1- Confirm that

$$\left[i\hbar\frac{\partial}{\partial t_1} - \hat{H}_0(x_1) - V_H(x_1,t_1)\right]G(x_1,t_1,x_2,t_2) + i\int v(x_1,x_3)\times$$
$$\times G(x_1,t_1,x_3,t_1^+)G(x_3,t_1,x_2,t_2)dx_3 = \hbar\delta(x_1-x_2)\delta(t_1-t_2)$$

is equivalent to the Hartree-Fock equation.

2- Show that

$$P(\boldsymbol{q},\omega) = \frac{-i}{(2\pi)^4}\int G(\boldsymbol{q}+\boldsymbol{k},\omega+\omega')G(\boldsymbol{k},\omega')\Gamma(\boldsymbol{q}+\boldsymbol{k},\omega+\omega';-\boldsymbol{k},-\omega')d\boldsymbol{k}d\omega'$$

is indeed the transform of

$$P(1,2) = -i/\hbar\int G(1,3)\Gamma(3,4,2)G(4,1^+)d[3]d[4]\ .$$

3- Draw the polarization diagrams to the third order on the same basis as the self-energy diagrams of Figure 8.1 of the textbook.

4- One can use

$$V_q\sum_{\boldsymbol{k}}\frac{f_{\boldsymbol{k}-\boldsymbol{q}}-f_{\boldsymbol{k}}}{\hbar(\omega_q+\epsilon_{\boldsymbol{k}-\boldsymbol{q}}-\epsilon_{\boldsymbol{k}})} = 1\ ,$$

to determine the plasma eigenfrequencies $\omega = \omega_q$. Show that, for a three dimensional plasma, in the long wave-length limit, $(\boldsymbol{q}\to 0)$, using $f_{\boldsymbol{k}-\boldsymbol{q}} - f_{\boldsymbol{k}} \simeq -\boldsymbol{q}.\boldsymbol{\nabla}_{\boldsymbol{k}}f_{\boldsymbol{k}}$, one obtains

$$\epsilon(0,\omega) = 1 - \frac{\omega_{pl}^2}{\omega^2}\ ,$$

i.e. the classical (or Drude) dielectric function ($\omega_{pl} = \omega_{\boldsymbol{q}\to 0}$).

There will two other problems which I will let you know on next sunday.

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