Find $u(r,t)$.

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r} \quad ; \quad \left\{\begin{matrix} u(r,0)=1-\exp(1-r) \quad , \quad u_t(r,0)=1-r \\ u(r=1,t)=0 \end{matrix}\right\}$$

Answer (in brief):

$$u(r,t)=\sum_{n=1}^{\infty} J_0\left(\sqrt{\lambda_n}\,r\right)\left[\left(\frac{1}{c\sqrt{\lambda_n}}\right)\frac{\int_0^1 (1-\zeta)\zeta\, J_0\left(\sqrt{\lambda_n}\,\zeta\right)d\zeta}{\frac{1}{2}\left\{\left[J_0\left(\sqrt{\lambda_n}\right)\right]^2+\left[J_1\left(\sqrt{\lambda_n}\right)\right]^2\right\}}\sin\left(c\sqrt{\lambda_n}\,t\right)+\frac{\int_0^1 \left[1-\exp(1-\zeta)\right]\zeta\, J_0\left(\sqrt{\lambda_n}\,\zeta\right)d\zeta}{\frac{1}{2}\left\{\left[J_0\left(\sqrt{\lambda_n}\right)\right]^2+\left[J_1\left(\sqrt{\lambda_n}\right)\right]^2\right\}}\cos\left(c\sqrt{\lambda_n}\,t\right)\right]$$

where, $J_0\left(\sqrt{\lambda}\right)=0 \to \quad \lambda=\lambda_n \equiv \text{known} \quad ; \quad \lambda_1 \cong 5.7831, \quad \lambda_2 \cong 30.4712, \quad \lambda_3 \cong 74.8870, \quad \cdots$