Chapter 1

1. **THINK** In this problem we're given the radius of Earth, and asked to compute its circumference, surface area and volume.

EXPRESS Assuming Earth to be a sphere of radius

$$R_E = \left(6.37 \times 10^6 \text{ m}\right)\left(10^{-3} \text{ km/m}\right) = 6.37 \times 10^3 \text{ km},$$

the corresponding circumference, surface area and volume are:

$$C = 2\pi R_E, \quad A = 4\pi R_E^2, \quad V = \frac{4\pi}{3} R_E^3.$$

The geometric formulas are given in Appendix E.

ANALYZE (a) Using the formulas given above, we find the circumference to be

$$C = 2\pi R_E = 2\pi(6.37 \times 10^3 \text{ km}) = 4.00 \times 10^4 \text{ km}.$$

(b) Similarly, the surface area of Earth is

$$A = 4\pi R_E^2 = 4\pi\left(6.37 \times 10^3 \text{ km}\right)^2 = 5.10 \times 10^8 \text{ km}^2,$$

(c) and its volume is

$$V = \frac{4\pi}{3} R_E^3 = \frac{4\pi}{3}\left(6.37 \times 10^3 \text{ km}\right)^3 = 1.08 \times 10^{12} \text{ km}^3.$$

LEARN From the formulas given, we see that $C \sim R_E$, $A \sim R_E^2$, and $V \sim R_E^3$. The ratios of volume to surface area, and surface area to circumference are $V/A = R_E/3$ and $A/C = 2R_E$.

2. The conversion factors are: $1 \text{ gry} = 1/10 \text{ line}$, $1 \text{ line} = 1/12 \text{ inch}$ and 1 point = 1/72 inch. The factors imply that

1 gry = (1/10)(1/12)(72 points) = 0.60 point.

Thus, 1 gry^2 = (0.60 point)^2 = 0.36 point^2, which means that $0.50 \text{ gry}^2 = 0.18 \text{ point}^2$.

3. The metric prefixes (micro, pico, nano, …) are given for ready reference on the inside front cover of the textbook (see also Table 1–2).

(a) Since 1 km = 1 × 10^3 m and 1 m = 1 × 10^6 μm,

$$1\text{km} = 10^3\text{ m} = \left(10^3\text{ m}\right)\left(10^6\ \mu\text{m/m}\right) = 10^9\ \mu\text{m}.$$

The given measurement is 1.0 km (two significant figures), which implies our result should be written as 1.0×10^9 μm.

(b) We calculate the number of microns in 1 centimeter. Since 1 cm = 10^{-2} m,

$$1\text{cm} = 10^{-2}\text{ m} = \left(10^{-2}\text{m}\right)\left(10^6\ \mu\text{m/m}\right) = 10^4\ \mu\text{m}.$$

We conclude that the fraction of one centimeter equal to 1.0 μm is 1.0×10^{-4}.

(c) Since 1 yd = (3 ft)(0.3048 m/ft) = 0.9144 m,

$$1.0\text{ yd} = \left(0.91\text{m}\right)\left(10^6\ \mu\text{m/m}\right) = 9.1 \times 10^5\ \mu\text{m}.$$

4. (a) Using the conversion factors 1 inch = 2.54 cm exactly and 6 picas = 1 inch, we obtain

$$0.80\text{ cm} = \left(0.80\text{ cm}\right)\left(\frac{1\text{ inch}}{2.54\text{ cm}}\right)\left(\frac{6\text{ picas}}{1\text{ inch}}\right) \approx 1.9\text{ picas}.$$

(b) With 12 points = 1 pica, we have

$$0.80\text{ cm} = \left(0.80\text{ cm}\right)\left(\frac{1\text{ inch}}{2.54\text{ cm}}\right)\left(\frac{6\text{ picas}}{1\text{ inch}}\right)\left(\frac{12\text{ points}}{1\text{ pica}}\right) \approx 23\text{ points}.$$

5. **THINK** This problem deals with conversion of furlongs to rods and chains, all of which are units for distance.

EXPRESS Given that $1\text{ furlong} = 201.168\text{ m}$, $1\text{ rod} = 5.0292\text{ m}$ and $1\text{ chain} = 20.117\text{ m}$, the relevant conversion factors are

$$1.0\text{ furlong} = 201.168\text{ m} = (201.168\ \cancel{\text{m}})\frac{1\text{ rod}}{5.0292\ \cancel{\text{m}}} = 40\text{ rods},$$

and

$$1.0\text{ furlong} = 201.168\text{ m} = (201.168\ \cancel{\text{m}})\frac{1\text{ chain}}{20.117\ \cancel{\text{m}}} = 10\text{ chains}.$$

Note the cancellation of m (meters), the unwanted unit.

ANALYZE Using the above conversion factors, we find

(a) the distance d in *rods* to be $d = 4.0\text{ furlongs} = (4.0\text{ fu}\cancel{\text{r}}\text{longs})\dfrac{40\text{ rods}}{1\text{ fu}\cancel{\text{r}}\text{long}} = 160\text{ rods},$

(b) and in *chains* to be $d = 4.0 \text{ furlongs} = (4.0 \text{ fu}\cancel{\text{r}}\text{longs})\dfrac{10 \text{ chains}}{1 \text{ fu}\cancel{\text{r}}\text{long}} = 40 \text{ chains}.$

LEARN Since 4 furlongs is about 800 m, this distance is approximately equal to 160 rods ($1 \text{ rod} \approx 5 \text{ m}$) and 40 chains ($1 \text{ chain} \approx 20 \text{ m}$). So our results make sense.

6. We make use of Table 1-6.

(a) We look at the first ("cahiz") column: 1 fanega is equivalent to what amount of cahiz? We note from the already completed part of the table that 1 cahiz equals a dozen fanega. Thus, 1 fanega = $\frac{1}{12}$ cahiz, or 8.33×10^{-2} cahiz. Similarly, "1 cahiz = 48 cuartilla" (in the already completed part) implies that 1 cuartilla = $\frac{1}{48}$ cahiz, or 2.08×10^{-2} cahiz. Continuing in this way, the remaining entries in the first column are 6.94×10^{-3} and 3.47×10^{-3}.

(b) In the second ("fanega") column, we find 0.250, 8.33×10^{-2}, and 4.17×10^{-2} for the last three entries.

(c) In the third ("cuartilla") column, we obtain 0.333 and 0.167 for the last two entries.

(d) Finally, in the fourth ("almude") column, we get $\frac{1}{2}$ = 0.500 for the last entry.

(e) Since the conversion table indicates that 1 almude is equivalent to 2 medios, our amount of 7.00 almudes must be equal to 14.0 medios.

(f) Using the value (1 almude = 6.94×10^{-3} cahiz) found in part (a), we conclude that 7.00 almudes is equivalent to 4.86×10^{-2} cahiz.

(g) Since each decimeter is 0.1 meter, then 55.501 cubic decimeters is equal to 0.055501 m^3 or 55501 cm^3. Thus, 7.00 almudes = $\frac{7.00}{12}$ fanega = $\frac{7.00}{12}(55501 \text{ cm}^3) = 3.24 \times 10^4 \text{ cm}^3$.

7. We use the conversion factors found in Appendix D.

$$1 \text{ acre} \cdot \text{ft} = (43{,}560 \text{ ft}^2) \cdot \text{ft} = 43{,}560 \text{ ft}^3$$

Since 2 in. = (1/6) ft, the volume of water that fell during the storm is

$$V = (26 \text{ km}^2)(1/6 \text{ ft}) = (26 \text{ km}^2)(3281 \text{ft/km})^2(1/6 \text{ ft}) = 4.66 \times 10^7 \text{ ft}^3.$$

Thus,

$$V = \frac{4.66 \times 10^7 \text{ ft}^3}{4.3560 \times 10^4 \text{ ft}^3 / \text{acre} \cdot \text{ft}} = 1.1 \times 10^3 \text{ acre} \cdot \text{ft}.$$

8. From Fig. 1-4, we see that 212 S is equivalent to 258 W and 212 – 32 = 180 S is equivalent to 216 – 60 = 156 Z. The information allows us to convert S to W or Z.

(a) In units of W, we have

$$50.0 \text{ S} = (50.0 \text{ S})\left(\frac{258 \text{ W}}{212 \text{ S}}\right) = 60.8 \text{ W}$$

(b) In units of Z, we have

$$50.0 \text{ S} = (50.0 \text{ S})\left(\frac{156 \text{ Z}}{180 \text{ S}}\right) = 43.3 \text{ Z}$$

9. The volume of ice is given by the product of the semicircular surface area and the thickness. The area of the semicircle is $A = \pi r^2/2$, where r is the radius. Therefore, the volume is

$$V = \frac{\pi}{2} r^2 z$$

where z is the ice thickness. Since there are 10^3 m in 1 km and 10^2 cm in 1 m, we have

$$r = (2000\,\text{km})\left(\frac{10^3\,\text{m}}{1\text{km}}\right)\left(\frac{10^2\,\text{cm}}{1\text{m}}\right) = 2000 \times 10^5 \text{ cm}.$$

In these units, the thickness becomes

$$z = 3000\,\text{m} = (3000\,\text{m})\left(\frac{10^2\,\text{cm}}{1\text{m}}\right) = 3000 \times 10^2 \text{ cm}$$

which yields $V = \frac{\pi}{2}\left(2000 \times 10^5 \text{ cm}\right)^2 \left(3000 \times 10^2 \text{ cm}\right) = 1.9 \times 10^{22} \text{ cm}^3$.

10. Since a change of longitude equal to $360°$ corresponds to a 24 hour change, then one expects to change longitude by $360°/24 = 15°$ before resetting one's watch by 1.0 h.

11. (a) Presuming that a French decimal day is equivalent to a regular day, then the ratio of weeks is simply 10/7 or (to 3 significant figures) 1.43.

(b) In a regular day, there are 86400 seconds, but in the French system described in the problem, there would be 10^5 seconds. The ratio is therefore 0.864.

12. A day is equivalent to 86400 seconds and a meter is equivalent to a million micrometers, so

$$\frac{(3.7\ \text{m})(10^{6}\ \mu\text{m/m})}{(14\ \text{day})(86400\ \text{s/day})} = 3.1\ \mu\text{m/s}.$$

13. The time on any of these clocks is a straight-line function of that on another, with slopes $\neq$ 1 and y-intercepts $\neq$ 0. From the data in the figure we deduce

$$t_C = \frac{2}{7}t_B + \frac{594}{7}, \quad t_B = \frac{33}{40}t_A - \frac{662}{5}.$$

These are used in obtaining the following results.

(a) We find

$$t'_B - t_B = \frac{33}{40}\left(t'_A - t_A\right) = 495\ \text{s}$$

when $t'_A - t_A = 600$ s.

(b) We obtain $t'_C - t_C = \frac{2}{7}\left(t'_B - t_B\right) = \frac{2}{7}(495) = 141$ s.

(c) Clock B reads $t_B = (33/40)(400) - (662/5) \approx 198$ s when clock A reads $t_A = 400$ s.

(d) From $t_C = 15 = (2/7)t_B + (594/7)$, we get $t_B \approx -245$ s.

14. The metric prefixes (micro (μ), pico, nano, …) are given for ready reference on the inside front cover of the textbook (also Table 1–2).

(a) $$1\ \mu\text{century} = \left(10^{-6}\ \text{century}\right)\left(\frac{100\ \text{y}}{1\ \text{century}}\right)\left(\frac{365\ \text{day}}{1\ \text{y}}\right)\left(\frac{24\ \text{h}}{1\ \text{day}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 52.6\ \text{min}.$$

(b) The percent difference is therefore

$$\frac{52.6\ \text{min} - 50\ \text{min}}{52.6\ \text{min}} = 4.9\%.$$

15. A week is 7 days, each of which has 24 hours, and an hour is equivalent to 3600 seconds. Thus, two weeks (a fortnight) is 1209600 s. By definition of the micro prefix, this is roughly $1.21 \times 10^{12}\ \mu$s.

16. We denote the pulsar rotation rate f (for frequency).

$$f = \frac{1\ \text{rotation}}{1.55780644887275 \times 10^{-3}\ \text{s}}$$

(a) Multiplying f by the time-interval t = 7.00 days (which is equivalent to 604800 s, if we ignore *significant figure* considerations for a moment), we obtain the number of rotations:

$$N = \left(\frac{1\text{ rotation}}{1.55780644887275\times10^{-3}\text{ s}}\right)(604800\text{ s}) = 388238218.4$$

which should now be rounded to 3.88×10^8 rotations since the time-interval was specified in the problem to three significant figures.

(b) We note that the problem specifies the *exact* number of pulsar revolutions (one million). In this case, our unknown is t, and an equation similar to the one we set up in part (a) takes the form $N = ft$, or

$$1\times10^6 = \left(\frac{1\text{ rotation}}{1.55780644887275\times10^{-3}\text{ s}}\right)t$$

which yields the result t = 1557.80644887275 s (though students who do this calculation on their calculator might not obtain those last several digits).

(c) Careful reading of the problem shows that the time-uncertainty *per revolution* is $\pm3\times10^{-17}\text{s}$. We therefore expect that as a result of one million revolutions, the uncertainty should be $(\pm3\times10^{-17})(1\times10^6)= \pm3\times10^{-11}$ s .

17. **THINK** In this problem we are asked to rank 5 clocks, based on their performance as timekeepers.

EXPRESS We first note that none of the clocks advance by exactly 24 h in a 24-h period but this is not the most important criterion for judging their quality for measuring time intervals. What is important here is that the clock advance by the same (or nearly the same) amount in each 24-h period. The clock reading can then easily be adjusted to give the correct interval.

ANALYZE The chart below gives the corrections (in seconds) that must be applied to the reading on each clock for each 24-h period. The entries were determined by subtracting the clock reading at the end of the interval from the clock reading at the beginning.

Clocks C and D are both good timekeepers in the sense that each is consistent in its daily drift (relative to WWF time); thus, C and D are easily made “perfect” with simple and predictable corrections. The correction for clock C is less than the correction for clock D, so we judge clock C to be the best and clock D to be the next best. The correction that must be applied to clock A is in the range from 15 s to 17s. For clock B it is the range from –5 s to +10 s, for clock E it is in the range from –70 s to –2 s. After C and D, A has the smallest range of correction, B has the next smallest range, and E has the greatest range. From best to worst, the ranking of the clocks is C, D, A, B, E.

CLOCK	Sun. -Mon.	Mon. -Tues.	Tues. -Wed.	Wed. -Thurs.	Thurs. -Fri.	Fri. -Sat.
A	–16	–16	–15	–17	–15	–15
B	–3	+5	–10	+5	+6	–7
C	–58	–58	–58	–58	–58	–58
D	+67	+67	+67	+67	+67	+67
E	+70	+55	+2	+20	+10	+10

LEARN Of the five clocks, the readings in clocks A, B and E jump around from one 24-h period to another, making it difficult to correct them.

18. The last day of the 20 centuries is longer than the first day by

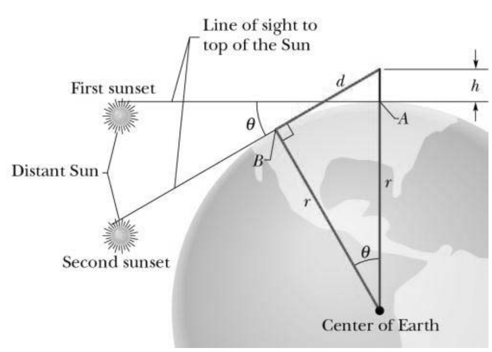
$$(20 \text{ century})\,(0.001 \text{ s/century}) = 0.02 \text{ s}.$$

The average day during the 20 centuries is (0 + 0.02)/2 = 0.01 s longer than the first day. Since the increase occurs uniformly, the cumulative effect T is

$$\begin{aligned} T &= (\text{average increase in length of a day})(\text{number of days}) \\ &= \left(\frac{0.01 \text{ s}}{\text{day}}\right)\left(\frac{365.25 \text{ day}}{\text{y}}\right)(2000 \text{ y}) \\ &= 7305 \text{ s} \end{aligned}$$

or roughly two hours.

19. When the Sun first disappears while lying down, your line of sight to the top of the Sun is tangent to the Earth's surface at point A shown in the figure. As you stand, elevating your eyes by a height h, the line of sight to the Sun is tangent to the Earth's surface at point B.



Let d be the distance from point B to your eyes. From the Pythagorean theorem, we have

$$d^2 + r^2 = (r+h)^2 = r^2 + 2rh + h^2$$

or $d^2 = 2rh + h^2$, where r is the radius of the Earth. Since $r \gg h$, the second term can be dropped, leading to $d^2 \approx 2rh$. Now the angle between the two radii to the two tangent points A and B is θ, which is also the angle through which the Sun moves about Earth during the time interval t = 11.1 s. The value of θ can be obtained by using

$$\frac{\theta}{360°} = \frac{t}{24\text{ h}}.$$

This yields

$$\theta = \frac{(360°)(11.1\text{ s})}{(24\text{ h})(60\text{ min/h})(60\text{ s/min})} = 0.04625°.$$

Using $d = r\tan\theta$, we have $d^2 = r^2\tan^2\theta = 2rh$, or

$$r = \frac{2h}{\tan^2\theta}$$

Using the above value for θ and h = 1.7 m, we have $r = 5.2\times10^6$ m.

20. (a) We find the volume in cubic centimeters

$$193\text{ gal} = (193\text{ gal})\left(\frac{231\text{ in}^3}{1\text{ gal}}\right)\left(\frac{2.54\text{ cm}}{1\text{ in}}\right)^3 = 7.31\times10^5\text{ cm}^3$$

and subtract this from 1×10^6 cm^3 to obtain 2.69×10^5 cm^3. The conversion gal $\rightarrow$ in^3 is given in Appendix D (immediately below the table of Volume conversions).

(b) The volume found in part (a) is converted (by dividing by (100 cm/m)^3) to 0.731 m^3, which corresponds to a mass of

$$\left(1000\ \text{kg/m}^3\right)\left(0.731\text{ m}^2\right) = 731\text{ kg}$$

using the density given in the problem statement. At a rate of 0.0018 kg/min, this can be filled in

$$\frac{731\text{kg}}{0.0018\text{ kg/min}} = 4.06\times10^5\text{ min} = 0.77\text{ y}$$

after dividing by the number of minutes in a year (365 days)(24 h/day) (60 min/h).

21. If M_E is the mass of Earth, m is the average mass of an atom in Earth, and N is the number of atoms, then $M_E = Nm$ or $N = M_E/m$. We convert mass m to kilograms using Appendix D (1 u = 1.661×10^{-27} kg). Thus,

$$N = \frac{M_E}{m} = \frac{5.98 \times 10^{24}\ \text{kg}}{(40\ \text{u})\left(1.661 \times 10^{-27}\ \text{kg/u}\right)} = 9.0 \times 10^{49}.$$

22. The density of gold is

$$\rho = \frac{m}{V} = \frac{19.32\ \text{g}}{1\ \text{cm}^3} = 19.32\ \text{g/cm}^3.$$

(a) We take the volume of the leaf to be its area A multiplied by its thickness z. With density $\rho = 19.32$ g/cm^3 and mass $m = 27.63$ g, the volume of the leaf is found to be

$$V = \frac{m}{\rho} = 1.430\ \text{cm}^3.$$

We convert the volume to SI units:

$$V = \left(1.430\ \text{cm}^3\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3 = 1.430 \times 10^{-6}\ \text{m}^3.$$

Since $V = Az$ with $z = 1 \times 10^{-6}$ m (metric prefixes can be found in Table 1–2), we obtain

$$A = \frac{1.430 \times 10^{-6}\ \text{m}^3}{1 \times 10^{-6}\ \text{m}} = 1.430\ \text{m}^2.$$

(b) The volume of a cylinder of length ℓ is $V = A\ell$ where the cross-section area is that of a circle: $A = \pi r^2$. Therefore, with $r = 2.500 \times 10^{-6}$ m and $V = 1.430 \times 10^{-6}$ m^3, we obtain

$$\ell = \frac{V}{\pi r^2} = 7.284 \times 10^4\ \text{m} = 72.84\ \text{km}.$$

23. **THINK** This problem consists of two parts: in the first part, we are asked to find the mass of water, given its volume and density; the second part deals with the mass flow rate of water, which is expressed as kg/s in SI units.

EXPRESS From the definition of density: $\rho = m/V$, we see that mass can be calculated as $m = \rho V$, the product of the volume of water and its density. With 1 g = 1×10^{-3} kg and 1 cm^3 = $(1 \times 10^{-2}\text{m})^3 = 1 \times 10^{-6}\text{m}^3$, the density of water in SI units (kg/m^3) is

$$\rho = 1\ \text{g/cm}^3 = \left(\frac{1\ \text{g}}{\text{cm}^3}\right)\left(\frac{10^{-3}\ \text{kg}}{\text{g}}\right)\left(\frac{\text{cm}^3}{10^{-6}\ \text{m}^3}\right) = 1 \times 10^3\ \text{kg/m}^3.$$

To obtain the flow rate, we simply divide the total mass of the water by the time taken to drain it.

ANALYZE (a) Using $m = \rho V$, the mass of a cubic meter of water is

$$m = \rho V = (1\times 10^3 \text{ kg/m}^3)(1\,\text{m}^3) = 1000\text{ kg}.$$

(b) The total mass of water in the container is

$$M = \rho V = (1\times 10^3 \text{ kg/m}^3)(5700 \text{ m}^3) = 5.70\times 10^6 \text{ kg},$$

and the time elapsed is t = (10 h)(3600 s/h) = 3.6×10^4 s. Thus, the mass flow rate R is

$$R = \frac{M}{t} = \frac{5.70\times 10^6 \text{ kg}}{3.6\times 10^4 \text{ s}} = 158 \text{ kg/s}.$$

LEARN In terms of volume, the drain rate can be expressed as

$$R' = \frac{V}{t} = \frac{5700 \text{ m}^3}{3.6\times 10^4 \text{ s}} = 0.158 \text{ m}^3/\text{s} \approx 42 \text{ gal/s}.$$

The greater the flow rate, the less time required to drain a given amount of water.

24. The metric prefixes (micro (μ), pico, nano, …) are given for ready reference on the inside front cover of the textbook (see also Table 1–2). The surface area A of each grain of sand of radius r = 50 μm = 50 × 10^{-6} m is given by $A = 4\pi(50\times 10^{-6})^2 = 3.14\times 10^{-8}$ m^2 (Appendix E contains a variety of geometry formulas). We introduce the notion of density, $\rho = m/V$, so that the mass can be found from $m = \rho V$, where ρ = 2600 kg/m^3. Thus, using $V = 4\pi r^3/3$, the mass of each grain is

$$m = \rho V = \rho\left(\frac{4\pi r^3}{3}\right) = \left(2600\ \frac{\text{kg}}{\text{m}^3}\right)\frac{4\pi\left(50\times 10^{-6}\text{ m}\right)^3}{3} = 1.36\times 10^{-9}\text{ kg}.$$

We observe that (because a cube has six equal faces) the indicated surface area is 6 m^2. The number of spheres (the grains of sand) N that have a total surface area of 6 m^2 is given by

$$N = \frac{6 \text{ m}^2}{3.14\times 10^{-8}\text{ m}^2} = 1.91\times 10^8.$$

Therefore, the total mass M is $M = Nm = \left(1.91\times 10^8\right)\left(1.36\times 10^{-9}\text{ kg}\right) = 0.260\text{ kg}.$

25. The volume of the section is (2500 m)(800 m)(2.0 m) = 4.0 × 10^6 m^3. Letting "d" stand for the thickness of the mud after it has (uniformly) distributed in the valley, then its volume there would be (400 m)(400 m)d. Requiring these two volumes to be equal, we can solve for d. Thus, d = 25 m. The volume of a small part of the mud over a patch of area of 4.0 m^2 is (4.0)d = 100 m^3. Since each cubic meter corresponds to a mass of

1900 kg (stated in the problem), then the mass of that small part of the mud is 1.9×10^{5} kg.

26. (a) The volume of the cloud is (3000 m)π(1000 m)^2 = 9.4 × 10^9 m^3. Since each cubic meter of the cloud contains from 50 × 10^6 to 500 × 10^6 water drops, then we conclude that the entire cloud contains from 4.7 × 10^{18} to 4.7 × 10^{19} drops. Since the volume of each drop is $\frac{4}{3}\pi(10 \times 10^{-6}\text{ m})^3 = 4.2 \times 10^{-15}\text{ m}^3$, then the total volume of water in a cloud is from 2×10^{3} to 2×10^{4} m^3.

(b) Using the fact that $1\text{ L} = 1\times10^{3}\text{cm}^{3} = 1\times10^{-3}\text{m}^{3}$, the amount of water estimated in part (a) would fill from 2×10^{6} to 2×10^{7} bottles.

(c) At 1000 kg for every cubic meter, the mass of water is from 2×10^{6} to 2×10^{7} kg. The coincidence in numbers between the results of parts (b) and (c) of this problem is due to the fact that each liter has a mass of one kilogram when water is at its normal density (under standard conditions).

27. We introduce the notion of density, $\rho = m/V$, and convert to SI units: 1000 g = 1 kg, and 100 cm = 1 m.

(a) The density ρ of a sample of iron is

$$\rho = \left(7.87\text{ g/cm}^3\right)\left(\frac{1\text{ kg}}{1000\text{ g}}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right)^3 = 7870\text{ kg/m}^3.$$

If we ignore the empty spaces between the close-packed spheres, then the density of an individual iron atom will be the same as the density of any iron sample. That is, if M is the mass and V is the volume of an atom, then

$$V = \frac{M}{\rho} = \frac{9.27 \times 10^{-26}\text{ kg}}{7.87 \times 10^{3}\text{ kg/m}^3} = 1.18 \times 10^{-29}\text{ m}^3.$$

(b) We set $V = 4\pi R^3/3$, where R is the radius of an atom (Appendix E contains several geometry formulas). Solving for R, we find

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3\left(1.18 \times 10^{-29}\text{ m}^3\right)}{4\pi}\right)^{1/3} = 1.41 \times 10^{-10}\text{ m}.$$

The center-to-center distance between atoms is twice the radius, or 2.82 × 10^{-10} m.

28. If we estimate the "typical" large domestic cat mass as 10 kg, and the "typical" atom (in the cat) as 10 u ≈ 2 × 10^{-26} kg, then there are roughly (10 kg)/(2 × 10^{-26} kg) ≈ 5 × 10^{26} atoms. This is close to being a factor of a thousand greater than Avogadro's number. Thus this is roughly a kilomole of atoms.

29. The mass in kilograms is

$$(28.9\ \text{piculs})\left(\frac{100\,\text{gin}}{1\,\text{picul}}\right)\left(\frac{16\,\text{tahil}}{1\,\text{gin}}\right)\left(\frac{10\,\text{chee}}{1\,\text{tahil}}\right)\left(\frac{10\ \text{hoon}}{1\ \text{chee}}\right)\left(\frac{0.3779\,\text{g}}{1\,\text{hoon}}\right)$$

which yields 1.747 × 10^6 g or roughly 1.75× 10^3 kg.

30. To solve the problem, we note that the first derivative of the function with respect to time gives the rate. Setting the rate to zero gives the time at which an extreme value of the variable mass occurs; here that extreme value is a maximum.

(a) Differentiating $m(t) = 5.00t^{0.8} - 3.00t + 20.00$ with respect to t gives

$$\frac{dm}{dt} = 4.00t^{-0.2} - 3.00.$$

The water mass is the greatest when $dm/dt = 0,$ or at $t = (4.00/3.00)^{1/0.2} = 4.21\,\text{s}.$

(b) At $t = 4.21\,\text{s},$ the water mass is

$$m(t = 4.21\,\text{s}) = 5.00(4.21)^{0.8} - 3.00(4.21) + 20.00 = 23.2\ \text{g}.$$

(c) The rate of mass change at $t = 2.00\ \text{s}$ is

$$\left.\frac{dm}{dt}\right|_{t=2.00\,\text{s}} = \left[4.00(2.00)^{-0.2} - 3.00\right]\text{g/s} = 0.48\ \text{g/s} = 0.48\frac{\text{g}}{\text{s}}\cdot\frac{1\,\text{kg}}{1000\ \text{g}}\cdot\frac{60\ \text{s}}{1\ \text{min}}$$
$$= 2.89\times10^{-2}\ \text{kg/min}.$$

d imilarly the rate of mass change at $t = 5.00\,\text{s}$ is

$$\left.\frac{dm}{dt}\right|_{t=2.00\,\text{s}} = \left[4.00\ 5.00\ ^{-0.2} - 3.00\right]\text{g s} = -0.101\,\text{g s} = -0.101\frac{\text{g}}{\text{s}}\cdot\frac{1\,\text{kg}}{1000\,\text{g}}\cdot\frac{60\,\text{s}}{1\ \text{min}}$$
$$= -6.05\times10^{-3}\ \text{kg min}.$$

31. he mass density of the candy is

$$\rho = \frac{m}{V} = \frac{0.0200\ \text{g}}{50.0\ \text{mm}^3} = 4.00\times10^{-4}\ \text{g mm}^3 = 4.00\times10^{-4}\ \text{kg cm}^3.$$

f e neglect the volume of the empty spaces bet een the candies then the total mass of the candies in the container hen filled to height h is $M = \rho Ah$ here $A =$ 14.0 cm 17.0 cm $= 238\ \text{cm}^2$ is the base area of the container that remains unchanged. hus the rate of mass change is given by

$$\frac{dM}{dt} = \frac{d\ \rho Ah}{dt} = \rho A\frac{dh}{dt} = \ 4.00\times10^{-4}\ \text{kg cm}^3\ \ 238\ \text{cm}^2\ \ 0.250\ \text{cm s}$$
$$= 0.0238\ \text{kg s} = 1.43\ \text{kg min.}$$

32. he total volume V of the real house is that of a triangular prism of height h = 3.0 m and base area A = 20 × 12 = 240 m^2 in addition to a rectangular bo height h = 6.0 m and same base . herefore

$$V = \frac{1}{2}hA + h'A = \left(\frac{h}{2} + h'\right)A = 1800\ \text{m}^3.$$

a ach dimension is reduced by a factor of 1 12 and e find

$$V_{\text{doll}} = \left(1800\ \text{m}^3\right)\left(\frac{1}{12}\right)^3 \approx 1.0\ \text{m}^3.$$

b n this case each dimension relative to the real house is reduced by a factor of 1 144. herefore

$$V_{\text{miniature}} = \left(1800\ \text{m}^3\right)\left(\frac{1}{144}\right)^3 \approx 6.0\times10^{-4}\ \text{m}^3.$$

33. **THINK** n this problem e are asked to differentiate bet een three types of tons *displacement* ton *freight* ton and *register* ton all of hich are units of volume.

EXPRESS he three different tons are defined in terms of *barrel bulk* ith 1 barrel bulk $= 0.1415\ \text{m}^3 = 4.0155$ . . bushels using 1 m^3 $=$ 28.378 . . bushels . hus in terms of . . bushels e have

$$1\ \text{displacement ton} = \ 7\ \text{barrels bulk}\ \times\left(\frac{4.0155\ \ .\ .\ \text{bushels}}{1\ \text{barrel bulk}}\right) = 28.108\ \ .\ .\ \text{bushels}$$
$$1\ \text{freight ton} = \ 8\ \text{barrels bulk}\ \times\left(\frac{4.0155\ \ .\ .\ \text{bushels}}{1\ \text{barrel bulk}}\right) = 32.124\ \ .\ .\ \text{bushels}$$
$$1\ \text{register ton} = \ 20\ \text{barrels bulk}\ \times\left(\frac{4.0155\ \ .\ .\ \text{bushels}}{1\ \text{barrel bulk}}\right) = 80.31\ \ .\ .\ \text{bushels}$$

ANALYZE a The difference between 73 “freight” tons and 73 “displacement” tons is

$$\Delta V = 73 \text{ freight tons} - \text{displacement tons} = 73\ 32.124 \quad . \ . \text{ bushels} - 28.108 \quad . \ . \text{ bushels}$$
$$= 293.168 \quad . \ . \text{ bushels} \approx 293 \quad . \ . \text{ bushels}$$

(b) Similarly, the difference between 73 “register” tons and 73 “displacement” tons is

$$\Delta V = 73 \text{ register tons} - \text{displacement tons} = 73\ 80.31 \quad . \ . \text{ bushels} - 28.108 \quad . \ . \text{ bushels}$$
$$= 3810.746 \quad . \ . \text{ bushels} \approx 3.81 \times 10^3 \quad . \ . \text{ bushels}$$

LEARN ith 1 register ton $>$ 1 freight ton $>$ 1 displacement ton e e pect the difference found in b to be greater than that in a . his is indeed the case.

34. he customer e pects a volume $V_1 = 20 \times 7056$ in^3 and receives $V_2 = 20 \times 5826$ in.^{3} the difference being $\Delta V = V_1 - V_2 = 24600$ in.^{3} or

$$\Delta V = \left(24600 \text{ in.}^3\right)\left(\frac{2.54\,\text{cm}}{1 \text{ inch}}\right)^3\left(\frac{1}{1000 \text{ cm}^3}\right) = 403$$

here ppendi has been used.

35. he first t o conversions are easy enough that a *formal* conversion is not especially called for but in the interest of *practice makes perfect* e go ahead and proceed formally

a $11 \text{ tuffets} = \left(11 \text{ tuffets}\right)\left(\frac{2 \text{ peck}}{1 \text{ tuffet}}\right) = 22 \text{ pecks}$.

b $11 \text{ tuffets} = \left(11 \text{ tuffets}\right)\left(\frac{0.50 \quad \text{mperial bushel}}{1 \text{ tuffet}}\right) = 5.5 \quad \text{mperial bushels}$.

c $11 \text{ tuffets} = \left(5.5 \quad \text{mperial bushel}\right)\left(\frac{36.3687}{1 \quad \text{mperial bushel}}\right) \approx 200$.

36. able 7 can be completed as follo s

a It should be clear that the first column (under “wey”) is the reciprocal of the first ro – so that $\frac{9}{10} = 0.900$ $\frac{3}{40} = 7.50 \times 10^{-2}$ and so forth. hus 1 pottle = 1.56×10^{-3} ey and 1 gill = 8.32×10^{-6} ey are the last t o entries in the first column.

b In the second column (under “chaldron”), clearly we have 1 chaldron = 1 chaldron (that is, the entries along the “diagonal” in the table must be 1’s). To find out how many chaldron are e ual to one bag e note that 1 ey = 10 9 chaldron = 40 3 bag so that $\frac{1}{12}$ chaldron = 1 bag. hus the ne t entry in that second column is $\frac{1}{12}$ = 8.33 $\times$ 10^{-2}. imilarly 1 pottle = 1.74 $\times$ 10^{-3} chaldron and 1 gill = 9.24 $\times$ 10^{-6} chaldron.

c n the third column under "bag" e have 1 chaldron = 12.0 bag 1 bag = 1 bag 1 pottle = 2.08 $\times$ 10^{-2} bag and 1 gill = 1.11 $\times$ 10^{-4} bag.

d n the fourth column under "pottle" e find 1 chaldron = 576 pottle 1 bag = 48 pottle 1 pottle = 1 pottle and 1 gill = 5.32 $\times$ 10^{-3} pottle.

e n the last column under "gill" e obtain 1 chaldron = 1.08 $\times$ 10^{5} gill 1 bag = 9.02 $\times$ 10^{3} gill 1 pottle = 188 gill and of course 1 gill = 1 gill.

f sing the information from part c 1.5 chaldron = 1.5 12.0 = 18.0 bag. nd since each bag is 0.1091 m^3 e conclude 1.5 chaldron = 18.0 0.1091 = 1.96 m^3.

37. he volume of one unit is 1 cm^3 = 1 $\times$ 10^{-6} m^3 so the volume of a mole of them is 6.02 $\times$ 10^{23} cm^3 = 6.02 $\times$ 10^{17} m^3. he cube root of this number gives the edge length 8.4×10^5 m^3. his is e uivalent to roughly 8 $\times$ 10^2 km.

38. a sing the fact that the area A of a rectangle is idth $\times$ (length e find

$$\begin{aligned} A_{\text{total}} &= (3.00\,\text{acre}) + (25.0\,\text{perch})(4.00\,\text{perch}) \\ &= (3.00\text{ acre})\left(\frac{(40\text{ perch})(4\text{ perch})}{1\text{acre}}\right) + 100\,\text{perch}^2 \\ &= 580\text{ perch}^2. \end{aligned}$$

e multiply this by the perch^2 $\rightarrow$ rood conversion factor 1 rood 40 perch^2 to obtain the ans er A_{total} = 14.5 roods.

b e convert our intermediate result in part a

$$A_{\text{total}} = \left(580\text{ perch}^2\right)\left(\frac{16.5\,\text{ft}}{1\text{perch}}\right)^2 = 1.58\times 10^5\ \text{ft}^2.$$

o e use the feet $\rightarrow$ meters conversion given in ppendi to obtain

$$A_{\text{total}} = \left(1.58\times 10^5\text{ ft}^2\right)\left(\frac{1\text{m}}{3.281\text{ft}}\right)^2 = 1.47\times 10^4\ \text{m}^2.$$

39. **THINK** his problem compares the . . gallon ith . . gallon t o non units for volume. he interpretation of the type of gallons hether . . or . . affects the amount of gasoline one calculates for traveling a given distance.

EXPRESS f the fuel consumption rate is R in miles gallon then the amount of gasoline in gallons needed for a trip of distance d in miles ould be

$$V \text{ gallon} = \frac{d \text{ miles}}{R \text{ miles gallon}}$$

ince the car as manufactured in . . the fuel consumption rate is calibrated based on U.K. gallon, and the correct interpretation should be “40 miles per U.K. gallon.” In U.K., one would think of gallon as U.K. gallon; however, in the U.S., the word “gallon” would naturally be interpreted as . . gallon. ote also that since 1 . . gallon = 4.5460900 and 1 . . gallon = 3.7854118 the relationship bet een the t o is

$$1 \text{ . . gallon} = 4.5460900 \left(\frac{1 \text{ . . gallon}}{3.7854118} \right) = 1.20095 \text{ . . gallons}$$

ANALYZE a he amount of gasoline actually re uired is

$$V' = \frac{750 \text{ miles}}{40 \text{ miles . . gallon}} = 18.75 \text{ . . gallons} \approx 18.8 \text{ . . gallons}$$

his means that the driver mistakenly believes that the car should need 18.8 . . gallons.

b sing the conversion factor found above this is e uivalent to

$$V' = (18.75 \text{ . . gallons}) \times \left(\frac{1.20095 \text{ . . gallons}}{1 \text{ . . gallon}} \right) \approx 22.5 \text{ . . gallons}$$

LEARN ne . . gallon is greater than one . gallon by roughly a factor of 1.2 in volume. herefore 40 mi . . gallon is less fuel efficient than 40 mi . . gallon.

40. uation 1 9 gives to very high precision the conversion from atomic mass units to kilograms. ince this problem deals ith the ratio of total mass 1.0 kg divided by the mass of one atom 1.0 u but converted to kilograms then the computation reduces to simply taking the reciprocal of the number given in . 1 9 and rounding off appropriately. hus the ans er is 6.0×10^{26}.

41. **THINK** his problem involves converting *cord* a non unit for volume to unit.

EXPRESS sing the e act conversion 1 in. = 2.54 cm = 0.0254 m for length e have

$$1\ \text{ft} = 12\ \text{in} = \ 12\ \text{in.}\ \times\left(\frac{0.0254\ \text{m}}{1\,\text{in}}\right) = 0.3048\ \text{m}\,.$$

hus $1\ \text{ft}^3 = \ 0.3048\ \text{m}^{\ 3} = 0.0283\ \text{m}^3$ for volume these results also can be found in ppendi .

ANALYZE he volume of a cord of ood is $V = \ 8\ \text{ft}\ \times\ 4\ \text{ft}\ \times\ 4\ \text{ft}\ = 128\ \text{ft}^3$. sing the conversion factor found above e obtain

$$V = 1\,\text{cord} = 128\ \text{ft}^3 = \ 128\ \text{ft}^3\ \times\left(\frac{0.0283\ \text{m}^3}{1\ \text{ft}^3}\right) = 3.625\ \text{m}^3$$

hich implies that $1\ \text{m}^3 = \left(\dfrac{1}{3.625}\right)\text{cord} = 0.276\ \text{cord} \approx 0.3\ \text{cord}\,.$

LEARN he un anted units ft^3 all cancel out as they should. n conversions units obey the same algebraic rules as variables and numbers.

42. a n atomic mass units the mass of one molecule is 16 1 1 u = 18 u. sing . 1 9 e find

$$18\text{u} = \ (18\text{u})\left(\frac{1.6605402 \times 10^{-27}\ \text{kg}}{1\text{u}}\right) \ = 3.0 \times 10^{-26}\ \text{kg}.$$

b e divide the total mass by the mass of each molecule and obtain the appro imate number of ater molecules

$$N \approx \frac{1.4 \times 10^{21}}{3.0 \times 10^{-26}} \ \approx\ 5 \times 10^{46}.$$

43. million milligrams comprise a kilogram so 2.3 kg eek is 2.3×10^6 mg eek. iguring 7 days a eek 24 hours per day 3600 second per hour e find 604800 seconds are e uivalent to one eek. hus 2.3×10^6 mg eek 604800 s eek = 3.8 mg s.

44. he volume of the ater that fell is

$$\begin{aligned} V &= \left(26\ \text{km}^2\right)\left(2.0\ \text{in.}\right) = \left(26\ \text{km}^2\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)^2 \left(2.0\ \text{in.}\right)\left(\frac{0.0254\ \text{m}}{1\ \text{in.}}\right) \\ &= \left(26 \times 10^6\ \ \text{m}^2\right)\left(0.0508\ \text{m}\right) \\ &= 1.3 \times 10^6\ \ \text{m}^3. \end{aligned}$$

e rite the mass per unit volume density of the ater as $\rho = \dfrac{m}{V} = 1 \times 10^3\ \ \text{kg/m}^3$.

he mass of the ater that fell is therefore given by $m = \rho V$

$$m = \left(1 \times 10^3\ \ \text{kg/m}^3\right)\left(1.3 \times 10^6\ \ \text{m}^3\right) = 1.3 \times 10^9\ \ \text{kg}.$$

45. he number of seconds in a year is 3.156×10^7. his is listed in ppendi and results from the product

$$365.25 \text{ day y} \quad 24 \text{ h day} \quad 60 \text{ min h} \quad 60 \text{ s min} .$$

a he number of shakes in a second is 10^8 therefore there are indeed more shakes per second than there are seconds per year.

b enoting the age of the universe as 1 u day or 86400 u sec then the time during hich humans have e isted is given by

$$\frac{10^6}{10^{10}} = 10^{-4} \text{ u day}$$

hich may also be e pressed as $\left(10^{-4} \text{ u day}\right)\left(\frac{86400 \text{ u sec}}{1 \text{ u day}}\right) = 8.6 \text{ u sec}.$

46. he volume removed in one year is $V = 75 \times 10^4 \text{ m}^2 \quad 26 \text{ m} \approx 2 \times 10^7 \text{ m}^3$ hich e convert to cubic kilometers $V = \left(2 \times 10^7 \text{ m}^3\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)^3 = 0.020 \text{ km}^3.$

47. **THINK** his problem involves e pressing the speed of light in astronomical units per minute.

EXPRESS e first convert meters to astronomical units and seconds to minutes using

$$1000 \text{ m} = 1 \text{ km} \quad 1 \quad = 1.50 \times 10^8 \text{ km} \quad 60 \text{ s} = 1 \text{ min}.$$

ANALYZE sing the conversion factors above the speed of light can be re ritten as

$$c = 3.0 \times 10^8 \text{ m s} = \left(\frac{3.0 \times 10^8 \text{ m}}{\text{s}}\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)\left(\frac{}{1.50 \times 10^8 \text{ km}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 0.12 \quad /\text{min}.$$

LEARN hen e pressed the speed of light c in min e readily see that it takes about 8.3 = 1 0.12 minutes for sunlight to reach the arth i.e. to travel a distance of 1 .

48. ince one atomic mass unit is $1 \text{u} = 1.66 \times 10^{-24}$ g see ppendi the mass of one mole of atoms is about $m = 1.66 \times 10^{-24} \text{ g} \quad 6.02 \times 10^{23} = 1 \text{g}$. n the other hand the mass of one mole of atoms in the common astern mole is

$$m' = \frac{75\text{ g}}{7.5} = 10\text{ g}$$

herefore in atomic mass units the average mass of one atom in the common astern mole is

$$\frac{m'}{N_A} = \frac{10\text{ g}}{6.02\times10^{23}} = 1.66\times10^{-23}\text{ g} = 10\text{ u.}$$

49. a uaring the relation 1 ken = 1.97 m and setting up the ratio e obtain

$$\frac{1\text{ ken}^2}{1\text{ m}^2} = \frac{1.97^2\text{ m}^2}{1\text{ m}^2} = 3.88.$$

b imilarly e find

$$\frac{1\text{ ken}^3}{1\text{ m}^3} = \frac{1.97^3\text{ m}^3}{1\text{ m}^3} = 7.65.$$

c he volume of a cylinder is the circular area of its base multiplied by its height. hus

$$\pi r^2 h = \pi (3.00)^2 (5.50) = 156\text{ ken}^3.$$

d f e multiply this by the result of part b e determine the volume in cubic meters 155.5 7.65 = 1.19×10^3 m^3.

50. ccording to ppendi a nautical mile is 1.852 km so 24.5 nautical miles ould be 45.374 km. lso according to ppendi a mile is 1.609 km so 24.5 miles is 39.4205 km. he difference is 5.95 km.

51. a or the minimum 43 cm case 9 cubits converts as follo s

$$9\text{cubits} = (9\text{cubits})\left(\frac{0.43\text{m}}{1\text{cubit}}\right) = 3.9\text{m}.$$

nd for the ma imum 53 cm case e have $9\text{cubits} = (9\text{cubits})\left(\frac{0.53\text{m}}{1\text{cubit}}\right) = 4.8\text{m}.$

b imilarly ith 0.43 m $\rightarrow$ 430 mm and 0.53 m $\rightarrow$ 530 mm e find 3.9×10^3 mm and 4.8×10^3 mm respectively.

c e can convert length and diameter first and then compute the volume or first compute the volume and then convert. e proceed using the latter approach here d is diameter and ℓ is length .

$$V_{\text{cylinder min}} = \frac{\pi}{4}\ell d^2 = 28\text{ cubit}^3 = \left(28\text{ cubit}^3\right)\left(\frac{0.43\,\text{m}}{1\text{ cubit}}\right)^3 = 2.2\text{ m}^3.$$

imilarly ith 0.43 m replaced by 0.53 m e obtain $V_{\text{cylinder ma}} = 4.2\text{ m}^3$.

52. Abbreviating wapentake as "wp" and assuming a hide to be 110 acres, we set up the ratio 25 p 11 barn along ith appropriate conversion factors

$$\frac{(25\quad\text{p})\left(\frac{100\text{ hide}}{1\quad\text{p}}\right)\left(\frac{110\text{ acre}}{1\text{ hide}}\right)\left(\frac{4047\text{ m}^2}{1\text{ acre}}\right)}{(11\text{ barn})\left(\frac{1\times10^{-28}\text{ m}^2}{1\text{ barn}}\right)} \approx 1\times10^{36}.$$

53. **THINK** he ob ective of this problem is to convert the arth un distance 1 to parsecs and light years.

EXPRESS o relate parsec pc to e note that hen θ is measured in radians it is e ual to the arc length s divided by the radius R. or a very large radius circle and small value of θ the arc may be appro imated as the straight line segment of length 1 . hus

$$\theta = 1\text{ arcsec} = (1\text{ arcsec})\left(\frac{1\text{ arcmin}}{60\text{ arcsec}}\right)\left(\frac{1^\circ}{60\text{ arcmin}}\right)\left(\frac{2\pi\text{ radian}}{360^\circ}\right) = 4.85\times10^{-6}\text{ rad}.$$

herefore one parsec is

$$1\text{ pc} = \frac{s}{\theta} = \frac{1}{4.85\times10^{-6}} = 2.06\times10^5 \quad .$$

e t e relate to light year ly . ince a year is about 3.16×10^7 s

$$1\,\text{ly} = (186\;000\,\text{mi/s})\left(3.16\times10^7\text{ s}\right) = 5.9\times10^{12}\text{ mi}.$$

ANALYZE a ince $1\text{ pc} = 2.06\times10^5$ inverting the relation gives

$$1 \quad = (1 \quad)\left(\frac{1\text{ pc}}{2.06\times10^5}\right) = 4.9\times10^{-6}\text{ pc}.$$

b iven that $1 \quad = 92.9\times10^6$ mi and $1\text{ ly} = 5.9\times10^{12}$ mi the t o e pressions together lead to

$$1 \quad = 92.9\times10^6\text{ mi} = (92.9\times10^6\text{ mi})\left(\frac{1\text{ ly}}{5.9\times10^{12}\text{ mi}}\right) = 1.57\times10^{-5}\text{ ly}.$$

LEARN ur results can be further combined to give $1\,\text{pc} = 3.2$ ly. rom the above e pression e readily see that it takes 1.57×10^{-5} y or about 8.3 min for unlight to travel a distance of 1 to reach the arth.

54. a sing ppendi e have 1 ft = 0.3048 m 1 gal = 231 in.^{3} and 1 in.^{3} = 1.639 × 10^{-2} . rom the latter t o items e find that 1 gal = 3.79 . hus the uantity 460 ft^2 gal becomes

$$460 \text{ ft}^2 \text{ gal} = \left(\frac{460 \text{ ft}^2}{\text{gal}}\right)\left(\frac{1 \text{ m}}{3.28 \text{ ft}}\right)^2\left(\frac{1 \text{ gal}}{3.79}\right) = 11.3 \text{ m}^2/ \quad .$$

b lso since 1 m^3 is e uivalent to 1000 our result from part a becomes

$$11.3 \text{ m}^2 \quad = \left(\frac{11.3 \text{ m}^2}{\quad}\right)\left(\frac{1000}{1 \text{ m}^3}\right) = 1.13 \times 10^4 \text{ m}^{-1}.$$

c he inverse of the original uantity is 460 ft^2 gal $^{-1}$ = 2.17 × 10^{-3} gal ft^2.

d he ans er in c represents the volume of the paint in gallons needed to cover a s uare foot of area. rom this e could also figure the paint thickness it turns out to be about a tenth of a millimeter as one sees by taking the reciprocal of the ans er in part b .

55. a he receptacle is a volume of 40 cm 40 cm 30 cm = 48000 cm^3 = 48 = 48 16 11.356 = 67.63 standard bottles hich is a little more than 3 nebuchadne ars the largest bottle indicated . he remainder 7.63 standard bottles is ust a little less than 1 methuselah. hus the ans er to part a is 3 nebuchadne ars and 1 methuselah.

b ince 1 methuselah.= 8 standard bottles then the e tra amount is 8 − 7.63 = 0.37 standard bottle.

c sing the conversion factor 16 standard bottles = 11.356 e have

$$0.37 \text{ standard bottle} = \quad 0.37 \text{ standard bottle} \left(\frac{11.356}{16 \text{ standard bottles}}\right) = 0.26 \quad .$$

56. he mass of the pig is 3.108 slugs or 3.108 14.59 = 45.346 kg. eferring no to the corn a . . bushel is 35.238 liters. hus a value of 1 for the *corn-hog ratio* ould be e uivalent to 35.238 45.346 = 0.7766 in the indicated metric units. herefore a value of 5.7 for the *ratio* corresponds to 5.7 0.777 ≈ 4.4 in the indicated metric units.

57. o alape o peppers have spiciness = 8000 and this amount multiplied by 400 the number of people is 3.2 ×10^6 hich is roughly ten times the value for a

single habanero pepper. ore precisely 10.7 habanero peppers ill provide that total re uired value.

58. n the simplest approach e set up a ratio for the total increase in *horizontal depth x* here Δx = 0.05 m is the increase in hori ontal depth per step

$$x = N_{\text{steps}}\,\Delta x = \left(\frac{4.57}{0.19}\right)(0.05\ \text{m}) = 1.2\ \text{m}.$$

o ever e can approach this more carefully by noting that if there are N = 4.57 .19 ≈ 24 rises then under normal circumstances e ould e pect N – 1 = 23 runs hori ontal pieces in that staircase. his ould yield 23 0.05 m = 1.15 m hich to t o significant figures agrees ith our first result.

59. he volume of the filled container is 24000 cm^3 = 24 liters hich using the conversion given in the problem is e uivalent to 50.7 pints . . he e pected number is therefore in the range from 1317 to 1927 tlantic oysters. nstead the number received is in the range from 406 to 609 acific oysters. his represents a shortage in the range of roughly 700 to 1500 oysters the ans er to the problem . ote that the minimum value in our ans er corresponds to the minimum tlantic minus the ma imum acific and the ma imum value corresponds to the ma imum tlantic minus the minimum acific.

60. a e reduce the stock amount to ritish teaspoons

$$\begin{aligned}
1\ \text{breakfastcup} &= 2 \times 8 \times 2 \times 2 = 64\ \text{teaspoons} \\
1\ \text{teacup} &= 8 \times 2 \times 2 = 32\ \text{teaspoons} \\
6\ \text{tablespoons} &= 6 \times 2 \times 2 = 24\ \text{teaspoons} \\
1\ \text{dessertspoon} &= 2\ \text{teaspoons}
\end{aligned}$$

hich totals to 122 ritish teaspoons or 122 . . teaspoons since li uid measure is being used. o ith one . cup e ual to 48 teaspoons upon dividing 122 48 ≈ 2.54 e find this amount corresponds to 2.5 . . cups plus a remainder of precisely 2 teaspoons. n other ords

122 . . teaspoons = 2.5 . . cups 2 . . teaspoons.

b or the nettle tops one half uart is still one half uart.

c or the rice one ritish tablespoon is 4 ritish teaspoons hich since dry goods measure is being used corresponds to 2 . . teaspoons.

d ritish saltspoon is $\frac{1}{2}$ ritish teaspoon hich corresponds since dry goods measure is again being used to 1 . . teaspoon.

Chapter

1. he speed assumed constant is v = 90 km h 1000 m km ⁄ 3600 s h = 25 m s. hus in 0.50 s the car travels a distance $d = vt$ = 25 m s 0.50 s ≈ 13 m.

