فصل ۷

۴-

$$A(a,b) = \delta(a,b) - \sum_p X(a,p)\Upsilon(p,b)$$

رابطه تصحیح شده:

$$S^{-1}(p,q) = \delta(p,q) - \sum_a \Upsilon(p,a)X(a,q)$$

$$A^{-1}(a,b) = \delta(a,b) + \sum_{p,q} X(a,p)S(p,q)\Upsilon(q,b)$$

$$\begin{aligned}\left(AA^{-1}\right)_{a,b} &= \sum_k A(a,k)A^{-1}(k,b) = \sum_k \delta(a,k)\delta(k,b) - \sum_{k,p}\delta(k,b)X(a,p)\Upsilon(p,k) \\ &+ \sum_k\{[\delta(a,k) - \sum_{p'} X(a,p')\Upsilon(p',k)]\sum_{p,q} X(k,p)S(p,q)\Upsilon(q,b)\}\end{aligned}$$

$$\begin{aligned}\left(AA^{-1}\right)_{a,b} &= \delta(a,b) - \sum_p X(a,p)\Upsilon(p,b) + \sum_{k,p,q}\delta(a,k)X(k,p)S(p,q)\Upsilon(q,b) \\ &- \sum_{p',p,q}\{\sum_k \Upsilon(p',k)X(k,p)\}X(a,p')S(p,q)\Upsilon(q,b)\end{aligned}$$

$$\begin{aligned}\left(AA^{-1}\right)_{a,b} &= \delta(a,b) - \sum_p X(a,p)\Upsilon(p,b) + \sum_{p,q} X(a,p)S(p,q)\Upsilon(q,b) \\ &- \sum_{p',p,q}\{\delta(p',p) - S^{-1}(p',p)\}X(a,p')S(p,q)\Upsilon(q,b)\end{aligned}$$

$$\left(AA^{-1}\right)_{a,b} = \delta(a,b) - \sum_p X(a,p)\Upsilon(p,b) + \sum_{p',p,q} X(a,p')S^{-1}(p',p)S(p,q)\Upsilon(q,b)$$

$$\left(AA^{-1}\right)_{a,b} = \delta(a,b) - \sum_p X(a,p)\Upsilon(p,b) + \sum_{p',q} X(a,p')\delta(p',q)\Upsilon(q,b) = \delta(a,b)$$

$$\Rightarrow \qquad AA^{-1} = 1$$

۵- می توان روابط را به سه دسته تقسیم نمود:

(الف)

$$f(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) = \int\int u(\boldsymbol{r}_1 - \boldsymbol{r}_3, t_1 - t_3)\, v(\boldsymbol{r}_3 - \boldsymbol{r}_2, t_3 - t_2)\, d\boldsymbol{r}_3 dt_3$$

$$\begin{aligned} f(\boldsymbol{r}_1 - \boldsymbol{r}_2, \omega) &= \int d(t_1 - t_2) e^{i\omega(t_1 - t_2)} f(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) \\ &= \int d(t_1 - t_2) e^{i\omega(t_1 - t_2)} \int\int u(\boldsymbol{r}_1 - \boldsymbol{r}_3, t_1 - t_3)\, v(\boldsymbol{r}_3 - \boldsymbol{r}_2, t_3 - t_2)\, d\boldsymbol{r}_3 dt_3 \\ &= \int d\boldsymbol{r}_3 \int e^{i\omega(t_1 - t_3)} u(\boldsymbol{r}_1 - \boldsymbol{r}_3, t_1 - t_3)\, d(t_1 - t_3) \times \\ &\quad \times \int e^{i\omega(t_3 - t_2)} v(\boldsymbol{r}_3 - \boldsymbol{r}_2, t_3 - t_2)\, d(t_2 - t_3) \\ &= \int d\boldsymbol{r}_3\, u(\boldsymbol{r}_1 - \boldsymbol{r}_3, \omega)\, v(\boldsymbol{r}_3 - \boldsymbol{r}_2, \omega) \end{aligned}$$

$$\begin{aligned} f(\boldsymbol{q}, \omega) &= \int d(\boldsymbol{r}_1 - \boldsymbol{r}_2)\, e^{-i\boldsymbol{q}.(\boldsymbol{r}_1 - \boldsymbol{r}_2)} \int u(\boldsymbol{r}_1 - \boldsymbol{r}_3, \omega)\, v(\boldsymbol{r}_3 - \boldsymbol{r}_2, \omega)\, d\boldsymbol{r}_3 \\ &= \int d(\boldsymbol{r}_1 - \boldsymbol{r}_3)\, e^{-i\boldsymbol{q}.(\boldsymbol{r}_1 - \boldsymbol{r}_3)}\, u(\boldsymbol{r}_1 - \boldsymbol{r}_3, \omega) \int e^{-i\boldsymbol{q}.(\boldsymbol{r}_3 - \boldsymbol{r}_2)}\, v(\boldsymbol{r}_3 - \boldsymbol{r}_2, \omega)\, d(\boldsymbol{r}_3 - \boldsymbol{r}_2) \\ &= u(\boldsymbol{q}, \omega)\, v(\boldsymbol{q}, \omega) \end{aligned}$$

