

Subject:

Year. 200 Month. Day.

$$g'(z) = \frac{1}{\sqrt{z}} \times z \sqrt{z^2+1} \quad g(z) = \frac{1}{\sqrt{z}} \cdot \frac{z}{z} (z^2+1)$$

$$\Rightarrow u(m, z) = \alpha^2 L_2 z + \frac{1}{z} (z^2+1)$$

حالت مستقیم: معادله $P(m, z) dx + Q(m, z) dz = 0$ کامل نباشد. فرض می‌کنیم فاکتور انتگرال به شکل $P(m, z) = \alpha^2 z^\beta$ این فاکتور را در معادله ضرب می‌کنیم و سپس شرط کامل بودن لایبرس می‌کنیم. اگر برای α و β جواب حقیقی بدست آید در این صورت معادله

این روش قابل حل میباشد.

$$EX: z(2-2\alpha z) dx - x dz = 0$$

$$\frac{\partial P}{\partial z} = 2-2\alpha z \quad \frac{\partial Q}{\partial x} = -1$$

$$\frac{\partial P}{\partial z} - \frac{\partial Q}{\partial x} = 3-2\alpha z$$

$$\xrightarrow{\alpha^2 z^\beta} z(2-2\alpha z) dx - x dz = 0 \Rightarrow (2z - 2\alpha z^2) dx - \alpha z dz = 0$$

$$\Rightarrow (2\alpha^{\beta+1} z^{\beta+1} - 2\alpha^{\alpha+1} z^{\beta+2}) dx - (\alpha^{\alpha+1} z^\beta) dz = 0$$

$$\frac{\partial P}{\partial z} - \frac{\partial Q}{\partial x} = 0 \quad \text{شرط کامل بودن} \Rightarrow \frac{\partial P}{\partial z} = \frac{\partial Q}{\partial x}$$

$$2\alpha^{\beta+1} (\beta+1) z^\beta - 2\alpha^{\alpha+1} (\beta+2) z^{\beta+1} = (\alpha+1) \alpha^{\alpha+1} z^\beta$$

$$2(\beta+1) \alpha^{\beta+1} z^\beta - 2(\beta+2) \alpha^{\alpha+1} z^{\beta+1} = (\alpha+1) \alpha^{\alpha+1} z^\beta$$

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$$\begin{cases} 2\beta + 2 = -(\alpha + 1) \\ 2 + \beta = 0 \end{cases} \quad \begin{cases} -2 = -\alpha - 1 \\ \beta = -2 \end{cases} \quad \begin{cases} \alpha = 1 \\ \beta = -2 \end{cases} \quad f(x, y) = x y^{-2}$$

با استفاده از فاکتور انتگرال معادله دیفرانسیل زیر را حل کنید.

$$y (y^2 - 2x^2) dx + x (2y^2 - x^2) dy = 0$$

$$\frac{\partial P}{\partial y} = 2y^2 - 2x^2 \quad \frac{\partial Q}{\partial x} = 2y^2 - 2x^2$$

$$x^{\alpha} y^{\beta} \rightarrow x \left[(y^2 - 2x^2) dx + (2xy^2 - x^2) dy = 0 \right]$$

$$\left(x^{\alpha} y^{\beta+2} - 2x^{\alpha+2} y^{\beta+1} \right) dx + \left(2x^{\alpha+1} y^{\beta+2} - x^{\alpha+2} y^{\beta} \right) dy = 0$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \Rightarrow$$

$$(\beta+2) x^{\alpha} y^{\beta+1} - 2(\beta+1) x^{\alpha+2} y^{\beta} = 2(1+\alpha) x^{\alpha} y^{\beta+2} - (\alpha+2) x^{\alpha+2} y^{\beta}$$

$$\begin{cases} \beta+2 = 2(1+\alpha) \\ (\alpha+2) = 2(\beta+1) \end{cases} \quad \begin{cases} \beta+2 = 2+2\alpha \\ \alpha+2 = 2\beta+2 \end{cases} \quad \begin{cases} 2\alpha - \beta = 1 \\ (-\alpha + 2\beta = 1) \times 2 \end{cases} \quad \begin{cases} 2\alpha - \beta = 1 \\ -2\alpha + 4\beta = 2 \\ +2\beta = 3 \quad \beta = 1 \\ \alpha = 1 \end{cases}$$

$$f(x, y) = x y \quad \text{فاکتور انتگرال گیری}$$