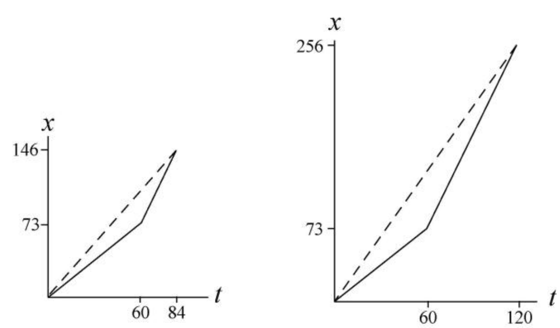
2. a sing the fact that time = distance velocity hile the velocity is constant e find

$$v_{\text{avg}} = \frac{73.2\text{ m} + 73.2\text{ m}}{\frac{73.2\text{ m}}{1.22\text{ m s}} + \frac{73.2\text{ m}}{3.05\text{ m}}} = 1.74\text{ m s.}$$

b sing the fact that distance = vt hile the velocity v is constant e find

$$v_{\text{avg}} = \frac{1.22\text{ m s } 60\text{ s } + 3.05\text{ m s } 60\text{ s}}{120\text{ s}} = 2.14\text{ m s.}$$

c he graphs are sho n belo ith meters and seconds understood . he first consists of t o solid line segments the first having a slope of 1.22 and the second having a slope of 3.05. he slope of the dashed line represents the average velocity in both graphs . he second graph also consists of t o solid line segments having the same slopes as before — the main difference compared to the first graph being that the stage involving higher speed motion lasts much longer.



3. **THINK** his one dimensional kinematics problem consists of t o parts and e are asked to solve for the average velocity and average speed of the car.

EXPRESS ince the trip consists of t o parts let the displacements during first and second parts of the motion be Δx_1 and Δx_2 and the corresponding time intervals be Δt_1 and Δt_2 respectively. o because the problem is one dimensional and both displacements are in the same direction the total displacement is simply $\Delta x = \Delta x_1$ Δx_2 and the total time for the trip is $\Delta t = \Delta t_1$ Δt_2. sing the definition of average velocity given in . 2 2 e have

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}.$$

o find the average speed e note that during a time Δt if the velocity remains a positive constant then the speed is e ual to the magnitude of velocity and the distance is e ual to the magnitude of displacement ith $d = \Delta x = v\Delta t$.

ANALYZE

a uring the first part of the motion the displacement is Δx_1 = 40 km and the time taken is

$$t_1 = \frac{40 \text{ km}}{30 \text{ km} \quad \text{h}} = 1.33 \text{ h}.$$

imilarly during the second part of the trip the displacement is Δx_2 = 40 km and the time interval is

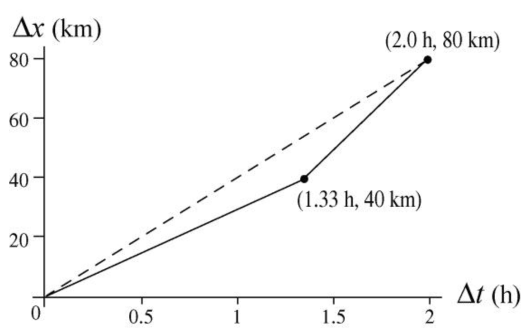
$$t_2 = \frac{40 \text{ km}}{60 \text{ km} \quad \text{h}} = 0.67 \text{ h}.$$

he total displacement is $\Delta x = \Delta x_1 \quad \Delta x_2$ = 40 km 40 km = 80 km and the total time elapsed is $\Delta t = \Delta t_1 \quad \Delta t_2$ = 2.00 h. onse uently the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{80 \text{ km}}{2.0 \text{ h}} = 40 \text{ km} \quad \text{h}.$$

b n this case the average speed is the same as the magnitude of the average velocity $s_{\text{avg}} = 40$ km h.

c he graph of the entire trip sho n belo consists of t o contiguous line segments the first having a slope of 30 km h and connecting the origin to $\Delta t_1 \ \Delta x_1$ = 1.33 h 40 km and the second having a slope of 60 km h and connecting $\Delta t_1 \ \Delta x_1$ to $\Delta t, \Delta x$ = 2.00 h 80 km .



rom the graphical point of vie the slope of the dashed line dra n from the origin to $\Delta t, \Delta x$ represents the average velocity.

LEARN he average velocity is a vector uantity that depends only on the net displacement also a vector bet een the starting and ending points.

4. verage speed as opposed to average velocity relates to the total distance as opposed to the net displacement. he distance D up the hill is of course the same as the distance do n the hill and since the speed is constant during each stage of the

motion e have speed = D t. hus the average speed is

$$\frac{D_{\text{up}} + D_{\text{do n}}}{t_{\text{up}} + t_{\text{do n}}} = \frac{2D}{\frac{D}{v_{\text{up}}} + \frac{D}{v_{\text{do n}}}}$$

hich after canceling D and plugging in v_{up} = 40 km h and $v_{\text{do n}}$ = 60 km h yields 48 km h for the average speed.

5. **THINK** n this one dimensional kinematics problem, we're given the position function x t and asked to calculate the position and velocity of the ob ect at a later time.

EXPRESS he position function is given as x t = 3 m s t – 4 m s^2 t^2 1 m s^3 t^3. he position of the ob ect at some instant t_0 is simply given by x t_0 . or the time interval $t_1 \le t \le t_2$ the displacement is $\Delta x = x\ t_2 - x\ t_1$. imilarly using . 2 2 the average velocity for this time interval is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x\ t_2 - x\ t_1}{t_2 - t_1}.$$

ANALYZE a lugging in t = 1 s into x t yields

$$x\ 1\text{ s} = 3\text{ m s } 1\text{ s} - 4\text{ m s}^2\ 1\text{ s}^2\ \ 1\text{ m s}^3\ 1\text{ s}^3 = 0.$$

b ith t = 2 s e get x 2 s = 3 m s 2 s – 4 m s^2 2 s ^{2} 1 m s^3 2 s ^{3} = –2 m.

c ith t = 3 s e have x 3 s = 3 m s 3 s – 4 m s^2 3 s ^{2} 1 m s^3 3 s ^{3} = 0 m.

d imilarly plugging in t = 4 s gives

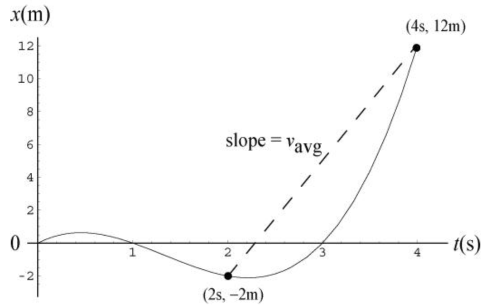
$$x\ 4\text{ s} = 3\text{ m s } 4\text{ s} - 4\text{ m s}^2\ 4\text{ s}^2\ \ 1\text{ m s}^3\ 4\text{ s}^3 = 12\text{ m}.$$

e he position at t = 0 is x = 0. hus the displacement bet een t = 0 and t = 4 s is $\Delta x = x\ 4\text{ s} - x\ 0 = 12\text{ m} - 0 = 12\text{ m}$.

f he position at t = 2 s is subtracted from the position at t = 4 s to give the displacement $\Delta x = x\ 4\text{ s} - x\ 2\text{ s} = 12\text{ m} - -2\text{ m} = 14\text{ m}$. hus the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14\text{ m}}{2\text{ s}} = 7\text{ m s}.$$

g he position of the ob ect for the interval $0 \le t \le 4$ is plotted belo . he straight line dra n from the point at t, x = 2 s –2 m to 4 s 12 m ould represent the average velocity ans er for part f .



LEARN ur graphical representation illustrates once again that the average velocity for a time interval depends only on the net displacement bet een the starting and ending points.

6. uber's speed is

$$v_0 = \ 200 \text{ m} \quad 6.509 \text{ s} \ = 30.72 \text{ m s} = 110.6 \text{ km h}$$

here e have used the conversion factor 1 m s = 3.6 km h. ince hittingham beat uber by 19.0 km h his speed is v_1 = 110.6 km h 19.0 km h = 129.6 km h or 36 m s 1 km h = 0.2778 m s . hus using . 2 2 the time through a distance of 200 m for hittingham is

$$\Delta t = \frac{\Delta x}{v_1} = \frac{200 \text{ m}}{36 \text{ m s}} = 5.554 \text{ s}.$$

7. ecogni ing that the gap bet een the trains is closing at a constant rate of 60 km h the total time that elapses before they crash is t = 60 km 60 km h = 1.0 h. uring this time the bird travels a distance of $x = vt$ = 60 km h 1.0 h = 60 km.

8. he amount of time it takes for each person to move a distance L ith speed v_s is $\Delta t = L \ \ v_s$. ith each additional person the depth increases by one body depth d

a he rate of increase of the layer of people is

$$R = \frac{d}{\Delta t} = \frac{d}{L \ \ v_s} = \frac{dv_s}{L} = \frac{0.25 \text{ m} \ \ 3.50 \text{ m s}}{1.75 \text{ m}} = 0.50 \text{ m s}$$

b he amount of time re uired to reach a depth of $D = 5.0$ m is

$$t = \frac{D}{R} = \frac{5.0 \text{ m}}{0.50 \text{ m s}} = 10 \text{ s}$$

9. onverting to seconds the running times are t_1 = 147.95 s and t_2 = 148.15 s respectively. f the runners ere e ually fast then

$$s_{\text{avg}_1} = s_{\text{avg}_2} \quad \Rightarrow \quad \frac{L_1}{t_1} = \frac{L_2}{t_2}.$$

rom this e obtain

$$L_2 - L_1 = \left(\frac{t_2}{t_1} - 1\right) L_1 = \left(\frac{148.15}{147.95} - 1\right) L_1 = 0.00135 L_1 \approx 1.4 \text{ m}$$

here e set $L_1 \approx 1000$ m in the last step. hus if L_1 and L_2 are no different than about 1.4 m then runner 1 is indeed faster than runner 2. o ever if L_1 is shorter than L_2 by more than 1.4 m then runner 2 ould actually be faster.

10. et v_w be the speed of the ind and v_c be the speed of the car.

a uppose during time interval t_1 the car moves in the same direction as the ind. hen the effective speed of the car is given by $v_{eff\,1} = v_c + v_w$ and the distance traveled is $d = v_{eff\,1} t_1 = \; v_c + v_w \; t_1$. n the other hand for the return trip during time interval t_2 the car moves in the opposite direction of the ind and the effective speed ould be $v_{eff\,2} = v_c - v_w$. he distance traveled is $d = v_{eff\,2} t_2 = \; v_c - v_w \; t_2$. he t o e pressions can be re ritten as

$$v_c + v_w = \frac{d}{t_1} \quad \text{and} \quad v_c - v_w = \frac{d}{t_2}$$

dding the t o e uations and dividing by t o e obtain $v_c = \frac{1}{2}\left(\frac{d}{t_1} + \frac{d}{t_2}\right)$. hus method 1 gives the car's speed v_c a in indless situation.

b f method 2 is used the result ould be

$$v_c' = \frac{d}{t_1 + t_2 \quad 2} = \frac{2d}{t_1 + t_2} = \frac{2d}{\dfrac{d}{v_c + v_w} + \dfrac{d}{v_c - v_w}} = \frac{v_c^2 - v_w^2}{v_c} = v_c \left[1 - \left(\frac{v_w}{v_c}\right)^2\right].$$

he fractional difference is

$$\frac{v_c - v_c'}{v_c} = \left(\frac{v_w}{v_c}\right)^2 = \; 0.0240 \; ^2 = 5.76 \times 10^{-4}.$$

11. he values used in the problem statement make it easy to see that the first part of the trip at 100 km h takes 1 hour and the second part at 40 km h also takes 1 hour. pressed in decimal form the time left is 1.25 hour and the distance that remains is 160 km. hus a speed v = 160 km 1.25 h = 128 km h is needed.

12. a et the fast and the slo cars be separated by a distance d at t = 0. f during the time interval $t = L \;\; v_s = $ 12.0 m 5.0 m s = 2.40 s in hich the slo car has moved a distance of $L = 12.0$ m the fast car moves a distance of $vt = d + L$ to oin the line of slo cars then the shock ave ould remain stationary. he condition implies a separation of

$$d = vt - L = \; 25 \text{ m s} \;\; 2.4 \text{ s} \; - 12.0 \text{ m} = 48.0 \text{ m}.$$

b et the initial separation at $t = 0$ be $d = 96.0$ m. t a later time t the slo and the fast cars have traveled $x = v_s t$ and the fast car oins the line by moving a distance $d + x$. rom

$$t = \frac{x}{v_s} = \frac{d+x}{v}$$

e get

$$x = \frac{v_s}{v - v_s} d = \frac{5.00 \text{ m s}}{25.0 \text{ m s} - 5.00 \text{ m s}} \ 96.0 \text{ m} \ = 24.0 \text{ m}$$

hich in turn gives $t = \ 24.0 \text{ m} \quad 5.00 \text{ m s} \ = 4.80 \text{ s}$. ince the rear of the slo car pack has moved a distance of $\Delta x = x - L = 24.0 \text{ m} - 12.0 \text{ m} = 12.0 \text{ m}$ do nstream the speed of the rear of the slo car pack or e uivalently the speed of the shock ave is

$$v_{\text{shock}} = \frac{\Delta x}{t} = \frac{12.0 \text{ m}}{4.80 \text{ s}} = 2.50 \text{ m s}.$$

c ince $x > L$ the direction of the shock ave is do nstream.

13. a enoting the travel time and distance from an ntonio to ouston as T and D respectively the average speed is

$$s_{\text{avg1}} = \frac{D}{T} = \frac{55 \text{ km h} \ \ T \ 2 \ + \ 90 \text{ km h} \ \ T \ \ 2}{T} = 72.5 \text{ km h}$$

hich should be rounded to 73 km h.

b sing the fact that time = distance speed hile the speed is constant e find

$$s_{\text{avg2}} = \frac{D}{T} = \frac{D}{\frac{D \ \ 2}{55 \text{ km h}} + \frac{D \ \ 2}{90 \text{ km h}}} = 68.3 \text{ km h}$$

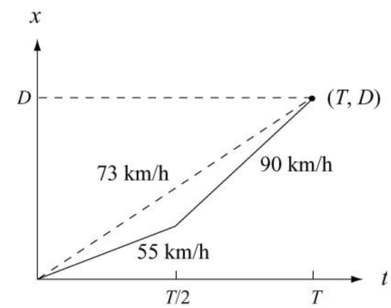
hich should be rounded to 68 km h.

c he total distance traveled $2D$ must not be confused ith the net displacement ero . e obtain for the t o ay trip

$$s_{\text{avg}} = \frac{2D}{\frac{D}{72.5 \text{ km h}} + \frac{D}{68.3 \text{ km h}}} = 70 \text{ km h}.$$

d ince the net displacement vanishes the average velocity for the trip in its entirety is ero.

e n asking for a *sketch* the problem is allo ing the student to arbitrarily set the distance D the intent is *not* to make the student go to an atlas to look it up the student can ust as easily arbitrarily set T instead of D as ill be clear in the follo ing discussion. e briefly describe the graph ith kilometers per hour understood for the slopes t o contiguous line segments the first having a slope of 55 and connecting the origin to $t_1 \ x_1 \ = \ T \ 2 \ \ 55T \ 2$ and the second having a slope of 90 and connecting $t_1 \ x_1$ to T, D here $D = \ 55 \quad 90 \ T \ 2$. he average velocity from the graphical point of vie is the slope of a line dra n from the origin to T, D . he graph not dra n to scale is depicted belo



14. sing the general property $\frac{d}{dx}$e p $bx = b$e p bx e rite

$$v = \frac{dx}{dt} = \left(\frac{d\ 19t}{dt}\right) \cdot e^{-t} + \ 19t \ \cdot \left(\frac{de^{-t}}{dt}\right) \ .$$

f a concern develops about the appearance of an argument of the e ponential $-t$ apparently having units then an e plicit factor of 1 T here T = 1 second can be inserted and carried through the computation hich does not change our ans er . he result of this differentiation is

$$v = 16\ 1 - t\ e^{-t}$$

ith t and v in units s and m s respectively . e see that this function is ero hen t = 1 s. o that e kno *when* it stops e find out *where* it stops by plugging our result t = 1 into the given function x = 16te^{-t} ith x in meters. herefore e find x = 5.9 m.

15. e use . 2 4 to solve the problem.

a he velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt}\ \ 4 - 12t + 3t^2\ \ = -12 + 6t.$$

hus at t = 1 s the velocity is v = –12 6 1 = –6 m s.

b ince $v < 0$ it is moving in the $-x$ direction at t = 1 s.

c t t = 1 s the *speed* is v = 6 m s.

d or 0 < t < 2 s v decreases until it vanishes. or 2 < t < 3 s v increases from ero to the value it had in part c . hen v is larger than that value for t > 3 s.

e es since v smoothly changes from negative values consider the t = 1 result to positive note that as $t \to \ \infty$ e have $v \to \ \infty$. ne can check that v = 0 hen $t = 2$ s.

f o. n fact from v = –12 6t e kno that $v > 0$ for $t > 2$ s.

16. e use the functional notation x t v t and a t in this solution here the latter t o uantities are obtained by differentiation

$$v(t) = \frac{dx(t)}{dt} = -12t \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -12$$

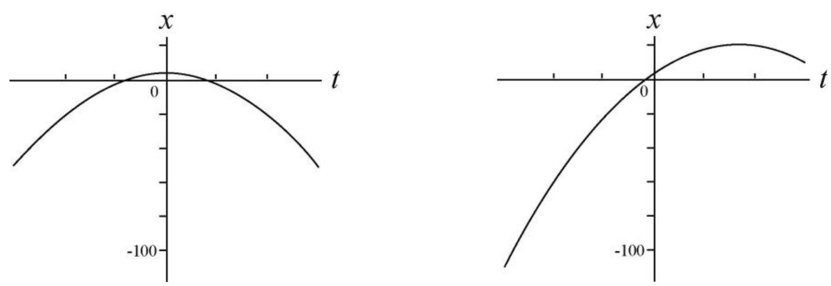
ith units understood.

a rom v t = 0 e find it is momentarily at rest at t = 0.

b e obtain x 0 = 4.0 m.

c and d e uiring x t = 0 in the e pression x t = 4.0 – 6.0t^2 leads to t = ±0.82 s for the times hen the particle can be found passing through the origin.

e e sho both the asked for graph on the left) as well as the "shifted" graph that is relevant to part f . n both cases the time a is is given by –3 ≤ t ≤ 3 units understood .



f e arrived at the graph on the right sho n above by adding 20t to the x t e pression.

g amining here the slopes of the graphs become ero it is clear that the shift causes the v = 0 point to correspond to a larger value of x the top of the second curve sho n in part e is higher than that of the first .

17. e use . 2 2 for average velocity and . 2 4 for instantaneous velocity and ork ith distances in centimeters and times in seconds.

a e plug into the given e uation for x for t = 2.00 s and t = 3.00 s and obtain x_2 = 21.75 cm and x_3 = 50.25 cm respectively. he average velocity during the time interval 2.00 ≤ t ≤ 3.00 s is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{50.25\text{ cm} - 21.75\text{ cm}}{3.00\text{ s} - 2.00\text{ s}}$$

hich yields v_{avg} = 28.5 cm s.

b he instantaneous velocity is $v = \frac{dx}{dt} = 4.5t^2$ hich at time t = 2.00 s yields v = 4.5 2.00 ^{2} = 18.0 cm s.

c t t = 3.00 s the instantaneous velocity is v = 4.5 3.00 ^{2} = 40.5 cm s.

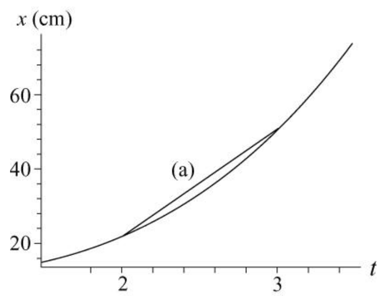
d t t = 2.50 s the instantaneous velocity is v = 4.5 2.50 ^{2} = 28.1 cm s.

e et t_m stand for the moment hen the particle is mid ay bet een x_2 and x_3 that is hen the particle is at x_m = x_2 x_3 2 = 36 cm . herefore

$$x_m = 9.75 + 1.5 t_m^3 \quad \Rightarrow \quad t_m = 2.596$$

in seconds. hus the instantaneous speed at this time is v = 4.5 2.596 ^{2} = 30.3 cm s.

f he ans er to part a is given by the slope of the straight line bet een t = 2 and t = 3 in this x vs t plot. he ans ers to parts b c d and e correspond to the slopes of tangent lines not sho n but easily imagined to the curve at the appropriate points.



18. a aking derivatives of x t = 12t^2 – 2t^3 e obtain the velocity and the acceleration functions

$$v\ t\ = 24t - 6t^2 \quad \text{and} \quad a\ t\ = 24 - 12t$$

ith length in meters and time in seconds. lugging in the value t = 3 yields x 3 = 54 m.

b imilarly plugging in the value t = 3 yields v 3 = 18 m s.

c or t = 3 a 3 = –12 m s^2.

d t the ma imum x e must have v = 0 eliminating the t = 0 root the velocity e uation reveals t = 24 6 = 4 s for the time of ma imum x. lugging t = 4 into the e uation for x leads to x = 64 m for the largest x value reached by the particle.

e rom d e see that the x reaches its ma imum at t = 4.0 s.

f ma imum v re uires a = 0 hich occurs hen t = 24 12 = 2.0 s. his inserted into the velocity e uation gives v_{ma} = 24 m s.

g rom f e see that the ma imum of v occurs at t = 24 12 = 2.0 s.

h n part e the particle as momentarily motionless at t = 4 s. he acceleration at that time is readily found to be 24 – 12 4 = –24 m s^2.

i he *average velocity* is defined by . 2 2 so e see that the values of x at t = 0 and t = 3 s are needed these are respectively x = 0 and x = 54 m found in part a . hus

$$v_{\text{avg}} = \frac{54-0}{3-0} = 18 \text{ m s}.$$

19. **THINK** n this one dimensional kinematics problem, we're given the speed of a particle at t o instants and asked to calculate its average acceleration.

EXPRESS e represent the initial direction of motion as the x direction. he average acceleration over a time interval $t_1 \le t \le t_2$ is given by . 2 7

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v\ t_2\ - v\ t_1}{t_2 - t_1}.$$

ANALYZE et v_1 = 18 m s at $t_1 = 0$ and v_2 = –30 m s at t_2 = 2.4 s. sing . 2 7 e find

$$a_{\text{avg}} = \frac{v\ t_2\ - v\ t_1}{t_2 - t_1} = \frac{-30 \text{ m s}\ -\ +1 \text{ m s}}{2.4 \text{ s} - 0} = -20 \text{ m s}^2.$$

LEARN he average acceleration has magnitude 20 m s^2 and is in the opposite direction to the particle's initial velocity. This makes sense because the velocity of the particle is decreasing over the time interval. ith $t_1 = 0$ the velocity of the particle as a function of time can be ritten as

$$v = v_0 + at = 18 \text{ m s} - 20 \text{ m s}^2\ t.$$

20. e use the functional notation $x\ t$ $v\ t$ and $a\ t$ and find the latter t o uantities by differentiating

$$v(t) = \frac{dx(t)}{t} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

ith units understood. hese e pressions are used in the parts that follo .

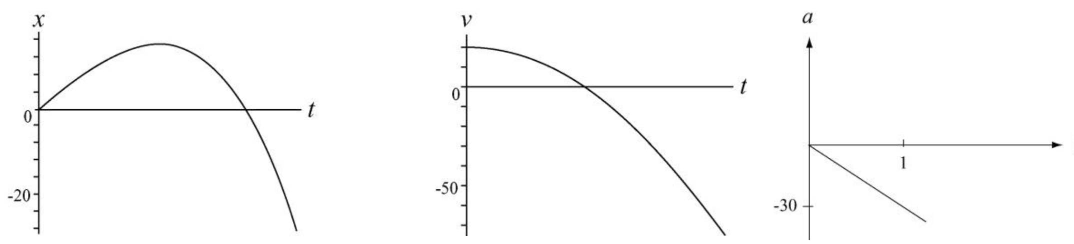
a rom $0 = -15t^2 + 20$ e see that the only positive value of t for hich the particle is momentarily stopped is $t = \sqrt{20\ \ 15} = 1.2$ s.

b rom 0 = – 30t e find a 0 = 0 that is it vanishes at t = 0 .

c t is clear that $a\ t$ = – 30t is negative for t 0.

d he acceleration $a\ t$ = – 30t is positive for t 0.

e he graphs are sho n belo . units are understood.



21. e use . 2 2 average velocity and . 2 7 average acceleration . egarding our coordinate choices the initial position of the man is taken as the origin and his direction of motion during 5 min $\le t \le$ 10 min is taken to be the positive x direction. e also use the fact that $\Delta x = v\Delta t$ hen the velocity is constant during a time interval Δt .

a he entire interval considered is Δt = 8 – 2 = 6 min hich is e uivalent to 360 s hereas the sub interval in hich he is *moving* is only $\Delta t' = 8 - 5 = 3\text{min} = 180$ s. is position at t = 2 min is x = 0 and his position at t = 8 min is $x = v\Delta t' =$ 2.2 180 $= 396$ m . herefore

$$v_{\text{avg}} = \frac{396\text{ m} - 0}{360\text{ s}} = 1.10\text{ m s}.$$

b he man is at rest at t = 2 min and has velocity v = 2.2 m s at t = 8 min. hus keeping the ans er to 3 significant figures

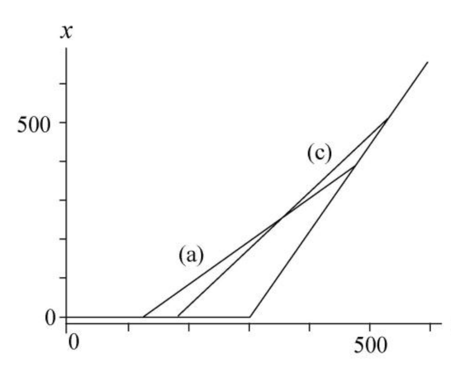
$$a_{\text{avg}} = \frac{2.2\text{ m s} - 0}{360\text{ s}} = 0.00611\text{ m s}^2 .$$

c o the entire interval considered is Δt = 9 – 3 = 6 min 360 s again hereas the sub interval in hich he is moving is $\Delta t' = 9 - 5 = 4\text{min} = 240$ s . is position at $t = 3$ min is x = 0 and his position at t = 9 min is $x = v\Delta t' =$ 2.2 240 $= 528$ m . herefore

$$v_{\text{avg}} = \frac{528\text{ m} - 0}{360\text{ s}} = 1.47\text{ m s}.$$

d he man is at rest at t = 3 min and has velocity v = 2.2 m s at t = 9 min. onse uently a_{avg} = 2.2 360 = 0.00611 m s^2 ust as in part b .

e he hori ontal line near the bottom of this x vs t graph represents the man standing at x = 0 for 0 $\le t$ 300 s and the linearly rising line for 300 $\le t \le$ 600 s represents his constant velocity motion. he lines represent the ans ers to part a and c in the sense that their slopes yield those results.



he graph of v vs t is not sho n here but ould consist of two horizontal “steps” (one at v = 0 for 0 $\le t$ 300 s and the ne t at v = 2.2 m s for 300 $\le$

$t \le 600$ s . he indications of the average accelerations found in parts b and d would be dotted lines connecting the "steps" at the appropriate t values the slopes of the dotted lines representing the values of a_{avg} .

22. n this solution e make use of the notation x t for the value of x at a particular t. he notations v t and a t have similar meanings.

a ince the unit of ct^2 is that of length the unit of c must be that of length time^2 or m s^2 in the system.

b ince bt^3 has a unit of length b must have a unit of length time^3 or m s^3.

c hen the particle reaches its ma imum or its minimum coordinate its velocity is ero. ince the velocity is given by $v = dx\ dt = 2ct - 3bt^2$ $v = 0$ occurs for $t = 0$ and for

$$t = \frac{2c}{3b} = \frac{2\ 3.0\text{ m}\ \ \text{s}^2}{3\ 2.0\text{ m}\ \ \text{s}^3} = 1.0\text{ s} .$$

or $t = 0$ $x = x_0 = 0$ and for $t = 1.0$ s $x = 1.0$ m x_0. ince e seek the ma imum e re ect the first root $t = 0$ and accept the second $t = 1$s .

d n the first 4 s the particle moves from the origin to $x = 1.0$ m turns around and goes back to

$$x\ 4\text{ s} = 3.0\text{ m}\ \ \text{s}^2\ \ 4.0\text{ s}\ ^2 - 2.0\text{ m}\ \ \text{s}^3\ \ 4.0\text{ s}\ ^3 = -80\text{ m} .$$

he total path length it travels is 1.0 m 1.0 m 80 m = 82 m.

e ts displacement is $\Delta x = x_2 - x_1$ here $x_1 = 0$ and $x_2 = -80$ m. hus $\Delta x = -80$ m .

he velocity is given by $v = 2ct - 3bt^2 =$ 6.0 m s^2 t – 6.0 m s^3 t^2.

f lugging in $t = 1$ s e obtain

$$v\ 1\text{ s} = 6.0\text{ m s}^2\ \ 1.0\text{ s} - 6.0\text{ m s}^3\ \ 1.0\text{ s}\ ^2 = 0.$$

g imilarly $v\ 2\text{ s} = 6.0\text{ m s}^2\ \ 2.0\text{ s} - 6.0\text{ m s}^3\ \ 2.0\text{ s}\ ^2 = -12\text{ m s}$.

h $v\ 3\text{ s} = 6.0\text{ m s}^2\ \ 3.0\text{ s} - 6.0\text{ m s}^3\ \ 3.0\text{ s}\ ^2 = -36\text{ m s}$.

i $v\ 4\text{ s} = 6.0\text{ m s}^2\ \ 4.0\text{ s} - 6.0\text{ m s}^3\ \ 4.0\text{ s}\ ^2 = -72\text{ m s}$.

he acceleration is given by $a = dv\ dt = 2c - 6b =$ 6.0 m s^2 – 12.0 m s^3 t.

lugging in $t = 1$ s e obtain $a\ 1\text{ s} = 6.0\text{ m s}^2 - 12.0\text{ m s}^3\ \ 1.0\text{ s} = -6.0\text{ m s}^2$.

k $a\ 2\text{ s} = 6.0\text{ m s}^2 - 12.0\text{ m s}^3\ \ 2.0\text{ s} = -18\text{ m s}^2$.

l $a\ 3\text{ s} = 6.0\text{ m s}^2 - 12.0\text{ m s}^3\ 3.0\text{ s} = -30\text{ m s}^2$.

m $a\ 4\text{ s} = 6.0\text{ m s}^2 - 12.0\text{ m s}^3\ 4.0\text{ s} = -42\text{ m s}^2$.

23. **THINK** he electron undergoes a constant acceleration. iven the final speed of the electron and the distance it has traveled e can calculate its acceleration.

EXPRESS ince the problem involves constant acceleration the motion of the electron can be readily analy ed using the e uations given in able 2 1

$$v = v_0 + at \qquad 2-11$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad 2-15$$

$$v^2 = v_0^2 + 2a\ x - x_0 \qquad 2-16$$

he acceleration can be found by solving . 2 16.

ANALYZE ith $v_0 = 1.50\times10^5$ m s $v = 5.70\times10^6$ m s x_0 = 0 and x = 0.010 m e find the average acceleration to be

$$a = \frac{v^2 - v_0^2}{2x} = \frac{5.7\times10^6\text{ m s}^{\ 2} - 1.5\times10^5\text{ m s}^{\ 2}}{2\ 0.010\text{ m}} = 1.62\times10^{15}\text{ m s}^2.$$

LEARN t is al ays a good idea to apply other e uations in able 2 1 not used for solving the problem as a consistency check. or e ample since e no kno the value of the acceleration using . 2 11 the time it takes for the electron to reach its final speed ould be

$$t = \frac{v - v_0}{a} = \frac{5.70\times10^6\text{ m s} - 1.5\times10^5\text{ m s}}{1.62\times10^{15}\text{ m s}^2} = 3.426\times10^{-9}\text{ s}$$

ubstituting the value of t into . 2 15 the distance the electron travels is

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 1.5\times10^5\text{ m s}\ \ 3.426\times10^{-9}\text{ s} + \frac{1}{2}\ 1.62\times10^{15}\text{ m s}^2\ \ 3.426\times10^{-9}\text{ s}^{\ 2}$$

$$= 0.01\text{ m}$$

his is hat as given in the problem statement. o e kno the problem has been solved correctly.

24. n this problem e are given the initial and final speeds and the displacement and are asked to find the acceleration. e use the constant acceleration e uation given in . 2 16 $v^2 = v_0^2\ \ 2a\ x - x_0$.

a iven that $v_0 = 0$ $v = 1.6$ m s and $\Delta x = 5.0\mu$m the acceleration of the spores during the launch is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{1.6\text{ m s}\ ^2}{2\ 5.0\times10^{-6}\text{ m}} = 2.56\times10^5\text{ m s}^2 = 2.6\times10^4 g$$

b uring the speed reduction stage the acceleration is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{0-\ 1.6\text{ m s}\ ^2}{2\ 1.0\times10^{-3}\text{ m}} = -1.28\times10^3\text{ m s}^2 = -1.3\times10^2 g$$

he negative sign means that the spores are decelerating.

25. e separate the motion into t o parts and take the direction of motion to be positive. n part 1 the vehicle accelerates from rest to its highest speed e are given v_0 = 0 v = 20 m s and a = 2.0 m s^2. n part 2 the vehicle decelerates from its highest speed to a halt e are given v_0 = 20 m s v = 0 and a = –1.0 m s^2 negative because the acceleration vector points opposite to the direction of motion .

a rom able 2 1 e find t_1 the duration of part 1 from v = v_0 at. n this ay $20 = 0 + 2.0t_1$ yields t_1 = 10 s. e obtain the duration t_2 of part 2 from the same e uation. hus 0 = 20 –1.0 t_2 leads to t_2 = 20 s and the total is t = t_1 t_2 = 30 s.

b or part 1 taking x_0 = 0 e use the e uation v^2 = v_0^2 2a $x - x_0$ from able 2 1 and find

$$x = \frac{v^2 - v_0^2}{2a} = \frac{20\text{ m s}\ ^2 -\ 0\ ^2}{2\ 2.0\text{ m s}^2} = 100\text{ m}.$$

his position is then the *initial* position for part 2 so that hen the same e uation is used in part 2 e obtain

$$x - 100\text{ m} = \frac{v^2 - v_0^2}{2a} = \frac{0\ ^2 -\ 20\text{ m s}\ ^2}{2\ -1.0\text{ m s}^2}.$$

hus the final position is x = 300 m. hat this is also the total distance traveled should be evident the vehicle did not backtrack or reverse its direction of motion .

26. he constant acceleration condition permits the use of able 2 1.

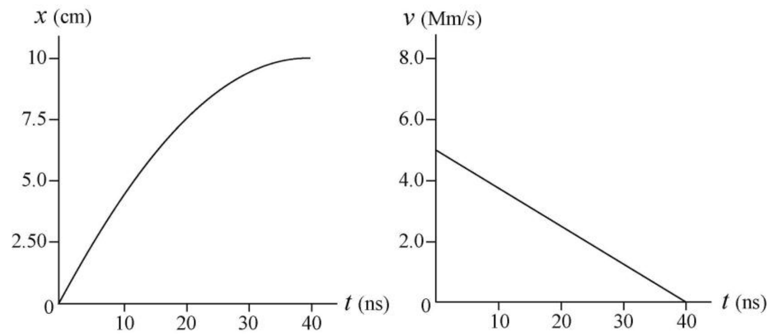
a etting v = 0 and x_0 = 0 in $v^2 = v_0^2 + 2a\ x - x_0$ e find

$$x = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\frac{5.00\times10^6\ ^2}{-1.25\times10^{14}} = 0.100\text{ m}.$$

ince the muon is slo ing the initial velocity and the acceleration must have opposite signs.

b elo are the time plots of the position x and velocity v of the muon from the moment it enters the field to the time it stops. he computation in part a made no reference to t so that other e uations from able 2 1 such as $v = v_0 + at$ and

$x = v_0 t + \frac{1}{2}at^2$ are used in making these plots.



27. e use $v = v_0$ at ith $t = 0$ as the instant hen the velocity e uals 9.6 m s.

a ince e ish to calculate the velocity for a time *before* $t = 0$ e set $t = -2.5$ s. hus . 2 11 gives

$$v = \; 9.6 \text{ m} \;\; \text{s} \;\; + \left(3.2 \text{ m} \;\; \text{s}^2\right) \;\; -2.5 \text{ s} \;\; = 1.6 \text{ m} \;\; \text{s}.$$

b o $t =$ 2.5 s and e find $v = \; 9.6 \text{ m} \;\; \text{s} \;\; + \left(3.2 \text{ m} \;\; \text{s}^2\right) \;\; 2.5 \text{ s} \;\; = 18 \text{ m} \;\; \text{s}.$

28. e take x in the direction of motion so $v_0 =$ 24.6 m s and $a = -$ 4.92 m s^2. e also take $x_0 = 0$.

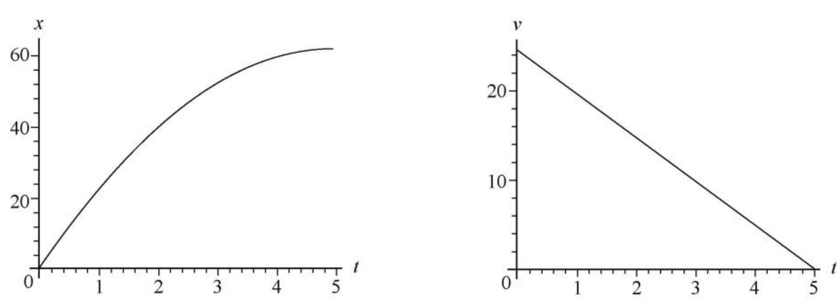
a he time to come to a halt is found using . 2 11

$$0 = v_0 + at \;\Rightarrow\; t = \frac{24.6 \text{ m} \;\; \text{s}}{-4.92 \text{ m} \;\; \text{s}^2} = 5.00 \text{ s}.$$

b lthough several of the e uations in able 2 1 ill yield the result e choose . 2 16 since it does not depend on our ans er to part a .

$$0 = v_0^2 + 2ax \;\Rightarrow\; x = -\frac{24.6 \;\; \text{m} \;\; \text{s} \;\; ^2}{2\left(-4.92 \;\; \text{m} \;\; \text{s}^2\right)} = 61.5 \text{ m}.$$

c sing these results e plot $v_0 t + \frac{1}{2}at^2$ the x graph sho n ne t on the left and v_0 at the v graph on the right over $0 \le t \le 5$ s ith units understood.



29. e assume the periods of acceleration duration t_1 and deceleration duration t_2 are periods of constant a so that able 2 1 can be used. aking the direction of motion to be x then a_1 = 1.22 m s^2 and a_2 = –1.22 m s^2. e use units so the velocity at t = t_1 is v = 305 60 = 5.08 m s.

a e denote Δx as the distance moved during t_1 and use . 2 16

$$v^2 = v_0^2 + 2a_1\Delta x \;\Rightarrow\; \Delta x = \frac{5.08\text{ m s}\;^2}{2\;1.22\text{ m s}^2} = 10.59\text{ m} \approx 10.6\text{ m}.$$

b sing . 2 11 e have

$$t_1 = \frac{v - v_0}{a_1} = \frac{5.08\text{ m s}}{1.22\text{ m s}^2} = 4.17\text{ s}.$$

he deceleration time t_2 turns out to be the same so that t_1 t_2 = 8.33 s. he distances traveled during t_1 and t_2 are the same so that they total to 2 10.59 m = 21.18 m. his implies that for a distance of 190 m – 21.18 m = 168.82 m the elevator is traveling at constant velocity. his time of constant velocity motion is

$$t_3 = \frac{168.82\text{ m}}{5.08\text{ m s}} = 33.21\text{ s}.$$

herefore the total time is 8.33 s 33.21 s ≈ 41.5 s.

30. e choose the positive direction to be that of the initial velocity of the car implying that a 0 since it is slo ing do n . e assume the acceleration is constant and use able 2 1.

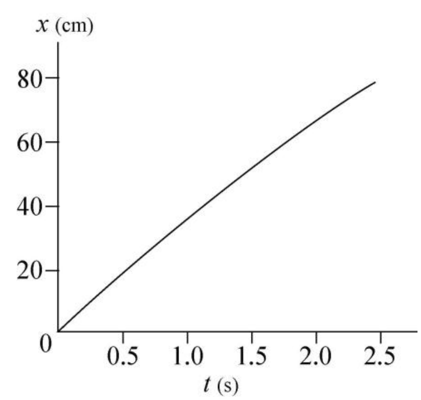
a ubstituting v_0 = 137 km h = 38.1 m s v = 90 km h = 25 m s and a = –5.2 m s^2 into $v = v_0$ at e obtain

$$t = \frac{25\text{ m s} - 38\text{ m s}}{-5.2\text{ m s}^2} = 2.5\text{ s}.$$

b e take the car to be at x = 0 hen the brakes are applied at time t = 0 . hus the coordinate of the car as a function of time is given by

$$x = (38\text{ m s})t + \frac{1}{2}(-5.2\text{ m s}^2)t^2$$

in units. his function is plotted from t = 0 to t = 2.5 s on the graph to the right. e have not sho n the v vs t graph here it is a descending straight line from v_0 to v.



31. **THINK** he rocket ship undergoes a constant acceleration from rest and e ant to kno the time elapsed and the distance traveled hen the rocket reaches a certain speed.

EXPRESS ince the problem involves constant acceleration the motion of the rocket can be readily analy ed using the e uations in able 2 1

$$v = v_0 + at \qquad 2-11$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad 2-15$$

$$v^2 = v_0^2 + 2a\ x - x_0 \qquad 2-16$$

ANALYZE a iven that $a = 9.8$ m s^2 $v_0 = 0$ and $v = 0.1c = 3.0\times10^7$ m s e can solve $v = v_0 + at$ for the time

$$t = \frac{v - v_0}{a} = \frac{3.0\times10^7\ \text{m s} - 0}{9.8\ \text{m s}^2} = 3.1\times10^6\ \text{s}$$

hich is about 1.2 months. o it takes 1.2 months for the rocket to reach a speed of 0.1c starting from rest ith a constant acceleration of 9.8 m s^2.

b o calculate the distance traveled during this time interval e evaluate $x = x_0 + v_0 t + \frac{1}{2}at^2$ ith x_0 = 0 and $v_0 = 0$. he result is

$$x = \frac{1}{2}\left(9.8\ \text{m s}^2\right)\ 3.1\times10^6\text{s}\ ^2 = 4.6\times10^{13}\ \text{m}.$$

LEARN n solving parts a and b e did not use . 2 16 $v^2 = v_0^2 + 2a\ x - x_0$. his e uation can be used to check our ans ers. he final velocity based on this e uation is

$$v = \sqrt{v_0^2 + 2a\ x - x_0} = \sqrt{0 + 2\ 9.8\ \text{m s}^2\ \ 4.6\times10^{13}\ \text{m} - 0} = 3.0\times10^7\ \text{m s}$$

hich is hat as given in the problem statement. o e kno the problems have been solved correctly.

32. he acceleration is found from . 2 11 or suitably interpreted . 2 7 .

$$a = \frac{\Delta v}{\Delta t} = \frac{(1020\ \text{km}\ \ \text{h})\left(\dfrac{1000\ \text{m}\ \ \text{km}}{3600\ \text{s}\ \ \text{h}}\right)}{1.4\ \text{s}} = 202.4\ \text{m}\ \ \text{s}^2\ .$$

n terms of the gravitational acceleration g this is e pressed as a multiple of 9.8 m s^2 as follo s

$$a = \left(\frac{202.4\ \text{m s}^2}{9.8\ \text{m s}^2}\right)g = 21g\ .$$

33. **THINK** he car undergoes a constant negative acceleration to avoid impacting a barrier. iven its initial speed e ant to kno the distance it has traveled and the time elapsed prior to the impact.

EXPRESS ince the problem involves constant acceleration the motion of the car can be readily analy ed using the e uations in able 2 1

$$v = v_0 + at \qquad 2-11$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad 2-15$$

$$v^2 = v_0^2 + 2a\ x - x_0 \qquad 2-16$$

e take x_0 = 0 and v_0 = 56.0 km h = 15.55 m s to be the initial position and speed of the car. olving . 2 15 ith t = 2.00 s gives the acceleration a. nce a is kno n the speed of the car upon impact can be found by using . 2 11.

ANALYZE a sing . 2 15 e find the acceleration to be

$$a = \frac{2\ x - v_0 t}{t^2} = \frac{2[\ 24.0\text{ m}\ -\ 15.55\text{ m s}\ \ 2.00\text{ s}\ ]}{2.00\text{ s}\ ^2} = -3.56\text{ m s}^2$$

or $a = 3.56$ m s^2. he negative sign indicates that the acceleration is opposite to the direction of motion of the car the car is slo ing do n.

b he speed of the car at the instant of impact is

$$v = v_0 + at = 15.55\text{ m s} + \ -3.56\text{ m s}^2\ \ 2.00\text{ s}\ = 8.43\text{ m s}$$

hich can also be converted to 30.3 km h.

LEARN n solving parts a and b e did not use . 1 16. his e uation can be used as a consistency check. he final velocity based on this e uation is

$$v = \sqrt{v_0^2 + 2a\ x - x_0} = \sqrt{\ 15.55\text{ m s}\ ^2 + 2\ -3.56\text{ m s}^2\ \ 24\text{ m} - 0} = 8.43\text{ m s}$$

hich is hat as calculated in b . his indicates that the problems have been solved correctly.

34. et d be the 220 m distance bet een the cars at t = 0 and v_1 be the 20 km h = 50 9 m s speed corresponding to a passing point of x_1 = 44.5 m and v_2 be the 40 km h =100 9 m s speed corresponding to a passing point of x_2 = 76.6 m of the red car. e have t o e uations based on . 2 17

$$d - x_1 = v_0 t_1 \quad \frac{1}{2}\ a\ t_1^2 \qquad \text{here } t_1 = x_1 / v_1$$

$$d - x_2 = v_0 t_2 \quad \frac{1}{2}\ a\ t_2^2 \qquad \text{here } t_2 = x_2\ / v_2$$

e simultaneously solve these e uations and obtain the follo ing results a he initial velocity of the green car is v_0 = – 13.9 m s. or roughly – 50 km h the negative sign means that it's along the $-x$ direction .

b he corresponding acceleration of the car is a = – 2.0 m s^2 the negative sign means that it's along the $-x$ direction .

35. he positions of the cars as a function of time are given by

$$x_r \; t \; = x_{r0} + \frac{1}{2} a_r t^2 = \; -35.0 \text{ m} \; + \frac{1}{2} a_r t^2$$
$$x_g \; t \; = x_{g0} + v_g t = \; 270 \text{ m} \; - \; 20 \text{ m s} \; t$$

here e have substituted the velocity and not the speed for the green car. he t o cars pass each other at $t = 12.0\,\text{s}$ hen the graphed lines cross. his implies that

$$270 \text{ m} \; - \; 20 \text{ m s} \; 12.0\,\text{s} \; = 30 \text{ m} = \; -35.0 \text{ m} \; + \frac{1}{2} a_r \; 12.0\,\text{s} \; ^2$$

hich can be solved to give $a_r = 0.90$ m s^2.

36. a uation 2 15 is used for part 1 of the trip and . 2 18 is used for part 2

$\Delta x_1 = v_{01}\, t_1 \quad \frac{1}{2}\, a_1\, t_1^2$ here a_1 = 2.25 m s^2 and $\Delta x_1 = \frac{900}{4}$ m

$\Delta x_2 = v_2\, t_2 - \frac{1}{2}\, a_2\, t_2^2$ here a_2 = –0.75 m s^2 and $\Delta x_2 = \frac{3\ 900}{4}$ m

n addition $v_{01} = v_2 = 0$. olving these e uations for the times and adding the results gives $t = t_1 + t_2 = 56.6$ s.

b uation 2 16 is used for part 1 of the trip

$$v^2 = \; v_{01} \; ^2 \quad 2 a_1 \Delta x_1 = 0 \quad 2 \; 2.25 \left(\frac{900}{4}\right) = 1013 \text{ m}^2 \text{ s}^2$$

hich leads to v = 31.8 m s for the ma imum speed.

37. a rom the figure e see that x_0 = –2.0 m. rom able 2 1 e can apply

$$x - x_0 = v_0 t \quad \tfrac{1}{2} a t^2$$

ith t = 1.0 s and then again ith t = 2.0 s. his yields t o e uations for the t o unkno ns v_0 and a

$$0.0-(-2.0\text{ m})=v_0(1.0\text{ s})+\frac{1}{2}a(1.0\text{ s})^2$$
$$6.0\text{ m}-(-2.0\text{ m})=v_0\ (2.0\text{ s})+\frac{1}{2}a(2.0\text{ s})^2.$$

olving these simultaneous e uations yields the results v_0 = 0 and a = 4.0 m s^2.

b he fact that the ans er is positive tells us that the acceleration vector points in the x direction.

38. e assume the train accelerates from rest $v_0 = 0$ and $x_0 = 0$ at $a_1 = +1.34$ m s^2 until the mid ay point and then decelerates at $a_2 = -1.34$ m s^2 until it comes to a stop $(v_2 = 0)$ at the ne t station. he velocity at the midpoint is v_1 hich occurs at x_1 = 806 2 = 403m.

a uation 2 16 leads to

$$v_1^2 = v_0^2 + 2a_1x_1 \quad\Rightarrow\quad v_1 = \sqrt{2\left(1.34\text{ m s}^2\right)(403\text{ m})} = 32.9\text{ m s}.$$

b he time t_1 for the accelerating stage is using . 2 15

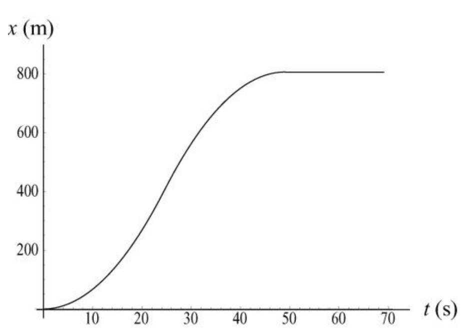
$$x_1 = v_0t_1 + \frac{1}{2}a_1t_1^2 \quad\Rightarrow\quad t_1 = \sqrt{\frac{2(403\text{ m})}{1.34\text{ m s}^2}} = 24.53\text{ s}.$$

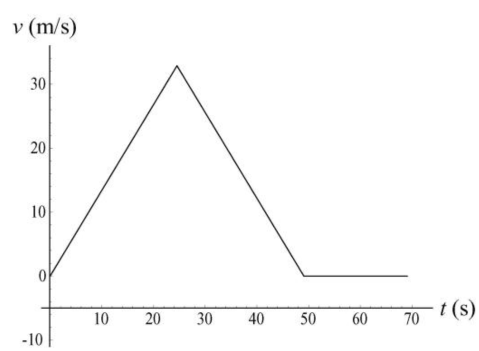
ince the time interval for the decelerating stage turns out to be the same e double this result and obtain t = 49.1 s for the travel time bet een stations.

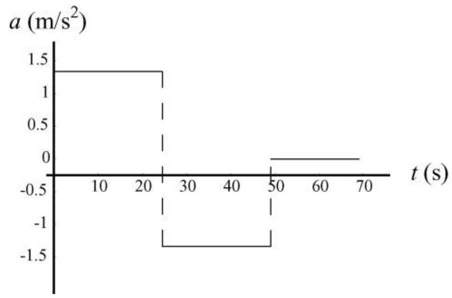
(c) With a "dead time" of 20 s, we have $T = t$ 20 = 69.1 s for the total time bet een start ups. hus . 2 2 gives

$$v_{\text{avg}} = \frac{806\text{ m}}{69.1\text{ s}} = 11.7\text{ m s}.$$

d he graphs for x v and a as a function of t are sho n belo . he third graph a t consists of three horizontal "steps" — one at 1.34 m s^2 during 0 t 24.53 s and the ne t at –1.34 m s^2 during 24.53 s t < 49.1 s and the last at zero during the "dead time" 49.1 s < t 69.1 s .







39. a e note that v = 12 6 = 2 m s ith t o significant figures understood . herefore ith an initial x value of 20 m car ill be at x = 28 m hen t = 4 s. his must be the value of x for car at that time e use . 2 15

$$28 \text{ m} = 12 \text{ m s } t \quad \tfrac{1}{2} a \quad t^2 \qquad \text{here } t = 4.0 \text{ s .}$$

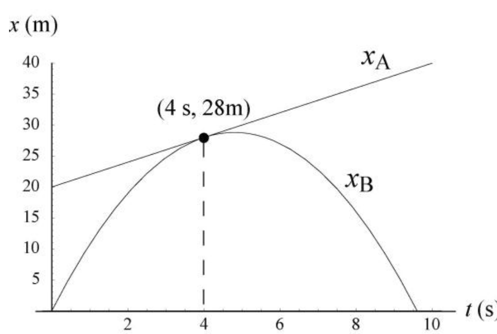
his yields a = – 2.5 m s^2.

b he uestion is using the value obtained for a in part a are there other values of t besides t = 4 s such that x = x he re uirement is

$$20 \quad 2t = 12t \quad \tfrac{1}{2} a \quad t^2$$

here a = –5 2. here are t o distinct roots unless the discriminant $\sqrt{10^2 - 2 \; -20 \quad a}$ is ero. n our case it <u>is</u> ero – hich means there is only one root. he cars are side by side only once at t = 4 s.

c sketch is sho n belo . t consists of a straight line x tangent to a parabola x at t = 4.



d e only care about <u>real</u> roots hich means 10^2 – 2 –20 a ≥ 0. f a 5 2 then there are no real solutions to the e uation the cars are never side by side.

e ere e have 10^2 – 2 –20 a 0 ⇒ t o real roots. he cars are side by side at t o different times.