(ب)

$$\begin{aligned} f(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) &= \int\int\int\int u(\boldsymbol{r}_1 - \boldsymbol{r}_3, t_1 - t_3)\, v(\boldsymbol{r}_3 - \boldsymbol{r}_4, t_3 - t_4) \times \\ &\quad \times w(\boldsymbol{r}_4 - \boldsymbol{r}_2, t_4 - t_2)\, d\boldsymbol{r}_3 dt_3\, d\boldsymbol{r}_4 dt_4 \end{aligned}$$

$$f(\boldsymbol{r}_1 - \boldsymbol{r}_2, \omega) = \int\int u(\boldsymbol{r}_1 - \boldsymbol{r}_3, \omega)\, v(\boldsymbol{r}_3 - \boldsymbol{r}_4, \omega)\, w(\boldsymbol{r}_4 - \boldsymbol{r}_2, \omega)\, d\boldsymbol{r}_3\, d\boldsymbol{r}_4$$

$$f(\boldsymbol{q}, \omega) = u(\boldsymbol{q}, \omega)\, v(\boldsymbol{q}, \omega)\, w(\boldsymbol{q}, \omega)$$

(ج)

$$\begin{aligned} f(\boldsymbol{r}_1 - \boldsymbol{r}_2, t_1 - t_2) &= \int\int\int\int d\boldsymbol{r}_3 dt_3\, d\boldsymbol{r}_4 dt_4\, u(\boldsymbol{r}_1 - \boldsymbol{r}_3, t_1 - t_3) \times \\ &\quad \times v(\boldsymbol{r}_3 - \boldsymbol{r}_2, \boldsymbol{r}_4 - \boldsymbol{r}_2, t_3 - t_2, t_4 - t_2)\, w(\boldsymbol{r}_1 - \boldsymbol{r}_4, t_1 - t_4) \end{aligned}$$

$$
\begin{aligned}
f(\boldsymbol{r}_1-\boldsymbol{r}_2,\omega) &= \int\int\int\int\int d(t_1-t_2)e^{i\omega(t_1-t_2)}u(\boldsymbol{r}_1-\boldsymbol{r}_3,t_1-t_3)\times\\
&\times v(\boldsymbol{r}_3-\boldsymbol{r}_2,\boldsymbol{r}_4-\boldsymbol{r}_2,t_3-t_2,t_4-t_2)\times\\
&\times w(\boldsymbol{r}_4-\boldsymbol{r}_1,t_4-t_1)\; d\boldsymbol{r}_3dt_3\; d\boldsymbol{r}_4dt_4
\end{aligned}
$$

$$
\begin{aligned}
f(\boldsymbol{r}_1-\boldsymbol{r}_2,\omega) &= \int d\boldsymbol{r}_3\int d\boldsymbol{r}_4\int\int\int e^{i\omega(t_1-t_3)}e^{i\omega(t_3-t_2)}u(\boldsymbol{r}_1-\boldsymbol{r}_3,t_1-t_3)\times\\
&\times v(\boldsymbol{r}_3-\boldsymbol{r}_2,\boldsymbol{r}_4-\boldsymbol{r}_2,t_3-t_2,t_4-t_2)\times\\
&\times w(\boldsymbol{r}_4-\boldsymbol{r}_1,t_4-t_1)\; d\boldsymbol{r}_3dt_3\; d\boldsymbol{r}_4dt_4
\end{aligned}
$$

$$
\mathcal{F}(t_1-t_2) = \int\int u(t_1,t_3)\; v(t_3,t_4,t_2)\; w(t_4,t_1)dt_3\; dt_4
$$

$$
\mathcal{F}(\omega) = \int e^{i\omega(t_1-t_2)}\mathcal{F}(t_1-t_2)\; d(t_1-t_2)
$$

$$
\begin{aligned}
\mathcal{F}(\omega) &= \int\int\int e^{i\omega(t_1-t_3)}e^{i\omega(t_3-t_2)}u(t_1,t_3)v(t_3,t_4,t_2)\times\\
&\times w(t_4,t_1)\; d(t_1-t_2)\; dt_3\; dt_4
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}(\omega) &= \int\int\int e^{i\omega(t_1-t_3)}e^{i\omega(t_3-t_2)}u(t_1,t_3)\; v(t_3,t_4,t_2)\times\\
&\times\Big\{\frac{1}{2\pi}\int w(\omega')e^{-i\omega'(t_1-t_4)}d\omega'\Big\}\; d(t_1-t_2)\; dt_3\; dt_4
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}(\omega) &= \frac{1}{2\pi}\int d\omega' w(\omega')\int\int\int e^{i(\omega-\omega')(t_1-t_3)}e^{i(\omega-\omega')(t_3-t_2)}u(t_1-t_3)\times\\
&\times v(t_3-t_2,t_4-t_2)\; e^{i\omega'(t_4-t_2)}\; d(t_1-t_3)\; d(t_3-t_2)\; d(t_4-t_2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}(\omega) &= \frac{1}{2\pi}\int d\omega' w(\omega')\int d(t_1-t_3)\; e^{i(\omega-\omega')(t_1-t_3)}u(t_1-t_3)\int d(t_4-t_2)\times\\
&\times e^{i\omega'(t_4-t_2)}\Big\{\int d(t_3-t_2)\; e^{i(\omega-\omega')(t_3-t_2)}v(t_3-t_2,t_4-t_2)\Big\}
\end{aligned}
$$

$$
\mathcal{F}(\omega) = \frac{1}{2\pi}\int d\omega' w(\omega')\; u(\omega-\omega')\int d(t_4-t_2)\; e^{i\omega'(t_4-t_2)}\; v(\omega-\omega',t_4-t_2)
$$

$$
\mathcal{F}(\omega) = \frac{1}{2\pi}\int d\omega' w(\omega')\; u(\omega-\omega')\; v(\omega-\omega',\omega')
$$

بنابراین:

$$
f(\boldsymbol{q},\omega) = \Big(\frac{1}{2\pi}\Big)^4\int\int u(\boldsymbol{q}-\boldsymbol{k},\omega-\omega')\; v(\boldsymbol{q}-\boldsymbol{k},\omega-\omega';\boldsymbol{k},\omega')\; w(\boldsymbol{k},\omega')\; d\boldsymbol{k}d\omega'
$$