دانشگاه صنعتی شاهرود

درس ریاضی 1

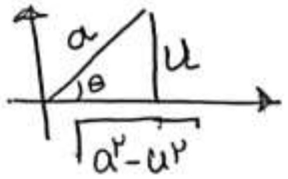
مدرّس : دکترمغاری

حالت خاص :

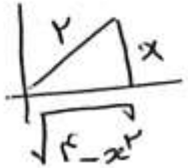
حالت اول : $\sqrt{a^2-u^2}$ u متغیر و a عدد

در این حالت :

$$\Longrightarrow u = a\sin\theta$$
$$du = a\cos\theta \, d\theta$$



$$\int \frac{\sqrt{4-x^2}}{x^2}\,dx \xrightarrow[dx = 2\cos\theta \, d\theta]{x = 2\sin\theta}$$



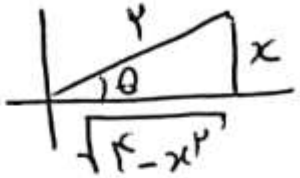
$\sqrt{۴-x^{۲}}$

$$\int \frac{\sqrt{۴-(۴\sin^{۲}\theta)}}{۴\sin^{۲}\theta}\,(۲\cos\theta\, d\theta)$$

$$= \int \frac{۲\cos\theta}{۴\sin^{۲}\theta}(۲\cos\theta)\,d\theta = \int \frac{\cos^{۲}\theta}{\sin^{۲}\theta}\,d\theta$$

$$= \int \left((\cot^{۲}\theta+۱)-۱\right)d\theta = -\cot\theta-\theta$$

$\cot\theta = \dfrac{\text{مجاور}}{\text{مقابل}}$



$$= \frac{-\left(\sqrt{۴-x^{۲}}\right)}{x} - \sin^{-۱}\left(\frac{x}{۲}\right)$$

$$\int \frac{dx}{(۵-۴x-x^۲)^{\frac{۳}{۲}}} = \int \frac{dx}{\left(۹-(x+۲)^۲\right)^{\frac{۳}{۲}}}$$

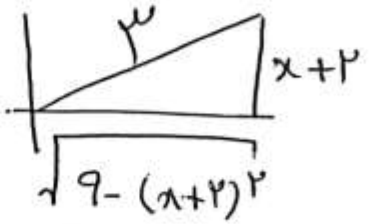
$$۵-۴x-x^۲ = ۵-(x^۲+۴x)$$

$$۵-\left((x+۲)^۲-۴\right)$$

$$= ۹-(x+۲)^۲$$

$$x+۲ = ۳\sin\theta$$

$$dx = ۳\cos\theta \, d\theta$$



$$= \int \frac{dx}{\left(۹-(x+۲)^۲\right)^{\frac{۳}{۲}}} = \int \frac{۳\cos\theta \, d\theta}{\left(۹-(۹\sin^۲\theta)\right)^{۳/۲}}$$

$$x+2=3\sin\theta$$
$$dx=3\cos\theta\,d\theta$$

$$=\int \frac{dx}{\left(9-(x+2)^2\right)^{\frac{3}{2}}}=\int \frac{3\cos\theta\,d\theta}{\left(9-(9\sin^2\theta)\right)^{\frac{3}{2}}}$$

$$=\int \frac{3\cos\theta\,d\theta}{9^{\frac{3}{2}}\left(\cos^2\theta\right)^{\frac{3}{2}}}=\int \frac{1}{9\cos^2\theta}\,d\theta$$

$$=\frac{1}{9}\int\left(1+\mathrm{tg}^2\theta\right)d\theta=\frac{1}{9}\,\mathrm{tag}\,\theta$$