40. e take the direction of motion as x so a = –5.18 m s^2 and e use units so v_0 = 55 1000 3600 = 15.28 m s.

a he velocity is constant during the reaction time T so the distance traveled during it is

$$d_r = v_0 T - \ 15.28 \text{ m s} \ \ 0.75 \text{ s} \ = 11.46 \text{ m}.$$

e use . 2 16 ith $v = 0$ to find the distance d_b traveled during braking

$$v^2 = v_0^2 + 2ad_b \ \Rightarrow \ d_b = -\frac{15.28 \text{ m s} \ ^2}{2\left(-5.18 \text{ m s}^2\right)}$$

hich yields $d_b = 22.53$ m. hus the total distance is d_r d_b = 34.0 m hich means that the driver *is* able to stop in time. nd if the driver ere to continue at v_0 the car ould enter the intersection in t = 40 m 15.28 m s = 2.6 s hich is barely enough time to enter the intersection before the light turns hich many people ould consider an acceptable situation.

b n this case the total distance to stop found in part a to be 34 m is greater than the distance to the intersection so the driver cannot stop ithout the front end of the car being a couple of meters into the intersection. nd the time to reach it at constant speed is 32 15.28 = 2.1 s hich is too long the light turns in 1.8 s . he driver is caught bet een a rock and a hard place.

41. he displacement Δx) for each train is the "area" in the graph (since the displacement is the integral of the velocity . ach area is triangular and the area of a triangle is 1 2 base height . hus the absolute value of the displacement for one train 1 2 40 m s 5 s = 100 m and that of the other train is 1 2 30 m s 4 s = 60 m. The initial "gap" between the trains was 200 m, and according to our displacement computations the gap has narro ed by 160 m. hus the ans er is 200 – 160 = 40 m.

42. a ote that 110 km h is e uivalent to 30.56 m s. uring a t o second interval you travel 61.11 m. he decelerating police car travels using . 2 15 51.11 m. n light of the fact that the initial "gap" between cars was 25 m, this means the gap has narro ed by 10.0 m – that is to a distance of 15.0 m bet een cars.

b irst e add 0.4 s to the considerations of part a . uring a 2.4 s interval you travel 73.33 m. he decelerating police car travels using . 2 15 58.93 m during that time. he initial distance bet een cars of 25 m has therefore narro ed by 14.4 m. hus at the start of your braking call it t_0 the gap bet een the cars is 10.6 m. he speed of the police car at t_0 is 30.56 – 5 2.4 = 18.56 m s. ollision occurs at time t hen $x_{\text{you}} = x_{\text{police}}$ e choose coordinates such that your position is $x = 0$ and the police car's position is x = 10.6 m at t_0 . . 2 15 becomes for each car

$$x_{\text{police}} - 10.6 = 18.56 \ t - t_0 \ - \tfrac{1}{2} \ 5 \ \ t - t_0 \ ^2$$

$$x_{\text{you}} = 30.56 \ t - t_0 \ - \tfrac{1}{2} \ 5 \ \ t - t_0 \ ^2 \ .$$

ubtracting e uations e find

$$10.6 = \ 30.56 - 18.56 \ \ t - t_0 \ \ \Rightarrow \ 0.883 \text{ s} = t - t_0.$$

t that time your speed is 30.56 $a\ t - t_0$ = 30.56 – 5 0.883 ≈ 26 m s or 94 km h .

43. n this solution e elect to ait until the last step to convert to units. onstant acceleration is indicated so use of able 2 1 is permitted. e start ith . 2 17 and denote the train's initial velocity as v_t and the locomotive's velocity as v_ℓ hich is also the final velocity of the train if the rear end collision is barely avoided . e note that the distance Δx consists of the original gap bet een them D as ell as the for ard distance traveled during this time by the locomotive $v_\ell t$. herefore

$$\frac{v_t + v_\ell}{2} = \frac{\Delta x}{t} = \frac{D + v_\ell t}{t} = \frac{D}{t} + v_\ell .$$

e no use . 2 11 to eliminate time from the e uation. hus

$$\frac{v_t + v_\ell}{2} = \frac{D}{(v_\ell - v_t)\ \ a} + v_\ell$$

hich leads to

$$a = \left(\frac{v_t + v_\ell}{2} - v_\ell\right)\left(\frac{v_\ell - v_t}{D}\right) = -\frac{1}{2D}(v_\ell - v_t)^2 .$$

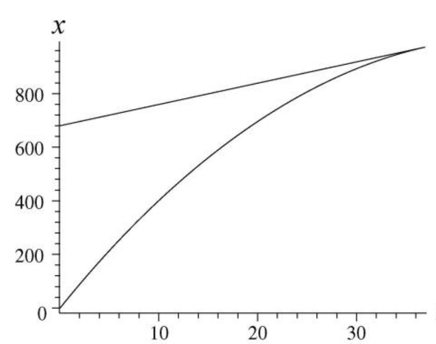
ence

$$a = -\frac{1}{2\ 0.676\text{ km}}\left(29\frac{\text{km}}{\text{h}} - 161\frac{\text{km}}{\text{h}}\right)^2 = -12888\text{ km}\ \ \text{h}^2$$

hich e convert as follo s

$$a = \left(-12888\text{ km}\ \ \text{h}^2\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2 = -0.994\text{ m}\ \ \text{s}^2$$

so that its *magnitude* is a = 0.994 m s^2. graph is sho n here for the case here a collision is ust avoided x along the vertical a is is in meters and t along the hori ontal a is is in seconds . he top straight line sho s the motion of the locomotive and the bottom curve sho s the motion of the passenger train.



he other case here the collision is not uite avoided ould be similar e cept that the slope of the bottom curve ould be greater than that of the top line at the point here they meet.

44. e neglect air resistance hich ustifies setting $a = -g$ = –9.8 m s^2 taking *down* as the $-y$ direction for the duration of the motion. e are allo ed to use able 2 1 ith Δy replacing Δx because this is constant acceleration motion. he ground level is taken to correspond to the origin of the y a is.

a sing $y = v_0 t - \frac{1}{2} g t^2$ ith y = 0.544 m and t = 0.200 s e find

$$v_0 = \frac{y + \; gt^2 \;\; 2}{t} = \frac{0.544\text{ m} + \;\; 9.8\text{ m s}^2 \;\;\; 0.200\text{ s} \;\; ^2 \;\; 2}{0.200\text{ s}} = 3.70\text{ m s} \;.$$

b he velocity at y = 0.544 m is

$$v \;\; = v_0 - gt = 3.70\text{ m s} - \;\; 9.8\text{ m s}^2 \;\;\; 0.200\text{ s} \;\; = 1.74 \;\; \text{m s} \;.$$

c sing $v^2 = v_0^2 - 2gy$ ith different values for y and v than before e solve for the value of y corresponding to ma imum height here v = 0 .

$$y = \frac{v_0^2}{2g} = \frac{\;\; 3.7\text{ m s} \;\; ^2}{2 \;\; 9.8\text{ m s}^2} = 0.698\text{ m.}$$

 hus the armadillo goes 0.698 – 0.544 = 0.154 m higher.

45. **THINK** s the ball travels vertically up ard its motion is under the influence of gravitational acceleration. he kinematics is one dimensional.

EXPRESS e neglect air resistance for the duration of the motion bet een “launching” and “landing”), so a = –g = –9.8 m s^2 e take do n ard to be the –y direction . e use the e uations in able 2 1 ith Δy replacing Δx because this is a = constant motion

$$v = v_0 - gt \qquad\qquad 2-11$$

$$y - y_0 = v_0 t - \frac{1}{2} gt^2 \qquad\qquad 2-15$$

$$v^2 = v_0^2 - 2g \;\; y - y_0 \qquad\qquad 2-16$$

 e set y_0 = 0. pon reaching the ma imum height y the speed of the ball is momentarily ero v = 0 . herefore e can relate its initial speed v_0 to y via the e uation $0 = v^2 = v_0^2 \;\; - 2gy$. he time it takes for the ball to reach ma imum height is given by $v = v_0 - gt = 0$ or $t = v_0 \;\; g$. herefore for the entire trip from the time it leaves the ground until the time it returns to the ground the total flight time is $T = 2t = 2v_0 \;\; g$.

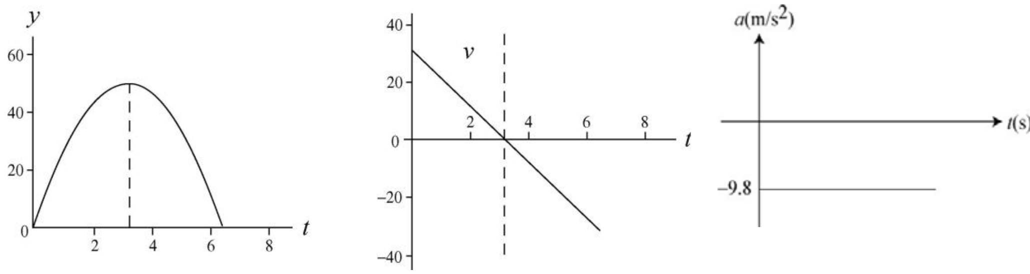
ANALYZE a t the highest point v = 0 and $v_0 = \sqrt{2gy}$. ith y = 50 m e find the initial speed of the ball to be

$$v_0 = \sqrt{2gy} = \sqrt{2 \;\; 9.8\text{ m s}^2 \;\;\; 50\text{ m} \;\;} = 31.3\text{ m s.}$$

b sing the result from a for v_0 the total flight time of the ball is

$$T = \frac{2v_0}{g} = \frac{2 \;\; 31.3\text{ m s}}{9.8\text{ m s}^2} = 6.39\text{ s}$$

c he plots of y v and a as a function of time are sho n belo . he acceleration graph is a hori ontal line at –9.8 m s^2. t t = 3.19 s y = 50 m.



LEARN n calculating the total flight time of the ball e could have used . 2 15. t $t = T > 0$ the ball returns to its original position $y = 0$. herefore

$$y = v_0 T - \frac{1}{2} g T^2 = 0 \;\; \Rightarrow \;\; T = \frac{2v_0}{g}$$

46. eglect of air resistance ustifies setting a = –g = –9.8 m s^2 here *down* is our –y direction for the duration of the fall. his is constant acceleration motion and e may use able 2 1 ith Δy replacing Δx .

a sing . 2 16 and taking the negative root since the final velocity is do n ard e have

$$v = -\sqrt{v_0^2 - 2g\Delta y} = -\sqrt{0 - 2\;\, 9.8\text{ m s}^2 \;\; -1700\text{ m}\;} = -183\text{ m s}.$$

ts magnitude is therefore 183 m s.

b o but it is hard to make a convincing case ithout more analysis. e estimate the mass of a raindrop to be about a gram or less so that its mass and speed from part a ould be less than that of a typical bullet hich is good ne s. ut the fact that one is dealing ith *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. f e factor in air resistance the final speed is smaller of course and e return to the relatively healthy situation ith hich e are familiar.

47. **THINK** he rench is in free fall ith an acceleration a = –g = –9.8 m s^2.

EXPRESS e neglect air resistance hich ustifies setting a = –g = –9.8 m s^2 taking *down* as the –y direction for the duration of the fall. his is constant acceleration motion hich ustifies the use of able 2 1 ith Δy replacing Δx

$$v = v_0 - gt \qquad\qquad 2-11$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad\qquad 2-15$$

$$v^2 = v_0^2 - 2g\;\, y - y_0 \qquad\qquad 2-16$$

ince the rench had an initial speed v_0 = 0 kno ing its speed of impact allo s us to apply . 2 16 to calculate the height from hich it as dropped.

ANALYZE a sing $v^2 = v_0^2 + 2a\Delta y$ e find the initial height to be

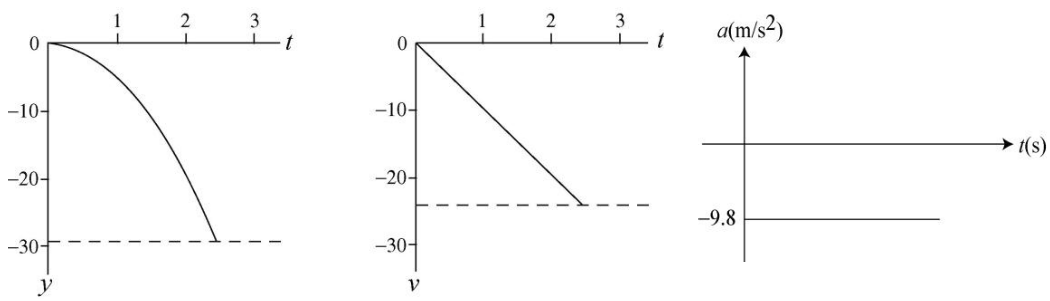
$$\Delta y = \frac{v_0^2 - v^2}{2a} = \frac{0 - \;\; -24\text{ m s} \;\;^2}{2 \;\; -9.8\text{ m s}^2} = 29.4\text{ m}.$$

o that it fell through a height of 29.4 m.

b olving $v = v_0 - gt$ for time e obtain a flight time of

$$t = \frac{v_0 - v}{g} = \frac{0 - \;\; -24\text{ m s}}{9.8\text{ m s}^2} = 2.45\text{ s}.$$

c units are used in the graphs and the initial position is taken as the coordinate origin. he acceleration graph is a hori ontal line at –9.8 m s^2.



LEARN s the rench falls ith $a = -g < 0$ its speed increases but its velocity becomes more negative as indicated by the second graph above.

48. e neglect air resistance hich ustifies setting $a = -g = -9.8$ m s^2 taking *down* as the $-y$ direction for the duration of the fall. his is constant acceleration motion hich ustifies the use of able 2 1 ith Δy replacing Δx .

a oting that $\Delta y = y - y_0 = -30$ m e apply . 2 15 and the uadratic formula ppendi to compute t

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t = \frac{v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{g}$$

hich ith $v_0 = -12$ m s since it is do n ard leads upon choosing the positive root so that t 0 to the result

$$t = \frac{-12\text{ m s} + \sqrt{\;\; -12\text{ m s} \;\;^2 - 2 \;\; 9.8\text{ m s}^2 \;\; -30\text{ m}}}{9.8\text{ m s}^2} = 1.54\text{ s}.$$

b nough information is no kno n that any of the e uations in able 2 1 can be used to obtain v ho ever the one e uation that does *not* use our result from part a is . 2 16

$$v = \sqrt{v_0^2 - 2g\Delta y} = 27.1\text{ m s}$$

here the positive root has been chosen in order to give *speed* hich is the magnitude of the velocity vector .

49. **THINK** n this problem a package is dropped from a hot air balloon hich is ascending vertically up ard. e analy e the motion of the package under the influence of gravity.

EXPRESS e neglect air resistance hich ustifies setting $a = -g = -9.8$ m s^2 taking *down* as the $-y$ direction for the duration of the motion. his allo s us to use able 2 1 ith Δy replacing Δx

$$v = v_0 - gt \qquad 2-11$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad 2-15$$

$$v^2 = v_0^2 - 2g \ \ y - y_0 \qquad 2-16$$

e place the coordinate origin on the ground and note that the initial velocity of the package is the same as the velocity of the balloon $v_0 =$ 12 m s and that its initial coordinate is $y_0 =$ 80 m. he time it takes for the package to hit the ground can be found by solving . 2 15 ith $y = 0$.

ANALYZE a e solve $0 = y = y_0 + v_0 t - \frac{1}{2} g t^2$ for time using the uadratic formula choosing the positive root to yield a positive value for t

$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12 \text{ m s} + \sqrt{\ 12 \text{ m s}\ ^2 + 2\left(9.8 \text{ m s}^2\right)(80 \text{ m})}}{9.8 \text{ m s}^2} = 5.45 \text{ s}.$$

b he speed of the package hen it hits the ground can be calculated using . 2 11. he result is

$$v = v_0 - gt = 12 \text{ m s} - \ 9.8 \text{ m s}^2 \ \ 5.447 \text{ s} \ = -41.38 \text{ m s}.$$

ts final *speed* is 41.38 m s.

LEARN ur ans ers can be readily verified by using . 2 16 hich as not used in either a or b . he e uation leads to

$$v = -\sqrt{v_0^2 - 2g \ \ y - y_0 \ } = -\sqrt{\ 12 \text{ m s}\ ^2 - 2 \ 9.8 \text{ m s}^2 \ \ 0 - 80 \text{ m} \ } = -41.38 \text{ m s}$$

hich agrees ith that calculated in b .

50. he y coordinate of pple 1 obeys $y - y_{o1} = -\frac{1}{2} g t^2$ here $y = 0$ hen $t = 2.0$ s. his allo s us to solve for y_{o1} and e find $y_{o1} = 19.6$ m.

he graph for the coordinate of pple 2 hich is thro n apparently at $t = 1.0$ s ith velocity v_2 is

$$y - y_{02} = v_2\ t - 1.0\ - \frac{1}{2} g\ \ t - 1.0\ ^2$$

here $y_{02} = y_{01}$ = 19.6 m and here $y = 0$ hen t = 2.25 s. hus e obtain $v_2| = 9.6$ m s appro imately.

51. a ith up ard chosen as the y direction e use . 2 11 to find the initial velocity of the package

$$v = v_0 \quad at \quad \Rightarrow \quad 0 = v_0 - \ 9.8\text{ m s}^2\ \ 2.0\text{ s}$$

hich leads to v_0 = 19.6 m s. o e use . 2 15

$$\Delta y = \ 19.6\text{ m s}\ \ 2.0\text{ s} \qquad \frac{1}{2}\ -9.8\text{ m s}^2\ \ 2.0\text{ s}\ ^2 \approx 20\text{ m}\ .$$

We note that the "2.0 s" in this second computation refers to the time interval 2 t 4 in the graph (whereas the "2.0 s" in the first computation referred to the $0 < t$ 2 time interval sho n in the graph .

b In our computation for part (b), the time interval ("6.0 s") refers to the $2 < t$ 8 portion of the graph

$$\Delta y = \ 19.6\text{ m s}\ \ 6.0\text{ s} \qquad \frac{1}{2}\ -9.8\text{ m s}^2\ \ 6.0\text{ s}\ ^2 \approx -59\text{ m}$$

or $\Delta y = 59$ m .

52. The full extent of the bolt's fall is given by

$$y - y_0 = -\frac{1}{2} g\, t^2$$

here $y - y_0$ = –90 m if up ard is chosen as the positive y direction . hus the time for the full fall is found to be t = 4.29 s. he first 80 of its free fall distance is given by $-72 = -g\,\tau^2$ 2 hich re uires time τ = 3.83 s.

a hus the final 20 of its fall takes $t - \tau$ = 0.45 s.

b e can find that speed using $v = -g\tau$. herefore v = 38 m s appro imately.

c imilarly $v_{final} = -\ g\ t \quad \Rightarrow \quad v_{final}$ = 42 m s.

53. **THINK** his problem involves t o ob ects a key dropped from a bridge and a boat moving at a constant speed. e look for conditions such that the key ill fall into the boat.

EXPRESS he speed of the boat is constant given by $v_b = d\ t$ here d is the distance of the boat from the bridge hen the key is dropped 12 m and t is the time the key takes in falling.

o calculate t e take the time to be ero at the instant the key is dropped e compute the time t hen y = 0 using $y = y_0 + v_0 t - \frac{1}{2}gt^2$ ith $y_0 = 45$ m. nce t is kno n the speed of the boat can be readily calculated.

ANALYZE ince the initial velocity of the key is ero the coordinate of the key is given by $y_0 = \frac{1}{2}gt^2$. hus the time it takes for the key to drop into the boat is

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2\ 45\text{ m}}{9.8\text{ m s}^2}} = 3.03\text{ s}.$$

herefore the speed of the boat is $v_b = \dfrac{12\text{ m}}{3.03\text{ s}} = 4.0$ m s.

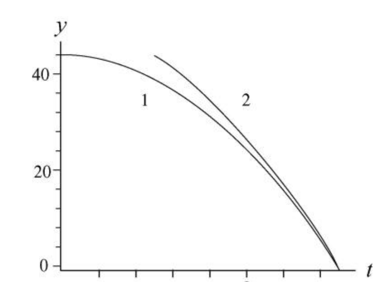
LEARN rom the general e pression $v_b = \dfrac{d}{t} = \dfrac{d}{\sqrt{2y_0\ g}} = d\sqrt{\dfrac{g}{2y_0}}$ e see that $v_b \sim 1\ \sqrt{y_0}$. his agrees ith our intuition that the lo er the height from hich the key is dropped the greater the speed of the boat in order to catch it.

54. a e neglect air resistance hich ustifies setting $a = -g = -9.8$ m s^2 taking *down* as the $-y$ direction for the duration of the motion. e are allo ed to use . 2 15 ith Δy replacing Δx because this is constant acceleration motion. e use primed variables e cept t ith the first stone hich has ero initial velocity and unprimed variables ith the second stone ith initial do n ard velocity $-v_0$ so that v_0 is being used for the initial *speed* . units are used throughout.

$$\Delta y' = 0(t) - \frac{1}{2}gt^2$$

$$\Delta y = (-v_0)(t-1) - \frac{1}{2}g(t-1)^2$$

ince the problem indicates $\Delta y' = \Delta y = -43.9$ m e solve the first e uation for t finding t = 2.99 s and use this result to solve the second e uation for the initial speed of the second stone



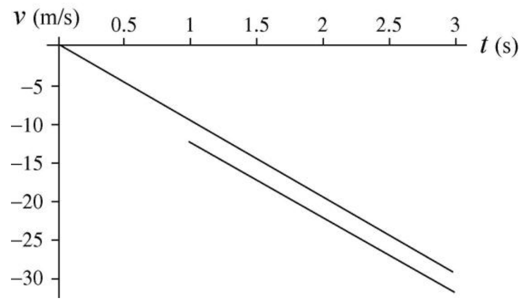
$$-43.9\text{ m} = (-v_0)(1.99\text{ s}) - \frac{1}{2}(9.8\text{ m s}^2)(1.99\text{ s})^2$$

hich leads to v_0 = 12.3 m s.

b he velocity of the stones are given by

$$v'_y = \frac{d\ \Delta y'}{dt} = -gt \qquad v_y = \frac{d\ \Delta y}{dt} = -v_0 - g\ \ t-1$$

he plot is sho n belo



55. **THINK** he free falling moist clay ball strikes the ground ith a non ero speed and it undergoes deceleration before coming to rest.

EXPRESS uring contact ith the ground its average acceleration is given by $a_{\text{avg}} = \frac{\Delta v}{\Delta t}$ here Δv is the change in its velocity during contact ith the ground and $\Delta t = 20.0\times 10^{-3}$ s is the duration of contact. hus e must first find the velocity of the ball ust before it hits the ground $y = 0$.

ANALYZE a o to find the velocity ust *before* contact e take $t = 0$ to be hen it is dropped. sing . 2 16 ith $y_0 = 15.0$ m e obtain

$$v = -\sqrt{v_0^2 - 2g\ \ y - y_0\ } = -\sqrt{0-2\ \ 9.8\text{ m s}^2\ \ \ 0-15\text{ m}\ } = -17.15\text{ m s}$$

here the negative sign is chosen since the ball is traveling do n ard at the moment of contact. onse uently the average acceleration during contact ith the ground is

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{0-\ \ -17.1\text{ m s}}{20.0\times 10^{-3}\text{ s}} = 857\text{ m s}^2.$$

b he fact that the result is positive indicates that this acceleration vector points up ard.

LEARN ince Δt is very small it is not surprising to have a very large acceleration to stop the motion of the ball. n later chapters e shall see that the acceleration is directly related to the magnitude and direction of the force e erted by the ground on the ball during the course of collision.

56. e use . 2 16

$$v\ ^2 = v\ ^2\ \ \ 2a\ y\ \ -y$$

ith $a = -9.8$ m s^2 $y\ \ -y\ = 0.40$ m and $v\ = \frac{1}{3}\ v$. t is then straightfor ard to solve $v\ = 3.0$ m s appro imately.

57. he average acceleration during contact ith the floor is $a_{\text{avg}} =\ \ v_2 - v_1\ \ \ \Delta t$

here v_1 is its velocity ust before striking the floor v_2 is its velocity ust as it leaves the floor and Δt is the duration of contact ith the floor 12×10^{-3} s .

a aking the y a is to be positively up ard and placing the origin at the point here the ball is dropped e first find the velocity ust before striking the floor using $v_1^2 = v_0^2 - 2gy$. ith $v_0 = 0$ and $y = -4.00$ m the result is

$$v_1 = -\sqrt{-2gy} = -\sqrt{-2\ 9.8\text{ m s}^2\quad -4.00\text{ m}} = -8.85\text{ m s}$$

here the negative root is chosen because the ball is traveling do n ard. o find the velocity ust after hitting the floor as it ascends ithout air friction to a height of 2.00 m e use $v^2 = v_2^2 - 2g\ y - y_0$ ith $v = 0$ $y = -2.00$ m it ends up t o meters *below* its initial drop height and $y_0 = -4.00$ m. herefore

$$v_2 = \sqrt{2g\ y - y_0} = \sqrt{2\ 9.8\text{ m s}^2\quad -2.00\text{ m} + 4.00\text{ m}} = 6.26\text{ m s} .$$

onse uently the average acceleration is

$$a_{\text{avg}} = \frac{v_2 - v_1}{\Delta t} = \frac{6.26\text{ m s} - \ -8.85\text{ m s}}{12.0 \times 10^{-3}\text{ s}} = 1.26 \times 10^3\text{ m s}^2.$$

b he positive nature of the result indicates that the acceleration vector points up ard. n a later chapter this ill be directly related to the magnitude and direction of the force e erted by the ground on the ball during the collision.

58. e choose *down* as the y direction and set the coordinate origin at the point here it as dropped hich is hen e start the clock . e denote the 1.00 s duration mentioned in the problem as $t - t'$ here t is the value of time hen it lands and t' is one second prior to that. he corresponding distance is $y - y' = 0.50h$ here y denotes the location of the ground. n these terms y is the same as h so e have $h - y' = 0.50h$ or $0.50h = y'$.

a e find t' and t from . 2 15 ith $v_0 = 0$

$$y' = \frac{1}{2}gt'^2 \Rightarrow t' = \sqrt{\frac{2y'}{g}}$$
$$y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} .$$

lugging in $y = h$ and $y' = 0.50h$ and dividing these t o e uations e obtain

$$\frac{t'}{t} = \sqrt{\frac{2(0.50h)\ \ g}{2h\ \ g}} = \sqrt{0.50} .$$

etting $t' = t - 1.00$ units understood and cross multiplying e find

$$t - 1.00 = t\sqrt{0.50} \;\Rightarrow\; t = \frac{1.00}{1 - \sqrt{0.50}}$$

hich yields t = 3.41 s.

b lugging this result into $y = \frac{1}{2}gt^2$ e find h = 57 m.

(c) In our approach, we did not use the quadratic formula, but we did "choose a root" hen e assumed in the last calculation in part a that $\sqrt{0.50}$ = 0.707 instead of –0.707. f e had instead let $\sqrt{0.50}$ = –0.707 then our ans er for t ould have been roughly 0.6 s hich ould imply that t' = t – 1 ould e ual a negative number indicating a time *before* it as dropped hich certainly does not fit ith the physical situation described in the problem.

59. e neglect air resistance hich ustifies setting $a = -g$ = –9.8 m s^2 taking *down* as the $-y$ direction for the duration of the motion. e are allo ed to use able 2 1 ith Δy replacing Δx because this is constant acceleration motion. he ground level is taken to correspond to the origin of the y a is.

a he time drop 1 leaves the no le is taken as t = 0 and its time of landing on the floor t_1 can be computed from . 2 15 ith v_0 = 0 and y_1 = –2.00 m.

$$y_1 = -\frac{1}{2}gt_1^2 \;\Rightarrow\; t_1 = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2\;\;-2.00\text{ m}}{9.8\text{ m s}^2}} = 0.639\text{ s}\,.$$

t that moment the fourth drop begins to fall and from the regularity of the dripping e conclude that drop 2 leaves the no le at t = 0.639 3 = 0.213 s and drop 3 leaves the no le at t = 2 0.213 s = 0.426 s. herefore the time in free fall up to the moment drop 1 lands for drop 2 is $t_2 = t_1$ – 0.213 s = 0.426 s. ts position at the moment drop 1 strikes the floor is

$$y_2 = -\frac{1}{2}gt_2^2 = -\frac{1}{2}\;\;9.8\text{ m s}^2\;\;0.426\text{ s}\;\;^2 = -0.889\text{ m}$$

or about 89 cm belo the no le.

b he time in free fall up to the moment drop 1 lands for drop 3 is $t_3 = t_1$ –0.426 s = 0.213 s. ts position at the moment drop 1 strikes the floor is

$$y_3 = -\frac{1}{2}gt_3^2 = -\frac{1}{2}\;\;9.8\text{ m s}^2\;\;0.213\text{ s}\;\;^2 = -0.222\text{ m}$$

or about 22 cm belo the no le.

60. To find the "launch" velocity of the rock, we apply . 2 11 to the ma imum height here the speed is momentarily ero

$$v = v_0 - gt \quad \Rightarrow \quad 0 = v_0 - \left(9.8\text{ m s}^2\right)\left(2.5\text{ s}\right)$$

so that v_0 = 24.5 m s ith y up . o e use . 2 15 to find the height of the to er taking y_0 = 0 at the ground level

$$y - y_0 = v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad y - 0 = \left(24.5\text{ m s}\right)\left(1.5\text{ s}\right) - \frac{1}{2}\left(9.8\text{ m s}^2\right)\left(1.5\text{ s}\right)^2 .$$

hus e obtain y = 26 m.

61. e choose *down* as the y direction and place the coordinate origin at the top of the building hich has height H . uring its fall the ball passes ith velocity v_1 the top of the indo hich is at y_1 at time t_1 and passes the bottom hich is at y_2 at time t_2. e are told $y_2 - y_1$ = 1.20 m and $t_2 - t_1$ = 0.125 s. sing . 2 15 e have

$$y_2 - y_1 = v_1\left(t_2 - t_1\right) + \frac{1}{2}g\left(t_2 - t_1\right)^2$$

hich immediately yields

$$v_1 = \frac{1.20\text{ m} - \frac{1}{2}\left(9.8\text{ m s}^2\right)\left(0.125\text{ s}\right)^2}{0.125\text{ s}} = 8.99\text{ m s}.$$

rom this . 2 16 ith v_0 = 0 reveals the value of y_1

$$v_1^2 = 2gy_1 \quad \Rightarrow \quad y_1 = \frac{8.99\text{ m s}^{\;2}}{2\;9.8\text{ m s}^2} = 4.12\text{ m}.$$

t reaches the ground y_3 = H at t_3. ecause of the symmetry e pressed in the problem (“upward flight is a reverse of the fall’’) we know that $t_3 - t_2$ = 2.00 2 = 1.00 s. nd this means $t_3 - t_1$ = 1.00 s 0.125 s = 1.125 s. o . 2 15 produces

$$y_3 - y_1 = v_1\ t_3 - t_1\ + \frac{1}{2}g\ t_3 - t_1\ ^2$$

$$y_3 - 4.12\text{ m} = 8.99\text{ m s}\ \ 1.125\text{ s} + \frac{1}{2}\ 9.8\text{ m s}^2\ \ 1.125\text{ s}\ ^2$$

hich yields y_3 = H = 20.4 m.

62. he height reached by the player is y = 0.76 m here e have taken the origin of the y a is at the floor and y to be up ard .

a he initial velocity v_0 of the player is

$$v_0 = \sqrt{2gy} = \sqrt{2\ 9.8\text{ m s}^2\ \ 0.76\text{ m}} = 3.86\text{ m s}.$$

his is a conse uence of . 2 16 here velocity v vanishes. s the player reaches y_1

= 0.76 m – 0.15 m = 0.61 m his speed v_1 satisfies $v_0^2 - v_1^2 = 2gy_1$ hich yields

$$v_1 = \sqrt{v_0^2 - 2gy_1} = \sqrt{3.86 \text{ m s}\ ^2 - 2\ 9.80 \text{ m s}^2\ \ 0.61 \text{ m}} = 1.71 \text{ m s} .$$

he time t_1 that the player spends *ascending* in the top Δy_1 = 0.15 m of the ump can no be found from . 2 17

$$\Delta y_1 = \frac{1}{2}(v_1 + v)t_1 \Rightarrow t_1 = \frac{2(0.15 \text{ m})}{1.71 \text{ m s} + 0} = 0.175 \text{ s}$$

hich means that the total time spent in that top 15 cm both ascending and descending is 2 0.175 s = 0.35 s = 350 ms.

b he time t_2 hen the player reaches a height of 0.15 m is found from . 2 15

$$0.15 \text{ m} = v_0 t_2 - \frac{1}{2} g t_2^2 = 3.86 \text{ m s}\ t_2 - \frac{1}{2}\ 9.8 \text{ m s}^2\ t_2^2$$

hich yields using the uadratic formula taking the smaller of the t o positive roots t_2 = 0.041 s = 41 ms hich implies that the total time spent in that bottom 15 cm both ascending and descending is 2 41 ms = 82 ms.

63. he time t the pot spends passing in front of the indo of length L = 2.0 m is 0.25 s each ay. e use v for its velocity as it passes the top of the indo going up . hen ith $a = -g$ = –9.8 m s^2 taking *down* to be the $-y$ direction . 2 18 yields

$$L = vt - \frac{1}{2}gt^2 \quad \Rightarrow \quad v = \frac{L}{t} - \frac{1}{2}gt.$$

he distance H the pot goes above the top of the indo is therefore using . 2 16 ith the *final velocity* being ero to indicate the highest point

$$H = \frac{v^2}{2g} = \frac{(L\ \ t - gt\ \ 2)^2}{2g} = \frac{(2.00 \text{ m}\ \ 0.25 \text{ s} - \ 9.80 \text{ m s}^2\ \ 0.25 \text{ s}\ \ \ 2)^2}{2\ 9.80 \text{ m s}^2} = 2.34 \text{ m}.$$

64. he graph sho s y = 25 m to be the highest point here the speed momentarily vanishes). The neglect of "air friction" (or whatever passes for that on the distant planet is certainly reasonable due to the symmetry of the graph.

a o find the acceleration due to gravity g_p on that planet e use . 2 15 ith y up

$$y - y_0 = vt + \frac{1}{2}g_p t^2 \quad \Rightarrow \quad 25 \text{ m} - 0 = (0)(2.5 \text{ s}) + \frac{1}{2}g_p (2.5 \text{ s})^2$$

so that g_p = 8.0 m s^2.

b hat same ma point on the graph can be used to find the initial velocity.

$$y - y_0 = \frac{1}{2}(v_0 + v)t \quad \Rightarrow \quad 25\text{ m} - 0 = \frac{1}{2}\,(v_0 + 0)(2.5\text{ s})$$

herefore v_0 = 20 m s.

65. he key idea here is that the speed of the head and the torso as ell at any given time can be calculated by finding the area on the graph of the head's acceleration versus time as sho n in . 2 26

$$v_1 - v_0 \;=\; \begin{pmatrix}\text{area bet een the acceleration curve}\\ \text{and the time a is from } t_0 \text{ } t\text{o } t_1\end{pmatrix}$$

a rom ig. 2.15a e see that the head begins to accelerate from rest v_0 = 0 at t_0 = 110 ms and reaches a ma imum value of 90 m s^2 at t_1 = 160 ms. he area of this region is

$$\text{area} = \frac{1}{2}\;160 - 110\;\times 10^{-3}\text{s}\cdot\left(90\text{ m s}^2\right) = 2.25\text{ m s}$$

hich is e ual to v_1 the speed at t_1.

b o compute the speed of the torso at t_1=160 ms e divide the area into 4 regions rom 0 to 40 ms region has ero area. rom 40 ms to 100 ms region has the shape of a triangle ith area

$$\text{area} \;= \frac{1}{2}\;0.0600\text{ s}\;\;50.0\text{ m s}^2\;\; = 1.50\text{ m s}\,.$$

rom 100 to 120 ms region has the shape of a rectangle ith area

$$\text{area} \;=\; 0.0200\text{ s}\;\;50.0\text{ m s}^2\;\; = 1.00\text{ m s}.$$

rom 110 to 160 ms region has the shape of a trape oid ith area

$$\text{area} \;=\; \frac{1}{2}\;0.0400\text{ s}\;\;50.0\;+\;20.0\;\text{ m s}^2 = \;1.40\text{ m s}.$$

ubstituting these values into . 2 26 ith v_0 = 0 then gives

$$v_1 - 0 = 0 + 1.50\text{ m s}\;\;1.00\text{ m s}\;\;1.40\text{ m s} = 3.90\text{ m s}$$

or $v_1 = 3.90$ m s.

66. he key idea here is that the position of an ob ect at any given time can be calculated by finding the area on the graph of the ob ect's velocity versus time as sho n in . 2 30

$$x_1 - x_0 \;=\; \begin{pmatrix}\text{area bet een the velocity curve}\\ \text{and the time a is from } t_0 \text{ } t\text{o } t_1\end{pmatrix}.$$

a o compute the position of the fist at t = 50 ms e divide the area in ig. 2 37 into t o regions. rom 0 to 10 ms region has the shape of a triangle ith area

$$\text{area} = \frac{1}{2}\ 0.010\ \text{s} \quad 2\ \text{m s} = 0.01\ \text{m.}$$

rom 10 to 50 ms region has the shape of a trape oid ith area

$$\text{area} = \frac{1}{2}\ 0.040\ \text{s} \quad 2 \quad 4\ \ \text{m s} = 0.12\ \text{m.}$$

ubstituting these values into . 2 30 ith x_0 = 0 then gives

$$x_1 - 0 = 0 + 0.01\ \text{m} \quad 0.12\ \text{m} = 0.13\ \text{m}$$

or $x_1 = 0.13$ m.

b he speed of the fist reaches a ma imum at t_1 = 120 ms. rom 50 to 90 ms region C has the shape of a trape oid ith area

$$\text{area} = \frac{1}{2}\ 0.040\ \text{s} \quad 4 \quad 5\ \ \text{m s} = 0.18\ \text{m.}$$

rom 90 to 120 ms region has the shape of a trape oid ith area

$$\text{area} = \frac{1}{2}\ 0.030\ \text{s} \quad 5 \quad 7.5\ \ \text{m s} = 0.19\ \text{m.}$$

ubstituting these values into . 2 30 ith x_0 = 0 then gives

$$x_1 - 0 = 0 + 0.01\ \text{m} \quad 0.12\ \text{m} \quad 0.18\ \text{m} \quad 0.19\ \text{m} = 0.50\ \text{m}$$

or $x_1 = 0.50$ m.

67. he problem is solved using . 2 31

$$v_1 - v_0 = \begin{pmatrix} \text{area bet een the acceleration curve} \\ \text{and the time a is from } t_0 \text{ to } t_1 \end{pmatrix}$$

o compute the speed of the unhelmeted bare head at t_1 = 7.0 ms e divide the area under the a vs. t graph into 4 regions rom 0 to 2 ms region has the shape of a triangle ith area

$$\text{area} = \frac{1}{2}\ 0.0020\ \text{s} \quad 120\ \text{m s}^2 = 0.12\ \text{m s.}$$

rom 2 ms to 4 ms region has the shape of a trape oid ith area

$$\text{area} = \frac{1}{2}\ 0.0020\ \text{s} \quad 120 \quad 140\ \ \text{m s}^2 = 0.26\ \text{m s.}$$

rom 4 to 6 ms region has the shape of a trape oid ith area

$$\text{area} = \frac{1}{2}\ 0.0020\text{ s}\quad 140\quad 200\ \text{m s}^2 = 0.34\text{ m s.}$$

rom 6 to 7 ms region has the shape of a triangle ith area

$$\text{area} = \frac{1}{2}\ 0.0010\text{ s}\quad 200\text{ m s}^2 = 0.10\text{ m s.}$$

ubstituting these values into . 2 31 ith v_0=0 then gives

$$v_{\text{unhelmeted}} = 0.12\text{ m s} + 0.26\text{ m s} + 0.34\text{ m s} + 0.10\text{ m s} = 0.82\text{ m s.}$$

arrying out similar calculations for the helmeted head e have the follo ing results rom 0 to 3 ms region has the shape of a triangle ith area

$$\text{area} = \frac{1}{2}\ 0.0030\text{ s}\quad 40\text{ m s}^2 = 0.060\text{ m s.}$$

rom 3 ms to 4 ms region has the shape of a rectangle ith area

$$\text{area} = 0.0010\text{ s}\quad 40\text{ m s}^2 = 0.040\text{ m s.}$$

rom 4 to 6 ms region has the shape of a trape oid ith area

$$\text{area} = \frac{1}{2}\ 0.0020\text{ s}\quad 40\quad 80\ \text{m s}^2 = 0.12\text{ m s.}$$

rom 6 to 7 ms region has the shape of a triangle ith area

$$\text{area} = \frac{1}{2}\ 0.0010\text{ s}\quad 80\text{ m s}^2 = 0.040\text{ m s.}$$

ubstituting these values into . 2 31 ith v_0 = 0 then gives

$$v_{\text{helmeted}} = 0.060\text{ m s} + 0.040\text{ m s} + 0.12\text{ m s} + 0.040\text{ m s} = 0.26\text{ m s.}$$

hus the difference in the speed is

$$\Delta v = v_{\text{unhelmeted}} - v_{\text{helmeted}} = 0.82\text{ m s} - 0.26\text{ m s} = 0.56\text{ m s.}$$

68. his problem can be solved by noting that velocity can be determined by the graphical integration of acceleration versus time. he speed of the tongue of the salamander is simply e ual to the area under the acceleration curve

$$v = \text{area} = \frac{1}{2}\ 10^{-2}\text{ s}\ \ 100\text{ m s}^2 + \frac{1}{2}\ 10^{-2}\text{ s}\ \ 100\text{ m s}^2 + 400\text{ m s}^2 + \frac{1}{2}\ 10^{-2}\text{ s}\ \ 400\text{ m s}^2$$
$$= 5.0\text{ m s.}$$

69. ince $v = dx \quad dt$. 2 4 then $\Delta x = \int v\, dt$ hich corresponds to the area under the v vs t graph. ividing the total area A into rectangular base $\times$ height and triangular $\left(\frac{1}{2}\text{base}\times\text{height}\right)$ areas e have

$$\begin{aligned} A &= A_{0<t<2} + A_{2<t<10} + A_{10<t<12} + A_{12<t<16} \\ &= \frac{1}{2}\; 2 \;\; 8 \; + \; 8 \;\; 8 \; + \left(\; 2 \;\; 4 \; + \frac{1}{2}\; 2 \;\; 4 \; \right) + \; 4 \;\; 4 \end{aligned}$$

ith units understood. n this ay e obtain Δx = 100 m.

70. o solve this problem e note that velocity is e ual to the time derivative of a position function as ell as the time integral of an acceleration function ith the integration constant being the initial velocity. hus the velocity of particle 1 can be ritten as

$$v_1 = \frac{dx_1}{dt} = \frac{d}{dt}\left(6.00t^2 + 3.00t + 2.00\right) = 12.0t + 3.00 .$$

imilarly the velocity of particle 2 is

$$v_2 = v_{20} + \int a_2 dt = 20.0 + \int \; -8.00t \;\; dt = 20.0 - 4.00t^2 .$$

he condition that $v_1 = v_2$ implies

$$12.0t + 3.00 = 20.0 - 4.00t^2 \quad \Rightarrow \quad 4.00t^2 + 12.0t - 17.0 = 0$$

hich can be solved to give taking positive root $t = \; -3 + \sqrt{26} \quad 2 = 1.05\,\text{s}$. hus the velocity at this time is $v_1 = v_2 = 12.0\; 1.05 \; + 3.00 = 15.6\,\text{m}\;\text{s}$.

71. a he derivative ith respect to time of the given e pression for x yields the "velocity" of the spot:

$$v \;\; t \; = 9 - \tfrac{9}{4}\, t^2$$

ith 3 significant figures understood. t is easy to see that v = 0 hen t = 2.00 s.

b t t = 2 s x = 9 2 – 2 ^{3} = 12. hus the location of the spot hen v = 0 is 12.0 cm from left edge of screen.

c he derivative of the velocity is $a = -\,\tfrac{9}{2}\, t$ hich gives an acceleration of -9.00 cm m^2 negative sign indicating left ard hen the spot is 12 cm from the left edge of screen.

d ince v 0 for times less than t = 2 s then the spot had been moving right ard.

e s implied by our ans er to part c it moves left ard for times immediately after t = 2 s. n fact the e pression found in part a guarantees that for all t 2 v 0 (that is, until the clock is “reset” by reaching an edge).

f s the discussion in part e sho s the edge that it reaches at some t 2 s cannot be the right edge it is the left edge x = 0 . olving the e pression given in the problem statement ith x = 0 for positive t yields the ans er the spot reaches the left edge at $t = \sqrt{12}$ s ≈ 3.46 s.

72. e adopt the convention fre uently used in the te t that up is the positive y direction.

a t the highest point in the tra ectory v = 0. hus ith t = 1.60 s the e uation $v = v_0 - gt$ yields v_0 = 15.7 m s.

b ne e uation that is not dependent on our result from part a is $y - y_0 = vt \quad \frac{1}{2}gt^2$ this readily gives $y_{ma} - y_0$ = 12.5 m for the highest ma point measured relative to here it started the top of the building .

c o e use our result from part a and plug into $y - y_0 = v_0t \quad \frac{1}{2}gt^2$ ith t = 6.00 s and y = 0 the ground level . hus e have

$$0 - y_0 = \ 15.68 \text{ m s} \ \ 6.00 \text{ s} \ - \frac{1}{2} \ \ 9.8 \text{ m s}^2 \ \ 6.00 \text{ s}\ ^2.$$

herefore y_0 the height of the building is e ual to 82.3 m.

73. e denote the re uired time as t assuming the light turns green hen the clock reads ero. y this time the distances traveled by the t o vehicles must be the same.

a enoting the acceleration of the automobile as a and the constant speed of the truck as v then

$$\Delta x = \left(\frac{1}{2}at^2\right)_{\text{car}} = (vt)_{\text{truck}}$$

hich leads to

$$t = \frac{2v}{a} = \frac{2(9.5 \text{ m s})}{2.2 \text{ m s}^2} = 8.6 \text{ s} .$$

herefore

$$\Delta x = vt = (9.5 \text{ m s})(8.6 \text{ s}) = 82 \text{ m} .$$

b he speed of the car at that moment is

$$v_{\text{car}} = at = \left(2.2 \text{ m s}^2\right)(8.6 \text{ s}) = 19 \text{ m s} .$$

74. f the plane ith velocity v maintains its present course and if the terrain continues its up ard slope of 4.3 then the plane ill strike the ground after traveling

$$\Delta x = \frac{h}{\tan\theta} = \frac{35\text{ m}}{\tan 4.3^\circ} = 465.5\text{ m} \approx 0.465\text{ km}.$$

his corresponds to a time of flight found from . 2 2 ith $v = v_{avg}$ since it is constant

$$t = \frac{\Delta x}{v} = \frac{0.465\text{ km}}{1300\text{ km h}} = 0.000358\text{ h} \approx 1.3\text{ s}.$$

his then estimates the time available to the pilot to make his correction.

75. e denote t_r as the reaction time and t_b as the braking time. he motion during t_r is of the constant velocity call it v_0 type. hen the position of the car is given by

$$x = v_0 t_r + v_0 t_b + \frac{1}{2} a t_b^2$$

here v_0 is the initial velocity and a is the acceleration hich e e pect to be negative valued since e are taking the velocity in the positive direction and e kno the car is decelerating . *After* the brakes are applied the velocity of the car is given by $v = v_0$ at_b. sing this e uation ith $v = 0$ e eliminate t_b from the first e uation and obtain

$$x = v_0 t_r - \frac{v_0^2}{a} + \frac{1}{2}\frac{v_0^2}{a} = v_0 t_r - \frac{1}{2}\frac{v_0^2}{a}.$$

e rite this e uation for each of the initial velocities

$$x_1 = v_{01} t_r - \frac{1}{2}\frac{v_{01}^2}{a} \qquad x_2 = v_{02} t_r - \frac{1}{2}\frac{v_{02}^2}{a}.$$

olving these e uations simultaneously for t_r and a e get

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02}(v_{02} - v_{01})}$$

and

$$a = -\frac{1}{2}\frac{v_{02} v_{01}^2 - v_{01} v_{02}^2}{v_{02} x_1 - v_{01} x_2}.$$

a ubstituting x_1 = 56.7 m v_{01} = 80.5 km h = 22.4 m s x_2 = 24.4 m and v_{02} = 48.3 km h = 13.4 m s e find

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02}\ v_{02} - v_{01}} = \frac{13.4\text{ m s}^2\ 56.7\text{ m} - 22.4\text{ m s}^2\ 24.4\text{ m}}{22.4\text{ m s}\ 13.4\text{ m s}\ 13.4\text{ m s} - 22.4\text{ m s}}$$
$$= 0.74\text{ s}.$$

b imilarly substituting x_1 = 56.7 m v_{01} = 80.5 km h = 22.4 m s x_2 = 24.4 m and

v_{02} = 48.3 km h = 13.4 m s gives

$$a = -\frac{1}{2}\frac{v_{02}v_{01}^2 - v_{01}v_{02}^2}{v_{02}x_1 - v_{01}x_2} = -\frac{1}{2}\frac{13.4\text{ m s}\ \ 22.4\text{ m s}^{\ 2} - \ 22.4\text{ m s}\ \ 13.4\text{ m s}^{\ 2}}{13.4\text{ m s}\ \ 56.7\text{ m} \ - \ 22.4\text{ m s}\ \ 24.4\text{ m}}$$
$$= -6.2\text{ m s}^2.$$

he *magnitude* of the deceleration is therefore 6.2 m s^2. lthough rounded off values are displayed in the above substitutions hat e have input into our calculators are the "exact" values (such as $v_{02} = \frac{161}{12}$ m s .

76. a constant velocity is e ual to the ratio of displacement to elapsed time. hus for the vehicle to be traveling at a constant speed v_p over a distance D_{23} the time delay should be $t = D_{23}\ \ v_p$.

b he time re uired for the car to accelerate from rest to a cruising speed v_p is $t_0 = v_p\ \ a$. uring this time interval the distance traveled is $\Delta x_0 = at_0^2\ \ 2 = v_p^2\ \ 2a$. he car then moves at a constant speed v_p over a distance $D_{12} - \Delta x_0 - d$ to reach intersection 2 and the time elapsed is $t_1 = \ D_{12} - \Delta x_0 - d\ \ \ v_p$. hus the time delay at intersection 2 should be set to

$$t_{\text{total}} = t_r + t_0 + t_1 = t_r + \frac{v_p}{a} + \frac{D_{12} - \Delta x_0 - d}{v_p} = t_r + \frac{v_p}{a} + \frac{D_{12} - \ v_p^2\ \ 2a\ - d}{v_p}$$
$$= t_r + \frac{1}{2}\frac{v_p}{a} + \frac{D_{12} - d}{v_p}$$

77. **THINK** he speed of the rod changes due to a non ero acceleration.

EXPRESS ince the problem involves constant acceleration the motion of the rod can be readily analy ed using the e uations given in able 2 1. e take x to be in the direction of motion so

$$v = (60\text{ km}\ \ \text{h})\left(\frac{1000\text{ m}\ \ \text{km}}{3600\text{ s}\ \ \text{h}}\right) = +16.7\text{ m}\ \ \text{s}$$

and a 0. he location here the rod starts from rest $v_0 = 0$ is taken to be $x_0 = 0$.

ANALYZE a sing . 2 7 e find the average acceleration to be

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{16.7\text{ m s} - 0}{5.4\text{ s} - 0} = 3.09\text{ m s}^2.$$

b ssuming constant acceleration $a = a_{\text{avg}} = 3.09\text{ m s}^2$ the total distance traveled during the 5.4 s time interval is

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}\ 3.09\text{ m s}^2\ \ 5.4\text{ s}\ ^2 = 45\text{ m}$$

c sing . 2 15 the time re uired to travel a distance of x = 250 m is

$$x = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(250\text{ m})}{3.1\text{ m s}^2}} = 12.73\text{ s}$$

LEARN he displacement of the rod as a function of time can be ritten as $x\ t\ = \frac{1}{2}\ 3.09\text{ m s}^2\ t^2$. ote that e could have chosen . 2 17 to solve for b

$$x = \frac{1}{2}(v_0 + v)\,t = \frac{1}{2}(16.7\text{ m s})(5.4\text{ s}) = 45\text{ m}.$$

78. e take the moment of applying brakes to be t = 0. he deceleration is constant so that able 2 1 can be used. ur primed variables such as $v_0' = 72$ km h = 20 m s refer to one train moving in the x direction and located at the origin hen t = 0 and unprimed variables refer to the other moving in the $-x$ direction and located at x_0 = 950 m hen t = 0 . e note that the acceleration vector of the unprimed train points in the *positive* direction even though the train is slo ing do n its initial velocity is v_0 = –144 km h = –40 m s. ince the primed train has the lo er initial speed it should stop sooner than the other train ould ere it not for the collision . sing 2 16 it should stop meaning $v' = 0$ at

$$x' = \frac{(v')^2 - (v_0')^2}{2a'} = \frac{0 -\ \ 20\text{ m s}\ ^2}{-2\text{ m s}^2} = 200\text{ m}\ .$$

he speed of the other train hen it reaches that location is

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{(-40\text{ m s})^2 + 2\left(1.0\text{ m s}^2\right)(200\text{ m} - 950\text{ m})}$$
$$= 10\text{ m s}$$

using 2 16 again. pecifically its velocity at that moment ould be –10 m s since it is still traveling in the $-x$ direction hen it crashes. f the computation of v had failed meaning that a negative number ould have been inside the s uare root then e ould have looked at the possibility that there as no collision and e amined ho far apart they finally ere. concern that can be brought up is hether the primed train collides before it comes to rest this can be studied by computing the time it stops . 2 11 yields t = 20 s and seeing here the unprimed train is at that moment . 2 18 yields x = 350 m still a good distance a ay from contact .

79. he y coordinate of iton 1 obeys $y - y_{01} = -\frac{1}{2}\,g\,t^2$ here y = 0 hen t = 3.0 s. his allo s us to solve for y_{01} and e find y_{01} = 44.1 m. he graph for the coordinate of iton 2 hich is thro n apparently at t = 1.0 s ith velocity v_1 is

$$y - y_{02} = v_1\ t{-}1.0\ - \tfrac{1}{2} g\ \ t - 1.0\ ^2$$

here $y_{02} = y_{01}$ 10 = 54.1 m and here again $y = 0$ hen $t = 3.0$ s. hus e obtain $v_1| = 17$ m s appro imately.

