{6.17}{2} = -162.04$$

$$\gamma_1 = 180 - 162.04 = 17.96$$

$$\phi_m = \bar{\gamma} - \gamma_1 + 5 = 50 - 17.96 + 5 = 37.04$$

$$\sin 37.04 = \frac{1 - \alpha}{1 + \alpha} \rightarrow \alpha = 0.25$$

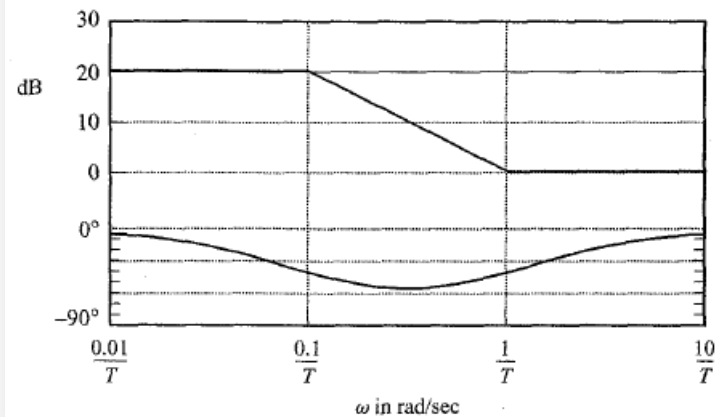
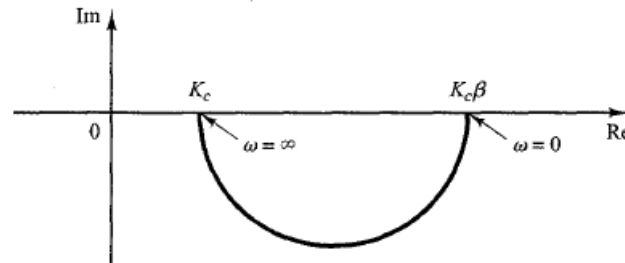
$$|G_1(j\bar{\omega}_{gc})| = \alpha \Rightarrow \frac{4 \times 10}{\bar{\omega}_{gc} \sqrt{\bar{\omega}_{gc}^2 + 4}} = 0.5 \Rightarrow \bar{\omega}_{gc} = \omega_m = 8.9 \text{ rad/sec}$$

$$T = \frac{1}{\omega_m \sqrt{\alpha}} = 0.23 \Rightarrow \frac{1}{T} = 4.4 \Rightarrow \frac{1}{\alpha T} = 18.22 \rightarrow k_c = \frac{k}{\alpha} = 41.7$$

$$G_c(s) = 41.7 \frac{s + 4.4}{s + 18.22}$$

Characteristics of Lag Compensators:

$$G_c(s) = k_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} = k_c \beta \frac{T s + 1}{\beta T s + 1} \quad (\beta > 1)$$





Thanks for your attention!