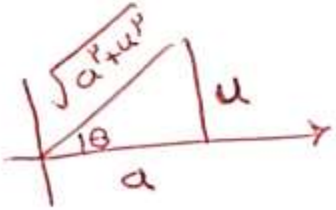
$$\mathrm{tag}\,\theta=\frac{\text{مقابل}\ \theta}{\text{مجاور}\ \theta}$$

$$=\frac{1}{9}\left(\frac{x+2}{\sqrt{9-(x+2)^2}}\right)$$

حالت دوم :

$$\sqrt{a^2+u^2}$$

$$u = a\,\mathrm{tag}\,\theta$$



$$\int \frac{dx}{x\sqrt{x^2+1}} \xrightarrow{x = 1\,\mathrm{tag}\,\theta}$$

$$dx = (\mathrm{tag}^2\theta+1)\,d\theta$$

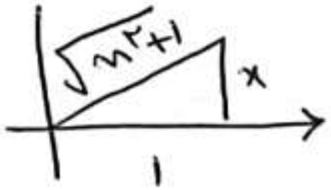
$$dx = (\sec^2\theta)\,d\theta$$

$$\int \frac{(\sec^2\theta)\,d\theta}{\mathrm{tag}\,\theta\sqrt{1+\mathrm{tg}^2\theta}} = \int \frac{\sec^2\theta}{\mathrm{tag}\,\theta\,\sec\theta}\,d\theta$$

$$= \int \frac{\sec\theta}{\mathrm{tag}\,\theta}\, d\theta = \int \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\, d\theta = \int \frac{1}{\sin\theta}\, d\theta$$

$$= \int \csc\theta\, d\theta = \ln\left(\csc\theta - \cot\theta\right)$$

$$= \ln\left(\frac{\sqrt{x^2+1}}{x} - \frac{1}{x}\right)$$



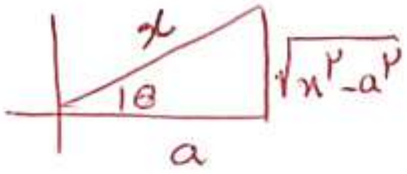
$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sin\theta = \frac{\text{مقابل } \theta}{\text{وتر}} \qquad \cot\theta = \frac{\text{مجاور } \theta}{\text{مقابل به } \theta}$$

حالت سوم :

$$\sqrt{x^2 - a^2}$$

$$x = a\sec\theta$$

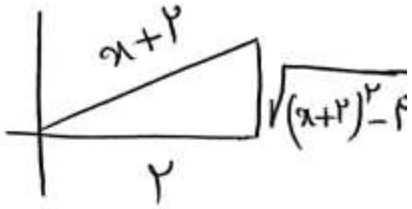


$$dx = a\sec\theta\, tg\theta\, d\theta$$

$$\int \frac{dx}{\sqrt{4x + x^2}} = \int \frac{dx}{\sqrt{(x+2)^2 - 4}}$$

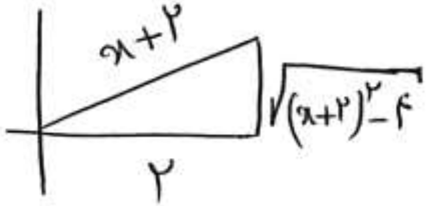
$$x + 2 = 2\sec\theta$$

$$dx = 2\sec\theta\, tg\theta\, d\theta$$



$$x+۲ = ۲\sec\theta$$

$$dx = ۲\sec\theta\, tg\theta\, d\theta$$



$$\int \frac{dx}{\sqrt{(x+۲)^۲-۴}} \qquad \int \frac{۲\sec\theta\, tag\theta\, d\theta}{\sqrt{۴\sec^۲\theta - ۴}} = \int \frac{۲\sec\theta\, tag\theta\, d\theta}{۲\, tag\theta}$$

$$= \int \sec\theta\, d\theta = Ln(\sec\theta + tag\theta)$$

$$= Ln\left(\frac{x+۲}{۲} - \frac{\sqrt{(x+۲)^۲-۴}}{۲} \right)$$

موفق باشید