80. e take x in the direction of motion. e use subscripts 1 and 2 for the data. hus $v_1 =$ 30 m s $v_2 =$ 50 m s and $x_2 - x_1 =$ 160 m.

a sing these subscripts . 2 16 leads to

$$a = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{50 \text{ m s}\ ^2 - \ 30 \text{ m s}\ ^2}{2(160 \text{ m})} = 5.0 \text{ m s}^2 .$$

b e find the time interval corresponding to the displacement $x_2 - x_1$ using . 2 17

$$t_2 - t_1 = \frac{2(x_2 - x_1)}{v_1 + v_2} = \frac{2(160 \text{ m})}{30 \text{ m s} + 50 \text{ m s}} = 4.0 \text{ s} .$$

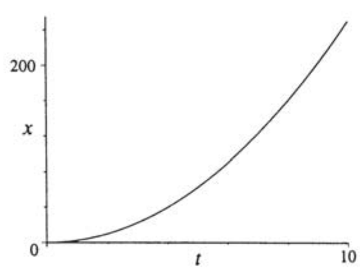
c ince the train is at rest $v_0 = 0$ hen the clock starts e find the value of t_1 from . 2 11

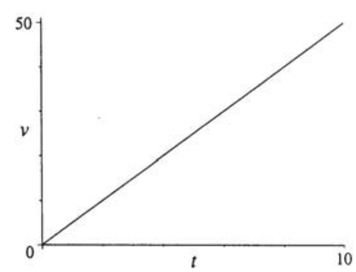
$$v_1 = v_0 + at_1 \ \ \Rightarrow \ \ t_1 = \frac{30 \text{ m s}}{5.0 \text{ m s}^2} = 6.0 \text{ s} .$$

d he coordinate origin is taken to be the location at hich the train as initially at rest so $x_0 = 0$. hus e are asked to find the value of x_1. lthough any of several e uations could be used e choose . 2 17

$$x_1 = \frac{1}{2}(v_0 + v_1)t_1 = \frac{1}{2}(30 \text{ m s})(6.0 \text{ s}) = 90 \text{ m} .$$

e he graphs are sho n belo ith units understood.





81. **THINK** he particle undergoes a *non-constant* acceleration along the x a is. n integration is re uired to calculate velocity.

EXPRESS ith a non constant acceleration $a\ t\ = dv\ \ dt$ the velocity of the particle at time t_1 is given by . 2 27 $v_1 = v_0 + \int_{t_0}^{t_1} a\ t\ dt$ here v_0 is the velocity at time t_0. n our situation e have $a = 5.0t$. n addition e also kno that $v_0 = 17$ m s at $t_0 = 2.0$ s.

ANALYZE ntegrating from t = 2 s to variable t = 4 s the acceleration to get the velocity and using the values given in the problem leads to

$$v = v_0 + \int_{t_0}^{t} adt = v_0 + \int_{t_0}^{t} 5.0t\ dt = v_0 + \frac{1}{2}\ 5.0\ \ t^2 - t_0^2\ = 17\ \ \tfrac{1}{2}\ 5.0\ \ 4^2 - 2^2\ = 47 \text{ m s.}$$

LEARN he velocity of the particle as a function of t is

$$v\ t\ = v_0 + \frac{1}{2}\ 5.0\ \ t^2 - t_0^2\ = 17 + \frac{1}{2}\ 5.0\ \ t^2 - 4\ = 7 + 2.5t^2$$

in units m s . ince the acceleration is linear in t e e pect the velocity to be uadratic in t and the displacement to be cubic in t.

82. he velocity v at t = 6 units and t o significant figures understood is $v_{\text{given}} + \int_{-2}^{6} adt$. uick ay to implement this is to recall the area of a triangle $\frac{1}{2}$ base height . he result is v = 7 m s 32 m s = 39 m s.

83. he ob ect once it is dropped v_0 = 0 is in free fall $a = -g$ = –9.8 m s^2 if e take *down* as the $-y$ direction and e use . 2 15 repeatedly.

a he positive distance D from the lo er dot to the mark corresponding to a certain reaction time t is given by $\Delta y = -D = -\frac{1}{2}gt^2$ or $D = gt^2$ 2. hus for $t_1 = 50.0$ ms

$$D_1 = \frac{(9.8 \text{ m s}^2)(50.0 \times 10^{-3} \text{ s})^2}{2} = 0.0123 \text{ m} = 1.23 \text{ cm.}$$

b or t_2 = 100 ms $D_2 = \dfrac{(9.8 \text{m s}^2)\ (100 \times 10^{-3} \text{ s})^2}{2} = 0.049 \text{ m} = 4D_1$.

c or t_3 = 150 ms $D_3 = \dfrac{(9.8 \text{m s}^2)\ (150 \times 10^{-3} \text{ s})^2}{2} = 0.11 \text{ m} = 9D_1$.

d or t_4 = 200 ms $D_4 = \dfrac{(9.8 \text{m s}^2)\ (200 \times 10^{-3} \text{ s})^2}{2} = 0.196 \text{ m} = 16D_1$.

e or t_4 = 250 ms $D_5 = \dfrac{(9.8 \text{ m s}^2)(250 \times 10^{-3} \text{ s})^2}{2} = 0.306 \text{ m} = 25D_1$.

84. e take the direction of motion as x take x_0 = 0 and use units so v = 1600 1000 3600 = 444 m s.

a uation 2 11 gives 444 = a 1.8 or a = 247 m s^2. e e press this as a multiple of g by setting up a ratio

$$a = \left(\frac{247 \text{ m s}^2}{9.8 \text{ m s}^2} \right) g = 25g.$$

b uation 2 17 readily yields

$$x = \frac{1}{2}(v_0 + v)\, t = \frac{1}{2}(444 \text{ m s})(1.8 \text{ s}) = 400 \text{ m}.$$

85. et D be the distance up the hill. hen

$$\text{average speed} = \frac{\text{total distance traveled}}{\text{total time of travel}} = \frac{2D}{\dfrac{D}{20 \text{ km h}} \quad \dfrac{D}{35 \text{ km h}}} \approx 25 \text{ km h}.$$

86. e obtain the velocity by integration of the acceleration

$$v - v_0 = \int_0^t 6.1 - 1.2t' \; dt'.$$

engths are in meters and times are in seconds. he student is encouraged to look at the discussion in ection 2 7 to better understand the manipulations here.

a he result of the above calculation is $v = v_0 + 6.1t - 0.6t^2$ here the problem states that v_0 = 2.7 m s. he ma imum of this function is found by kno ing hen its derivative the acceleration is ero a = 0 hen t = 6.1 1.2 = 5.1 s and plugging that value of t into the velocity e uation above. hus e find $v = 18$ m s.

b e integrate again to find x as a function of t

$$x - x_0 = \int_0^t v\, dt' = \int_0^t v_0 + 6.1t' - 0.6t'^2 \; dt' = v_0 t + 3.05t^2 - 0.2t^3.$$

ith x_0 = 7.3 m e obtain x = 83 m for t = 6. his is the correct ans er but one has the right to orry that it might not be after all the problem asks for the total distance traveled and $x - x_0$ is ust the *displacement* . f the cyclist backtracked then his total distance ould be greater than his displacement. hus e might ask did he backtrack o do so ould re uire that his velocity be momentarily ero at some point as he reversed his direction of motion . e could solve the above uadratic e uation for velocity for a positive value of t here v = 0 if e did e ould find that at t = 10.6 s a reversal does indeed happen. o ever in the time interval e are concerned ith in our problem ($0 \le t \le 6$ s), there is no reversal and the displacement is the same as the total distance traveled.

87. **THINK** In this problem we're given two different speeds, and asked to find the difference in their travel times.

EXPRESS he time is takes to travel a distance d ith a speed v_1 is $t_1 = d \; v_1$. imilarly ith a speed v_2 the time ould be $t_2 = d \; v_2$. he t o speeds in this problem are

$$v_1 = 55 \text{ mi h} = 55 \text{ mi h} \; \frac{1609 \text{ m mi}}{3600 \text{ s h}} = 24.58 \text{ m s}$$

$$v_2 = 65 \text{ mi h} = 65 \text{ mi h} \; \frac{1609 \text{ m mi}}{3600 \text{ s h}} = 29.05 \text{ m s}$$

ANALYZE ith $d = 700 \text{ km} = 7.0 \times 10^5 \text{ m}$ the time difference bet een the t o is

$$\Delta t = t_1 - t_2 = d \left(\frac{1}{v_1} - \frac{1}{v_2} \right) = 7.0 \times 10^5 \text{ m} \left(\frac{1}{24.58 \text{ m s}} - \frac{1}{29.05 \text{ m s}} \right) = 4383 \text{ s}$$

$$= 73 \text{ min}$$

or about 1.2 h.

LEARN he travel time as reduced from 7.9 h to 6.9 h. riving at higher speed ithin the legal limit reduces travel time.

88. he acceleration is constant and e may use the e uations in able 2 1.

a aking the first point as coordinate origin and time to be ero hen the car is there e apply . 2 17

$$x = \frac{1}{2} \left(v + v_0 \right) t = \frac{1}{2} \left(15.0 \text{ m s} + v_0 \right) \left(6.00 \text{ s} \right).$$

ith x = 60.0 m hich takes the direction of motion as the x direction e solve for the initial velocity v_0 = 5.00 m s.

b ubstituting v = 15.0 m s v_0 = 5.00 m s and t = 6.00 s into $a = \; v - v_0 \;\; t$. 2 11 e find a = 1.67 m s^2.

c ubstituting v = 0 in $v^2 = v_0^2 + 2ax$ and solving for x e obtain

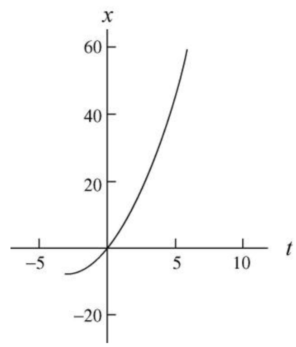
$$x = -\frac{v_0^2}{2a} = -\frac{5.00 \text{ m s}^{\;2}}{2\left(1.67 \text{ m s}^2\right)} = -7.50 \text{ m}$$

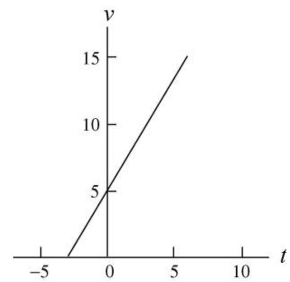
or $x \;\; = 7.50$ m.

d he graphs re uire computing the time hen v = 0 in hich case e use $v = v_0 \;\; at'$ = 0. hus

$$t' = \frac{-v_0}{a} = \frac{-5.00 \text{ m s}}{1.67 \text{ m s}^2} = -3.0 \text{ s}$$

indicates the moment the car as at rest. units are understood.





89. **THINK** n this problem e e plore the connection bet een the ma imum height an ob ect reaches under the influence of gravity and the total amount of time it stays in air.

EXPRESS eglecting air resistance and setting $a = -g = -9.8$ m s^2 taking *down* as the $-y$ direction for the duration of the motion e analy e the motion of the ball using able 2 1 ith Δy replacing Δx . e set $y_0 = 0$. pon reaching the ma imum height H the speed of the ball is momentarily ero $v = 0$. herefore e can relate its initial speed v_0 to H via the e uation

$$0 = v^2 = v_0^2 \; - 2gH \quad \Rightarrow \quad v_0 = \sqrt{2gH} \, .$$

he time it takes for the ball to reach ma imum height is given by $v = v_0 - gt = 0$ or $t = v_0 \;\; g = \sqrt{2H \;\; g}$.

ANALYZE f e ant the ball to spend t ice as much time in air as before i.e. $t' = 2t$ then the ne ma imum height H' it must reach is such that $t' = \sqrt{2H' \;\; g}$. olving for H' e obtain

$$H' = \frac{1}{2}gt'^2 = \frac{1}{2}g \;\; 2t \;\; ^2 = 4\left(\frac{1}{2}gt^2\right) = 4H \, .$$

LEARN ince $H \sim t^2$ doubling t means that H must increase fourfold. ote also that for $t' = 2t$ the initial speed must be t ice the original speed $v_0' = 2v_0$.

90. a sing the fact that the area of a triangle is $\frac{1}{2}$ base height and the fact that the integral corresponds to the area under the curve e find from t = 0 through t = 5 s the integral of v ith respect to t is 15 m. ince e are told that x_0 = 0 then e conclude that x = 15 m hen t = 5.0 s.

b e see directly from the graph that v = 2.0 m s hen t = 5.0 s.

c ince $a = dv \; dt$ = slope of the graph e find that the acceleration during the interval 4 t 6 is uniformly e ual to –2.0 m s^2.

d hinking of $x \;\; t$ in terms of accumulated area on the graph e note that x 1 = 1 m using this and the value found in part a . 2 2 produces

$$v_{\text{avg}} = \frac{x\ 5\ \ - x\ 1}{5-1} = \frac{15\text{ m} - 1\text{ m}}{4\text{ s}} = 3.5\text{ m s}.$$

e rom . 2 7 and the values $v\ t$ e read directly from the graph e find

$$a_{\text{avg}} = \frac{v\ 5\ \ - v\ 1}{5-1} = \frac{2\text{ m s} - 2\text{ m s}}{4\text{ s}} = 0.$$

91. aking the y direction *downward* and $y_0 = 0$ e have $y = v_0 t + \frac{1}{2} g t^2$ hich ith $v_0 = 0$ yields $t = \sqrt{2y\ \ g}$.

a or this part of the motion $y_1 = 50$ m so that $t_1 = \sqrt{\frac{2\ 50\text{ m}}{9.8\text{ m s}^2}} = 3.2\text{ s}$.

b or this ne t part of the motion e note that the total displacement is $y_2 = 100$ m. herefore the total time is

$$t_2 = \sqrt{\frac{2\ 100\text{ m}}{9.8\text{ m s}^2}} = 4.5\text{ s} .$$

he difference bet een this and the ans er to part a is the time re uired to fall through that second 50 m distance $\Delta t = t_2 - t_1 = 4.5\text{ s} - 3.2\text{ s} = 1.3\text{ s}$.

92. irection of x is implicit in the problem statement. he initial position hen the clock starts is $x_0 = 0$ here $v_0 = 0$ the end of the speeding up motion occurs at $x_1 = 1100\ 2 = 550$ m and the sub ay train comes to a halt $v_2 = 0$ at $x_2 = 1100$ m.

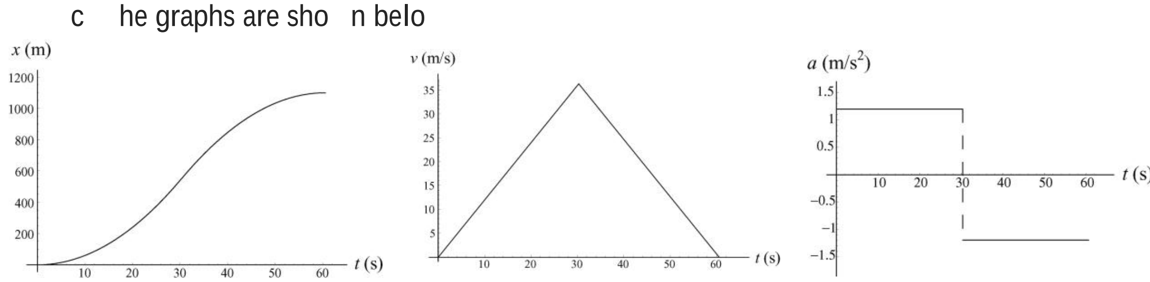
a sing . 2 15 the sub ay train reaches x_1 at

$$t_1 = \sqrt{\frac{2x_1}{a_1}} = \sqrt{\frac{2(550\text{ m})}{1.2\text{ m s}^2}} = 30.3\text{ s} .$$

he time interval $t_2 - t_1$ turns out to be the same value most easily seen using . 2 18 so the total time is $t_2 = 2\ 30.3\ = 60.6$ s.

b ts ma imum speed occurs at t_1 and e uals $v_1 = v_0 + a_1 t_1 = 36.3\text{ m}\ \ \text{s}$.

c he graphs are sho n belo



93. e neglect air resistance hich ustifies setting $a = -g = -9.8$ m s^2 taking *down* as the $-y$ direction) for the duration of the stone's motion. We are allowed to use Table 2 1 ith Δx replaced by y because the ball has constant acceleration motion and e choose $y_0 = 0$.

a e apply . 2 16 to both measurements ith units understood.

$$v_B^2 = v_0^2 - 2gy_B \;\Rightarrow\; \left(\frac{1}{2}v\right)^2 + 2g\left(y_A + 3\right) = v_0^2$$

$$v_A^2 = v_0^2 - 2gy_A \;\Rightarrow\; v^2 + 2gy_A = v_0^2$$

e e uate the t o e pressions that each e ual v_0^2 and obtain

$$\frac{1}{4}v^2 + 2gy_A + 2g(3) = v^2 + 2gy_A \;\Rightarrow\; 2g(3) = \frac{3}{4}v^2$$

hich yields $v = \sqrt{2g(4)} = 8.85$ m s.

b n ob ect moving up ard at A ith speed $v = 8.85$ m s ill reach a ma imum height $y - y_A = v^2$ $2g = 4.00$ m above point A this is again a conse uence of . 2 16 now with the "final" velocity set to zero to indicate the highest point). hus the top of its motion is 1.00 m above point B.

94. e neglect air resistance hich ustifies setting $a = -g = -9.8$ m s^2 taking *down* as the $-y$ direction for the duration of the motion. e are allo ed to use able 2 1 ith Δy replacing Δx because this is constant acceleration motion. he ground level is taken to correspond to the origin of the y a is. he total time of fall can be computed from . 2 15 using the uadratic formula .

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \;\Rightarrow\; t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

ith the positive root chosen. ith $y = 0$ $v_0 = 0$ and $y_0 = h = 60$ m e obtain

$$t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}} = 3.5 \text{ s} .$$

Thus, "1.2 s earlier" means we are examining where the rock is at $t = 2.3$ s

$$y - h = v_0 \; 2.3\,\text{s} \; - \frac{1}{2}g \; 2.3\,\text{s} \;^2 \;\Rightarrow\; y = 34 \text{ m}$$

here e again use the fact that $h = 60$ m and $v_0 = 0$.

95. **THINK** his problem involves analy ing a plot describing the position of an iceboat as function of time. he boat has a non ero acceleration due to the ind.

EXPRESS ince e are told that the acceleration of the boat is constant the e uations of able 2 1 can be applied. o ever the challenge here is that v_0 v and a are not e plicitly given. ur strategy to deduce these values is to apply the kinematic e uation $x - x_0 = v_0 t + \frac{1}{2}at^2$ to a variety of points on the graph and solve for the unkno ns from the simultaneous e uations.

ANALYZE a rom the graph e pick t o points on the curve t x = 2.0 s 16 m and 3.0 s 27 m . he corresponding simultaneous e uations are

$$16\text{ m} - 0 = v_0\ 2.0\text{ s} \quad \frac{1}{2}\ a\ 2.0\text{ s}^{\ 2}$$

$$27\text{ m} - 0 = v_0\ 3.0\text{ s} \quad \frac{1}{2}\ a\ 3.0\text{ s}^{\ 2}$$

olving the e uations lead to the values v_0 = 6.0 m s and a = 2.0 m s^2.

b rom able 2 1

$$x - x_0 = vt - \frac{1}{2}at^2 \quad \Rightarrow \quad 27\text{ m} - 0 = v\ 3.0\text{ s} \ - \frac{1}{2}\ 2.0\text{ m s}^2\ \ 3.0\text{ s}^{\ 2}$$

hich leads to v = 12 m s.

c ssuming the ind continues during $3.0 \le t \le 6.0$, we apply $x - x_0 = v_0 t \quad \frac{1}{2}at^2$ to this interval here v_0 = 12.0 m s from part b to obtain

$$\Delta x = \ 12.0\text{ m s}\ \ 3.0\text{ s} \quad \frac{1}{2}\ 2.0\text{ m s}^2\ \ 3.0\text{ s}^{\ 2} = 45\text{ m}.$$

LEARN y using the results obtained in a the position and velocity of the iceboat as a function of time can be ritten as

$$x\ t\ = \ 6.0\text{ m s}\ t + \frac{1}{2}\ 2.0\text{ m s}^2\ t^2 \quad \text{and} \quad v\ t\ = \ 6.0\text{ m s}\ + \ 2.0\text{ m s}^2\ t.$$

ne can readily verify that the same ans ers are obtained for b and c using the above e pressions for $x\ t$ and $v\ t$.

96. a et the height of the diving board be h. e choose *down* as the y direction and set the coordinate origin at the point here it as dropped hich is hen e start the clock . hus $y = h$ designates the location here the ball strikes the ater. et the depth of the lake be D and the total time for the ball to descend be T. he speed of the ball as it reaches the surface of the lake is then $v = \sqrt{2gh}$ from . 2 16 and the time for the ball to fall from the board to the lake surface is t_1 = $\sqrt{2h\ \ g}$ from . 2 15 . o the time it spends descending in the lake at constant velocity v is

$$t_2 = \frac{D}{v} = \frac{D}{\sqrt{2gh}}.$$

hus $T = t_1 \quad t_2 = \sqrt{\frac{2h}{g}} \quad \frac{D}{\sqrt{2gh}}$ hich gives

$$D = T\sqrt{2gh} - 2h = (4.80\text{ s})\sqrt{(2)(9.80\text{ m s}^2)(5.20\text{ m})} - 2(5.20\text{ m}) = 38.1\text{ m}.$$

b sing . 2 2 the magnitude of the average velocity is

$$v_{avg} = \frac{D+h}{T} = \frac{38.1\text{ m} + 5.20\text{ m}}{4.80\text{ s}} = 9.02\text{ m s}$$

c n our coordinate choices a positive sign for v_{avg} means that the ball is going do n ard. f ho ever up ard had been chosen as the positive direction then this ans er in b ould turn out negative valued.

d e find v_0 from $\Delta y = v_0 t + \frac{1}{2} g t^2$ ith $t = T$ and $\Delta y = h \quad D$. hus

$$v_0 = \frac{h+D}{T} - \frac{gT}{2} = \frac{5.20\text{ m} + 38.1\text{ m}}{4.80\text{ s}} - \frac{(9.8\text{ m s}^2)(4.80\text{ s})}{2} = 14.5\text{ m s}$$

e ere in our coordinate choices the negative sign means that the ball is being thro n up ard.

97. e choose *down* as the y direction and use the e uations of able 2 1 replacing x ith y ith $a = \quad g$ $v_0 = 0$ and $y_0 = 0$. e use subscript 2 for the elevator reaching the ground and 1 for the half ay point.

a uation 2 16 $v_2^2 = v_0^2 + 2a(y_2 - y_0)$ leads to

$$v_2 = \sqrt{2gy_2} = \sqrt{2(9.8\text{ m s}^2)(120\text{ m})} = 48.5\text{ m s}.$$

b he time at hich it strikes the ground is using . 2 15

$$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2(120\text{ m})}{9.8\text{ m s}^2}} = 4.95\text{ s}.$$

c o . 2 16 in the form $v_1^2 = v_0^2 + 2a(y_1 - y_0)$ leads to

$$v_1 = \sqrt{2gy_1} = \sqrt{2\ 9.8\ \text{ m s}^2\ \ 60\text{ m}} = 34.3\text{m s}.$$

d he time at hich it reaches the half ay point is using . 2 15

$$t_1 = \sqrt{\frac{2y_1}{g}} = \sqrt{\frac{2\ 60\text{ m}}{9.8\text{ m s}^2}} = 3.50\text{ s .}$$

98. aking y to be up ard and placing the origin at the point from hich the ob ects are dropped then the location of diamond 1 is given by $y_1 = -\frac{1}{2}gt^2$ and the location of diamond 2 is given by $y_2 = -\frac{1}{2}g(t-1)^2$. e are starting the clock hen the first ob ect is dropped. e ant the time for hich $y_2 - y_1$ = 10 m. herefore

$$-\frac{1}{2}g(t-1)^2 + \frac{1}{2}gt^2 = 10 \quad \Rightarrow \quad t = (10\ \ g) + 0.5 = 1.5\text{ s.}$$

99. ith y up ard e have y_0 = 36.6 m and y = 12.2 m. herefore using . 2 18 the last e uation in able 2 1 e find

$$y - y_0 = vt + \frac{1}{2}gt^2 \quad \Rightarrow \quad v = -22.0\text{ m s}$$

at t = 2.00 s. he term *speed* refers to the magnitude of the velocity vector so the ans er is v = 22.0 m s.

100. uring free fall e ignore the air resistance and set $a = -g = -9.8$ m s^2 here e are choosing *down* to be the $-y$ direction. he initial velocity is ero so that . 2 15 becomes $\Delta y = -\frac{1}{2}gt^2$ here Δy represents the *negative* of the distance d she has fallen. hus e can rite the e uation as $d = \frac{1}{2}gt^2$ for simplicity.

a he time t_1 during hich the parachutist is in free fall is using . 2 15 given by

$$d_1 = 50\text{ m} = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.80\text{ m}\ \ \text{s}^2)\,t_1^2$$

hich yields t_1 = 3.2 s. he *speed* of the parachutist ust before he opens the parachute is given by the positive root $v_1^2 = 2gd_1$ or

$$v_1 = \sqrt{2gh_1} = \sqrt{(2)(9.80\text{ m}\ \ \text{s}^2)(50\text{ m})} = 31\text{ m}\ \ \text{s.}$$

f the final speed is v_2 then the time interval t_2 bet een the opening of the parachute and the arrival of the parachutist at the ground level is

$$t_2 = \frac{v_1 - v_2}{a} = \frac{31\text{ m}\ \ \text{s} - 3.0\text{ m}\ \ \text{s}}{2\text{ m}\ \ \text{s}^2} = 14\text{ s.}$$

his is a result of . 2 11 here *speeds* are used instead of the negative valued velocities so that final velocity minus initial velocity turns out to e ual initial speed minus final speed e also note that the acceleration vector for this part of the motion is positive since it points up ard opposite to the direction of motion — hich makes it a deceleration . he total time of flight is therefore t_1 t_2 = 17 s.

b he distance through hich the parachutist falls after the parachute is opened is given by

$$d=\frac{v_1^2-v_2^2}{2a}=\frac{(31\text{m s})^2-(3.0\text{ m s})^2}{(2)(2.0\text{ m s}^2)}\approx 240\text{ m}.$$

n the computation e have used . 2 16 ith both sides multiplied by –1 hich changes the negative valued Δy into the positive d on the left hand side and s itches the order of v_1 and v_2 on the right hand side . hus the fall begins at a height of h = 50 $d \approx$ 290 m.

101. e neglect air resistance hich ustifies setting $a = -g = -9.8$ m s^2 taking do n as the $-y$ direction for the duration of the motion. e are allo ed to use able 2 1 ith Δy replacing Δx because this is constant acceleration motion. he ground level is taken to correspond to y = 0.

a ith $y_0 = h$ and v_0 replaced ith $-v_0$. 2 16 leads to

$$v=\sqrt{\ -v_0\ ^2-2g\ y-y_0\ }=\sqrt{v_0^2+2gh}\,.$$

he positive root is taken because the problem asks for the speed the *magnitude* of the velocity .

b e use the uadratic formula to solve . 2 15 for t ith v_0 replaced ith $-v_0$

$$\Delta y\ =\ -v_0t-\frac{1}{2}gt^2\ \Rightarrow\ t=\frac{-v_0+\sqrt{\ -v_0\ ^2-2g\Delta y}}{g}$$

here the positive root is chosen to yield t 0. ith y = 0 and $y_0 = h$ this becomes

$$t=\frac{\sqrt{v_0^2+2gh}-v_0}{g}.$$

c f it ere thro n up ard ith that speed from height h then in the absence of air friction it ould return to height h ith that same do n ard speed and ould therefore yield the same final speed before hitting the ground as in part a . n important perspective related to this is treated later in the book in the conte t of energy conservation .

d aving to travel up before it starts its descent certainly re uires more time than in part b . he calculation is uite similar ho ever e cept for no having v_0 in the e uation here e had put in $-v_0$ in part b . he details follo

$$\Delta y=v_0t-\frac{1}{2}gt^2\quad\Rightarrow\quad t=\frac{v_0+\sqrt{v_0^2-2g\Delta y}}{g}$$

ith the positive root again chosen to yield t 0. ith y = 0 and y_0 = h e obtain

$$t = \frac{\sqrt{v_0^2 + 2gh} + v_0}{g}.$$

102. e assume constant velocity motion and use . 2 2 ith v_{avg} = v 0 . herefore

$$\Delta x = v\Delta t = \left(303 \frac{\text{km}}{\text{h}} \left(\frac{1000\text{ m}\quad\text{km}}{3600\text{ s}\quad\text{h}}\right)\right) \left(100 \times 10^{-3}\text{ s}\right) = 8.4\text{ m}.$$

103. ssuming the hori ontal velocity of the ball is constant the hori ontal displacement is $\Delta x = v\Delta t$ here Δx is the hori ontal distance traveled Δt is the time and v is the hori ontal velocity. onverting v to meters per second e have 160 km h = 44.4 m s. hus

$$\Delta t = \frac{\Delta x}{v} = \frac{18.4\text{ m}}{44.4\text{ m}\quad\text{s}} = 0.414\text{ s}.$$

he velocity unit conversion implemented above can be figured "from basics" (1000 m = 1 km 3600 s = 1 h or found in ppendi .

104. n this solution e make use of the notation x t for the value of x at a particular t. hus x t = 50t 10t^2 ith units meters and seconds understood.

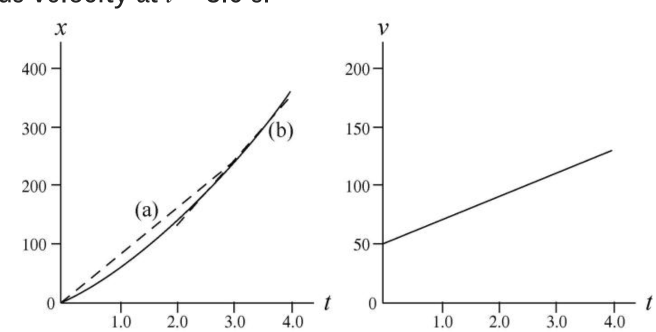
a he average velocity during the first 3 s is given by

$$v_{avg} = \frac{x\ 3\ - x\ 0}{\Delta t} = \frac{50\ \ 3\ +\ 10\ \ 3\ ^2 - 0}{3} = 80\text{ m}\quad\text{s}.$$

b he instantaneous velocity at time t is given by v = dx dt = 50 20t in units. t t = 3.0 s v = 50 20 3.0 = 110 m s.

c he instantaneous acceleration at time t is given by a = dv dt = 20 m s^2. t is constant so the acceleration at any time is 20 m s^2.

d and e he graphs that follo sho the coordinate x and velocity v as functions of time ith units understood. he dashed line marked a in the first graph runs from t = 0 x = 0 to t = 3.0s x = 240 m. ts slope is the average velocity during the first 3s of motion. he dashed line marked b is tangent to the x curve at t = 3.0 s. ts slope is the instantaneous velocity at t = 3.0 s.



105. e take x in the direction of motion so v_0 = 30 m s v_1 = 15 m s and a 0. he acceleration is found from . 2 11 a = $v_1 - v_0$ t_1 here t_1 = 3.0 s. his gives a = –5.0 m s^2. he displacement hich in this situation is the same as the distance traveled to the point it stops v_2 = 0 is using . 2 16

$$v_2^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad \Delta x = -\frac{30 \text{ m s}^{\ 2}}{2 \ -5 \text{ m s}^2} = 90 \text{ m}.$$

106. he problem consists of t o constant acceleration parts part 1 ith v_0 = 0 v = 6.0 m s x = 1.8 m and x_0 = 0 if e take its original position to be the coordinate origin and part 2 ith v_0 = 6.0 m s v = 0 and a_2 = –2.5 m s^2 negative because e are taking the positive direction to be the direction of motion .

a e can use . 2 17 to find the time for the first part

$$x - x_0 = \frac{1}{2} v_0 \quad v \ \ t_1 \quad \Rightarrow \quad 1.8 \text{ m} - 0 = \frac{1}{2} 0 \quad 6.0 \text{ m s} \ \ t_1$$

so that t_1 = 0.6 s. nd . 2 11 is used to obtain the time for the second part

$$v = v_0 + a_2 t_2 \quad \Rightarrow \quad 0 = 6.0 \text{ m s} \quad -2.5 \text{ m s}^2 \ t_2$$

from hich t_2 = 2.4 s is computed. hus the total time is t_1 t_2 = 3.0 s.

b e already kno the distance for part 1. e could find the distance for part 2 from several of the e uations but the one that makes no use of our part a results is . 2 16

$$v^2 = v_0^2 + 2a_2\Delta x_2 \quad \Rightarrow \quad 0 = 6.0 \text{ m s}^{\ 2} \quad 2 \ -2.5 \text{ m s}^2 \ \Delta x_2$$

hich leads to Δx_2 = 7.2 m. herefore the total distance traveled by the shuffleboard disk is 1.8 7.2 m = 9.0 m.

107. he time re uired is found from . 2 11 or suitably interpreted . 2 7 . irst e convert the velocity change to units

$$\Delta v = 100 \text{ km} \ \ \text{h} \left(\frac{1000 \text{ m} \ \ \text{km}}{3600 \text{ s} \ \ \text{h}} \right) = 27.8 \text{ m} \ \ \text{s}.$$

hus $\Delta t = \Delta v \ a$ = 27.8 50 = 0.556 s.

108. rom able 2 1 $v^2 - v_0^2 = 2a\Delta x$ is used to solve for a. ts minimum value is

$$a_{\text{min}} = \frac{v_2 - v_0^2}{2\Delta x_{\text{ma}}} = \frac{360 \text{ km} \ \ \text{h}^{\ 2}}{2 \ 1.80 \text{ km}} = 36000 \text{ km} \ \ \text{h}^2$$

hich converts to 2.78 m s^2.

109. a or the automobile Δv = 55 – 25 = 30 km h hich e convert to units

$$a = \frac{\Delta v}{\Delta t} = \frac{30 \text{ km h} \left(\frac{1000 \text{ m km}}{3600 \text{ s h}}\right)}{0.50 \text{ min } 60 \text{ s min}} = 0.28 \text{ m s}^2 .$$

b he change of velocity for the bicycle for the same time is identical to that of the car so its acceleration is also 0.28 m s^2.

110. onverting to units e have v = 3400 1000 3600 = 944 m s presumed constant and Δt = 0.10 s. hus $\Delta x = v\Delta t$ = 94 m.

111. his problem consists of t o parts part 1 ith constant acceleration so that the e uations in able 2 1 apply v_0 = 0 v = 11.0 m s x = 12.0 m and x_0 = 0 adopting the starting line as the coordinate origin and part 2 ith constant velocity so that $x - x_0 = vt$ applies ith v = 11.0 m s x_0 = 12.0 and x = 100.0 m.

a e obtain the time for part 1 from . 2 17

$$x - x_0 = \frac{1}{2}(v_0 + v)\, t_1 \;\Rightarrow\; 12.0 - 0 = \frac{1}{2}(0 + 11.0)t_1$$

so that t_1 = 2.2 s and e find the time for part 2 simply from 88.0 = 11.0 $t_2 \rightarrow t_2$ = 8.0 s. herefore the total time is t_1 t_2 = 10.2 s.

b ere the total time is re uired to be 10.0 s and e are to locate the point x_p here the runner s itches from accelerating to proceeding at constant speed. he e uations for parts 1 and 2 used above therefore become

$$x_p - 0 = \tfrac{1}{2}(0 + 11.0 \text{ m s})t_1$$
$$100.0 \text{ m} - x_p = (11.0 \text{ m s})(10.0 \text{ s} - t_1)$$

here in the latter e uation e use the fact that $t_2 = 10.0 - t_1$. olving the e uations for the t o unkno ns e find that t_1 = 1.8 s and x_p = 10.0 m.

112. he bullet starts at rest v_0 = 0 and after traveling the length of the barrel Δx=1.2 m emerges ith the given velocity v = 640 m s here the direction of motion is the positive direction. urning to the constant acceleration e uations in able 2 1 e use $\Delta x = \frac{1}{2}\ v_0 + v\ \ t$. hus e find t = 0.00375 s or 3.75 ms .

113. here is no air resistance hich makes it uite accurate to set $a = -g = -9.8$ m s^2 here do n ard is the $-y$ direction for the duration of the fall. e are allo ed to use able 2 1 ith Δy replacing Δx because this is constant acceleration motion in fact hen the acceleration changes during the process of catching the ball e ill again assume constant acceleration conditions in this case e have a_2 = 25g = 245 m s^2.

a he time of fall is given by . 2 15 ith v_0 = 0 and y = 0. hus

$$t=\sqrt{\frac{2y_0}{g}}=\sqrt{\frac{2\ 145\text{ m}}{9.8\text{ m s}^2}}=5.44\text{ s.}$$

b he final velocity for its free fall hich becomes the initial velocity during the catching process is found from . 2 16 other e uations can be used but they ould use the result from part a

$$v=-\sqrt{v_0^2-2g\ \ y-y_0}\ =-\sqrt{2gy_0}=-53.3\text{ m s}$$

here the negative root is chosen since this is a do n ard velocity. hus the speed is $v=53.3$ m s.

c or the catching process the ans er to part b plays the role of an *initial* velocity v_0 = –53.3 m s and the final velocity must become ero. sing . 2 16 e find

$$\Delta y_2=\frac{v^2-v_0^2}{2a_2}=\frac{-\ -53.3\text{ m s}\ ^2}{2\ \ 245\text{ m s}^2}=-5.80\text{ m}$$

or $\Delta y_2\ =5.80$ m. he negative value of Δy_2 signifies that the distance traveled hile arresting its motion is do n ard.

114. uring T_r the velocity v_0 is constant in the direction e choose as x and obeys v_0 = D_r T_r here e note that in units the velocity is v_0 = 200 1000 3600 = 55.6 m s. uring T_b the acceleration is opposite to the direction of v_0 hence for us a 0 until the car is stopped v = 0 .

a sing . 2 16 ith Δx_b = 170 m e find

$$v^2=v_0^2\ +2a\Delta x_b\quad\Rightarrow\quad a=-\frac{v_0^2}{2\Delta x_b}$$

hich yields a = 9.08 m s^2.

b e e press this as a multiple of g by setting up a ratio

$$a=\left(\frac{9.08\text{ m s}^2}{9.8\text{ m s}^2}\right)g=0.926g\ .$$

c e use . 2 17 to obtain the braking time

$$\Delta x_b=\frac{1}{2}\left(v_0\ +v\right)T_b\quad\Rightarrow\quad T_b=\frac{2(170\text{ m})}{55.6\text{ m s}}=6.12\text{ s}\ .$$

d e e press our result for T_b as a multiple of the reaction time T_r by setting up a ratio

$$T_b=\left(\frac{6.12\text{ s}}{400\times10^{-3}\text{ s}}\right)T_r\ =15.3T_r\ .$$

e ince T_b T_r most of the full time re uired to stop is spent in braking.

f e are only asked hat the *increase* in distance D is due to ΔT_r = 0.100 s so e simply have

$$\Delta D = v_0 \Delta T_r = (55.6 \text{ m s})(0.100 \text{ s}) = 5.56 \text{ m} .$$

115. he total time elapsed is $\Delta t = 2 \text{ h } 41 \text{ min} = 161 \text{ min}$ and the center point is displaced by $\Delta x = 3.70 \text{ m} = 370 \text{ cm}$. hus the average velocity of the center point is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{370 \text{ cm}}{161 \text{ min}} = 2.30 \text{ cm min}.$$

116. sing . 2 11 $v = v_0 + at$ e find the initial speed to be

$$v_0 = v - at = 0 - \ -3400 \ \ 9.8 \text{ m s}^2 \ \ 6.5 \times 10^{-3} \text{ s} \ = 216.6 \text{ m s}$$

117. he total number of days alked is including the first and the last day and leap year

$$N = 340 + 365 + 365 + 366 + 365 + 365 + 261 = 2427$$

hus the average speed of the alk is

$$s_{\text{avg}} = \frac{d}{\Delta t} = \frac{3.06 \times 10^7 \text{ m}}{2427 \text{ days} \ \ 86400 \text{ s day}} = 0.146 \text{ m s}.$$

118. a et d be the distance traveled. he average speed ith and ithout ings set as sails are $v_s = d \ \ t_s$ and $v_{ns} = d \ \ t_{ns}$ respectively. hus the ratio of the t o speeds is

$$\frac{v_s}{v_{ns}} = \frac{d \ \ t_s}{d \ \ t_{ns}} = \frac{t_{ns}}{t_s} = \frac{25.0 \text{ s}}{7.1 \text{ s}} = 3.52$$

b he difference in time e pressed in terms of v_s is

$$\Delta t = t_{ns} - t_s = \frac{d}{v_{ns}} - \frac{d}{v_s} = \frac{d}{v_s \ \ 3.52} - \frac{d}{v_s} = 2.52 \frac{d}{v_s} = 2.52 \frac{2.0 \text{ m}}{v_s} = \frac{5.04 \text{ m}}{v_s}$$

119. a ifferentiating $y \ t \ = \ 2.0 \text{ cm} \ \sin \ \pi t \ \ 4$ ith respect to t e obtain

$$v_y \ t \ = \frac{dy}{dt} = \left(\frac{\pi}{2} \text{ cm s} \right) \cos \ \pi t \ \ 4$$

he average velocity bet een t = 0 and t = 2.0 s is

$$v_{\text{avg}} = \frac{1}{2.0\text{ s}}\int_0^2 v_y dt = \frac{1}{2.0\text{ s}}\left(\frac{\pi}{2}\text{ cm s}\right)\int_0^2 \cos\left(\frac{\pi t}{4}\right)dt$$
$$= \frac{1}{2.0\text{ s}}\left(2\text{ cm}\right)\int_0^{\pi\ \ 2}\cos x\, dx = 1.0\text{ cm s}$$

b he instantaneous velocities of the particle at t = 0 1.0 s and 2.0 s are respectively

$$v_y\ \ 0\ = \left(\frac{\pi}{2}\text{ cm s}\right)\cos\ 0\ = \frac{\pi}{2}\text{ cm s}$$
$$v_y\ \ 1.0\text{ s}\ = \left(\frac{\pi}{2}\text{ cm s}\right)\cos\ \pi\ \ 4\ = \frac{\pi\sqrt{2}}{4}\text{ cm s}$$
$$v_y\ \ 2.0\text{ s}\ = \left(\frac{\pi}{2}\text{ cm s}\right)\cos\ \pi\ \ 2\ = 0$$

c ifferentiating $v_y\ \ t$ ith respect to t e obtain the follo ing e pression for acceleration

$$a_y\ \ t\ = \frac{dv_y}{dt} = \left(-\frac{\pi^2}{8}\text{ cm s}^2\right)\sin\ \pi t\ \ 4$$

he average acceleration bet een t = 0 and t = 2.0 s is

$$a_{\text{avg}} = \frac{1}{2.0\text{ s}}\int_0^2 a_y dt = \frac{1}{2.0\text{ s}}\left(-\frac{\pi^2}{8}\text{ cm s}^2\right)\int_0^2 \sin\left(\frac{\pi t}{4}\right)dt$$
$$= \frac{1}{2.0\text{ s}}\left(-\frac{\pi}{2}\text{ cm s}\right)\int_0^{\pi\ \ 2}\sin x\, dx = \frac{1}{2.0\text{ s}}\left(-\frac{\pi}{2}\text{ cm s}\right) = -\frac{\pi}{4}\text{ cm s}^2$$

d he instantaneous accelerations of the particle at t = 0 1.0 s and 2.0 s are respectively

$$a_y\ \ 0\ = \left(-\frac{\pi^2}{8}\text{ cm s}^2\right)\sin\ 0\ = 0$$
$$a_y\ \ 1.0\text{ s}\ = \left(-\frac{\pi^2}{8}\text{ cm s}^2\right)\sin\ \pi\ \ 4\ = -\frac{\pi^2\sqrt{2}}{16}\text{ cm s}^2$$
$$a_y\ \ 2.0\text{ s}\ = \left(-\frac{\pi^2}{8}\text{ cm s}^2\right)\sin\ \pi\ \ 2\ = -\frac{\pi^2}{8}\text{ cm s}^2$$

Chapter

1. **THINK** In this problem we're given the magnitude and direction of a vector in two dimensions and asked to calculate its x and y components.

EXPRESS he x and the y components of a vector $\vec{a}$ lying in the xy plane are given by

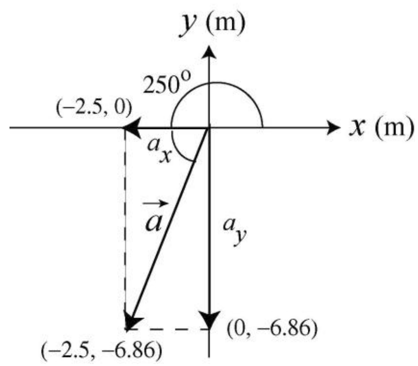
$$a_x = a\cos\theta \qquad a_y = a\sin\theta$$

here $a = \vec{a} = \sqrt{a_x^2 + a_y^2}$ is the magnitude and $\theta = \tan^{-1} a_y \ a_x$ is the angle bet een $\vec{a}$ and the positive x a is. iven that $\theta = 250°$ e see that the vector is in the third uadrant and e e pect both the x and the y components of $\vec{a}$ to be negative.

ANALYZE a he x component of $\vec{a}$ is

$$a_x = a\cos\theta = \ 7.3 \text{ m} \ \cos 250° = -2.50 \text{ m}$$

b and the y component is $a_y = a\sin\theta = \ 7.3 \text{ m} \ \sin 250° = -6.86 \text{ m} \approx -6.9 \text{ m}$. he results are depicted in the figure belo



LEARN n considering the variety of ays to compute these e note that the vector is 70 belo the $-x$ a is so the components could also have been found from

$$a_x = -\ 7.3 \text{ m} \ \cos 70° = -2.50 \text{ m} \qquad a_y = -\ 7.3 \text{ m} \ \sin 70° = -6.86 \text{ m}.$$

imilarly e note that the vector is 20 to the left from the $-y$ a is so one could also achieve the same results by using

$$a_x = -\ 7.3 \text{ m} \ \sin 20° = -2.50 \text{ m} \qquad a_y = -\ 7.3 \text{ m} \ \cos 20° = -6.86 \text{ m}.$$

s a consistency check e note that $\sqrt{a_x^2+a_y^2}=\sqrt{\;-2.50\text{ m}\;^2+\;-6.86\text{ m}\;^2}=7.3\text{ m}$ and $\tan^{-1}\left(a_y\;\;a_x\right)=\tan^{-1}\;\;-6.86\text{ m}\quad-2.50\text{ m}\;\;=250°$ hich are indeed the values given in the problem statement.

2. a ith r = 15 m and θ = 30 the x component of $\vec{r}$ is given by

$$r_x = r\cos\theta = \;15\text{ m}\;\cos 30\; = 13\text{ m}.$$

b imilarly the y component is given by $r_y = r\sin\theta = \;15\text{ m}\;\sin 30\; = 7.5\text{ m}.$

3. **THINK** In this problem we're given the x and y components a vector $\vec{A}$ in t o dimensions and asked to calculate its magnitude and direction.

EXPRESS vector $\vec{A}$ can be represented in the *magnitude-angle* notation $A\;\;\theta$ here

$$A=\sqrt{A_x^2+A_y^2}$$

is the magnitude and

$$\theta=\tan^{-1}\left(\frac{A_y}{A_x}\right)$$

is the angle $\vec{A}$ makes ith the positive x a is. iven that A_x = –25.0 m and A_y = 40.0 m the above formulas can be readily used to calculate A and θ.

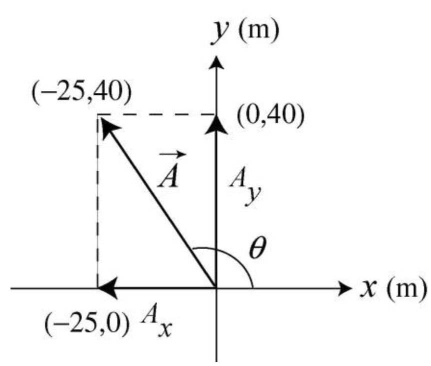
ANALYZE a he magnitude of the vector $\vec{A}$ is

$$A=\sqrt{A_x^2+A_y^2}=\sqrt{\;-25.0\text{ m}\;^2+\;40.0\text{ m}\;^2}=47.2\text{ m}$$

b ecalling that $\tan\theta = \tan\;\;\theta\;\;180$

$\tan^{-1}\;\;40.0\text{ m}\quad-25.0\text{ m}\;\;= -58\;$ or $122\;$.

oting that the vector is in the second uadrant by the signs of its x and y components e see that 122 is the correct ans er. he results are depicted in the figure to the right.



LEARN e can check our ans ers by noting that the x and the y components of $\vec{A}$ can be ritten as

$$A_x = A\cos\theta \qquad A_y = A\sin\theta.$$

ubstituting the results calculated above e obtain

$$A_x = 47.2 \text{ m} \cos 122° = -25.0 \text{ m} \quad A_y = 47.2 \text{ m} \sin 122° = +40.0 \text{ m}$$

hich indeed are the values given in the problem statement.

4. he angle described by a full circle is 360 = 2π rad hich is the basis of our conversion factor.

a $20.0° = (20.0°)\dfrac{2\pi \text{ rad}}{360°} = 0.349 \text{ rad}$.

b $50.0° = (50.0°)\dfrac{2\pi \text{ rad}}{360°} = 0.873 \text{ rad}$.

c $100° = (100°)\dfrac{2\pi \text{ rad}}{360°} = 1.75 \text{ rad}$.

d $0.330 \text{ rad} = (0.330 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 18.9°$.

e $2.10 \text{ rad} = (2.10 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 120°$.

f $7.70 \text{ rad} = (7.70 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 441°$.

5. he vector sum of the displacements $\vec{d}_{\text{storm}}$ and $\vec{d}_{\text{ne}}$ must give the same result as its originally intended displacement $\vec{d}_{\text{o}} = 120$ κm $\hat{}$ here east is $\hat{\imath}$ north is $\hat{}$. hus e rite

$$\vec{d}_{\text{storm}} = 100 \text{ κm } \hat{\imath} \quad \vec{d}_{\text{ne}} = A\hat{\imath} + B\hat{}.$$

a he e uation $\vec{d}_{\text{storm}} + \vec{d}_{\text{ne}} = \vec{d}_{\text{o}}$ readily yields A = –100 km and B = 120 km. he magnitude of $\vec{d}_{\text{ne}}$ is therefore e ual to $\vec{d}_{\text{ne}} = \sqrt{A^2 + B^2} = 156 \text{ km}$.

b he direction is

$$\tan^{-1} B\ A = -50.2 \text{ or } 180 \quad -50.2 = 129.8 .$$

e choose the latter value since it indicates a vector pointing in the second uadrant hich is hat e e pect here. he ans er can be phrased several e uivalent ays 129.8 counterclock ise from east or 39.8 est from north or 50.2 north from est.

6. a he height is $h = d \sin\theta$ here d = 12.5 m and θ = 20.0 . herefore h = 4.28 m.

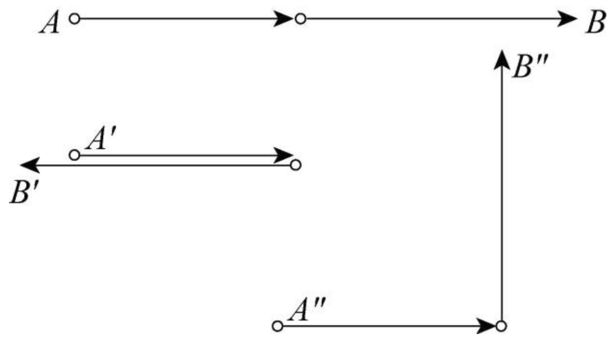
b he hori ontal distance is $d \cos\theta$ = 11.7 m.

7. a he vectors should be parallel to achieve a resultant 7 m long the unprimed case sho n belo

b anti parallel in opposite directions to achieve a resultant 1 m long primed case sho n

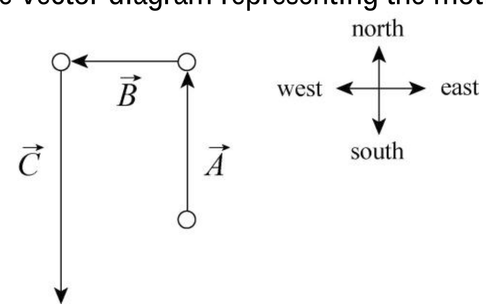
c and perpendicular to achieve a resultant $\sqrt{3^2+4^2} = 5$ m long the double primed case sho n .

n each sketch the vectors are sho n in a "head to tail" sketch but the resultant is not sho n. he resultant ould be a straight line dra n from beginning to end the beginning is indicated by A ith or ithout primes as the case may be and the end is indicated by B.



8. e label the displacement vectors $\vec{A}$ $\vec{B}$ and $\vec{C}$ and denote the result of their vector sum as $\vec{r}$. e choose *east* as the $\hat{\text{i}}$ direction x direction and *north* as the $\hat{\ }$ direction y direction . ll distances are understood to be in kilometers.

a he vector diagram representing the motion is sho n ne t



$\vec{A}$ = 3.1 km $\hat{\ }$

$\vec{B}$ = –2.4 km $\hat{\text{i}}$

$\vec{C}$ = –5.2 km $\hat{\ }$

b he final point is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = \text{ –2.4 κın } \hat{\text{i}} + \text{ –2.1 km } \hat{\ }$$

hose magnitude is

$$|\vec{r}| = \sqrt{(-2.4\text{ km})^2 + (-2.1\text{ km})^2} \approx 3.2\text{ km} \; .$$

c here are t o possibilities for the angle

$$\theta = \tan^{-1}\left(\frac{-2.1\text{ km}}{-2.4\text{ km}}\right) = 41° \text{ or } 221°.$$

e choose the latter possibility since $\vec{r}$ is in the third uadrant. t should be noted that many graphical calculators have polar $\leftrightarrow$ rectangular "shortcuts" that automatically produce the correct ans er for angle measured counterclock ise from the x a is . e may phrase the angle then as 221 counterclock ise from ast a phrasing that sounds peculiar at best or as 41 south from est or 49 est from south. he resultant $\vec{r}$ is not sho n in our sketch it ould be an arro directed from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

9. ll distances in this solution are understood to be in meters.

a $\vec{a}+\vec{b} =$ 4.0+ –1.0 $\hat{\text{i}}$+ –3.0 +1.0 $\hat{}$ + 1.0+4.0 $\hat{\text{k}}$= 3.0$\hat{\text{i}}$–2.0$\hat{}$+5.0$\hat{\text{k}}$ m.

b $\vec{a}-\vec{b} =$ 4.0– –1.0 $\hat{\text{i}}$+ –3.0 –1.0 $\hat{}$ + 1.0–4.0 $\hat{\text{k}}$= 5.0$\hat{\text{i}}$–4.0$\hat{}$–3.0$\hat{\text{k}}$ m.

c he re uirement $\vec{a}-\vec{b}+\vec{c}=0$ leads to $\vec{c}=\vec{b}-\vec{a}$ hich e note is the opposite of hat e found in part b . hus $\vec{c} =$ –5.0$\hat{\text{i}}$ + 4.0$\hat{}$ + 3.0$\hat{\text{k}}$ m.

