تمرین‌های تکمیلی (سری سوم):
معادلات دیفرانسیل با مشتقات جزئی

(1) $x\frac{\partial^2}{\partial x} + y\frac{\partial^2}{\partial y} = 2$

(2) $(x^2+y^2)\frac{\partial^2}{\partial x} + 2xy\frac{\partial^2}{\partial y} = (x+y)^3 2$

(3)
$$\begin{cases} \frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y} = 1 \\ u(x,x) = \frac{x}{2} \end{cases}$$

(4)
$$\begin{cases} x\frac{\partial u}{\partial x} + (y-1)\frac{\partial u}{\partial y} = u-2 \\ u(x,3) = x-2 \end{cases}$$

(5) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial^2 u}{\partial y^2} = 2y^2+2$

(6)
$$\begin{cases} \frac{\partial^2 u}{\partial x \partial y} = 1 \\ \left.\frac{\partial u}{\partial x}\right|_{y=0} = p \;;\; \left.\frac{\partial u}{\partial y}\right|_{x=0} = q \;;\; u(0,0) = p+q \end{cases}$$

(7)
$$\begin{cases} \frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2} \qquad t>0 \;;\; 0<x<2 \\ u(0,t) = 2 \qquad u(2,t) = 6 \qquad t>0 \\ u(x,0) = x \qquad 0<x<2 \end{cases}$$

حل کلی معادلات زیر:

(8) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = ye^x$

(9) $\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial x \partial y} + 2\frac{\partial^2 u}{\partial y^2} = 2x^2 - y^2$