10. he x, y and z components of $\vec{r}=\vec{c}+\vec{d}$ are respectively

a $r_x = c_x + d_x = 7.4\text{ m} + 4.4\text{ m} = 12\text{ m}$

b $r_y = c_y + d_y = -3.8\text{ m} - 2.0\text{ m} = -5.8\text{ m}$ and

c $r_z = c_z + d_z = -6.1\text{ m} + 3.3\text{ m} = -2.8\text{ m}$.

11. **THINK** his problem involves the addition of t o vectors $\vec{a}$ and $\vec{b}$. e ant to find the magnitude and direction of the resulting vector.

EXPRESS n t o dimensions a vector $\vec{a}$ can be ritten as in unit vector notation

$$\vec{a} = a_x\hat{\text{i}} + a_y\hat{} .$$

imilarly a second vector $\vec{b}$ can be e pressed as $\vec{b} = b_x\hat{\text{i}} + b_y\hat{}$. dding the t o vectors gives

$$\vec{r} = \vec{a}+\vec{b} = a_x + b_x \ \hat{\text{i}} + a_y + b_y \ \hat{} = r_x\hat{\text{i}} + r_y\hat{}$$

ANALYZE a iven that $\vec{a} =$ 4.0 m $\hat{\text{i}}$+ 3.0 m $\hat{}$ and $\vec{b} =$ –13.0 m $\hat{\text{i}}$+ 7.0 m $\hat{}$ e find the x and the y components of $\vec{r}$ to be

$$r_x = a_x \quad b_x = \; 4.0 \text{ m} \qquad -13 \text{ m} \; = -9.0 \text{ m}$$
$$r_y = a_y \quad b_y = \; 3.0 \text{ m} \qquad 7.0 \text{ m} \; = 10.0 \text{ m}.$$

hus $\vec{r} = \; -9.0\,\text{m} \; \hat{\text{i}} + \; 10\,\text{m} \; \hat{} \;.$

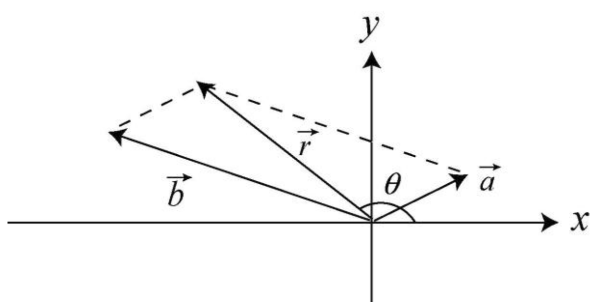
b he magnitude of $\vec{r}$ is $r = \vec{r} \; = \sqrt{r_x^2 + r_y^2} = \sqrt{\; -9.0 \text{ m} \;^2 + \; 10 \text{ m} \;^2} = 13 \text{ m}.$

c he angle bet een the resultant and the x a is is given by

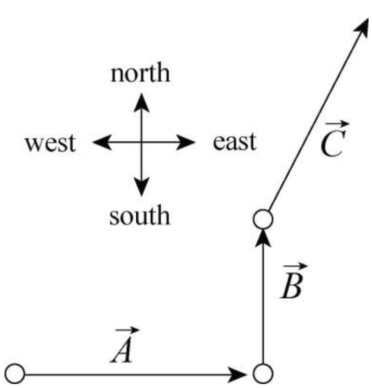
$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{10.0 \text{ m}}{-9.0 \text{ m}}\right) = -48^\circ \text{ or } 132^\circ.$$

 ince the x component of the resultant is negative and the y component is positive characteristic of the second uadrant e find the angle is 132 measured counterclock ise from x a is .

LEARN he addition of the t o vectors is depicted in the figure belo   not to scale . ndeed since $r_x < 0$ and $r_y > 0$ e e pect $\vec{r}$ to be in the second uadrant.



12. e label the displacement vectors $\vec{A}$ $\vec{B}$ and $\vec{C}$ and denote the result of their vector sum as $\vec{r}$. e choose *east* as the $\hat{\text{i}}$ direction x direction and *north* as the $\hat{}$ direction y direction . e note that the angle bet een $\vec{C}$ and the x a is is 60 . hus



$$\vec{A} \;=\; 50 \text{ km} \; \hat{\text{i}}$$
$$\vec{B} \;=\; 30 \text{ кm} \; \hat{}$$
$$\vec{C} \;=\; 25 \text{ km} \; \cos(60^\circ)\;\hat{\text{i}} \qquad 25 \text{ km} \; \sin(60^\circ)\;\hat{}$$

a he total displacement of the car from its initial position is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = 62.5 \text{ km } \hat{\mathrm{i}} + 51.7 \text{ km } \hat{}$$

hich means that its magnitude is

$$|\vec{r}| = \sqrt{62.5\,\text{km}^2 + 51.7\,\text{km}^2} = 81\text{ km.}$$

b he angle counterclock ise from x a is is $\tan^{-1}$ 51.7 km 62.5 km $= 40$ hich is to say that it points 40 *north of east*. lthough the resultant $\vec{r}$ is sho n in our sketch it ould be a direct line from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

13. e find the components and then add them as scalars not vectors . ith d = 3.40 km and θ = 35.0 e find $d\cos\theta$ $d\sin\theta$ = 4.74 km.

14. a umming the x components e have

$$20 \text{ m} \quad b_x - 20 \text{ m} - 60 \text{ m} = -140 \text{ m}$$

hich gives $b_x = -80$ m.

b umming the y components e have

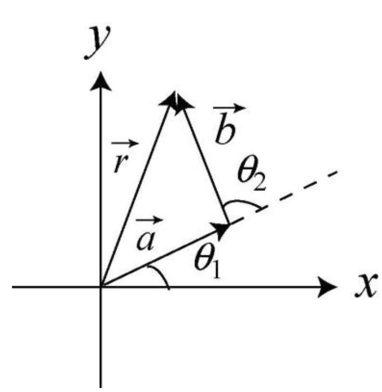
$$60 \text{ m} - 70 \text{ m} \quad c_y - 70 \text{ m} = 30 \text{ m}$$

hich implies c_y =110 m.

c sing the ythagorean theorem the magnitude of the overall displacement is given by $\sqrt{-140 \text{ m}^2 + 30 \text{ m}^2} \approx 143$ m.

d he angle is given by $\tan^{-1}$ 30 -140 $= -12°$ hich ould be 12° measured clock ise from the $-x$ a is or 168° measured counterclock ise from the x a is .

15. **THINK** his problem involves the addition of t o vectors $\vec{a}$ and $\vec{b}$ in t o dimensions. We're asked to find the components magnitude and direction of the resulting vector.



EXPRESS n t o dimensions a vector $\vec{a}$ can be ritten as in unit vector notation

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{} = a\cos\alpha\ \hat{\mathrm{i}} + a\sin\alpha\ \hat{}.$$

imilarly a second vector $\vec{b}$ can be e pressed as $\vec{b} = b_x\hat{\text{i}} + b_y\hat{} = b\cos\beta\ \hat{\text{i}} + b\sin\beta\ \hat{}$. rom the figure e have $\alpha = \theta_1$ and $\beta = \theta_1 + \theta_2$ since the angles are measured from the x a is and the resulting vector is

$$\vec{r} = \vec{a} + \vec{b} = a\cos\theta_1 + b\cos\ \theta_1 + \theta_2\ \ \hat{\text{i}} + a\sin\theta_1 + b\sin\ \theta_1 + \theta_2\ \ \hat{} = r_x\hat{\text{i}} + r_y\hat{}$$

ANALYZE a iven that $a = b = 10\text{ m}\ \ \theta_1 = 30°$ and $\theta_2 = 105°$ the x component of $\vec{r}$ is

$$r_x = a\cos\theta_1 + b\cos\ \theta_1 + \theta_2\ = \ 10\text{ m}\ \cos 30° + \ 10\text{ m}\ \cos\ 30° + 105°\ = 1.59\text{ m}$$

b imilarly the y component of $\vec{r}$ is

$$r_y = a\sin\theta_1 + b\sin\ \theta_1 + \theta_2\ = \ 10\text{ m}\ \sin 30° + \ 10\text{ m}\ \sin\ 30° + 105°\ = 12.1\text{ m}.$$

c he magnitude of $\vec{r}$ is $r = \ \vec{r}\ = \sqrt{\ 1.59\text{ m}\ ^2 + \ 12.1\text{ m}\ ^2} = 12.2\text{ m}.$

d he angle bet een $\vec{r}$ and the x a is is

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{12.1\text{ m}}{1.59\text{ m}}\right) = 82.5°.$$

LEARN s depicted in the figure the resultant $\vec{r}$ lies in the first uadrant. his is hat e e pect. ote that the magnitude of $\vec{r}$ can also be calculated by using la of cosine $\vec{a}\ \vec{b}$ and $\vec{r}$ form an isosceles triangle

$$r = \sqrt{a^2 + b^2 - 2ab\cos\ 180 - \theta_2\ } = \sqrt{\ 10\text{ m}\ ^2 + \ 10\text{ m}\ ^2 - 2\ 10\text{ m}\ \ 10\text{ m}\ \cos 75°}$$
$$= 12.2\text{ m}.$$

16. a $\vec{a} + \vec{b} = \ 3.0\hat{\text{i}} + 4.0\hat{}\ \ \text{m} + \ 5.0\hat{\text{i}} - 2.0\hat{}\ \ \text{m} = \ 8.0\text{ m}\ \hat{\text{i}} + \ 2.0\text{ m}\ \hat{}.$

b he magnitude of $\vec{a} + \vec{b}$ is

$$\ \vec{a} + \vec{b}\ = \sqrt{\ 8.0\text{ m}\ ^2 + \ 2.0\text{ m}\ ^2} = 8.2\text{ m}.$$

c he angle bet een this vector and the x a is is

$$\tan^{-1}\ \ 2.0\text{ m}\ \ \ 8.0\text{ m}\ \ = 14\ .$$

d $\vec{b} - \vec{a} = \ 5.0\hat{\text{i}} - 2.0\hat{}\ \ \text{m} - \ 3.0\hat{\text{i}} + 4.0\hat{}\ \ \text{m} = \ 2.0\text{ m}\ \hat{\text{i}} - \ 6.0\text{ m}\ \hat{}.$

e he magnitude of the difference vector $\vec{b} - \vec{a}$ is

$$\vec{b} - \vec{a} = \sqrt{ 2.0 \text{ m}^{2} + -6.0 \text{ m}^{2}} = 6.3 \text{ m}.$$

f he angle bet een this vector and the x a is is tan ^{1} –6.0 m 2.0 m = –72 . he vector is 72 *clockwise* from the a is defined by $\hat{\text{i}}$.

17. any of the operations are done efficiently on most modern graphical calculators using their built in vector manipulation and rectangular ↔ polar “shortcuts.” In this solution e employ the “traditional” methods (such as Eq. 3 6 . here the length unit is not displayed the unit meter should be understood.

a sing unit vector notation

$$\begin{aligned}
\vec{a} &= 50 \text{ m} \cos 30° \hat{\text{i}} + 50 \text{ m} \sin 30° \hat{\ } \\
\vec{b} &= 50 \text{ m} \cos 195° \hat{\text{i}} + 50 \text{ m} \sin 195° \hat{\ } \\
\vec{c} &= 50 \text{ m} \cos 315° \hat{\text{i}} + 50 \text{ m} \sin 315° \hat{\ } \\
\vec{a} + \vec{b} + \vec{c} &= 30.4 \text{ m} \hat{\text{i}} - 23.3 \text{ m} \hat{\ }.
\end{aligned}$$

he magnitude of this result is $\sqrt{ 30.4 \text{ m}^{2} + -23.3 \text{ m}^{2}} = 38 \text{ m}$.

b he t o possibilities presented by a simple calculation for the angle bet een the vector described in part a and the x direction are tan^{-1} –23.2 m 30.4 m = –37.5 and 180 –37.5 = 142.5 . he former possibility is the correct ans er since the vector is in the fourth uadrant indicated by the signs of its components . hus the angle is –37.5 hich is to say that it is 37.5 *clockwise* from the x a is. his is e uivalent to 322.5 counterclock ise from x.

c e find

$$\vec{a} - \vec{b} + \vec{c} = 43.3 - -48.3 + 35.4 \hat{\text{i}} - 25 - -12.9 + -35.4 \hat{\ } = 127 \hat{\text{i}} + 2.60 \hat{\ } \text{ m}$$

in unit vector notation. he magnitude of this result is

$$\vec{a} - \vec{b} + \vec{c} = \sqrt{ 127 \text{ m}^{2} + 2.6 \text{ m}^{2}} \approx 1.30 \times 10^{2} \text{ m}.$$

d he angle bet een the vector described in part c and the x a is is tan^{-1} 2.6 m 127 m $\approx 1.2°$.

e sing unit vector notation $\vec{d}$ is given by $\vec{d} = \vec{a} + \vec{b} - \vec{c} = -40.4\,\hat{i} + 47.4\,\hat{}$ m hich has a magnitude of $\sqrt{-40.4\text{ m}^{2} + 47.4\text{ m}^{2}} = 62$ m.

f he t o possibilities presented by a simple calculation for the angle bet een the vector described in part e and the x a is are $\tan^{-1} 47.4 \ -40.4 = -50.0°$ and $180° + -50.0° = 130°$. e choose the latter possibility as the correct one since it indicates that $\vec{d}$ is in the second uadrant indicated by the signs of its components .

18. f e ish to use . 3 5 in an unmodified fashion e should note that the angle bet een $\vec{C}$ and the x a is is 180 20.0 = 200 .

a he x and y components of $\vec{B}$ are given by

$$B_x = C_x - A_x = 15.0\text{ m} \cos 200 - 12.0\text{ m} \cos 40 = -23.3\text{ m}$$
$$B_y = C_y - A_y = 15.0\text{ m} \sin 200 - 12.0\text{ m} \sin 40 = -12.8\text{ m}.$$

onse uently its magnitude is $\vec{B} = \sqrt{-23.3\text{ m}^{2} + -12.8\text{ m}^{2}} = 26.6\text{ m}$.

b he t o possibilities presented by a simple calculation for the angle bet een $\vec{B}$ and the x a is are $\tan^{-1}$ –12.8 m –23.3 m = 28.9 and 180 28.9 = 209 . e choose the latter possibility as the correct one since it indicates that $\vec{B}$ is in the third uadrant indicated by the signs of its components . e note too that the ans er can be e uivalently stated as $-151°$.

19. a ith i directed for ard and directed left ard the resultant is 5.00 i 2.00 m . he magnitude is given by the ythagorean theorem $\sqrt{5.00\text{ m}^{2} + 2.00\text{ m}^{2}} = 5.385$ m ≈ 5.39 m.

b he angle is $\tan^{-1}$ 2.00 5.00 ≈ 21.8 left of for ard .

20. he desired result is the displacement vector in units of km $\vec{A}$ = 5.6 km 90 measured counterclock ise from the x a is or $\vec{A} = 5.6\text{ km}\,\hat{}$ here $\hat{}$ is the unit vector along the positive y a is north . his consists of the sum of t o displacements during the hiteout $\vec{B} = 7.8\text{ km}\ 50°$ or

$$\vec{B} = 7.8\text{ km}\ \cos 50°\hat{i} + \sin 50°\,\hat{} = 5.01\text{ km}\ \hat{i} + 5.98\text{ km}\ \hat{}$$

and the unkno n $\vec{C}$. hus $\vec{A} = \vec{B} + \vec{C}$.

a he desired displacement is given by $\vec{C} = \vec{A} - \vec{B} = -5.01\text{ km}\ \hat{\text{i}} - 0.38\text{ km}\ \hat{}$. he magnitude is $\sqrt{-5.01\text{ km}^{\ 2} + -0.38\text{ km}^{\ 2}} = 5.0\text{ km}$.

b he angle is $\tan^{-1}\ -0.38\text{ km}\ \ -5.01\text{ km}\ = 4.3°$ south of due est.

21. eading carefully e see that the x, y specifications for each "dart" are to be interpreted as $\Delta x\ \Delta y$ descriptions of the corresponding displacement vectors. e combine the different parts of this problem into a single e position.

a long the x a is e have ith the centimeter unit understood

$$30.0 + b_x - 20.0 - 80.0 = -140$$

hich gives $b_x = -70.0$ cm.

b long the y a is e have

$$40.0 - 70.0 + c_y - 70.0 = -20.0$$

hich yields $c_y = 80.0$ cm.

c he magnitude of the final location $-140\ \ -20.0$ is $\sqrt{-140^{\ 2} + -20.0^{\ 2}} = 141$ cm.

d ince the displacement is in the third uadrant the angle of the overall displacement is given by $\pi\ \ \tan^{-1}\ -20.0\ \ -140$ or 188 counterclock ise from the x a is or $-172°$ counterclock ise from the x a is .

22. ngles are given in 'standard' fashion so . 3 5 applies directly. e use this to rite the vectors in unit vector notation before adding them. o ever a very different looking approach using the special capabilities of most graphical calculators can be imagined. herever the length unit is not displayed in the solution belo the unit meter should be understood.

a llo ing for the different angle units used in the problem statement e arrive at

$$\vec{E} = 3.73\ \hat{\text{i}} + 4.70\ \hat{}$$

$$\vec{F} = 1.29\ \hat{\text{i}} - 4.83\ \hat{}$$

$$\vec{G} = 1.45\ \hat{\text{i}} + 3.73\ \hat{}$$

$$\vec{H} = -5.20\ \hat{\text{i}} + 3.00\ \hat{}$$

$$\vec{E} + \vec{F} + \vec{G} + \vec{H} = 1.28\ \hat{\text{i}}\ + 6.60\ \hat{}.$$

b he magnitude of the vector sum found in part a is $\sqrt{1.28\text{ m}^{\ 2} + 6.60\text{ m}^{\ 2}} = 6.72\text{ m}$.

c ts angle measured counterclock ise from the x a is is $\tan^{-1}$ 6.60 1.28 = 79.0 .

d sing the conversion factor π rad = 180° 79.0 = 1.38 rad.

23. he resultant along the y a is ith the same magnitude as $\vec{C}$ forms along ith $\vec{C}$ a side of an isosceles triangle ith $\vec{B}$ forming the base . f the angle bet een $\vec{C}$ and the y a is is $\theta = \tan^{-1}$ 3 4 $= 36.87°$ then it should be clear that referring to the magnitudes of the vectors $B = 2C\sin$ θ 2 . hus since C = 5.0 e find B = 3.2.

24. s a vector addition problem e e press the situation described in the problem statement as $\vec{A}$ $\vec{B}$ = 3A here $\vec{A}$ = A i and B = 7.0 m. ince i $\perp$ e may use the ythagorean theorem to e press B in terms of the magnitudes of the other t o vectors

$$B = \sqrt{\;3A\;^2 \quad A^2} \qquad \Rightarrow \qquad A = \frac{1}{\sqrt{10}} B \;=\; 2.2 \text{ m} .$$

25. he strategy is to find here the camel is $\vec{C}$ by adding the t o consecutive displacements described in the problem and then finding the difference bet een that location and the oasis $\vec{B}$. sing the magnitude angle notation

$$\vec{C} - \;\; 24 \angle -15° \quad 8.0 \angle 90° \;=\; 23.25 \angle 4.41°$$

so

$$\vec{B} - \vec{C} \;=\; 25 \angle 0° \;-\; 23.25 \angle 4.41° \;=\; 2.5 \angle -45°$$

hich is efficiently implemented using a vector capable calculator in polar mode. he distance is therefore 2.6 km.

26. he vector e uation is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. pressing $\vec{B}$ and $\vec{D}$ in unit vector notation e have $1.69\hat{\text{i}} + 3.63\hat{\ }$ m and $-2.87\hat{\text{i}} + 4.10\hat{\ }$ m respectively. here the length unit is not displayed in the solution belo the unit meter should be understood.

a dding corresponding components e obtain $\vec{R} =$ –3.18 m $\hat{\text{i}}$ + 4.72 m $\hat{\ }$.

b sing . 3 6 the magnitude is

$$|\vec{R}| = \sqrt{\;-3.18 \text{ m}\;^2 + \;4.72 \text{ m}\;^2} = 5.69 \text{ m}.$$

c he angle is

$$\theta = \tan^{-1}\left(\frac{4.72 \text{ m}}{-3.18 \text{ m}}\right) = -56.0° \qquad \text{ith } -x \text{ a is .}$$

f measured counterclock ise from x a is the angle is then $180° - 56.0° = 124°$. hus converting the result to polar coordinates e obtain

$$(-3.18\ 4.72) \rightarrow (5.69 \angle 124°)$$

27. olving the simultaneous e uations yields the ans ers

a $\vec{d}_1 = 4\,\vec{d}_3 = 8$ i 16 and

b $\vec{d}_2 = \vec{d}_3 = 2$ i 4 .

28. et $\vec{A}$ represent the first part of Beetle 1's trip (0.50 m east or 0.5 $\hat{\text{i}}$ and $\vec{C}$ represent the first part of Beetle 2's trip intended voyage (1.6 m at 50º north of east). For their respective second parts $\vec{B}$ is 0.80 m at 30 north of east and $\vec{D}$ is the unkno n. he final position of eetle 1 is

$$\vec{A}+\vec{B} = 0.5\text{ m }\hat{\text{i}} + 0.8\text{ m } \cos 30°\,\hat{\text{i}} + \sin 30°\ \hat{}\ = 1.19\text{ m }\hat{\text{i}} + 0.40\text{ m }\hat{}\,.$$

he e uation relating these is $\vec{A}+\vec{B} = \vec{C}+\vec{D}$ here

$$\vec{C} = 1.60\text{ m } \cos 50.0°\hat{\text{i}} + \sin 50.0°\hat{}\ = 1.03\text{ m }\hat{\text{i}} + 1.23\text{ m }\hat{}$$

a e find $\vec{D} = \vec{A}+\vec{B}-\vec{C} = 0.16\text{ m }\hat{\text{i}} + -0.83\text{ m }\hat{}$ and the magnitude is $D = 0.84$ m.

b he angle is $\tan^{-1}\ -0.83\ 0.16\ = -79°$ hich is interpreted to mean 79 south of east or 11 east of south .

29. et $l_0 = 2.0\text{ cm}$ be the length of each segment. he nest is located at the endpoint of segment w.

a sing unit vector notation the displacement vector for point A is

$$\vec{d}_A = \vec{w}+\vec{v}+\vec{i}+\vec{h} = l_0\ \cos 60°\hat{\text{i}} + \sin 60°\ \hat{}\ + \left(l_0\ \hat{}\right) + l_0\ \cos 120°\hat{\text{i}} + \sin 120°\ \hat{}\ + \left(l_0\ \hat{}\right)$$
$$= 2+\sqrt{3}\ l_0\ \hat{}\,.$$

herefore the magnitude of $\vec{d}_A$ is $\vec{d}_A\ = 2+\sqrt{3}\ \ 2.0\text{ cm } = 7.5\text{ cm}$.

b he angle of $\vec{d}_A$ is $\theta = \tan^{-1}\ d_{A\,y}\ \ d_{A\,x}\ = \tan^{-1}\ \infty\ = 90°$.

c imilarly the displacement for point B is

$$\vec{d}_B = \vec{w}+\vec{v}+\vec{j}+\vec{p}+\vec{o}$$
$$= l_0\ \cos 60°\hat{\text{i}} + \sin 60°\ \hat{}\ + \left(l_0\ \hat{}\right) + l_0\ \cos 60°\hat{\text{i}} + \sin 60°\ \hat{}\ + l_0\ \cos 30°\hat{\text{i}} + \sin 30°\ \hat{}\ + \left(l_0\ \hat{\text{i}}\right)$$
$$=\ 2+\sqrt{3}\ \ 2\ l_0\ \hat{\text{i}} +\ 3\ \ 2+\sqrt{3}\ \ l_0\ \hat{}.$$

herefore the magnitude of $\vec{d}_B$ is

$$\vec{d}_B\ = l_0\sqrt{\ 2+\sqrt{3}\ \ 2\ ^2 +\ 3\ \ 2+\sqrt{3}\ ^2} =\ 2.0\,\text{cm}\ \ 4.3\ = 8.6\,\text{cm}.$$

d he direction of $\vec{d}_B$ is

$$\theta_B = \tan^{-1}\left(\frac{d_{B\,y}}{d_{B\,x}}\right) = \tan^{-1}\left(\frac{3\ \ 2+\sqrt{3}}{2+\sqrt{3}\ \ 2}\right) = \tan^{-1}\ 1.13\ = 48°.$$

30. any of the operations are done efficiently on most modern graphical calculators using their built in vector manipulation and rectangular ↔ polar “shortcuts.” In this solution e employ the “traditional” methods (such as Eq. 3 6 .

a he magnitude of $\vec{a}$ is $a = \sqrt{\ 4.0\text{ m}\ ^2 +\ -3.0\text{ m}\ ^2} = 5.0\text{ m}.$

b he angle bet een $\vec{a}$ and the x a is is $\tan^{-1}$ –3.0 m 4.0 m = –37 . he vector is 37 *clockwise* from the a is defined by $\hat{\text{i}}$.

c he magnitude of $\vec{b}$ is $b = \sqrt{\ 6.0\text{ m}\ ^2 +\ 8.0\text{ m}\ ^2} = 10\text{ m}.$

d he angle bet een $\vec{b}$ and the x a is is $\tan^{-1}$ 8.0 m 6.0 m = 53 .

e $\vec{a}+\vec{b} =\ 4.0\text{ m} + 6.0\text{ m}\ \ \hat{\text{i}} +\ \ -3.0\text{ m} + 8.0\text{ m}\ \hat{}\ =\ 10\text{ m}\ \hat{\text{i}}\ +\ 5.0\text{ m}\ \hat{}.$ he magnitude of this vector is $\vec{a}+\vec{b}\ = \sqrt{\ 10\text{ m}\ ^2 +\ 5.0\text{ m}\ ^2} = 11\text{ m}$ e round to t o significant figures in our results.

f he angle bet een the vector described in part e and the x a is is $\tan^{-1}$ 5.0 m 10 m = 27 .

g $\vec{b}-\vec{a} =\ 6.0\text{ m} - 4.0\text{ m}\ \ \hat{\text{i}} +\ 8.0\text{ m} -\ -3.0\text{ m}\ \ \hat{}\ =\ 2.0\text{ m}\ \ \hat{\text{i}} +\ 11\text{ m}\ \hat{}.$ he magnitude of this vector is $\vec{b}-\vec{a}\ = \sqrt{\ 2.0\text{ m}\ ^2 +\ 11\text{ m}\ ^2} = 11\text{ m}$ hich is interestingly the same result as in part e e actly not ust to 2 significant figures this curious coincidence is made possible by the fact that $\vec{a} \perp \vec{b}$.

h he angle bet een the vector described in part g and the x a is is $\tan^{-1}$ 11 m 2.0 m = 80 .

i $\vec{a}-\vec{b}=$ 4.0 m −6.0 m $\hat{\text{i}}+$ −3.0 m −8.0 m $\hat{}$ = −2.0 m $\hat{\text{i}}+$ −11 m $\hat{}$. he magnitude of this vector is

$$\vec{a}-\vec{b}=\sqrt{\;-2.0\text{ m }^2+\;-11\text{ m }^2}=11\text{ m}.$$

he t o possibilities presented by a simple calculation for the angle bet een the vector described in part i and the x direction are $\tan^{-1}$ −11 m −2.0 m = 80 and 180 80 = 260 . he latter possibility is the correct ans er see part k for a further observation related to this result .

k ince $\vec{a}-\vec{b}=$ −1 $\vec{b}-\vec{a}$ they point in opposite anti parallel directions the angle bet een them is 180 .

31. a ith a = 17.0 m and θ = 56.0 e find $a_x = a\cos\theta$ = 9.51 m.

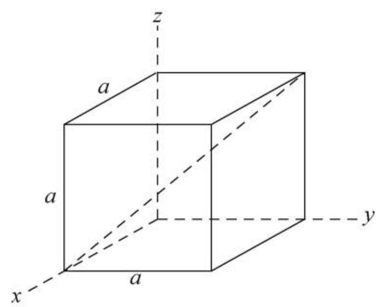
b imilarly $a_y = a\sin\theta$ = 14.1 m.

c he angle relative to the ne coordinate system is θ = 56.0 − 18.0 = 38.0 . hus $a'_x = a\cos\theta' = 13.4$ m.

d imilarly a'_y = $a\sin\theta$ = 10.5 m.

32. a s can be seen from igure 3 30 the point diametrically opposite the origin 0 0 0 has position vector $a\,\hat{\text{i}} + a\,\hat{} + a\,\hat{\text{k}}$ and this is the vector along the "body diagonal."

b rom the point a 0 0 hich corresponds to the position vector a the diametrically opposite point is 0 a, a ith the position vector $a\,\hat{} + a\,\hat{\text{k}}$. hus the vector along the line is the difference $-a\hat{\text{i}} + a\hat{} + a\hat{\text{k}}$.



c f the starting point is 0 a 0 ith the corresponding position vector $a\,\hat{}$ the diametrically opposite point is a 0, a ith the position vector $a\,\hat{\text{i}} + a\,\hat{\text{k}}$. hus the vector along the line is the difference $a\,\hat{\text{i}} - a\,\hat{} + a\,\hat{\text{k}}$.

d f the starting point is a a 0 ith the corresponding position vector $a\,\hat{\text{i}} + a\,\hat{}$ the diametrically opposite point is 0 0, a ith the position vector $a\,\hat{\text{k}}$. hus the vector along the line is the difference $-a\,\hat{\text{i}} - a\,\hat{} + a\,\hat{\text{k}}$.

e onsider the vector from the back lo er left corner to the front upper right corner. t is $a\,\hat{\text{i}} + a\,\hat{} + a\,\hat{\text{k}}$. e may think of it as the sum of the vector $a\,\hat{\text{i}}$ parallel to the x a is and the vector $a\,\hat{} + a\,\hat{\text{k}}$ perpendicular to the x a is. he tangent of the angle bet een the vector and the x a is is the perpendicular component divided by the parallel component. ince the magnitude of the perpendicular component is $\sqrt{a^2 + a^2} = a\sqrt{2}$ and the magnitude of the parallel component is a $\tan\theta = \left(a\sqrt{2}\right)$ $a = \sqrt{2}$. hus θ = 54.7°. he angle bet een the vector and each of the other t o ad acent sides the y and z a es is the same as is the angle bet een any of the other diagonal vectors and any of the cube sides ad acent to them.

f he length of any of the diagonals is given by $\sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$.

33. amining the figure e see that $\vec{a}$ $\vec{b}$ $\vec{c}$ = 0 here $\vec{a} \perp \vec{b}$.

a $|\,\vec{a} \times \vec{b}\,|$ = 3.0 4.0 = 12 since the angle bet een them is 90 .

b sing the ight and ule the vector $\vec{a} \times \vec{b}$ points in the $\hat{\text{i}} \times \hat{} = \hat{\text{k}}$ or the z direction.

c $\vec{a} \times \vec{c}$ = $\vec{a} \times -\vec{a} - \vec{b}$ = $-(\,\vec{a} \times \vec{b}$ = 12.

d he vector $-\vec{a} \times \vec{b}$ points in the $-\hat{\text{i}} \times \hat{} = -\hat{\text{k}}$ or the $-z$ direction.

e $\vec{b} \times \vec{c}$ = $\vec{b} \times -\vec{a} - \vec{b}$ = $-(\,\vec{b} \times \vec{a}$ = $(\,\vec{a} \times \vec{b}$ = 12.

f he vector points in the z direction as in part a .

34. e apply . 3 23 and . 3 27.

a $\vec{a} \times \vec{b}$ – $a_x b_y - a_y b_x$ $\hat{\text{k}}$ since all other terms vanish due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any z components. onse uently e obtain 3.0 4.0 – 5.0 2.0 $\hat{\text{k}} = 2.0\,\hat{\text{k}}$.

b $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$ yields 3.0 2.0 5.0 4.0 = 26.

c $\vec{a} + \vec{b} =$ 3.0 + 2.0 $\hat{\text{i}}$ + 5.0 + 4.0 $\hat{}$ $\Rightarrow$ $\vec{a}$ $\vec{b}$ $\cdot\,\vec{b}$ = 5.0 2.0 9.0 4.0 = 46.

d everal approaches are available. n this solution e ill construct a $\hat{b}$ unit vector and "dot" it (take the scalar product of it ith $\vec{a}$. n this case e make the desired unit vector by

$$\hat{b} = \frac{\vec{b}}{\vec{b}} = \frac{2.0\,\hat{\text{i}} + 4.0\,\hat{}}{\sqrt{2.0^{\ 2} + \ 4.0^{\ 2}}}.$$

e therefore obtain

$$a_b = \vec{a}\cdot\hat{b} = \frac{3.0 \ \ 2.0 \ + \ 5.0 \ \ 4.0}{\sqrt{2.0^{\ 2} + \ 4.0^{\ 2}}} = 5.8.$$

35. a he scalar or dot product is 4.50 7.30 cos 320 – 85.0 = – 18.8 .

b he vector or cross product is in the k direction by the right hand rule ith magnitude 4.50 7.30 sin 320 – 85.0 = 26.9 .

36. irst e re rite the given e pression as 4 $\vec{d}_{\text{plane}}$ $\vec{d}_{\text{cross}}$ here $\vec{d}_{\text{plane}}$ = $\vec{d}_1$ $\vec{d}_2$ and in the plane of $\vec{d}_1$ and $\vec{d}_2$ and $\vec{d}_{\text{cross}} = \vec{d}_1 \times \vec{d}_2$. oting that $\vec{d}_{\text{cross}}$ is perpendicular to the plane of $\vec{d}_1$ and $\vec{d}_2$ e see that the ans er must be 0 the scalar or dot product of perpendicular vectors is ero .

37. e apply . 3 23 and .3 27. f a vector capable calculator is used this makes a good e ercise for getting familiar ith those features. ere e briefly sketch the method.

a e note that $\vec{b}\times\vec{c} = -8.0\,\hat{\text{i}} + 5.0\,\hat{} + 6.0\,\hat{\text{k}}$. hus

$$\vec{a}\cdot \ \vec{b} \times \vec{c} \ - \ 3.0 \ \ -8.0 \ + \ 3.0 \ \ 5.0 \ + \ -2.0 \ \ 6.0 \ = \ -21.$$

b e note that $\vec{b}$ $\vec{c}$ = $1.0\,\hat{\text{i}} - 2.0\,\hat{}$ $3.0\,\hat{\text{k}}$. hus

$$\vec{a}\cdot \ \vec{b} + \vec{c} \ = \ 3.0 \ \ 1.0 \ + \ 3.0 \ \ -2.0 \ + \ -2.0 \ \ 3.0 \ = -9.0.$$

c inally

$$\begin{aligned}\vec{a}\times \ \vec{b} \ \ \vec{c} \ &= \ 3.0 \ \ 3.0 \ - \ -2.0 \ \ -2.0 \ \ \hat{\text{i}} + \ -2.0 \ \ 1.0 \ - \ 3.0 \ \ 3.0 \ \ \hat{} \\ &\quad + \ 3.0 \ \ -2.0 \ - \ 3.0 \ \ 1.0 \ \ \hat{\text{k}} \qquad . \\ &= \ 5\hat{\text{i}} - 11\hat{} - 9\hat{\text{k}}\end{aligned}$$

38. sing the fact that

$$\hat{\text{i}}\times\hat{} = \hat{\text{k}} \quad \hat{}\times\hat{\text{k}} = \hat{\text{i}} \quad \hat{\text{k}}\times\hat{\text{i}} = \hat{}$$

e obtain

$$2\vec{A} \times \vec{B} = 2\left(2.00\hat{\imath} + 3.00\hat{\jmath} - 4.00\hat{k}\right) \times \left(-3.00\hat{\imath} + 4.00\hat{\jmath} + 2.00\hat{k}\right)$$
$$= 44.0\hat{\imath} + 16.0\hat{\jmath} + 34.0\hat{k}.$$

e t making use of

$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$$
$$\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{k}\cdot\hat{\imath} = 0$$

e have

$$3\vec{C}\cdot\left(2\vec{A}\times\vec{B}\right) = 3\left(7.00\hat{\imath} - 8.00\hat{\jmath}\right)\cdot\left(44.0\hat{\imath} + 16.0\hat{\jmath} + 34.0\hat{k}\right)$$
$$= 3\ \ 7.00\ \ 44.0\ \ \ -8.00\ \ 16.0\ + 0\ \ 34.0\ \ = 540.$$

39. rom the definition of the dot product bet een $\vec{A}$ and $\vec{B}$ $\vec{A}\cdot\vec{B} = AB\cos\theta$ e have

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

ith $A = 6.00$ $B = 7.00$ and $\vec{A}\cdot\vec{B} = 14.0$ $\cos\theta = 0.333$ or $\theta = 70.5°$.

40. he displacement vectors can be ritten as in meters

$$\vec{d}_1 = 4.50\text{ m}\ \ \cos 63°\hat{\jmath} + \sin 63°\hat{k}\ = \ 2.04\text{ m}\ \hat{\jmath} + \ 4.01\text{ m}\ \hat{k}$$
$$\vec{d}_2 = 1.40\text{ m}\ \ \cos 30°\hat{\imath} + \sin 30°\hat{k}\ = \ 1.21\text{ m}\ \hat{\imath} + \ 0.70\text{ m}\ \hat{k}.$$

a he dot product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\vec{d}_1\cdot\vec{d}_2 = \ 2.04\hat{\jmath} + 4.01\hat{k}\ \cdot\ 1.21\hat{\imath} + 0.70\hat{k}\ = \ 4.01\hat{k}\ \cdot\ 0.70\hat{k}\ = 2.81\text{ m}^2.$$

b he cross product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\vec{d}_1\times\vec{d}_2 = \ 2.04\hat{\jmath} + 4.01\hat{k}\ \times\ 1.21\hat{\imath} + 0.70\hat{k}$$
$$= \ 2.04\ \ 1.21\ \ -\hat{k}\ \ \ \ 2.04\ \ 0.70\ \hat{\imath} + \ 4.01\ \ 1.21\ \hat{\jmath}$$
$$= \ 1.43\ \hat{\imath} + 4.86\hat{\jmath} - 2.48\hat{k}\ \text{ m}^2.$$

c he magnitudes of $\vec{d}_1$ and $\vec{d}_2$ are

$$d_1 = \sqrt{\ 2.04\text{ m}\ ^2 + \ 4.01\text{ m}\ ^2} = 4.50\text{ m}$$
$$d_2 = \sqrt{\ 1.21\text{ m}\ ^2 + \ 0.70\text{ m}\ ^2} = 1.40\text{ m}.$$

hus the angle bet een the t o vectors is

$$\theta = \cos^{-1}\left(\frac{\vec{d}_1 \cdot \vec{d}_2}{d_1 d_2}\right) = \cos^{-1}\left(\frac{2.81\ \text{m}^2}{4.50\ \text{m} \quad 1.40\ \text{m}}\right) = 63.5°.$$

41. **THINK** he angle bet een t o vectors can be calculated using the definition of scalar product.

EXPRESS ince the scalar product of t o vectors $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\cdot\vec{b} = ab\cos\phi = a_x b_x + a_y b_y + a_z b_z$$

the angle bet een them is given by

$$\cos\phi = \frac{a_x b_x \ + \ a_y b_y \ + \ a_z b_z}{ab} \quad \Rightarrow \quad \phi = \cos^{-1}\left(\frac{a_x b_x \ + \ a_y b_y \ + \ a_z b_z}{ab}\right).$$

nce the magnitudes and components of the vectors are kno n the angle ϕ can be readily calculated.

ANALYZE iven that $\vec{a} = \ 3.0\ \hat{\text{i}} + \ 3.0\ \hat{} + \ 3.0\ \hat{\text{k}}$ and $\vec{b} = \ 2.0\ \hat{\text{i}} + \ 1.0\ \hat{} + \ 3.0\ \hat{\text{k}}$ the magnitudes of the vectors are

$$a \ = \ \vec{a} \ = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{\ 3.0\ ^2 + \ 3.0\ ^2 + \ 3.0\ ^2} = 5.20$$

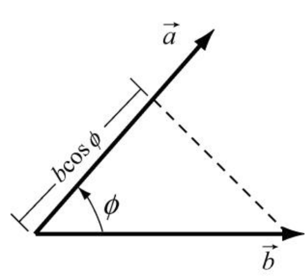
$$b \ = \ \vec{b} \ = \sqrt{b_x^2 + b_y^2 + b_z^2} = \sqrt{\ 2.0\ ^2 + \ 1.0\ ^2 + \ 3.0\ ^2} = 3.74.$$

he angle bet een them is found to be

$$\cos\phi = \frac{3.0 \quad 2.0 \ + \ 3.0 \quad 1.0 \ + \ 3.0 \quad 3.0}{5.20 \quad 3.74} = 0.926$$

or ϕ= 22 .

LEARN s the name implies the scalar product or dot product bet een t o vectors is a scalar uantity. t can be regarded as the product bet een the magnitude of one of the vectors and the scalar component of the second vector along the direction of the first one as illustrated belo see also in ig. 3 18 of the te t



$$\vec{a}\cdot\vec{b} = ab\cos\phi = \ a \quad b\cos\phi$$

42. he t o vectors are ritten as in unit of meters

$$\vec{d}_1 = 4.0\hat{\imath} \;\; 5.0\hat{} = d_{1x}\hat{\imath} + d_{1y}\hat{} \qquad \vec{d}_2 = -3.0\hat{\imath} \;\; 4.0\hat{} = d_{2x}\hat{\imath} + d_{2y}\hat{}$$

a he vector cross product gives

$$\vec{d}_1 \times \vec{d}_2 = \; d_{1x}d_{2y} - d_{1y}d_{2x} \; \hat{k} = \;\; 4.0 \;\; 4.0 \; - \; 5.0 \;\; -3.0 \;\; \hat{k} = 31 \; \hat{k}$$

b he scalar dot product gives

$$\vec{d}_1 \cdot \vec{d}_2 = d_{1x}d_{2x} + d_{1y}d_{2y} = \; 4.0 \;\; -3.0 \; + \; 5.0 \;\; 4.0 \; = 8.0.$$

c

$$\vec{d}_1 + \vec{d}_2 \;\; \cdot \vec{d}_2 = \vec{d}_1 \cdot \vec{d}_2 + d_2^2 = 8.0 + \; -3.0 \;^2 + \; 4.0 \;^2 = 33.$$

d ote that the magnitude of the d_1 vector is $\sqrt{16 \;\; 25}$ = 6.4. o the dot product is 6.4 5.0 cosθ = 8. ividing both sides by 32 and taking the inverse cosine yields θ = 75.5°. herefore the component of the d_1 vector along the direction of the d_2 vector is 6.4cos$\theta \approx$ 1.6.

43. **THINK** n this problem e are given three vectors $\vec{a}$ $\vec{b}$ and $\vec{c}$ on the xy plane and asked to calculate their components.

EXPRESS rom the figure e note that $\vec{c} \perp \vec{b}$ hich implies that the angle bet een $\vec{c}$ and the x a is is θ 90 . n unit vector notation the three vectors can be ritten as

$$\vec{a} = a_x\hat{\imath}$$
$$\vec{b} = b_x\hat{\imath} + b_y\hat{} = \; b\cos\theta \; \hat{\imath} + \; b\sin\theta \; \hat{}$$
$$\vec{c} = c_x\hat{\imath} + c_y\hat{} = \; c\cos \; \theta + 90° \;\; \hat{\imath} + \; c\sin \; \theta + 90° \;\; \hat{}.$$

he above e pressions allo us to evaluate the components of the vectors.

ANALYZE a he x component of $\vec{a}$ is $a_x = a$ cos 0 = a = 3.00 m.

b imilarly the y componnet of $\vec{a}$ is $a_y = a$ sin 0 = 0.

c he x component of $\vec{b}$ is $b_x = b$ cos 30 = 4.00 m cos 30 = 3.46 m

d and the y component is $b_y = b$ sin 30 = 4.00 m sin 30 = 2.00 m.

e he x component of $\vec{c}$ is $c_x = c$ cos 120 = 10.0 m cos 120 = –5.00 m f and the y component is $c_y = c \sin 30$ = 10.0 m sin 120 = 8.66 m.

g he fact that $\vec{c} = p\vec{a} + q\vec{b}$ implies

$$\vec{c} = c_x\hat{\text{i}} + c_y\hat{\ } = p\ \ a_x\hat{\text{i}}\ \ + q\ \ b_x\hat{\text{i}} + b_y\hat{\ }\ \ =\ \ pa_x + qb_x\ \ \hat{\text{i}} + qb_y\hat{\ }$$

or

$$c_x = pa_x + qb_x \qquad c_y = qb_y\,.$$

ubstituting the values found above e have

$$\begin{aligned} -5.00\text{ m} &= p\ \ 3.00\text{ m}\ \ + q\ \ 3.46\text{ m} \\ 8.66\text{ m} &= q\ \ 2.00\text{ m}\ \ . \end{aligned}$$

olving these e uations e find p = –6.67.

h imilarly $q = 4.33$ (note that it's easiest to solve for q first . he numbers p and q have no units.

LEARN his e ercise sho s that given t o non parallel vectors in t o dimensions the third vector can al ays be ritten as a linear combination of the first t o.

44. pplying . 3 23 $\vec{F} = q\vec{v} \times \vec{B}$ here q is a scalar becomes

$$F_x\hat{\text{i}} + F_y\hat{\ } + F_z\hat{\text{k}} = q\left(v_yB_z - v_zB_y\right)\hat{\text{i}} + q\left(v_zB_x - v_xB_z\right)\hat{\ } + q\left(v_xB_y - v_yB_x\right)\hat{\text{k}}$$

hich — plugging in values — leads to three e ualities

$$\begin{aligned} 4.0 &= 2\ \ 4.0B_z\ \ -\ \ 6.0B_y \\ -20 &= 2\ \ 6.0B_x\ \ -\ \ 2.0B_z \\ 12 &= 2\ \ 2.0B_y\ \ -\ \ 4.0B_x \end{aligned}$$

ince e are told that $B_x = B_y$ the third e uation leads to B_y = –3.0. nserting this value into the first e uation e find B_z = –4.0. hus our ans er is

$$\vec{B} = -3.0\,\hat{\text{i}} - 3.0\,\hat{\ } - 4.0\,\hat{\text{k}}.$$

45. he t o vectors are given by

$$\begin{aligned} \vec{A} &= 8.00\ \text{cos}130^\circ\hat{\text{i}} + \sin 130^\circ\hat{\ }\ \ = -5.14\,\hat{\text{i}} + 6.13\,\hat{\ } \\ \vec{B} &= B_x\hat{\text{i}} + B_y\hat{\ } = -7.72\,\hat{\text{i}} - 9.20\,\hat{\ }. \end{aligned}$$

a he dot product of $5\vec{A}\cdot\vec{B}$ is

$$5\vec{A}\cdot\vec{B} = 5\ \ -5.14\,\hat{\text{i}} + 6.13\,\hat{}\ \ \cdot\ \ -7.72\,\hat{\text{i}} - 9.20\,\hat{}\ \ = 5\ \ -5.14\ \ -7.72\ \ +\ \ 6.13\ \ -9.20$$
$$= -83.4.$$

b n unit vector notation

$$4\vec{A}\times 3\vec{B} = 12\vec{A}\times\vec{B} = 12\ \ -5.14\,\hat{\text{i}} + 6.13\,\hat{}\ \ \times\ \ -7.72\,\hat{\text{i}} - 9.20\,\hat{}\ \ = 12\ \ 94.6\,\hat{\text{k}}\ \ = 1.14\times 10^{3}\ \hat{\text{k}}$$

c e note that the a imuthal angle is undefined for a vector along the z a is. hus our result is "1.14×10^{3} θ not defined and ϕ = 0°."

d ince $\vec{A}$ is in the xy plane and $\vec{A}\times\vec{B}$ is perpendicular to that plane then the ans er is 90°.

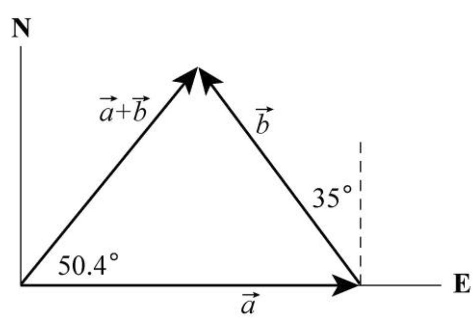
e learly $\vec{A}$ + 3.00 k = –5.14 i 6.13 3.00 k.

f he ythagorean theorem yields magnitude $A = \sqrt{5.14\ ^2 + 6.13\ ^2 + 3.00\ ^2} = 8.54$. he a imuthal angle is θ = 130° ust as it as in the problem statement $\vec{A}$ is the pro ection onto the xy plane of the ne vector created in part e . he angle measured from the z a is is

$$\phi = \cos^{-1}\ 3.00\ 8.54\ = 69.4°.$$

46. he vectors are sho n on the diagram. he x a is runs from est to east and the y a is runs from south to north. hen a_x = 5.0 m a_y = 0

$$b_x = -\ 4.0\text{ m}\ \ \sin 35\ \ = -2.29\text{ m}\quad b_y =\ 4.0\text{ m}\ \ \cos 35\ \ = 3.28\text{ m}.$$



a et $\vec{c} = \vec{a} + \vec{b}$. hen $c_x = a_x + b_x$ = 5.00 m – 2.29 m = 2.71 m and $c_y = a_y + b_y$ = 0 3.28 m = 3.28 m . he magnitude of c is

$$c = \sqrt{c_x^2 + c_y^2} = \sqrt{(2.71\text{m})^2 + (3.28\text{m})^2} = 4.2\text{ m}.$$

b he angle θ that $\vec{c} = \vec{a} + \vec{b}$ makes ith the x a is is

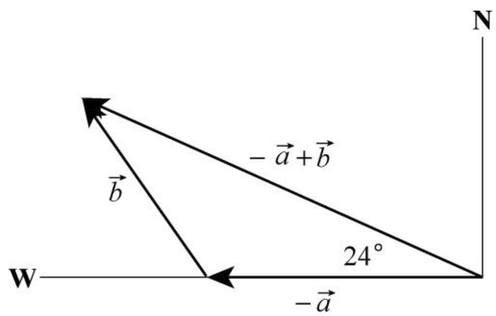
$$\theta = \tan^{-1}\left(\frac{c_y}{c_x}\right) = \tan^{-1}\left(\frac{3.28}{2.71}\right) = 50.5° \approx 50°.$$

he second possibility θ = 50.4 180 = 230.4 is re ected because it ould point in a direction opposite to $\vec{c}$.

c he vector $\vec{b} - \vec{a}$ is found by adding $-\vec{a}$ to $\vec{b}$. he result is sho n on the diagram to the right. et $\vec{c} = \vec{b} - \vec{a}.$ he components are

$$c_x = b_x - a_x = -2.29 \text{ m} - 5.00 \text{ m} = -7.29 \text{ m}$$
$$c_y = b_y - a_y = 3.28 \text{ m}.$$

he magnitude of $\vec{c}$ is $c = \sqrt{c_x^2 + c_y^2} = 8.0\,\text{m}$.



d he tangent of the angle θ that $\vec{c}$ makes ith the x a is east is

$$\tan\theta = \frac{c_y}{c_x} = \frac{3.28 \text{ m}}{-7.29 \text{ m}} = -4.50.$$

here are t o solutions –24.2 and 155.8 . s the diagram sho s the second solution is correct. he vector $\vec{c} = -\vec{a} + \vec{b}$ is 24 north of est.

47. oting that the given 130° is measured counterclock ise from the x a is the t o vectors can be ritten as

$$\vec{A} = 8.00\ \cos 130°\,\hat{\imath} + \sin 130°\,\hat{} \ = -5.14\,\hat{\imath} + 6.13\,\hat{}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{} = -7.72\,\hat{\imath} - 9.20\,\hat{}.$$

a he angle bet een the negative direction of the y a is $-\hat{}$ and the direction of $\vec{A}$ is

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot -\hat{\ }}{A}\right) = \cos^{-1}\left(\frac{-6.13}{\sqrt{-5.14^{\ 2} + \ 6.13^{\ 2}}}\right) = \cos^{-1}\left(\frac{-6.13}{8.00}\right) = 140°.$$

lternatively one may say that the $-y$ direction corresponds to an angle of 270° and the ans er is simply given by 270°–130° = 140°.

b ince the y a is is in the xy plane and $\vec{A}\times\vec{B}$ is perpendicular to that plane then the ans er is 90.0°.

c he vector can be simplified as

$$\vec{A}\times\ \vec{B}+3.00\hat{\text{k}}\ = \ -5.14\,\hat{\text{i}}+6.13\,\hat{\ }\ \times\ -7.72\,\hat{\text{i}}-9.20\,\hat{\ }+3.00\,\hat{\text{k}}$$
$$=18.39\,\hat{\text{i}}+15.42\,\hat{\ }+94.61\,\hat{\text{k}}$$

ts magnitude is $\vec{A}\times\ \vec{B}+3.00\hat{\text{k}}\ \ =97.6.$ he angle bet een the negative direction of the y a is $-\hat{\ }$ and the direction of the above vector is

$$\theta = \cos^{-1}\left(\frac{-15.42}{97.6}\right) = 99.1°.$$

48. here the length unit is not displayed the unit meter is understood.

a e first note that the magnitudes of the vectors are $a = \ \vec{a}\ = \sqrt{\ 3.2^{\ 2} + \ 1.6^{\ 2}} = 3.58$ and $b = \ \vec{b}\ = \sqrt{\ 0.50^{\ 2} + \ 4.5^{\ 2}} = 4.53$. o

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y = ab\cos\phi$$
$$3.2\ \ 0.50\ +\ 1.6\ \ 4.5\ =\ 3.58\ \ 4.53\ \cos\phi$$

hich leads to ϕ = 57 the inverse cosine is double valued as is the inverse tangent but e kno this is the right solution since both vectors are in the same uadrant .

b ince the angle measured from x for $\vec{a}$ is $\tan^{-1}$ 1.6 3.2 = 26.6 e kno the angle for $\vec{c}$ is 26.6 –90 = –63.4 the other possibility 26.6 90 ould lead to a c_x 0 . herefore

$$c_x = c\cos\ -63.4\ \ =\ 5.0\ \ 0.45\ = 2.2\text{ m.}$$

c lso $c_y = c\sin\ -63.4\ \ =\ 5.0\ \ -0.89\ = -4.5$ m.

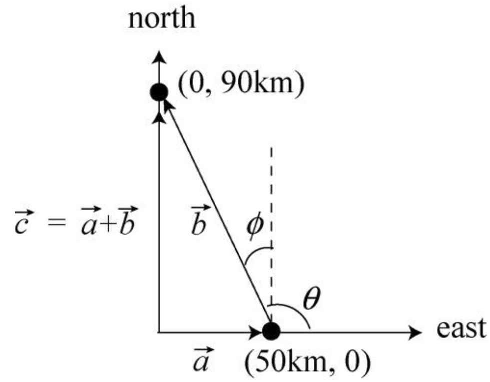
d nd e kno the angle for $\vec{d}$ to be 26.6 90 = 116.6 hich leads to

$$d_x = d\cos 116.6 = 5.0 \quad -0.45 = -2.2 \text{ m}.$$

e inally $d_y = d\sin 116.6 = 5.0 \quad 0.89 = 4.5$ m.

49. **THINK** his problem deals ith the displacement of a sailboat. e ant to find the displacement vector bet een t o locations.

EXPRESS he situation is depicted in the figure belo . et $\vec{a}$ represent the first part of his actual voyage 50.0 km east and $\vec{c}$ represent the intended voyage 90.0 km north . e look for a vector $\vec{b}$ such that $\vec{c} = \vec{a} + \vec{b}$.



ANALYZE a sing the ythagorean theorem the distance traveled by the sailboat is

$$b = \sqrt{50.0 \text{ km}^2 + 90.0 \text{ km}^2} = 103 \text{ km}.$$

b he direction is

$$\phi = \tan^{-1}\left(\frac{50.0 \text{ km}}{90.0 \text{ km}}\right) = 29.1°$$

est of north hich is e uivalent to 60.9° north of due est .

LEARN his problem could also be solved by first e pressing the vectors in unit vector notation $\vec{a} = 50.0$ km $\hat{i}$ $\vec{c} = 90.0$ km $\hat{j}$. his gives

$$\vec{b} = \vec{c} - \vec{a} = -50.0 \text{ km } \hat{i} + 90.0 \text{ km } \hat{j}.$$

he angle bet een $\vec{b}$ and the x a is is

$$\theta = \tan^{-1}\left(\frac{90.0 \text{ km}}{-50.0 \text{ km}}\right) = 119.1°.$$

he angle θ is related to ϕ by $\theta = 90° + \phi$.

50. he t o vectors $\vec{d}_1$ and $\vec{d}_2$ are given by $\vec{d}_1 = -d_1 \hat{}$ and $\vec{d}_2 = d_2 \hat{\text{i}}$.

a he vector $\vec{d}_2$ 4 $=$ d_2 4 $\hat{\text{i}}$ points in the x direction. he factor does not affect the result.

b he vector $\vec{d}_1$ –4 $=$ d_1 4 $\hat{}$ points in the y direction. The minus sign (with the "–4") does affect the direction $- -y = y$.

c $\vec{d}_1 \cdot \vec{d}_2 = 0$ since $\hat{\text{i}} \cdot \hat{} = 0$. he t o vectors are perpendicular to each other.

d $\vec{d}_1 \cdot \vec{d}_2$ 4 $=$ $\vec{d}_1 \cdot \vec{d}_2$ $4 = 0$ as in part c .

e $\vec{d}_1 \times \vec{d}_2 = -d_1 d_2$ $\hat{} \times \hat{\text{i}}$ $= d_1 d_2 \hat{\text{k}}$ in the z direction.

f $\vec{d}_2 \times \vec{d}_1 = -d_2 d_1$ $\hat{\text{i}} \times \hat{}$ $= -d_1 d_2 \hat{\text{k}}$ in the $-z$ direction.

g he magnitude of the vector in e is $d_1 d_2$.

h he magnitude of the vector in f is $d_1 d_2$.

i ince $d_1 \times \vec{d}_2$ 4 $=$ $d_1 d_2$ 4 $\hat{\text{k}}$ the magnitude is $d_1 d_2$ 4.

he direction of $\vec{d}_1 \times \vec{d}_2$ 4 $=$ $d_1 d_2$ 4 $\hat{\text{k}}$ is in the z direction.

51. lthough e think of this as a three dimensional movement it is rendered effectively t o dimensional by referring measurements to its ell defined plane of the fault.

a he magnitude of the net displacement is

$$\overrightarrow{AB} = \sqrt{AD^2 + AC^2} = \sqrt{17.0 \text{ m}^2 + 22.0 \text{ m}^2} = 27.8\text{m}.$$

b he magnitude of the vertical component of $\overrightarrow{AB}$ is AD sin 52.0 = 13.4 m.

52. he three vectors are

$$\vec{d}_1 = 4.0\hat{\text{i}} + 5.0\hat{} - 6.0\hat{\text{k}}$$
$$\vec{d}_2 = -1.0\hat{\text{i}} + 2.0\hat{} \quad 3.0\hat{\text{k}}$$
$$\vec{d}_3 = 4.0\hat{\text{i}} + 3.0\hat{} \quad 2.0\hat{\text{k}}$$

a $\vec{r} = \vec{d}_1 - \vec{d}_2 + \vec{d}_3 =$ 9.0 m $\hat{\mathrm{i}} +$ 6.0 m $\hat{} +$ –7.0 m $\hat{\mathrm{k}}$.

b he magnitude of $\vec{r}$ is $\vec{r} = \sqrt{9.0 \text{ m}^2 + 6.0 \text{ m}^2 + -7.0 \text{ m}^2} = 12.9$ m. he angle bet een $\vec{r}$ and the z a is is given by

$$\cos\theta = \frac{\vec{r} \cdot \hat{\mathrm{k}}}{\vec{r}} = \frac{-7.0 \text{ m}}{12.9 \text{ m}} = -0.543$$

hich implies $\theta = 123°$.

c he component of $\vec{d}_1$ along the direction of $\vec{d}_2$ is given by $d_{\parallel} = \vec{d}_1 \cdot \hat{\mathrm{u}} - d_1 \cos\varphi$ here φ is the angle bet een $\vec{d}_1$ and $\vec{d}_2$ and $\hat{\mathrm{u}}$ is the unit vector in the direction of $\vec{d}_2$. sing the properties of the scalar dot product e have

$$d_{\parallel} = d_1\left(\frac{\vec{d}_1 \cdot \vec{d}_2}{d_1 d_2}\right) = \frac{\vec{d}_1 \cdot \vec{d}_2}{d_2} = \frac{4.0 \quad -1.0 + 5.0 \quad 2.0 + -6.0 \quad 3.0}{\sqrt{-1.0^2 + 2.0^2 + 3.0^2}} = \frac{-12}{\sqrt{14}} = -3.2 \text{ m}.$$

d o e are looking for $d_{\perp}$ such that $d_1^2 = 4.0^2 + 5.0^2 + -6.0^2 = 77 = d_{\parallel}^2 + d_{\perp}^2$. rom c e have

$$d_{\perp} = \sqrt{77 \text{ m}^2 - -3.2 \text{ m}^2} = 8.2 \text{ m}.$$

his gives the magnitude of the perpendicular component and is consistent ith hat one ould get using . 3 24 but if more information such as the direction or a full specification in terms of unit vectors is sought then more computation is needed.

53. **THINK** his problem involves finding scalar and vector products bet een t o vectors $\vec{a}$ and $\vec{b}$.

EXPRESS e apply s. 3 20 and 3 24 to calculate the scalar and vector products bet een t o vectors

$$\vec{a} \cdot \vec{b} = ab \cos\phi$$

$$\vec{a} \times \vec{b} = ab \sin\phi.$$

ANALYZE a iven that $a = \vec{a} = 10$ $b = \vec{b} = 6.0$ and $\phi = 60°$ the scalar dot product of $\vec{a}$ and $\vec{b}$ is

$$\vec{a} \cdot \vec{b} = ab \cos\phi = 10 \quad 6.0 \ \cos 60° = 30.$$

b imilarly the magnitude of the vector cross product of the t o vectors is

$$\vec{a}\times\vec{b} \quad = ab\sin\phi = \; 10 \quad 6.0 \;\; \sin 60° = 52.$$

LEARN hen t o vectors $\vec{a}$ and $\vec{b}$ are parallel $\phi = 0$ their scalar and vector products are $\vec{a}\cdot\vec{b} = ab\cos\phi = ab$ and $\vec{a}\times\vec{b} \;= ab\sin\phi = 0$ respectively. o ever hen they are perpendicular $\phi = 90°$ e have $\vec{a}\cdot\vec{b} = ab\cos\phi = 0$ and $\vec{a}\times\vec{b} \;= ab\sin\phi = ab$.

54. rom the figure it is clear that $\vec{a} \quad \vec{b} \quad \vec{c} = 0$ here $\vec{a}\perp\vec{b}$.

a $\vec{a} \quad \vec{b} = 0$ since the angle bet een them is 90 .

b $\vec{a} \quad \vec{c} = \vec{a} \quad -\vec{a} - \vec{b} \quad = -|\vec{a}|^2 = -16$.

c imilarly $\vec{b} \quad \vec{c} = -9.0$.

55. e choose x east and y north and measure all angles in the “standard” way positive ones are counterclock ise from x. hus vector $\vec{d}_1$ has magnitude d_1 = 4.00 m ith the unit meter and direction θ_1 = 225 . lso $\vec{d}_2$ has magnitude d_2 = 5.00 m and direction θ_2 = 0 and vector $\vec{d}_3$ has magnitude d_3 = 6.00 m and direction θ_3 = 60 .

a he x component of $\vec{d}_1$ is $d_{1x} = d_1 \cos\theta_1 = -2.83$ m.

b he y component of $\vec{d}_1$ is $d_{1y} = d_1 \sin\theta_1 = -2.83$ m.

c he x component of $\vec{d}_2$ is $d_{2x} = d_2 \cos\theta_2 = 5.00$ m.

d he y component of $\vec{d}_2$ is $d_{2y} = d_2 \sin\theta_2 = 0$.

e he x component of $\vec{d}_3$ is $d_{3x} = d_3 \cos\theta_3 = 3.00$ m.

f he y component of $\vec{d}_3$ is $d_{3y} = d_3 \sin\theta_3 = 5.20$ m.

g he sum of x components is

$$d_x = d_{1x} \quad d_{2x} \quad d_{3x} = -2.83\text{ m} \quad 5.00\text{ m} \quad 3.00\text{ m} = 5.17\text{ m}.$$

h he sum of y components is

$$d_y = d_{1y} \quad d_{2y} \quad d_{3y} = -2.83\text{ m} \quad 0 \quad 5.20\text{ m} = 2.37\text{ m}.$$

i he magnitude of the resultant displacement is

$$d=\sqrt{d_x^2+d_y^2}=\sqrt{5.17\text{ m}^2+2.37\text{ m}^2}=5.69\text{ m}.$$

nd its angle is

$$\theta=\tan^{-1}\ 2.37\ 5.17\ =24.6$$

hich recalling our coordinate choices means it points at about 25 north of east.

k and l his ne displacement the direct line home hen vectorially added to the previous net displacement must give ero. hus the ne displacement is the negative or opposite of the previous net displacement. hat is it has the same magnitude 5.69 m but points in the opposite direction 25 south of est .

56. f e ish to use . 3 5 directly e should note that the angles for $\vec{Q}$ $\vec{R}$ and $\vec{S}$ are 100 250 and 310 respectively if they are measured counterclock ise from the x a is.

a sing unit vector notation ith the unit meter understood e have

$$\vec{P}=10.0\cos(25.0°)\,\hat{\text{i}}+10.0\sin(25.0°)\,\hat{}$$
$$\vec{Q}=12.0\cos(100°)\,\hat{\text{i}}+12.0\sin(100°)\,\hat{}$$
$$\vec{R}=8.00\cos(250°)\,\hat{\text{i}}+8.00\sin(250°)\,\hat{}$$
$$\vec{S}=9.00\cos(310°)\,\hat{\text{i}}+9.00\sin(310°)\,\hat{}$$
$$\vec{P}+\vec{Q}+\vec{R}+\vec{S}=10.0\text{ m}\ \hat{\text{i}}+1.63\text{ m}\ \hat{}$$

b he magnitude of the vector sum is $\sqrt{10.0\text{ m}^2+1.63\text{ m}^2}=10.2\text{ m}$.

c he angle is $\tan^{-1}\ 1.63\text{ m}\ 10.0\text{ m}\ \approx 9.24$ measured counterclock ise from the x a is.

57. **THINK** his problem deals ith addition and subtraction of t o vectors.

EXPRESS rom the problem statement e have

$$\vec{A}+\vec{B}=6.0\ \hat{\text{i}}+1.0\ \hat{}\qquad \vec{A}-\vec{B}=-4.0\ \hat{\text{i}}+7.0\ \hat{}$$

olving the simultaneous e uations gives $\vec{A}$ and $\vec{B}$.

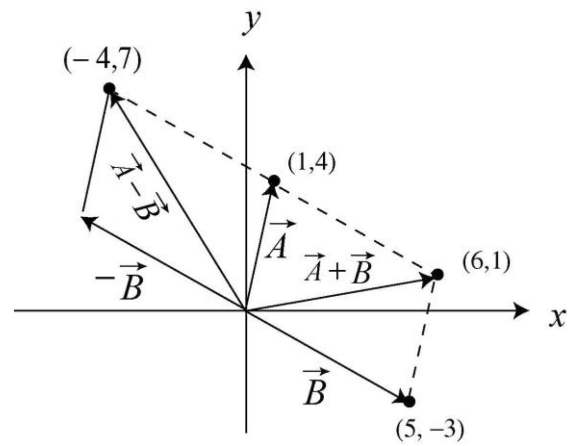
ANALYZE dding the above e uations and dividing by 2 leads to $\vec{A} = 1.0\ \hat{\imath} + 4.0\ \hat{}$.
he magnitude of $\vec{A}$ is

$$A = \ \vec{A}\ = \sqrt{A_x^2 + A_y^2} = \sqrt{\ 1.0\ ^2 + \ 4.0\ ^2} = 4.1$$

LEARN he vector $\vec{B}$ is $\vec{B} = \ 5.0\ \hat{\imath} + \ -3.0\ \hat{}$ and its magnitude is

$$B = \ \vec{B}\ = \sqrt{B_x^2 + B_y^2} = \sqrt{\ 5.0\ ^2 + \ -3.0\ ^2} = 5.8.$$

he results are summari ed in the figure to the right.



58. he vector can be ritten as $\vec{d} = \ 2.5\ \text{m}\ \hat{}$ here e have taken $\hat{}$ to be the unit vector pointing north.

a he magnitude of the vector $\vec{a} = 4.0\,\vec{d}$ is 4.0 2.5 m = 10 m.

b he direction of the vector $\vec{a} = 4.0\vec{d}$ is the same as the direction of $\vec{d}$ north .

c he magnitude of the vector $\vec{c} = -3.0\vec{d}$ is 3.0 2.5 m = 7.5 m.

d he direction of the vector $\vec{c} = -3.0\vec{d}$ is the opposite of the direction of $\vec{d}$. hus the direction of $\vec{c}$ is south.

59. eference to igure 3 18 and the accompanying material in that section is helpful. f e convert $\vec{B}$ to the magnitude angle notation as $\vec{A}$ already is e have $\vec{B} = (14.4 \angle 33.7°)$ appropriate notation especially if e are using a vector capable calculator in polar mode . here the length unit is not displayed in the solution the unit meter should be understood. n the magnitude angle notation rotating the a is by 20 amounts to subtracting that angle from the angles previously specified. hus $\vec{A} = (12.0 \angle 40.0°)'$ and $\vec{B} = \ 14.4 \angle 13.7°\ '$ here the 'prime' notation indicates that the description is in terms of the ne coordinates. onverting these results to $x,\ y$ representations e obtain

a $\vec{A} = \ 9.19\ \text{m}\ \ \hat{\imath}' + \ 7.71\ \text{m}\ \ \hat{}'$.

b imilarly $\vec{B} = 14.0$ m î′ + 3.41 m ĵ′.

60. he t o vectors can be found be solving the simultaneous e uations.

a f e add the e uations e obtain $2\vec{a} = 6\vec{c}$ hich leads to $\vec{a} = 3\vec{c} = 9\hat{\mathrm{i}} + 12\hat{\mathrm{j}}$.

b lugging this result back in e find $\vec{b} = \vec{c} = 3\hat{\mathrm{i}} + 4\hat{\mathrm{j}}$.

61. he three vectors given are

$$\begin{aligned}\vec{a} &= 5.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}} - 6.0\,\hat{\mathrm{k}}\\ \vec{b} &= -2.0\,\hat{\mathrm{i}} + 2.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}\\ \vec{c} &= 4.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}\end{aligned}$$

a he vector e uation $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ is

$$\begin{aligned}\vec{r} &= (5.0 - (-2.0) + 4.0)\hat{\mathrm{i}} + (4.0 - 2.0 + 3.0)\hat{\mathrm{j}} + (-6.0 - 3.0 + 2.0)\hat{\mathrm{k}}\\ &= 11\hat{\mathrm{i}} + 5.0\hat{\mathrm{j}} - 7.0\hat{\mathrm{k}}.\end{aligned}$$

b e find the angle from $+z$ by "dotting" (taking the scalar product) $\vec{r}$ ith $\hat{\mathrm{k}}$. oting that

$$r = |\vec{r}| = \sqrt{(11.0)^2 + (5.0)^2 + (-7.0)^2} = 14$$

. 3 20 ith . 3 23 leads to

$$\vec{r} \cdot \hat{\mathrm{k}} = -7.0 = (14)(1)\cos\phi \;\Rightarrow\; \phi = 120°.$$

c o find the component of a vector in a certain direction it is efficient to "dot" it (take the scalar product of it ith a unit vector in that direction. n this case e make the desired unit vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{-2.0\hat{\mathrm{i}} + 2.0\hat{\mathrm{j}} + 3.0\hat{\mathrm{k}}}{\sqrt{(-2.0)^2 + (2.0)^2 + (3.0)^2}}.$$

e therefore obtain

$$a_b = \vec{a} \cdot \hat{b} = \frac{(5.0)(-2.0) + (4.0)(2.0) + (-6.0)(3.0)}{\sqrt{(-2.0)^2 + (2.0)^2 + (3.0)^2}} = -4.9.$$

d ne approach if all e re uire is the magnitude is to use the vector cross product as the problem suggests another hich supplies more information is to subtract the result in part c multiplied by $\hat{b}$ from $\vec{a}$. e briefly illustrate both methods. e note that if

$a\cos\theta$ here θ is the angle bet een $\vec{a}$ and $\vec{b}$ gives a_b the component along $\hat{b}$ then e e pect $a\sin\theta$ to yield the orthogonal component

$$a\sin\theta = \frac{\left|\vec{a}\times\vec{b}\right|}{b} = 7.3$$

alternatively one might compute θ form part c and proceed more directly . he second method proceeds as follo s

$$\vec{a} - a_b\,\hat{b} = (5.0 - 2.35)\hat{\mathrm{i}} + (4.0 - (-2.35))\hat{\ } \quad ((-6.0) - (-3.53))\hat{\mathrm{k}}$$
$$= 2.65\hat{\mathrm{i}} + 6.35\hat{\ } - 2.47\hat{\mathrm{k}}$$

his describes the perpendicular part of $\vec{a}$ completely. o find the magnitude of this part e compute

$$\sqrt{2.65^{2} + 6.35^{2} + -2.47^{2}} = 7.3$$

hich agrees ith the first method.

62. e choose x east and y north and measure all angles in the "standard" way positive ones counterclock ise from x negative ones clock ise . hus vector $\vec{d}_1$ has magnitude d_1 = 3.66 ith the unit meter and three significant figures assumed and direction θ_1 = 90 . lso $\vec{d}_2$ has magnitude d_2 = 1.83 and direction θ_2 = –45 and vector $\vec{d}_3$ has magnitude d_3 = 0.91 and direction θ_3 = –135 . e add the x and y components respectively

$$x \quad d_1\cos\theta_1 + d_2\cos\theta_2 + d_3\cos\theta_3 = 0.65\text{ m}$$
$$y \quad d_1\sin\theta_1 + d_2\sin\theta_2 + d_3\sin\theta_3 = 1.7\text{ m.}$$

a he magnitude of the direct displacement the vector sum $\vec{d}_1 \quad \vec{d}_2 \quad \vec{d}_3$ is $\sqrt{0.65\text{ m}^{2} + 1.7\text{ m}^{2}} = 1.8$ m.

b he angle understood in the sense described above is $\tan^{-1}$ 1.7 0.65 = 69 . hat is the first putt must aim in the direction 69 north of east.

63. he three vectors are

$$\vec{d}_1 = -3.0\hat{\mathrm{i}} + 3.0\hat{\ } + 2.0\hat{\mathrm{k}}$$
$$\vec{d}_2 = -2.0\hat{\mathrm{i}} - 4.0\hat{\ } + 2.0\hat{\mathrm{k}}$$
$$\vec{d}_3 = 2.0\hat{\mathrm{i}} + 3.0\hat{\ } + 1.0\hat{\mathrm{k}}.$$

a ince $\vec{d}_2 + \vec{d}_3 = 0\hat{\text{i}} - 1.0\hat{\text{j}} + 3.0\hat{\text{k}}$ e have

$$\vec{d}_1 \cdot \; \vec{d}_2 + \vec{d}_3 \; = \; -3.0\hat{\text{i}} + 3.0\hat{\text{j}} + 2.0\hat{\text{k}} \;\cdot\; 0\hat{\text{i}} - 1.0\hat{\text{j}} + 3.0\hat{\text{k}}$$
$$= 0 - 3.0 \quad 6.0 = 3.0\ \text{m}^2.$$

b sing . 3 27 e obtain $\vec{d}_2 \times \vec{d}_3 = -10\hat{\text{i}} + 6.0\hat{\text{j}} + 2.0\hat{\text{k}}.$ hus

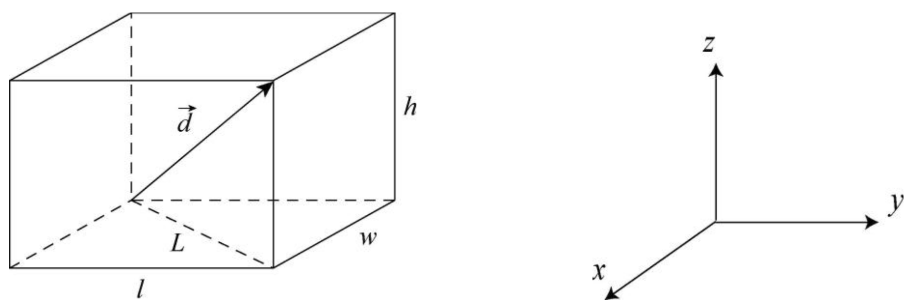
$$\vec{d}_1 \cdot \; \vec{d}_2 \times \vec{d}_3 \; = \; -3.0\hat{\text{i}} + 3.0\hat{\text{j}} + 2.0\hat{\text{k}} \;\cdot\; -10\hat{\text{i}} + 6.0\hat{\text{j}} + 2.0\hat{\text{k}}$$
$$= 30 + 18 + 4.0 = 52\ \text{m}^3.$$

c e found $\vec{d}_2 \quad \vec{d}_3$ in part a . se of . 3 27 then leads to

$$\vec{d}_1 \times \; \vec{d}_2 + \vec{d}_3 \; = \; -3.0\hat{\text{i}} + 3.0\hat{\text{j}} + 2.0\hat{\text{k}} \;\times\; 0\hat{\text{i}} - 1.0\hat{\text{j}} + 3.0\hat{\text{k}}$$
$$= \; 11\hat{\text{i}} \quad 9.0\hat{\text{j}} \quad 3.0\hat{\text{k}} \quad \text{m}^2$$

64. **THINK** his problem deals ith the displacement and distance traveled by a fly from one corner of a room to the diagonally opposite corner. he displacement vector is three dimensional.

EXPRESS he displacement of the fly is illustrated in the figure belo



coordinate system such as the one sho n above right allo s us to e press the displacement as a three dimensional vector.

ANALYZE a he magnitude of the displacement from one corner to the diagonally opposite corner is

$$d = \; \vec{d} \; = \sqrt{w^2 + l^2 + h^2}$$

ubstituting the values given e obtain

$$d = \; \vec{d} \; = \sqrt{w^2 + l^2 + h^2} = \sqrt{\;3.70\ \text{m}\;^2 + \;4.30\ \text{m}\;^2 + \;3.00\ \text{m}\;^2} = 6.42\ \text{m}.$$

b he displacement vector is along the straight line from the beginning to the end point of the trip. ince a straight line is the shortest distance bet een t o points the length of the path cannot be less than d the magnitude of the displacement.

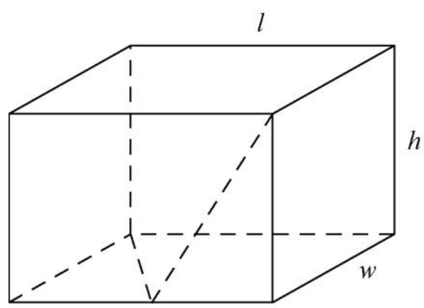
c he length of the path of the fly can be greater than d ho ever. he fly might for e ample cra l along the edges of the room. ts displacement ould be the same but the path length ould be $\ell + w + h = 11.0$ m.

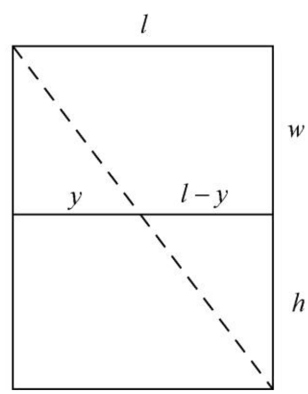
d he path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

e e take the x a is to be out of the page the y a is to be to the right and the z a is to be up ard as sho n in the figure above . hen the x component of the displacement is w = 3.70 m the y component of the displacement is 4.30 m and the z component is 3.00 m. hus the displacement vector can be ritten as

$$\vec{d} = \ 3.70 \text{ m} \ \hat{\text{i}} + \ 4.30 \text{ m} \ \hat{} + \ 3.00 \text{ m} \ \hat{\text{k}}.$$

f uppose the path of the fly is as sho n by the dotted lines on the diagram belo left . retend there is a hinge here the front all of the room oins the floor and lay the all do n as sho n above right .





he shortest alking distance bet een the lo er left back of the room and the upper right front corner is the dotted straight line sho n on the diagram. ts length is

$$s_{\text{min}} = \sqrt{(w+h)^2 + l^2} = \sqrt{(3.70 \text{ m} + 3.00 \text{ m})^2 + \ 4.30 \text{ m} \ ^2} = 7.96 \text{ m}.$$

LEARN o sho that the shortest path is indeed given by s_{min} e rite the length of the path as

$$s = \sqrt{y^2 + w^2} + \sqrt{\ l - y \ ^2 + h^2}.$$

he condition for minimum is given by

$$\frac{ds}{dy}=\frac{y}{\sqrt{y^2+w^2}}-\frac{l-y}{\sqrt{\;l-y\;^2+h^2}}=0\,.$$

little algebra sho s that the condition is satisfied hen $y=lw\quad w+h$ hich gives

$$s_{\text{min}}=\sqrt{w^2\left(1+\frac{l^2}{w+h\;^2}\right)}+\sqrt{h^2\left(1+\frac{l^2}{w+h\;^2}\right)}=\sqrt{\;w+h\;^2+l^2}\,.$$

ny other path ould be longer than 7.96 m.

65. a his is one e ample of an ans er –40 i – 20 25 k m ith i directed anti parallel to the first path directed anti parallel to the second path and k directed up ard in order to have a right handed coordinate system . ther e amples include 40 i 20 25 k m and 40i – 20 – 25 k m ith slightly different interpretations for the unit vectors . ote that the product of the components is positive in each e ample.

b sing the ythagorean theorem e have $\sqrt{\;40\text{ m}\;^2+\;20\text{ m}\;^2}$ = 44.7 m ≈ 45 m.

66. he vectors can be ritten as $\vec{a}=a\hat{\imath}$ and $\vec{b}=b\hat{\ }$ here $a\quad b>0$.

a e are asked to consider

$$\frac{\vec{b}}{d}=\left(\frac{b}{d}\right)\hat{\ }$$

in the case d 0. ince the coefficient of $\hat{\ }$ is positive then the vector points in the y direction.

b f ho ever d 0 then the coefficient is negative and the vector points in the $-y$ direction.

c ince cos 90 = 0 then $\vec{a}\cdot\vec{b}=0$ using . 3 20.

d ince $\vec{b}\quad d$ is along the y a is then by the same reasoning as in the previous part $\vec{a}\cdot\;\vec{b}\quad d\;=0$.

e y the right hand rule $\vec{a}\times\vec{b}$ points in the z direction.

f y the same rule $\vec{b}\times\vec{a}$ points in the $-z$ direction. e note that $\vec{b}\times\vec{a}=-\vec{a}\times\vec{b}$ is true in this case and uite generally.

g   ince sin 90   = 1   . 3 24 gives $\vec{a}\times\vec{b} = ab$ here a is the magnitude of $\vec{a}$.

h   lso $\vec{a} \times \vec{b} = \vec{b} \times \vec{a} = ab$.

i   ith d 0   e find that $\vec{a}\times \vec{b} \;\; d$ has magnitude $ab\,d$.

he vector $\vec{a}\times \vec{b} \;\; d$ points in the z direction.

67.   e note that the set of choices for unit vector directions has correct orientation for a right handed coordinate system). Students sometimes confuse "north" with "up", so it might be necessary to emphasi e that these are being treated as the mutually perpendicular directions of our real world, not just some "on the paper" or "on the blackboard" representation of it. Once the terminology is clear, these questions are basic to the definitions of the scalar dot and vector cross products.

a $\hat{i}\cdot\hat{k}=0$ since $\hat{i}\perp\hat{k}$

b $-\hat{k} \cdot -\hat{\ } =0$ since $\hat{k}\perp\hat{\ }$.

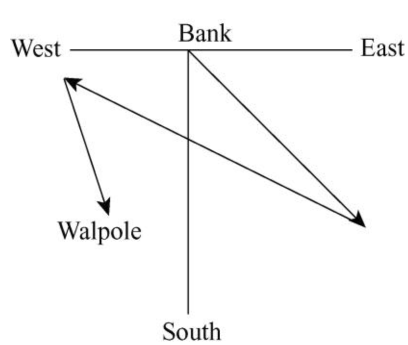
c $\hat{\ }\cdot -\hat{\ } =-1$.

d $\hat{k}\times\hat{\ }=-\hat{i}$ est .

e $-\hat{i} \times -\hat{\ } =+\hat{k}$ up ard .

f $-\hat{k} \times -\hat{\ } =-\hat{i}$ est .

68.   sketch of the displacements is sho n. he resultant not sho n   ould be a straight line from start   ank to finish   alpole .   ith a careful dra ing one should find that the resultant vector has length 29.5 km at 35   est of south.



69.   he point P is displaced vertically by $2R$ here R is the radius of the   heel. t is displaced hori ontally by half the circumference of the   heel or πR.   ince R = 0.450 m

the hori ontal component of the displacement is 1.414 m and the vertical component of the displacement is 0.900 m. f the x a is is hori ontal and the y a is is vertical the vector displacement in meters is $\vec{r} = \left(1.414\ \hat{\text{i}} \quad 0.900\ \hat{}\right)$. he displacement has a magnitude of

$$|\vec{r}| = \sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4} = 1.68\,\text{m}$$

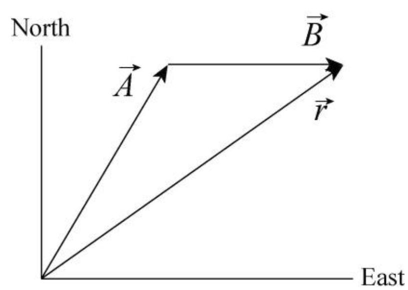
and an angle of

$$\tan^{-1}\left(\frac{2R}{\pi R}\right) = \tan^{-1}\left(\frac{2}{\pi}\right) = 32.5°$$

above the floor. n physics there are no “exact” measurements, yet that angle computation seemed to yield something *exact*. o ever there has to be some uncertainty in the observation that the heel rolled half of a revolution hich introduces some indefiniteness in our result.

70. he diagram sho s the displacement vectors for the t o segments of her alk labeled $\vec{A}$ and $\vec{B}$ and the total “final”) displacement vector, labeled $\vec{r}$. e take east to be the x direction and north to be the y direction. e observe that the angle bet een $\vec{A}$ and the x a is is 60 . here the units are not e plicitly sho n the distances are understood to be in meters. hus the components of $\vec{A}$ are A_x = 250 cos60 = 125 and A_y = 250 sin60 = 216.5. he components of $\vec{B}$ are B_x = 175 and B_y = 0. he components of the total displacement are

$r_x = A_x \quad B_x$ = 125 175 = 300
$r_y = A_y \quad B_y$ = 216.5 0 = 216.5.



a he magnitude of the resultant displacement is

$$\vec{r} = \sqrt{r_x^2 + r_y^2} = \sqrt{\ 300\text{ m}\ ^2 + \ 216.5\text{ m}\ ^2} = 370\,\text{m}.$$

b he angle the resultant displacement makes ith the x a is is

$$\tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{216.5\text{ m}}{300\text{ m}}\right) = 36°.$$

he direction is $36°$ north of due east.

c he total *distance* alked is d = 250 m 175 m = 425 m.

d he total distance alked is greater than the magnitude of the resultant displacement. he diagram sho s hy $\vec{A}$ and $\vec{B}$ are not collinear.

71. he vector $\vec{d}$ measured in meters can be represented as $\vec{d} =$ 3.0 m $-\hat{\ }$ here $-\hat{\ }$ is the unit vector pointing south. herefore $5.0\vec{d} = 5.0 \; -3.0 \text{ m} \;\hat{\ } = -15 \text{ m} \;\hat{\ }$.

a he positive scalar factor 5.0 affects the magnitude but not the direction. he magnitude of $5.0\vec{d}$ is 15 m.

b he ne direction of $5\vec{d}$ is the same as the old south.

he vector $-2.0\vec{d}$ can be ritten as $-2.0\vec{d} = \; 6.0 \text{ m} \;\hat{\ }$.

c he absolute value of the scalar factor $-2.0 \; = 2.0$ affects the magnitude. he ne magnitude is 6.0 m.

d he minus sign carried by this scalar factor reverses the direction so the ne direction is $+\hat{\ }$ or north.

72. The ant’s trip consists of three displacements:

$$\vec{d}_1 = \; 0.40 \text{ m} \;\; \cos 225°\,\hat{\mathrm{i}} + \sin 225°\,\hat{\ } \;\; = \; -0.28 \text{ m} \;\; \hat{\mathrm{i}} + \; -0.28 \text{ m} \;\; \hat{\ }$$
$$\vec{d}_2 = \; 0.50 \text{ m} \;\; \hat{\mathrm{i}}$$
$$\vec{d}_3 = \; 0.60 \text{ m} \;\; \cos 60°\,\hat{\mathrm{i}} + \sin 60°\,\hat{\ } \;\; = \; 0.30 \text{ m} \;\; \hat{\mathrm{i}} + \; 0.52 \text{ m} \;\; \hat{\ }$$

here the angle is measured ith respect to the positive x a is. e have taken the positive x and y directions to correspond to east and north respectively.

a he x component of $\vec{d}_1$ is $d_{1x} = \; 0.40 \text{ m} \; \cos 225° = -0.28 \text{ m}$.

b he y component of $\vec{d}_1$ is $d_{1y} = \; 0.40 \text{ m} \; \sin 225° = -0.28 \text{ m}$.

c he x component of $\vec{d}_2$ is $d_{2x} = 0.50 \text{ m}$.

d he y component of $\vec{d}_2$ is $d_{2y} = 0 \text{ m}$.

e he x component of $\vec{d}_3$ is $d_{3x} = 0.60\text{ m} \cos 60° = 0.30\text{ m}$.

f he y component of $\vec{d}_3$ is $d_{3y} = 0.60\text{ m} \sin 60° = 0.52\text{ m}$.

g he x component of the net displacement $\vec{d}_{\text{net}}$ is

$$d_{\text{net } x} = d_{1x} + d_{2x} + d_{3x} = -0.28\text{ m} + 0.50\text{ m} + 0.30\text{ m} = 0.52\text{ m}.$$

h he y component of the net displacement $\vec{d}_{\text{net}}$ is

$$d_{\text{net } y} = d_{1y} + d_{2y} + d_{3y} = -0.28\text{ m} + 0\text{ m} + 0.52\text{ m} = 0.24\text{ m}.$$

i he magnitude of the net displacement is

$$d_{\text{net}} = \sqrt{d^2_{\text{net } x} + d^2_{\text{net } y}} = \sqrt{0.52\text{ m}^2 + 0.24\text{ m}^2} = 0.57\text{ m}.$$

he direction of the net displacement is

$$\theta = \tan^{-1}\left(\frac{d_{\text{net } y}}{d_{\text{net } x}}\right) = \tan^{-1}\left(\frac{0.24\text{ m}}{0.52\text{ m}}\right) = 25° \quad \text{north of east}$$

f the ant has to return directly to the starting point the displacement ould be $-\vec{d}_{\text{net}}$.

k he distance the ant has to travel is $-\vec{d}_{\text{net}} = 0.57$ m.

l he direction the ant has to travel is $25°$ south of est .

73. e apply . 3 23 and . 3 27.

a $\vec{a} \times \vec{b} = a_x b_y - a_y b_x \;\hat{k}$ since all other terms vanish due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any z components. onse uently e obtain $3.0 \;\; 4.0 - 5.0 \;\; 2.0 \;\; \hat{k} = 2.0\hat{k}$.

b $\vec{a} \cdot b = a_x b_x \quad a_y b_y$ yields $3.0 \;\; 2.0 \quad 5.0 \;\; 4.0 = 26$.

c $\vec{a} + \vec{b} = 3.0 + 2.0 \;\hat{i} + 5.0 + 4.0 \;\hat{} \Rightarrow \quad \vec{a} \quad \vec{b} \cdot \vec{b} = 5.0 \;\; 2.0 \quad 9.0 \;\; 4.0 = 46$.

d everal approaches are available. n this solution e ill construct a $\hat{b}$ unit vector and "dot" it take the scalar product of it ith $\vec{a}$. n this case e make the desired unit vector by

$$\hat{b} = \frac{\vec{b}}{\vec{b}} = \frac{2.0\,\hat{\text{i}} + 4.0\,\hat{\ }}{\sqrt{2.0^{\,2} + \ 4.0^{\,2}}}\,.$$

e therefore obtain

$$a_b = \vec{a} \cdot \hat{b} = \frac{3.0 \quad 2.0 \ + \ 5.0 \quad 4.0}{\sqrt{2.0^{\,2} + \ 4.0^{\,2}}} = 5.81.$$

74. he t o vectors $\vec{a}$ and $\vec{b}$ are given by

$$\vec{a} = 3.20 \ \cos 63^\circ \hat{\ } + \sin 63^\circ \ \hat{\text{k}} \ = 1.45 \hat{\ } + 2.85 \ \hat{\text{k}}$$

$$\vec{b} = 1.40 \ \cos 48^\circ \ \hat{\text{i}} + \sin 48^\circ \ \hat{\text{k}} \ = 0.937 \hat{\text{i}} + 1.04 \ \hat{\text{k}}$$

he components of $\vec{a}$ are a_x = 0 a_y = 3.20 cos 63 = 1.45 and a_z = 3.20 sin 63 = 2.85.
he components of $\vec{b}$ are b_x = 1.40 cos 48 = 0.937 b_y = 0 and b_z = 1.40 sin 48 = 1.04.

a he scalar dot product is therefore

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = (0)(0.937) + (1.45)(0) + (2.85)(1.04) = 2.97.$$

b he vector cross product is

$$\begin{aligned}\vec{a} \times \vec{b} &= \left(a_y b_z - a_z b_y\right) \hat{\text{i}} \quad \left(\text{a}_z b_x - a_x b_z\right) \hat{\ } \quad \left(a_x b_y - a_y b_x\right) \hat{\text{k}} \\ &= \left((1.45)(1.04) - 0\right) \hat{\text{i}} \quad \left((2.85)(0.937) - 0\right) \hat{\ } + \left(0 - (1.45)(0.937)\right) \hat{\text{k}} \\ &= 1.51 \hat{\text{i}} \quad 2.67 \hat{\ } - 1.36 \hat{\text{k}}.\end{aligned}$$

c he angle θ bet een $\vec{a}$ and $\vec{b}$ is given by

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{ab}\right) = \cos^{-1}\left(\frac{2.97}{(3.20)(1.40)}\right) = 48.5^\circ.$$

75. e orient $\hat{\text{i}}$ east ard $\hat{\ }$ north ard and $\hat{\text{k}}$ up ard and use the follo ing fundamental products

$$\begin{aligned}\hat{\text{i}} \times \hat{\ } &= -\hat{\ } \times \hat{\text{i}} = \hat{\text{k}} \\ \hat{\ } \times \hat{\text{k}} &= -\hat{\text{k}} \times \hat{\ } = \hat{\text{i}} \\ \hat{\text{k}} \times \hat{\text{i}} &= -\hat{\text{i}} \times \hat{\text{k}} = \hat{\ }\end{aligned}$$

(a) “north cross west” = $\hat{\ } \times -\hat{\text{i}} = \hat{\text{k}}$ = “up.”

(b) "down dot south" = $-\hat{k} \cdot -\hat{} = 0$.

(c) "east cross up" = $\hat{i} \times \hat{k} = -\hat{}$ = "south."

(d) "west dot west" = $-\hat{i} \cdot -\hat{i} = 1$.

(e) "south cross south" = $-\hat{} \times -\hat{} = 0$.

76. et A denote the magnitude of $\vec{A}$ similarly for the other vectors. he vector e uation is $\vec{A} + \vec{B} = \vec{C}$ here B = 8.0 m and C = 2A. e are also told that the angle measured in the 'standard' sense for $\vec{A}$ is 0 and the angle for $\vec{C}$ is 90 hich makes this a right triangle hen dra n in a "head to tail" fashion) where B is the si e of the hypotenuse. sing the ythagorean theorem

$$B = \sqrt{A^2 + C^2} \Rightarrow 8.0 = \sqrt{A^2 + 4A^2}$$

hich leads to $A = 8 \quad \sqrt{5} = 3.6$ m.

77. e orient $\hat{i}$ east ard $\hat{}$ north ard and $\hat{k}$ up ard.

a he displacement is $\vec{d} = 1300 \text{ m } \hat{i} + 2200 \text{ m } \hat{} + -410 \text{ m } \hat{k}$.

b he displacement for the return portion is $\vec{d}' = -1300 \text{ m } \hat{i} - 2200 \text{ m } \hat{}$ and the magnitude is $d' = \sqrt{-1300 \text{ m}^{\ 2} + -2200 \text{ m}^{\ 2}} = 2.56 \times 10^3 \text{ m}$.

he net displacement is ero since his final position matches his initial position.

78. et $\vec{c} = \vec{b} \times \vec{a}$. hen the magnitude of $\vec{c}$ is $c = ab \sin \phi$. ince $\vec{c}$ is perpendicular to $\vec{a}$ the magnitude of $\vec{a} \times \vec{c}$ is ac. he magnitude of $\vec{a} \times \vec{b} \times \vec{a}$ is conse uently

$$\vec{a} \times \vec{b} \times \vec{a} = ac = a^2 b \sin \phi .$$

ubstituting the values given e obtain

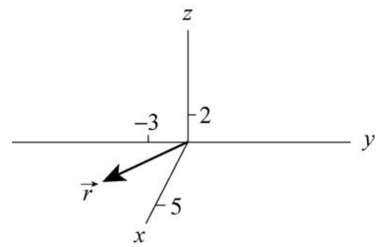
$$\vec{a} \times \vec{b} \times \vec{a} = a^2 b \sin \phi = 3.90^{\ 2} \ 2.70 \ \sin 63.0^\circ = 36.6 .$$

79. he area of a triangle is half the product of its base and altitude. he base is the side formed by vector $\vec{a}$. hen the altitude is $b \sin \phi$ and the area is $A = \frac{1}{2} ab \sin \phi = \frac{1}{2} \ \vec{a} \times \vec{b}$. ubstituting the values given e have

$$A = \frac{1}{2} ab \sin \phi = \frac{1}{2} \ 4.3 \ \ 5.4 \ \sin 46^\circ \approx 8.4 .$$

Chapter

1. a he magnitude of $\vec{r}$ is



$$\vec{r} = \sqrt{5.0 \text{ m}^2 + -3.0 \text{ m}^2 + 2.0 \text{ m}^2} = 6.2 \text{ m}.$$

b sketch is sho n. he coordinate values are in meters.

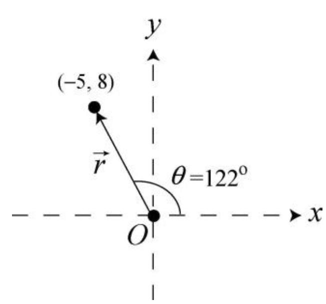
2. a he position vector according to . 4 1 is $\vec{r} = -5.0 \text{ m } \hat{\text{i}} \quad 8.0 \text{ m} \,\hat{}$.

b he magnitude is $\vec{r} = \sqrt{x^2 \quad y^2 \quad z^2} = \sqrt{-5.0 \text{ m}^2 + 8.0 \text{ m}^2 + 0 \text{ m}^2} = 9.4 \text{ m}.$

c any calculators have polar ↔ rectangular conversion capabilities that make this computation more efficient than hat is sho n belo . oting that the vector lies in the xy plane and using . 3 6 e obtain

$$\theta = \tan^{-1}\left(\frac{8.0 \text{ m}}{-5.0 \text{ m}}\right) = -58° \text{ or } 122°$$

here the latter possibility 122 measured counterclock ise from the x direction is chosen since the signs of the components imply the vector is in the second uadrant.



d he sketch is sho n to the right. he vector is 122 counterclock ise from the x direction.

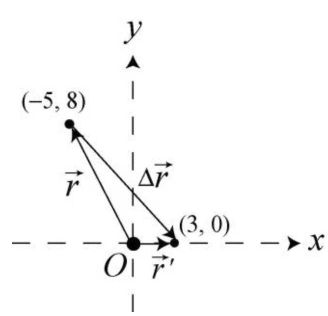
e he displacement is $\Delta\vec{r} = \vec{r}' - \vec{r}$ here $\vec{r}$ is given in part a and $\vec{r}' = 3.0 \text{ m } \hat{\text{i}}$. herefore $\Delta\vec{r} = 8.0 \text{ m } \hat{\text{i}} - 8.0 \text{ m} \,\hat{}$.

f he magnitude of the displacement is

$$\Delta\vec{r} = \sqrt{8.0 \text{ m}^2 + -8.0 \text{ m}^2} = 11 \text{ m}.$$

g he angle for the displacement using . 3 6 is

$$\tan^{-1}\left(\frac{8.0 \text{ m}}{-8.0 \text{ m}}\right) = -45° \text{ or } 135°$$



here e choose the former possibility –45 or 45 measured *clockwise* from x since the signs of the components imply the vector is in the fourth uadrant. sketch of $\Delta\vec{r}$ is sho n on the right.

3. he initial position vector $\vec{r}_0$ satisfies $\vec{r}-\vec{r}_0=\Delta\vec{r}$ hich results in

$$\vec{r}_0=\vec{r}-\Delta\vec{r}= 3.0\hat{} - 4.0\hat{k}\text{ m} - 2.0\hat{i} - 3.0\hat{} + 6.0\hat{k}\text{ m} = -2.0\text{ m }\hat{i} + 6.0\text{ m }\hat{} + -10\text{ m }\hat{k}.$$

4. e choose a coordinate system ith origin at the clock center and x right ard (toward the “3:00” position) and $+y$ upward (toward “12:00”).

a n unit vector notation e have $\vec{r}_1 = 10\text{ cm }\hat{i}$ and $\vec{r}_2 = -10\text{ cm }\hat{}$. hus . 4 2 gives

$$\Delta\vec{r}=\vec{r}_2-\vec{r}_1= -10\text{ cm }\hat{i} + -10\text{ cm }\hat{}.$$

he magnitude is given by $\Delta\vec{r} = \sqrt{-10\text{ cm}^2 + -10\text{ cm}^2} = 14\text{ cm}.$

b sing . 3 6 the angle is

$$\theta=\tan^{-1}\left(\frac{-10\text{ cm}}{-10\text{ cm}}\right)=45^\circ\text{ or }-135^\circ.$$

e choose -135° since the desired angle is in the third uadrant. n terms of the magnitude angle notation one may rite

$$\Delta\vec{r}=\vec{r}_2-\vec{r}_1= -10\text{ cm }\hat{i} + -10\text{ cm }\hat{} \rightarrow 14\text{cm}\angle -135^\circ .$$

c n this case e have $\vec{r}_1 = -10\text{ cm }\hat{}$ and $\vec{r}_2 = 10\text{ cm }\hat{}$ and $\Delta\vec{r} = 20\text{ cm }\hat{}$. hus $\Delta\vec{r} = 20\text{ cm}$.

d sing . 3 6 the angle is given by

$$\theta=\tan^{-1}\left(\frac{20\text{ cm}}{0\text{ cm}}\right)=90^\circ.$$

e n a full hour s eep the hand returns to its starting position and the displacement is ero.

f he corresponding angle for a full hour s eep is also ero.

5. **THINK** his problem deals ith the motion of a train in t o dimensions. he entire trip consists of three parts, and we're interested in the overall average velocity.

EXPRESS he average velocity of the entire trip is given by . 4 8 $\vec v_{\text{avg}} = \Delta\vec r \ \ \Delta t$ here the total displacement $\Delta\vec r = \Delta\vec r_1 + \Delta\vec r_2 + \Delta\vec r_3$ is the sum of three displacements each result of a constant velocity during a given time and $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$ is the total amount of time for the trip. e use a coordinate system ith x for ast and y for orth.

ANALYZE a n unit vector notation the first displacement is given by

$$\Delta\vec r_1 = \left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{40.0\ \text{min}}{60\ \text{min h}}\right)\hat{\text{i}} = \ 40.0\ \text{km}\ \hat{\text{i}}.$$

he second displacement has a magnitude of $60.0\,\frac{\text{km}}{\text{h}} \cdot \frac{20.0\ \text{min}}{60\ \text{min h}} = 20.0\ \text{km}$ and its direction is 40 north of east. herefore

$$\Delta\vec r_2 = \ 20.0\ \text{km}\ \ \cos 40.0^\circ\ \hat{\text{i}} + \ 20.0\ \text{km}\ \ \sin 40.0^\circ\ \hat{\ } = \ 15.3\ \text{km}\ \hat{\text{i}} + \ 12.9\ \text{km}\ \hat{\ }.$$

imilarly the third displacement is

$$\Delta\vec r_3 = -\left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{50.0\ \text{min}}{60\ \text{min h}}\right)\hat{\text{i}} = \ -50.0\ \text{km}\ \hat{\text{i}}.$$

hus the total displacement is

$$\begin{aligned}\Delta\vec r = \Delta\vec r_1 + \Delta\vec r_2 + \Delta\vec r_3 &= \ 40.0\ \text{km}\ \hat{\text{i}} + \ 15.3\ \text{km}\ \hat{\text{i}} + \ 12.9\ \text{km}\ \hat{\ } - \ 50.0\ \text{km}\ \hat{\text{i}} \\ &= \ 5.30\ \text{km}\ \hat{\text{i}} + \ 12.9\ \text{km}\ \hat{\ }.\end{aligned}$$

he time for the trip is $\Delta t = \ 40.0 \ \ 20.0 \ \ 50.0$ min = 110 min hich is e uivalent to 1.83 h. . 4 8 then yields

$$\vec v_{\text{avg}} = \frac{\ 5.30\ \text{km}\ \hat{\text{i}} + \ 12.9\ \text{km}\ \hat{\ }}{1.83\ \text{h}} = \ 2.90\ \text{km h}\ \hat{\text{i}} + \ 7.01\ \text{km h}\ \hat{\ }.$$

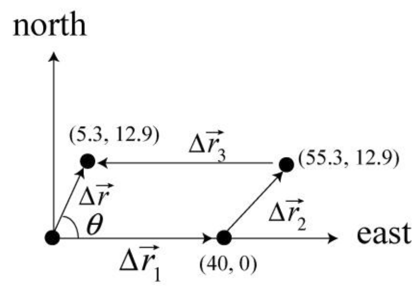
he magnitude of $\vec v_{\text{avg}}$ is $\ \vec v_{\text{avg}}\ = \sqrt{\ 2.90\ \text{km h}\ ^2 + \ 7.01\ \text{km h}\ ^2} = 7.59$ km h.

b he angle is given by

$$\theta = \tan^{-1}\left(\frac{v_{\text{avg}\ y}}{v_{\text{avg}\ x}}\right) = \tan^{-1}\left(\frac{7.01\ \text{km h}}{2.90\ \text{km h}}\right) = 67.5^\circ \quad \text{north of east}$$

or 22.5° east of due north.

LEARN he displacement of the train is depicted in the figure belo



ote that the net displacement $\Delta\vec{r}$ is found by adding $\Delta\vec{r}_1$ $\Delta\vec{r}_2$ and $\Delta\vec{r}_3$ vectorially.

6. o emphasi e the fact that the velocity is a function of time e adopt the notation $v\ t$ for $dx\ \ dt$.

a uation 4 10 leads to

$$v\ t\ = \frac{d}{dt}\ 3.00t\hat{\mathrm{i}}\ - 4.00t^{2\hat{}}\quad 2.00\hat{\mathrm{k}}\ =\ 3.00\ \mathrm{m\ s}\ \hat{\mathrm{i}}\ -\ 8.00\ \mathrm{m\ s}^2\ t\ \hat{}$$

b valuating this result at t = 2.00 s produces $\vec{v}$ = $3.00\hat{\mathrm{i}}\ -\ 16.0\hat{}$ m s.

c he speed at t = 2.00 s is $v\ = \vec{v}\ = \sqrt{\ 3.00\ \mathrm{m\ s}\ ^2 +\ \ -16.0\ \mathrm{m\ s}\ ^2} = 16.3$ m s.

d he angle of $\vec{v}$ at that moment is

$$\tan^{-1}\left(\frac{-16.0\ \mathrm{m\ s}}{3.00\ \mathrm{m\ s}}\right) = -79.4^\circ \text{ or } 101^\circ$$

here e choose the first possibility 79.4 measured *clockwise* from the x direction or 281 counterclock ise from x since the signs of the components imply the vector is in the fourth uadrant.

7. sing . 4 3 and . 4 8 e have

$$\vec{v}_{\mathrm{avg}} = \frac{-2.0\hat{\mathrm{i}}\quad 8.0\hat{}\ -\ 2.0\hat{\mathrm{k}}\ \ \mathrm{m}\ -\ \ 5.0\hat{\mathrm{i}}\ -\ 6.0\hat{}\quad 2.0\hat{\mathrm{k}}\ \ \mathrm{m}}{10\ \mathrm{s}} =\ -0.70\hat{\mathrm{i}}\quad 1.40\hat{}\ -\ 0.40\hat{\mathrm{k}}\ \ \mathrm{m\ s}.$$

8. ur coordinate system has $\hat{\mathrm{i}}$ pointed east and $\hat{}$ pointed north. he first displacement is $\vec{r}_{AB}$ = 483 km $\hat{\mathrm{i}}$ and the second is $\vec{r}_{BC}$ = -966 km $\hat{}$.

a he net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = 483 \text{ km } \hat{\text{i}} - 966 \text{ km } \hat{}$$

hich yields $\vec{r}_{AC} = \sqrt{483 \text{ km}^2 + -966 \text{ km}^2} = 1.08 \times 10^3$ km.

b he angle is given by

$$\theta = \tan^{-1}\left(\frac{-966 \text{ km}}{483 \text{ km}}\right) = -63.4°.$$

e observe that the angle can be alternatively e pressed as 63.4 south of east or 26.6 east of south.

c ividing the magnitude of $\vec{r}_{AC}$ by the total time 2.25 h gives

$$\vec{v}_{\text{avg}} = \frac{483 \text{ km } \hat{\text{i}} - 966 \text{ km } \hat{}}{2.25 \text{ h}} = 215 \text{ km h } \hat{\text{i}} - 429 \text{ km h } \hat{}$$

ith a magnitude $\vec{v}_{\text{avg}} = \sqrt{215 \text{ km h}^2 + -429 \text{ km h}^2} = 480$ km h.

d he direction of $\vec{v}_{\text{avg}}$ is 26.6 east of south same as in part b . n magnitude angle notation e ould have $\vec{v}_{\text{avg}} = 480 \text{ km h} \angle -63.4°$.

e ssuming the AB trip as a straight one and similarly for the BC trip then $\vec{r}_{AB}$ is the distance traveled during the AB trip and $\vec{r}_{BC}$ is the distance traveled during the BC trip. ince the average speed is the total distance divided by the total time it e uals

$$\frac{483 \text{ km} + 966 \text{ km}}{2.25 \text{ h}} = 644 \text{ km h}.$$

9. he x,y coordinates in meters of the points are A = 15 –15 B = 30 –45 C = 20 –15 and D = 45 45 . he respective times are t_A = 0 t_B = 300 s t_C = 600 s and t_D = 900 s. verage velocity is defined by . 4 8. ach displacement $\Delta\vec{r}$ is understood to originate at point A.

a he average velocity having the least magnitude 5.0 m 600 s is for the displacement ending at point C $\vec{v}_{\text{avg}} = 0.0083$ m s.

b he direction of $\vec{v}_{\text{avg}}$ is 0° measured counterclock ise from the x a is .

c he average velocity having the greatest magnitude $\sqrt{15\text{ m}^2+30\text{ m}^2}$ 300 s is for the displacement ending at point B $\vec{v}_{avg}=0.11$ m s.

d he direction of $\vec{v}_{avg}$ is 297° counterclock ise from x or –63° hich is e uivalent to measuring 63° *clockwise* from the x a is .

10. e differentiate $\vec{r}=5.00t\,\hat{\imath}+\;et+ft^2\;\hat{}$.

a The particle's motion is indicated by the derivative of $\vec{r}$ $\vec{v}$ = 5.00 i e $2ft$. he angle of its direction of motion is conse uently

$$\theta=\tan^{-1}\;v_y\;v_x\;=\tan^{-1}\;e\;\;2ft\;\;5.00\;.$$

he graph indicates θ_0 = 35.0° hich determines the parameter e

$$e=\;5.00\text{ m s}\;\tan 35.0°\;=3.50\text{ m s}.$$

b e note from the graph that θ = 0 hen t = 14.0 s. hus e $2ft=0$ at that time. his determines the parameter f

$$f=\frac{-e}{2t}=\frac{-3.5\text{ m s}}{2\;14.0\text{ s}}=-0.125\text{ m s}^2.$$

11. n parts b and c e use . 4 10 and . 4 16. or part d e find the direction of the velocity computed in part b since that represents the asked for tangent line.

a lugging into the given e pression e obtain

$$\vec{r}\Big|_{t=2.00}=\;2.00\;8\;-5.00\;2\;\;\hat{\imath}\;\;6.00-7.00\;16\;\;\hat{}\;=\;6.00\hat{\imath}\;-\;106\hat{}\;\;\text{m}$$

b aking the derivative of the given e pression produces

$$\vec{v}\;t\;=\;6.00t^2\;-\;5.00\;\;\hat{\imath}\;-\;28.0t^3\;\hat{}$$

here e have ritten $v\;t$ to emphasi e its dependence on time. his becomes at t = 2.00 s $\vec{v}$ = 19.0$\hat{\imath}$ – 224$\hat{}$ m s.

c ifferentiating the $\vec{v}\;t$ found above ith respect to t produces $12.0t\,\hat{\imath}-84.0t^2\,\hat{}$ hich yields $\vec{a}$ = 24.0$\hat{\imath}$–336$\hat{}$ m s^2 at t = 2.00 s.

d he angle of $\vec{v}$ measured from x is either

$$\tan^{-1}\left(\frac{-224\text{ m s}}{19.0\text{ m s}}\right) = -85.2° \text{ or } 94.8°$$

here e settle on the first choice –85.2 hich is e uivalent to 275 measured counterclock ise from the x a is since the signs of its components imply that it is in the fourth uadrant.

12. e adopt a coordinate system ith $\hat{i}$ pointed east and $\hat{}$ pointed north the coordinate origin is the flagpole. We "translate" the given information into unit vector notation as follo s

$$\vec{r}_o = 40.0\text{ m } \hat{i} \quad \text{and} \quad \vec{v}_o = -10.0\text{ m s } \hat{}$$

$$\vec{r} = 40.0\text{ m } \hat{} \quad \text{and} \quad \vec{v} = 10.0\text{ m s } \hat{i}.$$

a sing . 4 2 the displacement $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r} - \vec{r}_o = -40.0\text{ m } \hat{i} + 40.0\text{ m } \hat{}$$

ith a magnitude $\Delta\vec{r} = \sqrt{-40.0\text{ m}^2 + 40.0\text{ m}^2} = 56.6$ m.

b he direction of $\Delta\vec{r}$ is

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{40.0\text{ m}}{-40.0\text{ m}}\right) = -45.0° \text{ or } 135°.$$

ince the desired angle is in the second uadrant e pick 135° 45° north of due est . ote that the displacement can be ritten as $\Delta\vec{r} = \vec{r} - \vec{r}_o = (56.6\angle 135°)$ in terms of the magnitude angle notation.

c he magnitude of $\vec{v}_{avg}$ is simply the magnitude of the displacement divided by the time Δt = 30.0 s . hus the average velocity has magnitude 56.6 m 30.0 s = 1.89 m s.

d uation 4 8 sho s that $\vec{v}_{avg}$ points in the same direction as $\Delta\vec{r}$ that is 135° 45° north of due est .

e sing . 4 15 e have

$$\vec{a}_{avg} = \frac{\vec{v} - \vec{v}_o}{\Delta t} = 0.333\text{ m s}^2\ \hat{i} + 0.333\text{ m s}^2\ \hat{}.$$

he magnitude of the average acceleration vector is therefore e ual to $\vec{a}_{avg} = \sqrt{0.333\text{ m s}^{2\ 2} + 0.333\text{ m s}^{2\ 2}} = 0.471\text{ m s}^2$.

f he direction of $\vec{a}_{avg}$ is

$$\theta = \tan^{-1}\left(\frac{0.333\text{ m s}^2}{0.333\text{ m s}^2}\right) = 45° \text{ or } -135°.$$

ince the desired angle is no in the first uadrant e choose 45° and $\vec{a}_{avg}$ points north of due east.

13. **THINK** no ing the position of a particle as function of time allo s us to calculate its corresponding velocity and acceleration by taking time derivatives.

EXPRESS rom the position vector $\vec{r}\ t$ the velocity and acceleration of the particle can be found by differentiating $\vec{r}\ t$ ith respect to time

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}.$$

ANALYZE a aking the derivative of the position vector $\vec{r}\ t = \hat{\imath} + 4t^2\ \hat{} + t\hat{k}$ ith respect to time e have in units m s

$$\vec{v} = \frac{d}{dt}\ \hat{\imath} + 4t^2\ \hat{} + t\hat{k} = 8t\ \hat{} + \hat{k}.$$

b aking another derivative ith respect to time leads to in units m s^2

$$\vec{a} = \frac{d}{dt}\ 8t\ \hat{} + \hat{k} = 8\ \hat{}.$$

LEARN he particle undergoes constant acceleration in the y direction. his can be seen by noting that the y component of $\vec{r}\ t$ is $4t^2$ hich is uadratic in t.

14. e use . 4 15 ith $\vec{v}_1$ designating the initial velocity and $\vec{v}_2$ designating the later one.

a he average acceleration during the $\Delta t = 4$ s interval is

$$\vec{a}_{avg} = \frac{-2.0\hat{\imath} - 2.0\ \hat{}\ \ 5.0\hat{k}\ \text{ m s} - \ 4.0\hat{\imath} - 22\ \hat{}\ \ 3.0\hat{k}\ \text{ m s}}{4\text{ s}} = \ -1.5\text{ m s}^2\ \hat{\imath} + \ 0.5\text{ m s}^2\ \hat{k}.$$

b he magnitude of $\vec{a}_{avg}$ is $\sqrt{\ -1.5\text{ m s}^2\ ^2 + \ 0.5\text{ m s}^2\ ^2} = 1.6\text{ m s}^2$.

c ts angle in the xz plane measured from the x a is is one of these possibilities

$$\tan^{-1}\left(\frac{0.5\text{ m s}^2}{-1.5\text{ m s}^2}\right) = -18° \text{ or } 162°$$

here e settle on the second choice since the signs of its components imply that it is in the second uadrant.

15. **THINK** Given the initial velocity and acceleration of a particle, we're interested in finding its velocity and position at a later time.

EXPRESS ince the acceleration $\vec{a} = a_x\hat{\text{i}} + a_y\hat{} = -1.0\text{ m s}^2\ \hat{\text{i}} + -0.50\text{ m s}^2\ \hat{}$ is constant in both x and y directions e may use able 2 1 for the motion along each direction. his can be handled individually for x and y or together ith the unit vector notation for $\Delta\vec{r}$.

ince the particle started at the origin the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. he velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$ here $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the constant acceleration. long the x direction e have

$$x\ t\ = v_{0x}t + \frac{1}{2}a_x t^2 \qquad v_x\ t\ = v_{0x} + a_x t$$

imilarly along the y direction e get

$$y\ t\ = v_{0y}t + \frac{1}{2}a_y t^2 \qquad v_y\ t\ = v_{0y} + a_y t\ .$$

Known $v_{0x} = 3.0\text{ m s}$ $v_{0y} = 0$ $a_x = -1.0\text{ m s}^2$ $a_y = -0.5\text{ m s}^2$.

ANALYZE a ubstituting the values given the components of the velocity are

$$v_x\ t\ = v_{0x} + a_x t = \ 3.0\text{ m s}\ -\ 1.0\text{ m s}^2\ t$$
$$v_y\ t\ = v_{0y} + a_y t = -\ 0.50\text{ m s}^2\ t$$

hen the particle reaches its ma imum x coordinate at $t = t_m$ e must have $v_x = 0$. herefore $3.0 - 1.0t_m = 0$ or $t_m = 3.0$ s. he y component of the velocity at this time is

$$v_y\ t = 3.0\text{ s}\ = -\ 0.50\text{ m s}^2\ \ 3.0\ = -1.5\text{ m s}$$

hus $\vec{v}_m = \ -1.5\text{ m s}\ \hat{}$.

b t $t = 3.0$ s the components of the position are

$$x\ \ t=3.0\,\text{s}\ \ =v_{0x}t+\frac{1}{2}a_x t^2 =\ \ 3.0\text{ m s}\ \ 3.0\text{ s}\ +\frac{1}{2}\ \ -1.0\text{ m s}^2\ \ 3.0\text{ s}\ ^2=4.5\text{ m}$$
$$y\ \ t=3.0\,\text{s}\ \ =v_{0y}t+\frac{1}{2}a_y t^2 =0+\frac{1}{2}\ \ -0.5\text{ m s}^2\ \ 3.0\text{ s}\ ^2=-2.25\text{ m}$$

sing unit vector notation the results can be ritten as $\vec{r}_m =$ 4.50 m $\hat{\text{i}}$ – 2.25 m $\hat{}$.

LEARN he motion of the particle in this problem is t o dimensional and the kinematics in the x and y directions can be analy ed separately.

16. e make use of . 4 16.

a he acceleration as a function of time is

$$\vec{a}=\frac{d\vec{v}}{dt}=\frac{d}{dt}\left(\left(6.0t-4.0t^2\right)\hat{\text{i}}\quad 8.0\hat{}\right)=\left(6.0-8.0t\right)\hat{\text{i}}$$

in units. pecifically e find the acceleration vector at $t=3.0\,\text{s}$ to be $\left(6.0-8.0\ \ 3.0\ \ \right)\hat{\text{i}}=\ \ -18\text{ m s}^2\ \ \hat{\text{i}}$.

b he e uation is $\vec{a}=\left(6.0-8.0t\right)\hat{\text{i}}=0$ e find t = 0.75 s.

c ince the y component of the velocity v_y = 8.0 m s is never ero the velocity cannot vanish.

d ince speed is the magnitude of the velocity e have

$$v=\ \vec{v}\ \ =\sqrt{\left(6.0t-4.0t^2\right)^2+\left(8.0\right)^2}=10$$

in units m s . o solve for t e first s uare both sides of the above e uation follo ed by some rearrangement

$$\left(6.0t-4.0t^2\right)^2+64\ =\ 100\ \ \Rightarrow\ \ \left(6.0t-4.0t^2\right)^2=36$$

aking the s uare root of the ne e pression and making further simplification lead to

$$6.0t-4.0t^2=\pm 6.0\ \ \Rightarrow\ \ 4.0t^2-6.0t\pm 6.0=0$$

inally using the uadratic formula e obtain

$$t=\frac{6.0\pm\sqrt{36-4(4.0)(\pm 6.0)}}{2(8.0)}$$

here the re uirement of a real positive result leads to the uni ue ans er t = 2.2 s.

17. e find t by applying . 2 11 to motion along the y a is ith v_y = 0 characteri ing $y = y_{ma}$

$$0 = \ 12\text{ m s}\quad -2.0\text{ m s}^2\ t \ \Rightarrow\ t = 6.0\text{ s.}$$

hen . 2 11 applies to motion along the x a is to determine the ans er

$$v_x = \ 8.0\text{ m s}\quad 4.0\text{ m s}^2\ \ 6.0\text{ s}\ = 32\text{ m s.}$$

herefore the velocity of the cart hen it reaches $y = y_{ma}$ is 32 m s i.

18. e find t by solving $\Delta x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

$$12.0\text{ m} = 0 + \ 4.00\text{ m s}\ t + \frac{1}{2}\ 5.00\text{ m s}^2\ t^2$$

here e have used Δx = 12.0 m v_x = 4.00 m s and a_x = 5.00 m s^2. e use the uadratic formula and find t = 1.53 s. hen . 2 11 actually its analog in t o dimensions applies ith this value of t. herefore its velocity hen Δx = 12.00 m is

$$\begin{aligned}\vec{v} &= \vec{v}_0 + \vec{a}t = \ 4.00\text{ m s}\ \hat{\text{i}} + \ 5.00\text{ m s}^2\ \ 1.53\text{ s}\ \hat{\text{i}} + \ 7.00\text{ m s}^2\ \ 1.53\text{ s}\ \hat{} \\ &= \ 11.7\text{ m s}\ \hat{\text{i}} + \ 10.7\text{ m s}\ \hat{}.\end{aligned}$$

hus the magnitude of $\vec{v}$ is $\vec{v} = \sqrt{11.7\text{ m s}^{\ 2} + \ 10.7\text{ m s}^{\ 2}} = 15.8\text{ m s.}$

b he angle of $\vec{v}$ measured from x is

$$\tan^{-1}\left(\frac{10.7\text{ m s}}{11.7\text{ m s}}\right) = 42.6°.$$

19. e make use of . 4 16 and . 4 10.

sing $\vec{a} = 3t\hat{\text{i}} + 4t\hat{}$ e have in m s

$$\vec{v}\ t\ = \vec{v}_0 + \int_0^t \vec{a}\ dt = \ 5.00\hat{\text{i}} + 2.00\hat{}\ + \int_0^t \ 3t\hat{\text{i}} + 4t\hat{}\ \ dt = \left(5.00 + 3t^2\ \ 2\right)\hat{\text{i}} + \left(2.00 + 2t^2\right)\hat{}$$

ntegrating using . 4 10 then yields in meters

$$\vec{r}\ t\ = \vec{r}_0 + \int_0^t \vec{v}dt = \ 20.0\hat{\text{i}} + 40.0\hat{\ }\ + \int_0^t\ \ 5.00 + 3t^2\ \ 2\ \hat{\text{i}} +\ 2.00 + 2t^2\ \hat{\ }\ dt$$

$$= \ 20.0\hat{\text{i}} + 40.0\hat{\ }\ +\ 5.00t + t^3\ \ 2\ \hat{\text{i}} +\ 2.00t + 2t^3\ 3\ \hat{\ }$$

$$= \ 20.0 + 5.00t + t^3\ \ 2\ \hat{\text{i}} +\ 40.0 + 2.00t + 2t^3\ 3\ \hat{\ }$$

a t $t = 4.00$ s e have $\vec{r}\ t = 4.00\,\text{s}\ =\ 72.0\ \text{m}\ \hat{\text{i}} +\ 90.7\ \text{m}\ \hat{\ }$.

b $\vec{v}\ t = 4.00\,\text{s}\ =\ 29.0\ \text{m s}\ \hat{\text{i}} +\ 34.0\ \text{m s}\ \hat{\ }$. hus the angle bet een the direction of travel and x measured counterclock ise is $\theta = \tan^{-1}\ 34.0\,\text{m s}\ \ 29.0\,\text{m s}\ = 49.5°$.

20. he acceleration is constant so that use of able 2 1 for both the x and y motions is permitted. here units are not sho n units are to be understood. ollision bet een particles A and B re uires t o things. irst the y motion of B must satisfy using . 2 15 and noting that θ is measured from the y a is

$$y = \frac{1}{2}a_y t^2 \ \Rightarrow\ 30\ \text{m} = \frac{1}{2}\left[\ 0.40\ \text{m s}^2\ \ \cos\theta\right]t^2.$$

econd the x motions of A and B must coincide

$$vt = \frac{1}{2}a_x t^2 \ \Rightarrow\ 3.0\ \text{m s}\ t = \frac{1}{2}\left[\ 0.40\ \text{m s}^2\ \ \sin\theta\right]t^2.$$

e eliminate a factor of t in the last relationship and formally solve for time

$$t = \frac{2v}{a_x} = \frac{2\ 3.0\ \text{m s}}{0.40\ \text{m s}^2\ \ \sin\theta}.$$

his is then plugged into the previous e uation to produce

$$30\ \text{m} = \frac{1}{2}\left[\ 0.40\ \text{m s}^2\ \ \cos\theta\right]\left(\frac{2\ 3.0\ \text{m s}}{0.40\ \text{m s}^2\ \ \sin\theta}\right)^2$$

hich ith the use of $\sin^2\theta = 1 - \cos^2\theta$ simplifies to

$$30 = \frac{9.0}{0.20}\ \frac{\cos\theta}{1 - \cos^2\theta}\ \Rightarrow\ 1 - \cos^2\theta = \frac{9.0}{(0.20)(30)}\cos\theta.$$

e use the uadratic formula choosing the positive root to solve for $\cos\theta$

$$\cos\theta = \frac{-1.5 + \sqrt{1.5^2 - 4(1.0)(-1.0)}}{2} = \frac{1}{2}$$

hich yields $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$.

21. e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he initial velocity is hori ontal so that $v_{0_y} = 0$ and $v_{0x} = v_0 = 10$ m/s.

(a) With the origin at the initial point (where the dart leaves the thrower's hand), the y coordinate of the dart is given by $y = -\frac{1}{2}gt^2$ so that ith y = $-PQ$ e have $PQ = \frac{1}{2}\left(9.8 \text{ m s}^2\right)\left(0.19 \text{ s}\right)^2 = 0.18$ m.

b rom $x = v_0t$ e obtain x = 10 m s 0.19 s = 1.9 m.

22. e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable.

a ith the origin at the initial point edge of table the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$. f t is the time of flight and y = –1.20 m indicates the level at hich the ball hits the floor then

$$t = \sqrt{\frac{2(-1.20 \text{ m})}{-9.80 \text{ m s}^2}} = 0.495 \text{s}.$$

b he initial hori ontal velocity of the ball is $\vec{v} = v_0 \hat{\text{i}}$. ince x = 1.52 m is the hori ontal position of its impact point ith the floor e have $x = v_0t$. hus

$$v_0 = \frac{x}{t} = \frac{1.52 \text{ m}}{0.495 \text{ s}} = 3.07 \text{ m s}.$$

23. a rom . 4 22 ith $\theta_0 = 0$ the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\ 45.0 \text{ m}}{9.80 \text{ m s}^2}} = 3.03 \text{ s}.$$

b he hori ontal distance traveled is given by . 4 21

$$\Delta x = v_0 t = \ 250 \text{ m s} \ \ 3.03 \text{ s} \ = 758 \text{ m}.$$

c nd from . 4 23 e find

$$|v_y| = gt = \ 9.80 \text{ m s}^2 \ \ 3.03 \text{ s} \ = 29.7 \text{ m s}.$$

24. e use . 4 26

$$R_{ma} = \left(\frac{v_0^2}{g}\sin 2\theta_0\right)_{ma} = \frac{v_0^2}{g} = \frac{(9.50\,\text{m s})^2}{9.80\,\text{m s}^2} = 9.209\text{ m} \approx 9.21\text{m}$$

to compare ith o ell's long jump; the difference from R_{ma} is only ΔR = 9.21m – 8.95m = 0.259 m.

25. sing . 4 26 the take off speed of the umper is

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{9.80\text{ m s}^2 \quad 77.0\text{ m}}{\sin 2\ 12.0^\circ}} = 43.1\text{ m s}$$

26. e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. The coordinate origin is the throwing point (the stone's initial position . he x component of its initial velocity is given by $v_{0x} = v_0\cos\theta_0$ and the y component is given by $v_{0y} = v_0\sin\theta_0$ here v_0 = 20 m s is the initial speed and θ_0 = 40.0 is the launch angle.

a t t = 1.10 s its x coordinate is

$$x = v_0 t\cos\theta_0 = (20.0\text{ m s})(1.10\text{ s})\cos 40.0^\circ = 16.9\text{ m}$$

b ts y coordinate at that instant is

$$y = v_0 t\sin\theta_0 - \frac{1}{2}gt^2 = (20.0\text{m s})(1.10\text{s})\sin 40.0^\circ - \frac{1}{2}(9.80\text{m s}^2)(1.10\text{s})^2 = 8.21\text{ m.}$$

c t t' = 1.80 s its x coordinate is $x = (20.0\text{ m s})(1.80\text{ s})\cos 40.0^\circ = 27.6\text{ m.}$

d ts y coordinate at t' is

$$y = (20.0\text{m s})(1.80\text{s})\sin 40.0^\circ - \frac{1}{2}(9.80\text{m s}^2)(1.80\text{s}^2) = 7.26\text{m.}$$

e he stone hits the ground earlier than t = 5.0 s. o find the time hen it hits the ground solve $y = v_0 t\sin\theta_0 - \frac{1}{2}gt^2 = 0$ for t. e find

$$t = \frac{2v_0}{g}\sin\theta_0 = \frac{2(20.0\text{ m s})}{9.8\text{ m s}^2}\sin 40^\circ = 2.62\text{ s.}$$

ts x coordinate on landing is

$$x = v_0 t \cos\theta_0 = (20.0 \text{ m s})(2.62 \text{ s}) \cos 40° = 40.2 \text{ m}.$$

f ssuming it stays here it lands its vertical component at t = 5.00 s is y = 0.

27. e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he coordinate origin is at ground level directly belo the release point. e rite θ_0 = –30.0 since the angle sho n in the figure is measured clockwise from horizontal. We note that the initial speed of the decoy is the plane's speed at the moment of release v_0 = 290 km h hich e convert to units 290 1000 3600 = 80.6 m s.

a e use . 4 12 to solve for the time

$$\Delta x = v_0 \cos\theta_0 \quad t \quad \Rightarrow \quad t = \frac{700 \text{ m}}{80.6 \text{ m s} \ \cos\ -30.0°} = 10.0 \text{ s}.$$

b nd e use . 4 22 to solve for the initial height y_0

$$y - y_0 = v_0 \sin\theta_0 \quad t - \frac{1}{2} g t^2 \quad \Rightarrow \quad 0 - y_0 = -40.3 \text{ m s} \ \ 10.0 \text{ s} - \frac{1}{2} \ 9.80 \text{ m s}^2 \ \ 10.0 \text{ s} \ ^2$$

hich yields y_0 = 897 m.

28. a sing the same coordinate system assumed in . 4 22 e solve for $y = h$

$$h = y_0 + v_0 \sin\theta_0 t - \frac{1}{2} g t^2$$

hich yields h = 51.8 m for y_0 = 0 v_0 = 42.0 m s θ_0 = 60.0 and t = 5.50 s.

b he hori ontal motion is steady so $v_x = v_{0x} = v_0 \cos\theta_0$ but the vertical component of velocity varies according to . 4 23. hus the speed at impact is

$$v = \sqrt{(v_0 \cos\theta_0)^2 + (v_0 \sin\theta_0 - gt)^2} = 27.4 \text{ m s}.$$

c e use . 4 24 ith v_y = 0 and $y = H$

$$H = \frac{(v_0 \sin\theta_0)^2}{2g} = 67.5 \text{ m}.$$

29. e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he coordinate origin is at its initial position here it is launched . t ma imum height e observe v_y = 0 and denote $v_x = v$ hich is also e ual to v_{0x} . n this notation e have $v_0 = 5v$. e t e observe v_0 cos $\theta_0 = v_{0x} = v$ so that e arrive at an e uation here $v \neq 0$ cancels hich can be solved for θ_0

$$5v\ \cos\theta_0 = v \ \Rightarrow\ \theta_0 = \cos^{-1}\left(\frac{1}{5}\right) = 78.5°.$$

30. lthough e could use . 4 26 to find here it lands e choose instead to ork ith . 4 21 and . 4 22 for the soccer ball since these ill give information about here *and when* and these are also considered more fundamental than . 4 26. ith Δy = 0 e have

$$\Delta y = \ v_0 \sin\theta_0\ \ t - \frac{1}{2}gt^2 \ \Rightarrow\ t = \frac{19.5 \text{ m s}\ \ \sin 45.0°}{9.80 \text{ m s}^2\ \ \ 2} = 2.81 \text{ s}.$$

hen . 4 21 yields Δx = v_0 cos θ_0 t = 38.7 m. hus using . 4 8 the player must have an average velocity of

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{38.7 \text{ m}\ \ \hat{\text{i}} - \ 55 \text{ m}\ \ \hat{\text{i}}}{2.81 \text{ s}} = \ -5.8 \text{ m s}\ \ \hat{\text{i}}$$

hich means his average speed assuming he ran in only one direction is 5.8 m s.

31. e first find the time it takes for the volleyball to hit the ground. sing . 4 22 e have

$$y - y_0 = \ v_0 \sin\theta_0\ \ t - \frac{1}{2}gt^2 \ \Rightarrow\ 0 - 2.30 \text{ m} = \ -20.0 \text{ m s}\ \sin 18.0°\ t - \frac{1}{2}\ 9.80 \text{ m s}^2\ \ t^2$$

hich gives $t = 0.30$ s. hus the range of the volleyball is

$$R = \left(v_0 \cos\theta_0\right)t = \ 20.0 \text{ m s}\ \ \cos 18.0°\ 0.30 \text{ s}\ \ = 5.71 \text{ m}$$

n the other hand hen the angle is changed to $\theta_0' = 8.00°$ using the same procedure as sho n above e find

$$y - y_0 = \ v_0 \sin\theta_0'\ \ t' - \frac{1}{2}gt'^2 \ \Rightarrow\ 0 - 2.30 \text{ m} = \ -20.0 \text{ m s}\ \sin 8.00°\ t' - \frac{1}{2}\ 9.80 \text{ m s}^2\ \ t'^2$$

hich yields $t' = 0.46$ s and the range is

$$R' = \left(v_0 \cos\theta_0\right)t' = \ 20.0 \text{ m s}\ \ \cos 18.0°\ 0.46 \text{ s}\ \ = 9.06 \text{ m}$$

hus the ball travels an e tra distance of

$$\Delta R = R' - R = 9.06 \text{ m} - 5.71 \text{ m} = 3.35 \text{ m}$$

32. e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he coordinate origin is at the release point the initial position for the ball as it begins pro ectile motion in the sense of 4 5 and e let θ_0 be the angle of thro sho n in the figure . ince the hori ontal component of the velocity of the ball is $v_x = v_0 \cos 40.0$ the time it takes for the ball to hit the all is

$$t = \frac{\Delta x}{v_x} = \frac{22.0 \text{ m}}{25.0 \text{ m s} \ \cos 40.0°} = 1.15 \text{ s.}$$

a he vertical distance is

$$\Delta y = \ v_0 \sin\theta_0 \ t - \frac{1}{2}gt^2 = \ 25.0 \text{ m s} \ \sin 40.0° \ 1.15 \text{ s} \ -\frac{1}{2} \ 9.80 \text{ m s}^2 \ \ 1.15 \text{ s} \ ^2 = 12.0 \text{ m.}$$

b he hori ontal component of the velocity hen it strikes the all does not change from its initial value $v_x = v_0 \cos 40.0 = 19.2$ m s.

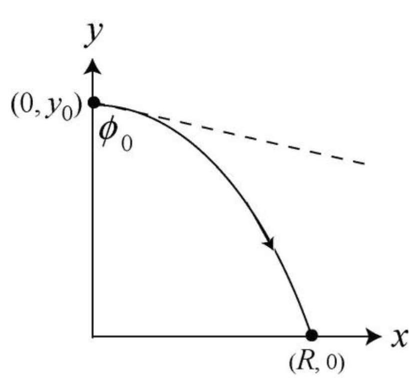
c he vertical component becomes using . 4 23

$$v_y = v_0 \sin\theta_0 - gt = \ 25.0 \text{ m s} \ \sin 40.0° - \ 9.80 \text{ m s}^2 \ \ 1.15 \text{ s} \ = 4.80 \text{ m s.}$$

d ince v_y 0 hen the ball hits the all it has not reached the highest point yet.

33. **THINK** This problem deals with projectile motion. We're interested in the horizontal displacement and velocity of the pro ectile before it strikes the ground.

EXPRESS e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he coordinate origin is at ground level directly belo the release point. e rite θ_0 = –37.0 for the angle measured from x since the angle $\phi_0 = 53.0°$ given in the problem is measured from the $-y$ direction. he initial setup of the problem is sho n in the figure to the right not to scale .



ANALYZE (a) The initial speed of the projectile is the plane's speed at the moment of release. iven that $y_0 = 730$ m and $y = 0$ at $t = 5.00$ s e use . 4 22 to find v_0

$$y-y_0 = \ v_0\sin\theta_0\ \ t-\frac{1}{2}gt^2 \ \Rightarrow\ 0-730\text{ m} = v_0\sin\ -37.0°\ \ 5.00\text{ s}\ -\frac{1}{2}\ 9.80\text{ m s}^2\ \ 5.00\text{ s}\ ^2$$

hich yields v_0 = 202 m s.

b he hori ontal distance traveled is

$$R = v_x t = \ v_0\cos\theta_0\ t = \ \ 202\text{ m s}\ \cos\ -37.0°\ \ \ 5.00\text{ s}\ = 806\text{ m}.$$

c he x component of the velocity ust before impact is

$$v_x = v_0\cos\theta_0 = \ 202\text{ m s}\ \cos\ -37.0°\ = 161\text{ m s}.$$

d he y component of the velocity ust before impact is

$$v_y = v_0\sin\theta_0 - gt = \ 202\text{ m s}\ \sin\ -37.0°\ -\ 9.80\text{ m s}^2\ \ 5.00\text{ s}\ = -171\text{ m s}.$$

LEARN n this pro ectile problem e analy ed the kinematics in the vertical and hori ontal directions separately since they do not affect each other. he x component of the velocity $v_x = v_0\cos\theta_0$, remains unchanged throughout since there's no horizontal acceleration.

34. a ince the y component of the velocity of the stone at the top of its path is ero its speed is

$$v = \sqrt{v_x^2+v_y^2} = v_x = v_0\cos\theta_0 = \ 28.0\text{ m s}\ \cos 40.0° = 21.4\text{ m s}.$$

b sing the fact that $v_y = 0$ at the ma imum height y_{ma} the amount of time it takes for the stone to reach y_{ma} is given by . 4 23

$$0 = v_y = v_0\sin\theta_0 - gt \ \Rightarrow\ t = \frac{v_0\sin\theta_0}{g}.$$

ubstituting the above e pression into . 4 22 e find the ma imum height to be

$$y_{ma} \ = \ v_0\sin\theta_0\ \ t-\frac{1}{2}gt^2 = v_0\sin\theta_0\left(\frac{v_0\sin\theta_0}{g}\right)-\frac{1}{2}g\left(\frac{v_0\sin\theta_0}{g}\right)^2 = \frac{v_0^2\sin^2\theta_0}{2g}.$$

o find the time the stone descends to $y = y_{ma}\ \ 2$ e solve the uadratic e uation given in . 4 22

$$y = \frac{1}{2}y_{ma}\ = \frac{v_0^2\sin^2\theta_0}{4g} = \ v_0\sin\theta_0\ \ t-\frac{1}{2}gt^2 \ \Rightarrow\ t_\pm = \frac{2\pm\sqrt{2}\ v_0\sin\theta_0}{2g}.$$

hoosing $t = t_+$ for descending e have

$$v_x = v_0 \cos\theta_0 = \ 28.0 \text{ m s} \ \cos 40.0° = 21.4 \text{ m s}$$

$$v_y = v_0 \sin\theta_0 - g \frac{2+\sqrt{2} \ v_0 \sin\theta_0}{2g} = -\frac{\sqrt{2}}{2} v_0 \sin\theta_0 = -\frac{\sqrt{2}}{2} \ 28.0 \text{ m s} \ \sin 40.0° = –12.7 \text{ m s}$$

hus the speed of the stone hen $y = y_{\text{ma}}$ 2 is

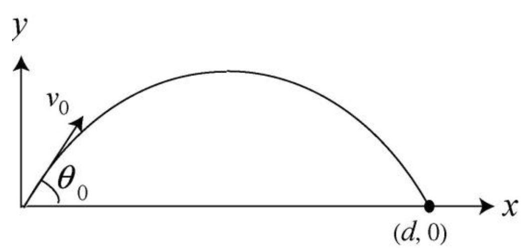
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\ 21.4 \text{ m s}\ ^2 + \ –12.7 \text{ m s}\ ^2} = 24.9 \text{ m s}.$$

c he percentage difference is

$$\frac{24.9 \text{ m s} – 21.4 \text{ m s}}{21.4 \text{ m s}} = 0.163 = 16.3 \quad .$$

35. **THINK** This problem deals with projectile motion of a bullet. We're interested in the firing angle that allo s the bullet to strike a target at some distance a ay.

EXPRESS e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he coordinate origin is at the end of the rifle the initial point for the bullet as it begins pro ectile motion in the sense of 4 5 and e let θ_0 be the firing angle. f the target is a distance d a ay then its coordinates are $x = d$, $y = 0$.



he pro ectile motion e uations lead to

$$d = \ v_0 \cos\theta_0 \ t \qquad 0 = v_0 t \sin\theta_0 - \frac{1}{2} g t^2$$

here θ_0 is the firing angle. he setup of the problem is sho n in the figure above scale e aggerated .

ANALYZE he time at hich the bullet strikes the target is given by $t = d \ \ v_0 \cos\theta_0$. liminating t leads to $2v_0^2 \sin\theta_0 \cos\theta_0 - gd = 0$. sing $\sin\theta_0 \cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$ e obtain

$$v_0^2 \sin \ \ 2\theta_0 \ = gd \ \Rightarrow \ \sin 2\theta_0 \ = \frac{gd}{v_0^2} = \frac{9.80 \text{ m s}^2 \ \ 45.7 \text{ m}}{460 \text{ m s} \ ^2}$$

hich yields $\sin 2\theta_0 \ = 2.11\times10^{-3}$ or θ_0 = 0.0606 . f the gun is aimed at a point a distance ℓ above the target then $\tan \theta_0 = \ell/d$ so that

$$\ell = d \tan\theta_0 = \ \ 45.7 \text{ m} \ \ \tan \ 0.0606° \ \ = 0.0484 \text{ m} = 4.84 \text{ cm.}$$

LEARN ue to the do n ard gravitational acceleration in order for the bullet to strike the target the gun must be aimed at a point slightly above the target.

36. e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he coordinate origin is at ground level directly belo the point here the ball as hit by the rac uet.

a e ant to kno ho high the ball is above the court hen it is at x = 12.0 m. irst . 4 21 tells us the time it is over the fence

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{12.0 \text{ m}}{(23.6 \text{ m s})\cos 0°} = 0.508 \text{ s.}$$

t this moment the ball is at a height above the court of

$$y = y_0 + (v_0 \sin \theta_0)\, t - \frac{1}{2} g t^2 = 1.10\text{m}$$

hich implies it does indeed clear the 0.90 m high fence.

b t t = 0.508 s the center of the ball is 1.10 m – 0.90 m = 0.20 m above the net.

c epeating the computation in part a ith θ_0 = –5.0 results in t = 0.510 s and $y = 0.040$ m hich clearly indicates that it cannot clear the net.

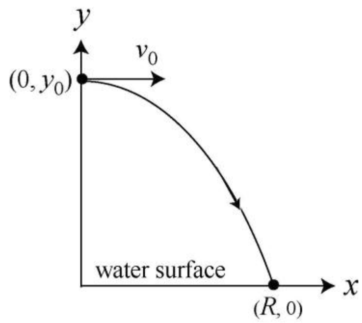
d n the situation discussed in part c the distance bet een the top of the net and the center of the ball at t = 0.510 s is 0.90 m – 0.040 m = 0.86 m.

37. **THINK** he tra ectory of the diver is a pro ectile motion. e are interested in the displacement of the diver at a later time.

EXPRESS he initial velocity has no vertical component $\theta_0 = 0$ but only an x component. s. 4 21 and 4 22 can be simplified to

$$x - x_0 = v_{0x}t$$
$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2.$$

here $x_0 = 0$ $v_{0x} = v_0 = +2.0$ m s and y_0 = 10.0 m taking the ater surface to be at $y = 0$. he setup of the problem is sho n in the figure belo .



ANALYZE a t $t = 0.80$ s the hori ontal distance of the diver from the edge is

$$x = x_0 + v_{0x}t = 0 + \ 2.0 \text{ m s} \ \ 0.80 \text{ s} \ = 1.60 \text{ m}.$$

b imilarly using the second e uation for the vertical motion e obtain

$$y = y_0 - \frac{1}{2}gt^2 = 10.0 \text{ m} - \frac{1}{2} \ 9.80 \text{ m s}^2 \ \ 0.80 \text{ s} \ ^2 = 6.86 \text{ m}.$$

c t the instant the diver strikes the ater surface y = 0. olving for t using the e uation $y = y_0 - \frac{1}{2}gt^2 = 0$ leads to

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2 \ 10.0 \text{ m}}{9.80 \text{ m s}^2}} = 1.43 \text{ s}.$$

uring this time the x displacement of the diver is $R = x =$ 2.00 m s 1.43 s = 2.86 m.

LEARN sing . 4 25 ith $\theta_0 = 0$ the tra ectory of the diver can also be ritten as

$$y = y_0 - \frac{gx^2}{2v_0^2} \ .$$

art c can also be solved by using this e uation

$$y = y_0 - \frac{gx^2}{2v_0^2} = 0 \ \Rightarrow \ x = R = \sqrt{\frac{2v_0^2 y_0}{g}} = \sqrt{\frac{2 \ 2.0 \text{ m s} \ ^2 \ 10.0 \text{ m}}{9.8 \text{ m s}^2}} = 2.86 \text{ m}.$$

38. n this pro ectile motion problem e have $v_0 = v_x$ = constant and hat is plotted is $v = \sqrt{v_x^2 + v_y^2}$. e infer from the plot that at t = 2.5 s the ball reaches its ma imum height here v_y = 0. herefore e infer from the graph that v_x = 19 m s.

a uring t = 5 s the hori ontal motion is $x - x_0 = v_x t$ = 95 m.

b ince $\sqrt{19 \text{ m s}^{2} + v_{0y}^2} = 31$ m s the first point on the graph e find $v_{0y} = 24.5$ m s. hus ith t = 2.5 s e can use $y_{ma} - y_0 = v_{0y}t - \frac{1}{2}gt^2$ or $v_y^2 = 0 = v_{0y}^{2} - 2g(y_{ma} - y_0)$ or $y_{ma} - y_0 = \frac{1}{2}\left(v_y + v_{0_y}\right)t$ to solve. ere e ill use the latter

$$y_{ma} - y_0 = \frac{1}{2} \; v_y + v_{0y} \; t \Rightarrow y_{ma} = \frac{1}{2} \; 0 + 24.5\text{m s} \; 2.5\text{ s} = 31 \text{ m}$$

here e have taken y_0 = 0 as the ground level.

39. ollo ing the hint e have the time reversed problem ith the ball thro n from the ground to ard the right at 60 measured counterclock ise from a right ard a is. e see in this time reversed situation that it is convenient to use the familiar coordinate system ith x as *rightward* and ith positive angles measured counterclock ise.

a he x e uation ith x_0 = 0 and x = 25.0 m leads to

$$25.0 \text{ m} = v_0 \cos 60.0 \quad 1.50 \text{ s}$$

so that v_0 = 33.3 m s. nd ith y_0 = 0 and $y = h$ 0 at t = 1.50 s e have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ here $v_{0y} = v_0 \sin 60.0$. his leads to h = 32.3 m.

b e have

$$v_x = v_{0x} = \; 33.3 \text{ m s} \cos 60.0 \; = 16.7 \text{ m s}$$
$$v_y = v_{0y} - gt = \; 33.3 \text{ m s} \sin 60.0 \; - \; 9.80 \text{ m s}^2 \; 1.50 \text{ s} \; = 14.2 \text{ m s}.$$

he magnitude of $\vec{v}$ is given by

$$\vec{v} = \sqrt{v_x^2 + v_y^2} = \sqrt{16.7 \text{ m s}^{2} + 14.2 \text{ m s}^{2}} = 21.9 \text{ m s}.$$

c he angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{14.2 \text{ m s}}{16.7 \text{ m s}}\right) = 40.4°.$$

d We interpret this result ("undoing" the time reversal as an initial velocity from the edge of the building of magnitude 21.9 m s ith angle do n from left ard of 40.4 .

40. a olving the uadratic e uation . 4 22

$$y - y_0 = v_0 \sin\theta_0 \ \ t - \frac{1}{2}gt^2 \ \Rightarrow \ 0 - 2.160 \text{ m} = \ 15.00 \text{ m s} \ \sin 45.00^\circ \ t - \frac{1}{2} \ 9.800 \text{ m s}^2 \ t^2$$

the total travel time of the shot in the air is found to be $t = 2.352\,\text{s}$. herefore the hori ontal distance traveled is

$$R = \left(v_0 \cos\theta_0\right)t = \ 15.00 \text{ m s} \ \cos 45.00^\circ \ 2.352 \text{ s} \ = 24.95 \text{ m}.$$

b sing the procedure outlined in a but for $\theta_0 = 42.00^\circ$ e have

$$y - y_0 = v_0 \sin\theta_0 \ \ t - \frac{1}{2}gt^2 \ \Rightarrow \ 0 - 2.160 \text{ m} = \ 15.00 \text{ m s} \ \sin 42.00^\circ \ t - \frac{1}{2} \ 9.800 \text{ m s}^2 \ t^2$$

and the total travel time is $t = 2.245\,\text{s}$. his gives

$$R = \left(v_0 \cos\theta_0\right)t = \ 15.00 \text{ m s} \ \cos 42.00^\circ \ 2.245 \text{ s} \ = 25.02 \text{ m}.$$

41. ith the rcher fish set to be at the origin the position of the insect is given by $x \ y$ here $x = R \ \ 2 = v_0^2 \sin 2\theta_0 \ \ 2g$ and y corresponds to the ma imum height of the parabolic tra ectory $y = y_{\text{ma}} = v_0^2 \sin^2\theta_0 \ \ 2g$. rom the figure e have

$$\tan\phi = \frac{y}{x} = \frac{v_0^2 \sin^2\theta_0 \ \ 2g}{v_0^2 \sin 2\theta_0 \ \ 2g} = \frac{1}{2}\tan\theta_0$$

iven that $\phi = 36.0^\circ$ e find the launch angle to be

$$\theta_0 = \tan^{-1}\left(2\tan\phi\right) = \tan^{-1}\left(2\tan 36.0^\circ\right) = \tan^{-1}\left(1.453\right) = 55.46^\circ \approx 55.5^\circ.$$

ote that θ_0 depends only on ϕ and is independent of d.

42. a sing the fact that the person as the pro ectile reaches the ma imum height over the middle heel located at $x = 23 \text{ m} + \ 23 \ \ 2 \ \ \text{m} = 34.5 \text{ m}$ e can deduce the initial launch speed from . 4 26

$$x = \frac{R}{2} = \frac{v_0^2 \sin 2\theta_0}{2g} \ \Rightarrow \ v_0 = \sqrt{\frac{2gx}{\sin 2\theta_0}} = \sqrt{\frac{2 \ 9.8 \text{ m s}^2 \ \ 34.5 \text{ m}}{\sin \ 2 \cdot 53^\circ}} = 26.5 \text{ m s}.$$

pon substituting the value to . 4 25 e obtain

$$y = y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + \ 23\text{ m}\ \tan 53° - \frac{9.8\text{ m s}^2\ \ 23\text{ m}\ ^2}{2\ 26.5\text{ m s}\ ^2\ \cos 53°\ ^2} = 23.3\text{ m}.$$

ince the height of the heel is $h_w = 18$ m the clearance over the first heel is $\Delta y = y - h_w = 23.3\text{ m} - 18\text{ m} = 5.3\text{ m}$.

b he height of the person hen he is directly above the second heel can be found by solving . 4 24. ith the second heel located at $x = 23\text{ m} + \ 23\ \ 2\ \ \text{m} = 34.5\text{ m}$ e have

$$y = y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + \ 34.5\text{ m}\ \tan 53° - \frac{9.8\text{ m s}^2\ \ 34.5\text{ m}\ ^2}{2\ 26.52\text{ m s}\ ^2\ \cos 53°\ ^2}$$
$$= 25.9\text{ m}.$$

herefore the clearance over the second heel is $\Delta y = y - h_w = 25.9\text{ m} - 18\text{ m} = 7.9\text{ m}$.

c he location of the center of the net is given by

$$0 = y - y_0 = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} \ \Rightarrow\ x = \frac{v_0^2\sin 2\theta_0}{g} = \frac{26.52\text{ m s}\ ^2\sin\ 2\cdot 53°}{9.8\text{ m s}^2} = 69\text{ m}.$$

43. e designate the given velocity $\vec{v} = \ 7.6$ m s $\hat{\text{i}} + \ 6.1$ m s $\hat{}$ as $\vec{v}_1$ as opposed to the velocity hen it reaches the ma height $\vec{v}_2$ or the velocity hen it returns to the ground $\vec{v}_3$ and take $\vec{v}_0$ as the launch velocity as usual. he origin is at its launch point on the ground.

a ifferent approaches are available but since it ill be useful for the rest of the problem to first find the initial y velocity that is ho e ill proceed. sing . 2 16 e have

$$v_{1y}^2 = v_{0y}^2 - 2g\Delta y \ \Rightarrow\ \ 6.1\text{ m s}\ ^2 = v_{0y}^2 - 2\ 9.8\text{ m s}^2\ \ 9.1\text{ m}$$

hich yields $v_{0\,y}$ = 14.7 m s. no ing that $v_{2\,y}$ must e ual 0 e use . 2 16 again but no ith $\Delta y = h$ for the ma imum height

$$v_{2y}^2 = v_{0y}^2 - 2gh \ \Rightarrow\ \ 0 = \ 14.7\text{ m s}\ ^2 - 2\ 9.8\text{ m s}^2\ \ h$$

hich yields h = 11 m.

b ecalling the derivation of . 4 26 but using $v_{0\,y}$ for v_0 sin θ_0 and v_{0x} for v_0 cos θ_0 e have

$$0 = v_{0y}t - \frac{1}{2}gt^2 \qquad R = v_{0x}t$$

hich leads to $R = 2v_{0x}v_{0y}$ g. oting that v_{0x} = v_{1x} = 7.6 m s e plug in values and obtain

$$R = 2\ 7.6\text{ m s}\ \ 14.7\text{ m s}\ \ \ 9.8\text{ m s}^2\ = 23\text{ m}.$$

c ince v_{3x} = v_{1x} = 7.6 m s and v_{3y} = – $v_{0\,y}$ = –14.7 m s e have

$$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2} = \sqrt{\ 7.6\text{ m s}\ ^2 + \ -14.7\text{ m s}\ ^2} = 17\text{ m s}.$$

d he angle measured from hori ontal for $\vec{v}_3$ is one of these possibilities

$$\tan^{-1}\left(\frac{-14.7\text{ m}}{7.6\text{ m}}\right) = -63° \text{ or } 117°$$

here e settle on the first choice –63 hich is e uivalent to 297 since the signs of its components imply that it is in the fourth uadrant.

44. e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he initial velocity is hori ontal so that v_{0y} = 0 and $v_{0x} = v_0 = 161$ km/h . onverting to units this is v_0 = 44.7 m s.

a ith the origin at the initial point (where the ball leaves the pitcher's hand), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$ and the x coordinate is given by $x = v_0t$. rom the latter e uation e have a simple proportionality bet een hori ontal distance and time hich means the time to travel half the total distance is half the total time. pecifically if x = 18.3 2 m then t = 18.3 2 m 44.7 m s = 0.205 s.

b nd the time to travel the ne t 18.3 2 m must also be 0.205 s. t can be useful to rite the hori ontal e uation as $\Delta x = v_0\Delta t$ in order that this result can be seen more clearly.

c sing the e uation $y = -\frac{1}{2}gt^2$ e see that the ball has reached the height of $-\frac{1}{2}\left(9.80\text{ m s}^2\right)\left(0.205\text{ s}\right)^2\ = 0.205$ m at the moment the ball is half ay to the batter.

(d) The ball's height when it reaches the batter is $-\frac{1}{2}\left(9.80\text{ m s}^2\right)\left(0.409\text{ s}\right)^2 = -0.820$m hich hen subtracted from the previous result implies it has fallen another 0.615 m. ince the value of y is not simply proportional to t e do not e pect e ual time intervals to correspond to e ual height changes in a physical sense this is due to the fact that the initial y velocity for the first half of the motion is not the same as the "initial" y velocity for the second half of the motion.

45. a et $m = \frac{d_2}{d_1} = 0.600$ be the slope of the ramp so $y = mx$ there. e choose our coordinate origin at the point of launch and use . 4 25. hus

$$y = \tan 50.0^\circ \; x - \frac{9.80 \text{ m s}^2 \; x^2}{2 \; 10.0 \text{ m s}^{\;\;2} \; \cos 50.0^\circ \;\;^2} = 0.600x$$

hich yields x = 4.99 m. his is less than d_1 so the ball *does* land on the ramp.

b sing the value of x found in part a e obtain $y = mx$ = 2.99 m. hus the ythagorean theorem yields a displacement magnitude of $\sqrt{x^2 \;\; y^2}$ = 5.82 m.

c he angle is of course the angle of the ramp $\tan^{-1} \; m$ = 31.0 .

46. sing the fact that $v_y = 0$ hen the player is at the ma imum height y_{ma} the amount of time it takes to reach y_{ma} can be solved by using . 4 23

$$0 = v_y = v_0 \sin\theta_0 - gt \quad \Rightarrow \; t_{\text{ma}} \;\; = \frac{v_0 \sin\theta_0}{g}.$$

ubstituting the above e pression into . 4 22 e find the ma imum height to be

$$y_{\text{ma}} \;\; = \;\; v_0 \sin\theta_0 \;\; t_{\text{ma}} \;\; - \frac{1}{2} g t_{\text{ma}}^2 \;\; = v_0 \sin\theta_0 \left(\frac{v_0 \sin\theta_0}{g} \right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta_0}{g} \right)^2 = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

o find the time hen the player is at $y = y_{\text{ma}} \;\; 2$ e solve the uadratic e uation given in . 4 22

$$y = \frac{1}{2} y_{\text{ma}} \;\; = \frac{v_0^2 \sin^2\theta_0}{4g} = \;\; v_0 \sin\theta_0 \;\; t - \frac{1}{2} g t^2 \quad \Rightarrow \; t_\pm = \frac{2 \pm \sqrt{2} \;\; v_0 \sin\theta_0}{2g}.$$

ith $t = t_-$ for ascending the amount of time the player spends at a height $y \geq y_{\text{ma}} \;\; 2$ is

$$\Delta t = t_{\text{ma}} \;\; - t_- = \frac{v_0 \sin\theta_0}{g} - \frac{2 - \sqrt{2} \;\; v_0 \sin\theta_0}{2g} = \frac{v_0 \sin\theta_0}{\sqrt{2} g} = \frac{t_{\text{ma}}}{\sqrt{2}} \quad \Rightarrow \; \frac{\Delta t}{t_{\text{ma}}} = \frac{1}{\sqrt{2}} = 0.707.$$

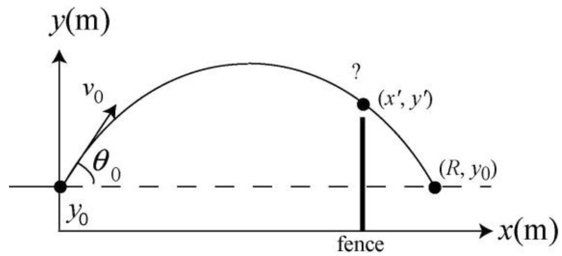
herefore the player spends about 70.7 of the time in the upper half of the ump. ote that the ratio $\Delta t \;\; t_{\text{ma}}$ is independent of v_0 and θ_0 even though Δt and t_{ma} depend on these uantities.

47. **THINK** he baseball undergoes projectile motion after being hit by the batter. We'd like to kno if the ball clears a high fence at some distance a ay.

EXPRESS e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he coordinate origin is at ground level directly belo impact point bet een bat and ball. n the absence of a fence ith $\theta_0 = 45°$ the hori ontal range same launch level is $R = 107$ m. e ant to kno ho high the ball is from the ground hen it is at $x' = 97.5$ m hich re uires kno ing the initial velocity. he tra ectory of the baseball can be described by . 4 25

$$y - y_0 = \tan\theta_0 \; x - \frac{gx^2}{2\; v_0 \cos\theta_0 \;^2}.$$

he setup of the problem is sho n in the figure belo not to scale .



ANALYZE a e first solve for the initial speed v_0. sing the range information $y = y_0$ hen $x = R$ and θ_0 = 45 . 4 25 gives

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.8\text{ m s}^2)(107\text{ m})}{\sin 2\cdot 45°}} = 32.4\text{ m s}.$$

hus the time at hich the ball flies over the fence is

$$x' = v_0\cos\theta_0 \; t' \;\Rightarrow\; t' = \frac{x'}{v_0\cos\theta_0} = \frac{97.5\text{ m}}{(32.4\text{ m s})\cos 45°} = 4.26\text{ s}.$$

t this moment the ball is at a height above the ground of

$$\begin{aligned} y' &= y_0 + (v_0\sin\theta_0)t' - \frac{1}{2}gt'^2 \\ &= 1.22\text{ m} + \;32.4\text{ m s}\;\sin 45° \;\; 4.26\text{ s} \; - \frac{1}{2}\; 9.8\text{ m s}^2 \;\; 4.26\text{ s}\;^2 \\ &= 9.88\text{ m} \end{aligned}$$

hich implies it does indeed clear the 7.32 m high fence.

b t $t' = 4.26$ s the center of the ball is 9.88 m – 7.32 m = 2.56 m above the fence.

LEARN sing the tra ectory e uation above one can sho that the minimum initial velocity re uired to clear the fence is given by

$$y'-y_0 = \tan\theta_0\ x' - \frac{gx'^2}{2\ v_0\cos\theta_0\ ^2}$$

or about 31.9 m s.

48. ollo ing the hint e have the time reversed problem ith the ball thro n from the roof to ard the left at 60 measured clock ise from a left ard a is. e see in this time reversed situation that it is convenient to take x as *leftward* ith positive angles measured clock ise. engths are in meters and time is in seconds.

a ith y_0 = 20.0 m and y = 0 at t = 4.00 s e have $y - y_0 = v_{0_y}t - \frac{1}{2}gt^2$ here $v_{0_y} = v_0 \sin 60°$. his leads to v_0 = 16.9 m s. his plugs into the x e uation $x - x_0 = v_{0x}t$ ith x_0 = 0 and $x = d$ to produce

$$d = \ 16.9 \text{ m s} \cos 60\ \ 4.00 \text{ s}\ = 33.7 \text{ m}.$$

b e have

$$v_x = v_{0x} = \ 16.9 \text{ m s}\ \cos 60.0° = 8.43 \text{ m s}$$
$$v_y = v_{0y} - gt = \ 16.9 \text{ m s}\ \sin 60.0° - \ 9.80 \text{m s}^2\ \ 4.00 \text{ s}\ = -24.6 \text{ m s}.$$

he magnitude of $\vec{v}$ is $\vec{v} = \sqrt{v_x^2 + v_y^2} = \sqrt{\ 8.43 \text{ m s}\ ^2 + \ -24.6 \text{ m s}\ ^2} = 26.0 \text{ m s}.$

c he angle relative to hori ontal is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-24.6 \text{ m s}}{8.43 \text{ m s}}\right) = -71.1°.$$

e may convert the result from rectangular components to magnitude angle representation

$$\vec{v} = \ 8.43\ -24.6\ \ \to\ 26.0 \angle -71.1°$$

and we now interpret our result ("undoing" the time reversal) as an initial velocity of magnitude 26.0 m s ith angle up from right ard of 71.1 .

49. **THINK** n this problem a football is given an initial speed and it undergoes pro ectile motion. We'd like to know the smallest and greatest angles at which a field goal can be scored.

EXPRESS e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he coordinate origin is at the point here the ball is kicked. e use x and y to denote the coordinates of the ball at the goalpost and try to find the kicking angle s θ_0 so that y = 3.44 m hen x = 50 m. riting the kinematic e uations for pro ectile motion

$$x = v_0 \cos\theta_0 \qquad y = v_0 t \sin\,\theta_0 - \tfrac{1}{2} g t^2$$

e see the first e uation gives $t = x\ v_0 \cos\,\theta_0$ and hen this is substituted into the second the result is

$$y = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\,\theta_0}.$$

ANALYZE ne may solve the above e uation by trial and error systematically trying values of θ_0 until you find the t o that satisfy the e uation. little manipulation ho ever ill give an algebraic solution sing the trigonometric identity

$$1 \quad \cos^2\,\theta_0 = 1 \quad \tan^2\,\theta_0$$

e obtain

$$\frac{1}{2}\frac{gx^2}{v_0^2}\tan^2\,\theta_0 - x\tan\theta_0 + y + \frac{1}{2}\frac{gx^2}{v_0^2} = 0$$

hich is a second order e uation for tan θ_0. o simplify riting the solution e denote

$$c = \frac{1}{2}gx^2 \quad v_0^2 = \frac{1}{2}\left(9.80\text{ m s}^2\right)\left(50\text{ m}\right)^2 \quad \left(25\text{ m s}\right)^2 = 19.6\text{ m}.$$

hen the second order e uation becomes $c\tan^2\,\theta_0 - x\tan\,\theta_0 \quad y \quad c = 0$. sing the uadratic formula e obtain its solution s .

$$\tan\,\theta_0 = \frac{x \pm \sqrt{x^2 - 4(y+c)c}}{2c} = \frac{50\text{ m} \pm \sqrt{\ 50\text{ m}\ ^2 - 4(3.44\text{ m} + 19.6\text{ m})(19.6\text{ m})}}{2(19.6\text{ m})}.$$

he t o solutions are given by tanθ_0 = 1.95 and tanθ_0 = 0.605. he corresponding first uadrant angles are θ_0 = 63 and θ_0 = 31 . hus

a he smallest elevation angle is θ_0 = 31 and

b he greatest elevation angle is θ_0 = 63 .

LEARN f kicked at any angle bet een 31 and 63 the ball ill travel above the cross bar on the goalposts.

50. e apply . 4 21 . 4 22 and . 4 23.

a rom $\Delta x = v_{0x} t$ e find $v_{0x} = 40\text{ m}\ \ 2\text{ s} = 20\text{ m s}$.

b rom $\Delta y = v_{0y} t - \tfrac{1}{2} g t^2$ e find $v_{0y} = \left(53\text{ m} + \tfrac{1}{2}\ 9.8\text{ m s}^2\ \ 2\text{ s}\ ^2\right)\ 2 = 36\text{ m s}$.

c rom $v_y = v_{0y} - gt'$ ith v_y = 0 as the condition for ma imum height e obtain $t' =$ 36 m s 9.8 m s^2 $=$3.7 s. uring that time the x motion is constant so $x' - x_0 =$ 20 m s 3.7 s $=$74 m.

51. a he skier umps up at an angle of $\theta_0 = 11.3°$ up from the hori ontal and thus returns to the launch level ith his velocity vector 11.3° belo the hori ontal. ith the sno surface making an angle of $\alpha = 9.0°$ do n ard ith the hori ontal the angle bet een the slope and the velocity vector is $\phi = \theta_0 - \alpha = 11.3° - 9.0° = 2.3°$.

b uppose the skier lands at a distance d do n the slope. sing . 4 25 ith $x = d\cos\alpha$ and $y = -d\sin\alpha$ the edge of the track being the origin e have

$$-d\sin\alpha = d\cos\alpha\,\tan\theta_0 - \frac{g\ d\cos\alpha\ ^2}{2v_0^2\cos^2\theta_0}.$$

olving for d e obtain

$$d = \frac{2v_0^2\cos^2\theta_0}{g\cos^2\alpha}\left(\cos\alpha\tan\theta_0 + \sin\alpha\right) = \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}\left(\cos\alpha\sin\theta_0 + \cos\theta_0\sin\alpha\right)$$

$$= \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}\sin\ \theta_0 + \alpha\ .$$

ubstituting the values given e find

$$d = \frac{2\ 10\text{ m s}\ ^2\cos 11.3°}{9.8\text{ m s}^2\ \ \cos^2\ 9.0°}\sin\ 11.3° + 9.0°\ \ = 7.117\text{ m}.$$

hich gives

$$y = -d\sin\alpha = -\ 7.117\text{ m}\ \sin\ 9.0°\ \ = -1.11\text{ m}.$$

herefore at landing the skier is appro imately 1.1 m belo the launch level.

c he time it takes for the skier to land is

$$t = \frac{x}{v_x} = \frac{d\cos\alpha}{v_0\cos\theta_0} = \frac{7.117\text{ m}\ \cos\ 9.0°}{10\text{ m s}\ \cos\ 11.3°} = 0.72\text{ s}.$$

sing . 4 23 the x and y components of the velocity at landing are

$$v_x = v_0\cos\theta_0 =\ 10\text{ m s}\ \cos\ 11.3°\ = 9.81\text{ m s}$$
$$v_y = v_0\sin\theta_0 - gt =\ 10\text{ m s}\ \sin\ 11.3°\ -\ 9.8\text{ m s}^2\ \ 0.72\text{ s}\ = -5.07\text{ m s}$$

hus the direction of travel at landing is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-5.07 \text{ m s}}{9.81 \text{ m s}}\right) = -27.3°.$$

or 27.3° below the horizontal. The result implies that the angle between the skier's path and the slope is $\phi = 27.3° - 9.0° = 18.3°$ or appro imately 18° to t o significant figures.

52. rom . 4 21 e find $t = x \ v_{0x}$. hen . 4 23 leads to

$$v_y = v_{0y} - gt = v_{0y} - \frac{gx}{v_{0x}}.$$

ince the slope of the graph is –0.500 e conclude

$$\frac{g}{v_{0x}} = \frac{1}{2} \Rightarrow v_{0x} = 19.6 \text{ m s}.$$

nd from the "y intercept" of the graph, we find v_{0y} = 5.00 m s. onse uently

$$\theta_0 = \tan^{-1} \ v_{0y} / v_{0x} \ = 14.3° \approx 14°.$$

53. et $y_0 = h_0$ = 1.00 m at x_0 = 0 hen the ball is hit. et $y_1 = h$ the height of the all and x_1 describe the point here it first rises above the all one second after being hit similarly $y_2 = h$ and x_2 describe the point here it passes back do n behind the all four seconds later. nd y_f = 1.00 m at $x_f = R$ is here it is caught. engths are in meters and time is in seconds.

a eeping in mind that v_x is constant e have $x_2 - x_1$ = 50.0 m = v_{1x} 4.00 s hich leads to v_{1x} = 12.5 m s. hus applied to the full si seconds of motion

$$x_f - x_0 = R = v_x \ 6.00 \text{ s} \ = 75.0 \text{ m}.$$

b e apply $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to the motion above the all

$$y_2 - y_1 = 0 = v_{1y}(4.00 \text{ s}) - \frac{1}{2}g(4.00 \text{ s})^2$$

and obtain v_{1y} = 19.6 m s. ne second earlier using $v_{1y} = v_{0y} - g$ 1.00 s e find $v_{0y} = 29.4$ m s. herefore the velocity of the ball ust after being hit is

$$\vec{v} = v_{0x}\hat{\text{i}} + v_{0y}\hat{} = \ 12.5 \text{ m s} \ \hat{\text{i}} + \ 29.4 \text{ m s} \ \hat{}$$

ts magnitude is $\vec{v} = \sqrt{12.5\text{ m s}^{2} \quad 29.4\text{ m s}^{2}} = 31.9\text{ m s}.$

c he angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{29.4\text{ m s}}{12.5\text{ m s}}\right) = 67.0°.$$

e interpret this result as a velocity of magnitude 31.9 m s ith angle up from right ard of 67.0 .

d uring the first 1.00 s of motion $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ yields

$$h = 1.0\text{ m} + (29.4\text{ m s})(1.00\text{ s}) - \tfrac{1}{2}(9.8\text{ m s}^2)(1.00\text{ s})^2 = 25.5\text{ m}.$$

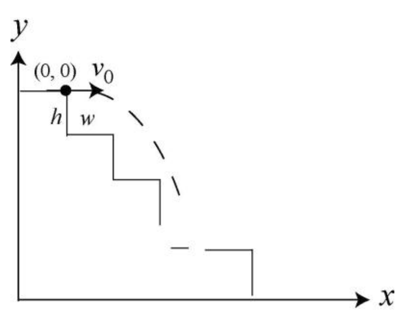
54. or $\Delta y = 0$. 4 22 leads to $t = 2v_0\sin\theta_0\ g$ hich immediately implies $t_{\text{ma}} = 2v_0\ g$ (which occurs for the "straight up" case: $\theta_0 = 90°$. hus

$$\tfrac{1}{2}t_{\text{ma}} = v_0\ g \Rightarrow \tfrac{1}{2} = \sin\theta_0.$$

herefore the half ma imum time flight is at angle θ_0 = 30.0°. ince the least speed occurs at the top of the tra ectory hich is here the velocity is simply the x component of the initial velocity $v_0\cos\theta_0 = v_0\cos30°$ for the half ma imum time flight then e need to refer to the graph in order to find v_0 – in order that e may complete the solution. n the graph e note that the range is 240 m hen θ_0 = 45.0°. uation 4 26 then leads to v_0 = 48.5 m s. he ans er is thus 48.5 m s cos30.0° = 42.0 m s.

55. **THINK** n this problem a ball rolls off the top of a stair ay ith an initial speed and we'd like to know on which step it lands first.

EXPRESS e denote h as the height of a step and w as the idth. o hit step n the ball must fall a distance nh and travel hori ontally a distance bet een $n-1$ w and nw. e take the origin of a coordinate system to be at the point here the ball leaves the top of the stair ay and e choose the y a is to be positive in the up ard direction as sho n in the figure.



he coordinates of the ball at time t are given by $x = v_{0x}t$ and $y = -\frac{1}{2}gt^2$ since $v_{0y} = 0$.

ANALYZE e e uate y to $-nh$ and solve for the time to reach the level of step n

$$t = \sqrt{\frac{2nh}{g}}.$$

he x coordinate then is

$$x = v_{0x}\sqrt{\frac{2nh}{g}} = \ 1.52 \text{ m s } \sqrt{\frac{2n\ 0.203 \text{ m}}{9.8 \text{ m s}^2}} = \ 0.309 \text{ m } \ \sqrt{n}.$$

he method is to try values of n until e find one for hich x w is less than n but greater than $n - 1$. or n = 1 x = 0.309 m and x w = 1.52 hich is greater than n. or n = 2 x = 0.437 m and x w = 2.15 hich is also greater than n. or n = 3 x = 0.535 m and x w = 2.64. o this is less than n and greater than $n - 1$ so the ball hits the third step.

LEARN o check the consistency of our calculation e can substitute n = 3 into the above e uations. he results are t = 0.353 s y = 0.609 m and x = 0.535 m. his indeed corresponds to the third step.

56. e apply . 4 35 to solve for speed v and . 4 34 to find acceleration a.

a ince the radius of arth is 6.37×10^6 m the radius of the satellite orbit is

$$r = \ 6.37 \times 10^6 \quad 640 \times 10^3 \quad \text{m} = 7.01 \times 10^6 \text{ m}.$$

herefore the speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi(7.01 \times 10^6 \text{ m})}{(98.0 \text{ min})(60 \text{ s} \quad \text{min})} = 7.49 \times 10^3 \text{ m} \quad \text{s}.$$

b he magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(7.49 \times 10^3 \text{ m} \quad \text{s})^2}{7.01 \times 10^6 \text{ m}} = 8.00 \text{ m} \quad \text{s}^2.$$

57. he magnitude of centripetal acceleration $a = v^2 \ r$ and its direction to ard the center of the circle form the basis of this problem.

a f a passenger at this location e periences $\vec{a} = 1.83 \text{ m} \quad \text{s}^2$ east then the center of the circle is *east* of this location. he distance is $r = v^2 \ a = \ 3.66 \text{ m s}^{\ 2} \ \ 1.83 \text{ m s}^2 \ = 7.32$ m.

b hus relative to the center the passenger at that moment is located 7.32 m to ard the est.

c f the direction of $\vec{a}$ e perienced by the passenger is no *south*—indicating that the center of the merry go round is south of him then relative to the center the passenger at that moment is located 7.32 m to ard the north.

58. a he circumference is $c = 2\pi r = 2\pi$ 0.15 m = 0.94 m.

b ith T = 60 s 1200 = 0.050 s the speed is $v = c\ T$ = 0.94 m 0.050 s = 19 m s. his is e uivalent to using . 4 35.

c he magnitude of the acceleration is $a = v^2\ r$ = 19 m s ^{2} 0.15 m = 2.4×10^3 m s^2.

d he period of revolution is 1200 rev min $^{-1}$ = 8.3×10^{-4} min hich becomes in units T = 0.050 s = 50 ms.

59. a ince the heel completes 5 turns each minute its period is one fifth of a minute or 12 s.

b he magnitude of the centripetal acceleration is given by $a = v^2\ R$ here R is the radius of the heel and v is the speed of the passenger. ince the passenger goes a distance $2\pi R$ for each revolution his speed is

$$v = \frac{2\pi(15\text{ m})}{12\text{ s}} = 7.85\text{ m s}$$

and his centripetal acceleration is $a = \frac{(7.85\text{ m s})^2}{15\text{ m}} = 4.1\text{ m s}^2$.

c hen the passenger is at the highest point his centripetal acceleration is do n ard to ard the center of the orbit.

d t the lo est point the centripetal acceleration is $a = 4.1$ m s^2 same as part b .

e he direction is up to ard the center of the orbit.

60. a uring constant speed circular motion the velocity vector is perpendicular to the acceleration vector at every instant. hus $\vec{v}$ $\vec{a}$ = 0.

b he acceleration in this vector at every instant points to ard the center of the circle hereas the position vector points from the center of the circle to the ob ect in motion. hus the angle bet een $\vec{r}$ and $\vec{a}$ is 180 at every instant so $\vec{r} \times \vec{a}$ = 0.

61. e apply . 4 35 to solve for speed v and . 4 34 to find centripetal acceleration a.

a $v = 2\pi r\ T = 2\pi$ 20 km 1.0 s = 126 km s = 1.3×10^5 m s.

b he magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(126\text{ km s})^2}{20\text{ km}} = 7.9 \times 10^5\text{ m s}^2.$$

c learly both v and a ill increase if T is reduced.

62. he magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(10\text{ m s})^2}{25\text{ m}} = 4.0\text{ m s}^2.$$

63. e first note that $\vec{a}_1$ the acceleration at t_1 = 2.00 s is perpendicular to $\vec{a}_2$ the acceleration at t_2=5.00 s by taking their scalar dot product

$$\vec{a}_1 \cdot \vec{a}_2 = \; 6.00\text{ m s}^2\ \hat{\text{i}}\quad 4.00\text{ m s}^2\ \hat{}\ \cdot\ 4.00\text{ m s}^2\ \hat{\text{i}}\quad -6.00\text{ m s}^2\ \hat{}\ =0.$$

ince the acceleration vectors are in the negative radial directions then the t o positions at t_1 and t_2 are a uarter circle apart or three uarters of a circle depending on hether one measures clock ise or counterclock ise . uick sketch leads to the conclusion that if the particle is moving counterclock ise as the problem states then it travels three uarters of a circumference in moving from the position at time t_1 to the position at time t_2 . etting T stand for the period then $t_2 - t_1$ = 3.00 s = 3T/4. his gives T = 4.00 s. he magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\;6.00\text{ m s}^2\ ^2 + \ 4.00\text{ m s}\ ^2} = 7.21\text{ m s}^2.$$

sing s. 4 34 and 4 35 e have $a = 4\pi^2 r\ \ T^2$ hich yields

$$r = \frac{aT^2}{4\pi^2} = \frac{\ 7.21\text{ m s}^2\ \ 4.00\text{ s}\ ^2}{4\pi^2} = 2.92\text{ m}.$$

64. hen traveling in circular motion ith constant speed the instantaneous acceleration vector necessarily points toward the center. Thus, the center is “straight up” from the cited point.

a ince the center is “straight up” from (4.00 m, 4.00 m), the x coordinate of the center is 4.00 m.

b To find out “how far up” we need to kno the radius. sing . 4 34 e find

$$r = \frac{v^2}{a} = \frac{(5.00\ \text{m s})^2}{12.5\ \ \text{m s}^2} = 2.00\ \text{m}.$$

hus the y coordinate of the center is 2.00 m 4.00 m = 6.00 m. hus the center may be ritten as $x\ \ y$ = 4.00 m 6.00 m .

65. ince the period of a uniform circular motion is $T = 2\pi r\ \ v$ here r is the radius and v is the speed the centripetal acceleration can be ritten as

$$a = \frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2}.$$

ased on this e pression e compare the magnitudes of the allet and purse accelerations and find their ratio is the ratio of r values. herefore a_{allet} = 1.50 a_{purse} . hus the allet acceleration vector is

$$a = 1.50\ \ 2.00\ \text{m s}^2\ \hat{\text{i}}\ \ \ 4.00\ \text{m s}^2\ \hat{}\ = 3.00\ \text{m s}^2\ \hat{\text{i}}\ \ \ 6.00\ \text{m s}^2\ \hat{}\ .$$

66. he fact that the velocity is in the y direction and the acceleration is in the x direction at t_1 = 4.00 s implies that the motion is clock ise. he position corresponds to the “9 00 position.” n the other hand the position at t_2 = 10.0 s is in the “6 00 position” since the velocity points in the $-x$ direction and the acceleration is in the y direction. he time interval $\Delta t = 10.0\,\text{s} - 4.00\,\text{s} = 6.00\ \text{s}$ is e ual to 3 4 of a period

$$6.00\ \text{s} = \frac{3}{4}T \ \ \Rightarrow\ \ T = 8.00\ \text{s}.$$

uation 4 35 then yields

$$r = \frac{vT}{2\pi} = \frac{3.00\,\text{m s}\ \ 8.00\,\text{s}}{2\pi} = 3.82\ \text{m}.$$

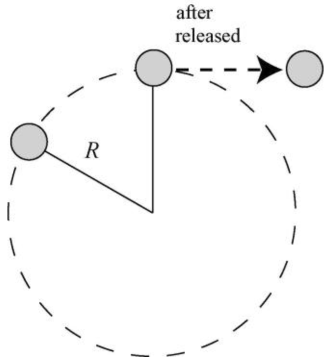
a he x coordinate of the center of the circular path is $x = 5.00\ \text{m} + 3.82\ \text{m} = 8.82\ \text{m}.$

b he y coordinate of the center of the circular path is $y = 6.00\ \text{m}.$

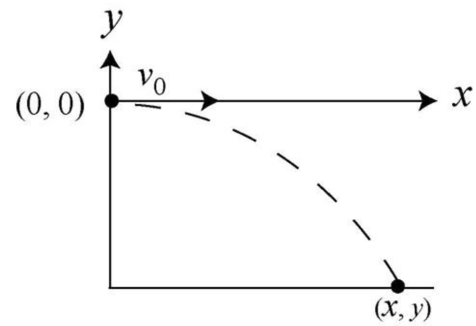
n other ords the center of the circle is at x,y = 8.82 m 6.00 m .

67. **THINK** n this problem e have a stone hirled in a hori ontal circle. fter the string breaks the stone undergoes pro ectile motion.

EXPRESS he stone moves in a circular path top vie sho n belo left initially but undergoes pro ectile motion after the string breaks side vie sho n belo right . ince $a = v^2\ \ R$ to calculate the centripetal acceleration of the stone e need to kno its speed during its circular motion this is also its initial speed hen it flies off . e use the kinematic e uations of pro ectile motion discussed in 4 6 to find that speed.



top vie



side vie

aking the y direction to be up ard and placing the origin at the point here the stone leaves its circular orbit then the coordinates of the stone during its motion as a pro ectile are given by $x = v_0 t$ and $y = -\frac{1}{2}gt^2$ since v_{0y} = 0 . t hits the ground at x = 10 m and $y = -2.0$ m.

ANALYZE ormally solving the y component e uation for the time e obtain $t = \sqrt{-2y \quad g}$ hich e substitute into the first e uation

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10\text{ m})\sqrt{-\frac{9.8\text{ m} \quad \text{s}^2}{2(-2.0\text{ m})}} = 15.7\text{ m} \quad \text{s}.$$

herefore the magnitude of the centripetal acceleration is

$$a = \frac{v_0^2}{R} = \frac{(15.7\text{ m} \quad \text{s})^2}{1.5\text{ m}} = 160\text{ m} \quad \text{s}^2.$$

LEARN he above e uations can be combined to give $a = \frac{gx^2}{-2yR}$. he e uation implies that the greater the centripetal acceleration the greater the initial speed of the pro ectile and the greater the distance traveled by the stone. his is precisely hat e e pect.

68. e note that after three seconds have elapsed $t_2 - t_1$ = 3.00 s the velocity for this ob ect in circular motion of period T is reversed e infer that it takes three seconds to reach the opposite side of the circle. hus T = 2 3.00 s = 6.00 s.

a sing . 4 35 $r = vT \quad 2\pi$ here $v = \sqrt{\quad 3.00\text{ m s} \quad ^2 + \quad 4.00\text{ m s} \quad ^2} = 5.00$ m s e obtain $r = 4.77$ m. The magnitude of the object's centripetal acceleration is therefore $a = v^2 \quad r$ = 5.24 m s^2.

b he average acceleration is given by . 4 15

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{-3.00\hat{\text{i}} - 4.00\hat{\ } \text{ m s} - 3.00\hat{\text{i}} + 4.00\hat{\ } \text{ m s}}{5.00\,\text{s} - 2.00\,\text{s}} = -2.00 \text{ m s}^2 \;\hat{\text{i}} \quad -2.67 \text{ m s}^2 \;\hat{\ }$$

hich implies $\vec{a}_{\text{avg}} = \sqrt{-2.00 \text{ m s}^2\;{}^2 + -2.67 \text{ m s}^2\;{}^2} = 3.33 \text{ m s}^2$.

69. e use . 4 15 first using velocities relative to the truck subscript t and then using velocities relative to the ground subscript g . e ork ith units so $20 \text{ km h} \to 5.6 \text{ m s}$ $30 \text{ km h} \to 8.3 \text{ m s}$ and $45 \text{ km h} \to 12.5 \text{ m s}$. e choose east as the $+\hat{\text{i}}$ direction.

a he velocity of the cheetah subscript c at the end of the 2.0 s interval is from . 4 44

$$\vec{v}_{\text{c t}} = \vec{v}_{\text{c g}} - \vec{v}_{\text{t g}} = 12.5 \text{ m s } \hat{\text{i}} - -5.6 \text{ m s } \hat{\text{i}} = 18.1 \text{ m s } \hat{\text{i}}$$

relative to the truck. ince the velocity of the cheetah relative to the truck at the beginning of the 2.0 s interval is $-8.3 \text{ m s } \hat{\text{i}}$ the average acceleration vector relative to the cameraman in the truck is

$$\vec{a}_{\text{avg}} = \frac{18.1 \text{ m s } \hat{\text{i}} - -8.3 \text{ m s } \hat{\text{i}}}{2.0\,\text{s}} = 13 \text{ m s}^2 \;\hat{\text{i}}$$

or $\vec{a}_{\text{avg}} = 13 \text{ m s}^2$.

b he direction of $\vec{a}_{\text{avg}}$ is $\hat{\text{i}}$ or east ard.

c he velocity of the cheetah at the start of the 2.0 s interval is from . 4 44

$$\vec{v}_{0\text{cg}} = \vec{v}_{0\text{ct}} + \vec{v}_{0\text{tg}} = -8.3 \text{ m s } \hat{\text{i}} + -5.6 \text{ m s } \hat{\text{i}} = -13.9 \text{ m s } \hat{\text{i}}$$

relative to the ground. he average acceleration vector relative to the cre member on the ground is

$$\vec{a}_{\text{avg}} = \frac{12.5 \text{ m s } \hat{\text{i}} - -13.9 \text{ m s } \hat{\text{i}}}{2.0\,\text{s}} = 13 \text{ m s}^2 \;\hat{\text{i}} \qquad \vec{a}_{\text{avg}} = 13 \text{ m s}^2$$

identical to the result of part a .

d he direction of $\vec{a}_{\text{avg}}$ is $\hat{\text{i}}$ or east ard.

70. e use . 4 44 noting that the upstream corresponds to the $\hat{\text{i}}$ direction.

a he subscript b is for the boat is for the ater and g is for the ground.

$$\vec{v}_{bg} = \vec{v}_{b} + \vec{v}_{g} = 14\text{ km h }\hat{i} + -9\text{ km h }\hat{i} = 5\text{ km h }\hat{i}.$$

hus the magnitude is $\vec{v}_{bg} = 5\text{ km h}$.

b he direction of $\vec{v}_{bg}$ is x or upstream.

c e use the subscript c for the child and obtain

$$\vec{v}_{cg} = \vec{v}_{cb} + \vec{v}_{bg} = -6\text{ km h }\hat{i} + 5\text{ km h }\hat{i} = -1\text{ km h }\hat{i}.$$

he magnitude is $\vec{v}_{cg} = 1\text{ km h}$.

d he direction of $\vec{v}_{cg}$ is $-x$ or do nstream.

71. hile moving in the same direction as the sidewalk's motion (covering a distance d relative to the ground in time t_1 = 2.50 s . 4 44 leads to

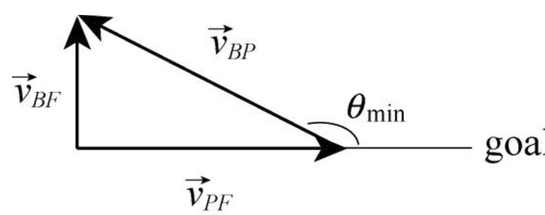
$$v_{\text{side alk}} \quad v_{\text{man running}} = \frac{d}{t_1}.$$

hile he runs back taking time t_2 = 10.0 s e have

$$v_{\text{side alk}} - v_{\text{man running}} = -\frac{d}{t_2}.$$

ividing these e uations and solving for the desired ratio e get $\frac{12.5}{7.5} = \frac{5}{3} = 1.67$.

72. e denote the velocity of the player ith $\vec{v}_{PF}$ and the relative velocity bet een the player and the ball be $\vec{v}_{BP}$. hen the velocity $\vec{v}_{BF}$ of the ball relative to the field is given by $\vec{v}_{BF} = \vec{v}_{PF} + \vec{v}_{BP}$. he smallest angle θ_{min} corresponds to the case hen $\vec{v}_{BF} \perp \vec{v}_{PF}$. ence



$$\theta_{\text{min}} = 180° - \cos^{-1}\left(\frac{|\vec{v}_{PF}|}{|\vec{v}_{BP}|}\right) = 180° - \cos^{-1}\left(\frac{4.0\text{ m s}}{6.0\text{ m s}}\right) = 130°.$$

73. e denote the police and the motorist ith subscripts p and m respectively. he coordinate system is indicated in ig. 4 46.

a he velocity of the motorist ith respect to the police car is

$$\vec{v}_{mp} = \vec{v}_{m} - \vec{v}_{p} = -60\text{ km h }\hat{j} - -80\text{ km h }\hat{i} = 80\text{ km h }\hat{i} - 60\text{ km h }\hat{j}.$$

b $\vec{v}_{mp}$ does happen to be along the line of sight. eferring to ig. 4 46 e find the vector pointing from one car to another is $\vec{r} =$ 800 m î – 600 m ˆ from M to P . ince the ratio of components in $\vec{r}$ is the same as in $\vec{v}_{mp}$ they must point the same direction.

c o they remain unchanged.

74. elocities are taken to be constant thus the velocity of the plane relative to the ground is $\vec{v}_{PG} =$ 55 km 1 4 hour ˆ = 220 km h ˆ . n addition

$$\vec{v}_{AG} = 42 \text{ km h } \cos 20° \hat{\text{i}} - \sin 20° \hat{\ } = 39 \text{ km h } \hat{\text{i}} - 14 \text{ km h } \hat{\ }.$$

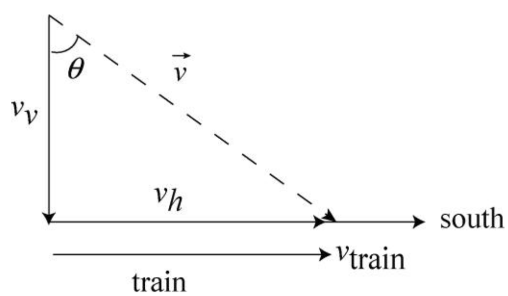
sing $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$ e have

$$\vec{v}_{PA} = \vec{v}_{PG} - \vec{v}_{AG} = -39 \text{ km h } \hat{\text{i}} + 234 \text{ km h } \hat{\ }.$$

hich implies $\vec{v}_{PA} = 237$ km h or 240 km h to t o significant figures.

75. **THINK** his problem deals ith relative motion in t o dimensions. aindrops appear to fall vertically by an observer on a moving train.

EXPRESS ince the raindrops fall vertically relative to the train the hori ontal component of the velocity of a raindrop v_h = 30 m s must be the same as the speed of the train i.e. $v_h = v_{\text{train}}$ see figure .



n the other hand if v_v is the vertical component of the velocity and θ is the angle bet een the direction of motion and the vertical then tan $\theta = v_h\ v_v$. no ing v_v and v_h allo s us to determine the speed of the raindrops.

ANALYZE ith $\theta = 70°$ e find the vertical component of the velocity to be

$$v_v = v_h \tan \theta = 30 \text{ m s } \tan 70 = 10.9 \text{ m s}.$$

herefore the speed of a raindrop is

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{30 \text{ m s }^2 + 10.9 \text{ m s }^2} = 32 \text{ m s}.$$

LEARN s long as the hori ontal component of the velocity of the raindrops coincides ith the speed of the train the passenger on board ill see the rain falling perfectly vertically.

76. he destination is $\vec{D}$ = 800 km here e orient a es so that y points north and x points east. his takes t o hours so the constant velocity of the plane relative to the ground is $\vec{v}_{\rm pg}$ = 400 km h . This must be the vector sum of the plane's velocity with respect to the air hich has *x,y* components 500cos70 500sin70 and the velocity of the air *wind* relative to the ground $\vec{v}_{\rm ag}$. hus

$$400 \text{ km h} = 500 \text{ km h} \cos 70 \ \mathrm{i} \quad 500 \text{ km h} \sin 70 \qquad \vec{v}_{\rm ag}$$

hich yields

$$\vec{v}_{\rm ag} = -171 \text{ km h i} - 70.0 \text{ km h} .$$

a he magnitude of $\vec{v}_{\rm ag}$ is $\vec{v}_{\rm ag} = \sqrt{-171 \text{ km h}^{\,2} + -70.0 \text{ km h}^{\,2}} = 185 \text{ km h}.$

b he direction of $\vec{v}_{\rm ag}$ is

$$\theta = \tan^{-1}\left(\frac{-70.0 \text{ km h}}{-171 \text{ km h}}\right) = 22.3° \quad \text{south of} \quad \text{est} .$$

77. **THINK** his problem deals ith relative motion in t o dimensions. no flakes falling vertically do n ard are seen to fall at an angle by a moving observer.

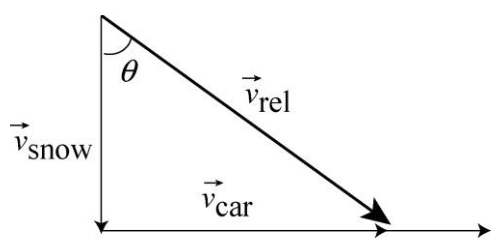
EXPRESS elative to the car the velocity of the sno flakes has a vertical component of $v_v = 8.0 \text{ m s}$ and a hori ontal component of $v_h = 50 \text{ km h} = 13.9 \text{ m s}$.

ANALYZE he angle θ from the vertical is found from

$$\tan\theta = \frac{v_h}{v_v} = \frac{13.9 \text{ m s}}{8.0 \text{ m s}} = 1.74$$

hich yields θ = 60 .

LEARN he problem can also be solved by e pressing the velocity relation in vector notation $\vec{v}_{\rm rel} = \vec{v}_{\rm car} + \vec{v}_{\rm sno}$ as sho n in the figure.



78. e make use of . 4 44 and . 4 45.

he velocity of eep P relative to A at the instant is

$$\vec{v}_{PA} = \; 40.0 \text{ m s} \;\; \cos 60°\hat{\text{i}} + \sin 60°\hat{} \;\; = \; 20.0 \text{ m s} \;\hat{\text{i}} + \; 34.6 \text{ m s} \;\hat{}.$$

imilarly the velocity of eep B relative to A at the instant is

$$\vec{v}_{BA} = \; 20.0 \text{ m s} \;\; \cos 30°\hat{\text{i}} + \sin 30°\hat{} \;\; = \; 17.3 \text{ m s} \;\hat{\text{i}} + \; 10.0 \text{ m s} \;\hat{}.$$

hus the velocity of P relative to B is

$$\vec{v}_{PB} = \vec{v}_{PA} - \vec{v}_{BA} = \; 20.0\hat{\text{i}} + 34.6\hat{} \;\; \text{m s} - \; 17.3\hat{\text{i}} + 10.0\hat{} \;\; \text{m s} = \; 2.68 \text{ m s} \;\hat{\text{i}} + \; 24.6 \text{ m s} \;\hat{}.$$

a he magnitude of $\vec{v}_{PB}$ is $\vec{v}_{PB} = \sqrt{\;2.68 \text{ m s}^{\;2} + \;24.6 \text{ m s}^{\;2}} = 24.8 \text{ m s}.$

b he direction of $\vec{v}_{PB}$ is $\theta = \tan^{-1} \;\; 24.6 \text{ m s} \;\;\; 2.68 \text{ m s} \;\; = 83.8°$ north of east or 6.2 east of north .

c he acceleration of P is

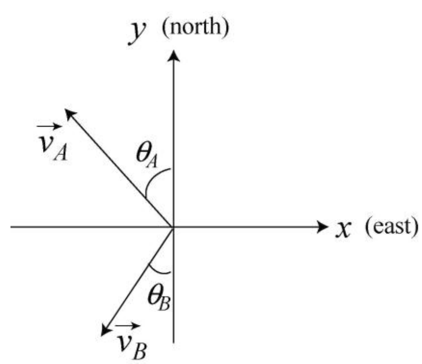
$$\vec{a}_{PA} = \; 0.400 \text{ m s}^2 \;\; \cos 60.0°\hat{\text{i}} + \sin 60.0°\hat{} \;\; = \; 0.200 \text{ m s}^2 \;\hat{\text{i}} + \; 0.346 \text{ m s}^2 \;\hat{}$$

and $\vec{a}_{PA} = \vec{a}_{PB}$. hus e have $\vec{a}_{PB} = 0.400 \text{ m s}^2$.

d he direction is 60.0° north of east or 30.0° east of north .

79. **THINK** his problem involves analy ing the relative motion of t o ships sailing in different directions.

EXPRESS iven that $\theta_A = 45°$ and $\theta_B = 40°$ as defined in the figure the velocity vectors relative to the shore for ships A and B are given by



$$\vec{v}_A = - \;\; v_A \cos 45° \;\; \hat{\text{i}} + \;\; v_A \sin 45° \;\; \hat{}$$
$$\vec{v}_B = - \;\; v_B \sin 40° \;\; \hat{\text{i}} - \;\; v_B \cos 40° \;\; \hat{}$$

ith v_A = 24 knots and v_B = 28 knots. e take east as $+\hat{\text{i}}$ and north as $\hat{}$.

he velocity of ship A relative to ship B is simply given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$.

ANALYZE a he relative velocity is

$$\vec{v}_{A\,B} = \vec{v}_A - \vec{v}_B = \quad v_B \sin 40° - v_A \cos 45° \;\hat{\text{i}} + \; v_B \cos 40° + v_A \sin 45° \;\hat{}$$
$$= \; 1.03 \text{ knots } \;\hat{\text{i}} + \; 38.4 \text{ knots } \hat{}$$

the magnitude of hich is $\vec{v}_{A\,B} = \sqrt{\;1.03 \text{ knots }^2 + \; 38.4 \text{ knots }^2} \approx 38.4$ knots.

b he angle θ_{AB} hich $\vec{v}_{A\,B}$ makes ith north is given by

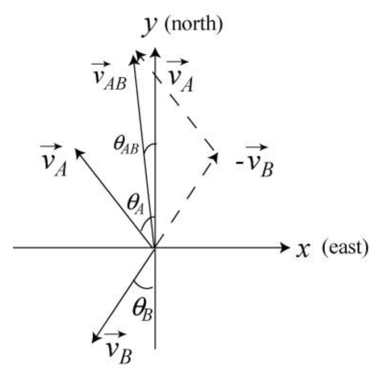
$$\theta_{AB} = \tan^{-1}\left(\frac{v_{AB\;x}}{v_{AB\;y}}\right) = \tan^{-1}\left(\frac{1.03 \text{ knots}}{38.4 \text{ knots}}\right) = 1.5°$$

hich is to say that $\vec{v}_{A\,B}$ points 1.5 east of north.

c ince the t o ships started at the same time their relative velocity describes at hat rate the distance bet een them is increasing. ecause the rate is steady e have

$$t = \frac{\Delta r_{AB}}{\vec{v}_{AB}} = \frac{160 \text{ nautical miles}}{38.4 \text{ knots}} = 4.2 \text{ h.}$$

d he velocity $\vec{v}_{A\,B}$ does not change ith time in this problem and $\vec{r}_{A\,B}$ is in the same direction as $\vec{v}_{A\,B}$ since they started at the same time. eversing the points of vie e have $\vec{v}_{A\,B} = -\vec{v}_{B\,A}$ so that $\vec{r}_{A\,B} = -\vec{r}_{B\,A}$ i.e. they are 180 opposite to each other . ence e conclude that B stays at a bearing of 1.5 est of south relative to A during the ourney neglecting the curvature of arth .



LEARN he relative velocity is depicted in the figure on the right. hen analy ing relative motion in t o dimensions a vector diagram such as the one sho n can be very helpful.

80. his is a classic problem involving t o dimensional relative motion. e align our coordinates so that *east* corresponds to x and *north* corresponds to y. e rite the vector addition e uation as $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$. e have $\vec{v}_{WG} = \; 2.0\angle 0°$ in the magnitude angle notation ith the unit m s understood or $\vec{v}_{WG} = 2.0\hat{\text{i}}$ in unit vector notation. e also have $\vec{v}_{BW} = \; 8.0\angle 120°$ here e have been careful to phrase the angle in the 'standard' way (measured counterclockwise from the $+x$ a is or $\vec{v}_{BW} = \; -4.0\hat{\text{i}} \;\; 6.9\hat{}$ m s.

a e can solve the vector addition e uation for $\vec{v}_{BG}$

$$\vec{v}_{BG} = v_{BW} + \vec{v}_{WG} = \ 2.0 \text{ m s } \hat{\text{i}} + \ -4.0\hat{\text{i}} \ \ 6.9\hat{\ } \ \text{ m s} = \ -2.0 \text{ m s } \hat{\text{i}} + \ 6.9 \text{ m s } \hat{\ }.$$

hus e find $\vec{v}_{BG} = 7.2$ m s.

b he direction of $\vec{v}_{BG}$ is $\theta = \tan^{-1}$ 6.9 m s –2.0 m s =106° measured counterclock ise from the x a is or 16 est of north.

c he velocity is constant and e apply $y - y_0 = v_y t$ in a reference frame. hus in the *ground* reference frame e have 200 m = 7.2 m s sin 106° $t \rightarrow t = 29$ s. ote f a student obtains “28 s,” then the student has probably neglected to take the y component properly a common mistake .

81. ere the subscript W refers to the ater. ur coordinates are chosen ith x being *east* and y being *north*. n these terms the angle specifying *east* ould be 0 and the angle specifying *south* ould be –90 or 270 . here the length unit is not displayed km is to be understood.

a e have $\vec{v}_{A\ W} = \vec{v}_{A\ B} + \vec{v}_{B\ W}$ so that

$$\vec{v}_{A\ B} = \ 22 \ \angle -90 \ \ - \ 40 \ \angle \ 37 \ \ = \ 56 \ \angle -125$$

in the magnitude angle notation conveniently done ith a vector capable calculator in polar mode . onverting to rectangular components e obtain

$$\vec{v}_{A\ B} = \ -32\text{km h } \ \hat{\text{i}} - \ 46 \text{ km h } \ \hat{\ } \ .$$

f course this could have been done in unit vector notation from the outset.

b ince the velocity components are constant integrating them to obtain the position is straightfor ard $\vec{r} - \vec{r}_0 = \int \vec{v}\, dt$

$$\vec{r} = \ 2.5 - 32t \ \ \hat{\text{i}} + \ 4.0 - 46t \ \ \hat{\ }$$

ith lengths in kilometers and time in hours.

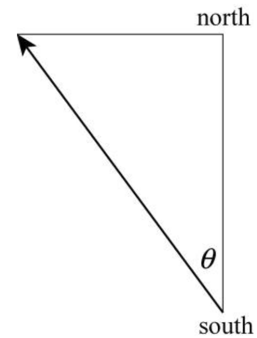
c he magnitude of this $\vec{r}$ is $r = \sqrt{\ 2.5 - 32t\ ^2 + \ 4.0 - 46t\ ^2}$. e minimi e this by taking a derivative and re uiring it to e ual ero — hich leaves us ith an e uation for t

$$\frac{dr}{dt} = \frac{1}{2} \frac{6286t - 528}{\sqrt{\ 2.5 - 32t\ ^2 + \ 4.0 - 46t\ ^2}} = 0$$

hich yields t = 0.084 h.

d lugging this value of t back into the e pression for the distance bet een the ships r e obtain r = 0.2 km. f course the calculator offers more digits $r = 0.225\ldots$), but they are not significant in fact the uncertainties implicit in the given data here should make the ship captains orry.

82. e construct a right triangle starting from the clearing on the south bank dra ing a line 200 m long due north *upward* in our sketch across the river and then a line due est upstream left ard in our sketch along the north bank for a distance 82 m + 1.1 m s t here the t dependent contribution is the distance that the river ill carry the boat do nstream during time t.



he hypotenuse of this right triangle the arro in our sketch also depends on t and on the boat's speed (relative to the water), and we set it e ual to the Pythagorean "sum" of the triangle's sides:

$$(4.0)t = \sqrt{200^2 + (82 + 1.1t)^2}$$

hich leads to a uadratic e uation for t

$$46724 + 180.4t - 14.8t^2 = 0.$$

b e solve for t first and find a positive value t = 62.6 s.

a he angle bet een the north ard 200 m leg of the triangle and the hypotenuse (which is measured "west of north") is then given by

$$\theta = \tan^{-1}\left(\frac{82 + 1.1t}{200}\right) = \tan^{-1}\left(\frac{151}{200}\right) = 37°.$$

83. e establish coordinates ith $\hat{\imath}$ pointing to the far side of the river perpendicular to the current and $\hat{}$ pointing in the direction of the current. e are told that the magnitude presumed constant of the velocity of the boat relative to the ater is $\vec{v}_{bw}$ = 6.4 km h. ts angle relative to the x a is is θ. ith km and h as the understood units the velocity of the ater relative to the ground is $\vec{v}_{wg} =$ 3.2 km h $\hat{}$.

(a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{v}_{bg} - v_{bg}\hat{\imath}$ here $v_{bg} > 0$ is unkno n. hus all $\hat{}$ components must cancel in the vector sum $\vec{v}_{bw}$ $\vec{v}_{wg}$ = $\vec{v}_{bg}$ hich means the $\vec{v}_{bw}$ sin θ = –3.2 km h $\hat{}$ so

$$\theta = \sin^{-1} \quad -3.2 \text{ km h} \quad 6.4 \text{ km h} \quad = -30 \ .$$

b sing the result from part a e find $v_{bg} = v_{bw}\cos\theta$ = 5.5 km h. hus traveling a distance of ℓ = 6.4 km re uires a time of 6.4 km 5.5 km h = 1.15 h or 69 min.

c f her motion is completely along the y a is as the problem implies then ith v_{wg} = 3.2 km h the ater speed e have

$$t_{\text{total}} = \frac{D}{v_{bw} \quad v_{wg}} \qquad \frac{D}{v_{bw} - v_{wg}} = 1.33 \text{ h}$$

here D = 3.2 km. his is e uivalent to 80 min.

d ince

$$\frac{D}{v_{bw} \quad v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = \frac{D}{v_{bw} - v_{wg}} + \frac{D}{v_{bw} + v_{wg}}$$

the ans er is the same as in the previous part that is $t_{\text{total}} = 80 \text{ min}$.

e he shortest time path should have $\theta = 0°$. his can also be sho n by noting that the case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = v_{bw}\cos\theta\ \hat{\text{i}} + \quad v_{bw}\sin\theta \quad v_{wg} \quad \hat{}$$

here the x component of $\vec{v}_{bg}$ must e ual l t. hus

$$t = \frac{l}{v_{bw}\cos\theta}$$

hich can be minimi ed using dt $d\theta$ = 0.

f he above e pression leads to t = 6.4 km 6.4 km h = 1.0 h or 60 min.

84. elative to the sled the launch velocity is $\vec{v}_{0\text{rel}} = v_{0x}$ i v_{0y} . Since the sled's motion is in the negative direction ith speed v_{s} note that e are treating v_{s} as a positive number, so the sled's velocity is actually $-v_{\text{s}}$ i then the launch velocity relative to the ground is $\vec{v}_0 = \ v_{0x} - v_{\text{s}}$ i v_{0y} . he hori ontal and vertical displacement relative to the ground are therefore

$$x_{\text{land}} - x_{\text{launch}} = \Delta x_{\text{bg}} = \ v_{0x} - v_{\text{s}} \ t_{\text{flight}}$$

$$y_{\text{land}} - y_{\text{launch}} = 0 = v_{0y}\, t_{\text{flight}} \quad \frac{1}{2} \ -g \ \ t_{\text{flight}}{}^{2}.$$

ombining these e uations leads to

$$\Delta x_{\text{bg}} = \frac{2v_{0x}v_{0y}}{g} - \left(\frac{2v_{0y}}{g}\right)v_s .$$

he first term corresponds to the "y intercept" on the graph and the second term in parentheses) corresponds to the magnitude of the "slope." From the figure e have

$$\Delta x_{bg} = 40 - 4v_s .$$

his implies v_{0y} = 4.0 s 9.8 m s^2 2 = 19.6 m s and that furnishes enough information to determine v_{0x}.

a v_{0x} = 40g 2v_{0y} = 40 m 9.8 m s^2 39.2 m s = 10 m s.

b s noted above v_{0y} = 19.6 m s.

c elative to the sled the displacement Δx_{bs} does not depend on the sled's speed, so Δx_{bs} = v_{0x} t_{flight} = 40 m.

d s in c relative to the sled the displacement Δx_{bs} does not depend on the sled's speed and Δx_{bs} = v_{0x} t_{flight} = 40 m.

85. sing displacement = velocity time for each constant velocity part of the trip along ith the fact that 1 hour = 60 minutes e have the follo ing vector addition e ercise using notation appropriate to many vector capable calculators

1667 m $\angle$ 0 1333 m $\angle$ –90 333 m $\angle$ 180 833 m $\angle$ –90 667 m $\angle$ 180 417 m $\angle$ –90 = 2668 m $\angle$ –76 .

a hus the magnitude of the net displacement is 2.7 km.

b ts direction is 76° clock ise relative to the initial direction of motion .

86. e use a coordinate system ith x east ard and y up ard.

a e note that 123 is the angle bet een the initial position and later position vectors so that the angle from x to the later position vector is 40 123 = 163 . n unit vector notation the position vectors are

$\vec{r}_1$ = 360 m cos 40° î 360 m sin 40° ĵ = 276 m î 231 m ĵ

$\vec{r}_2$ = 790 m cos 163° î 790 m sin 163° ĵ = –755 m î 231 m ĵ

respectively. onse uently e plug into . 4 3

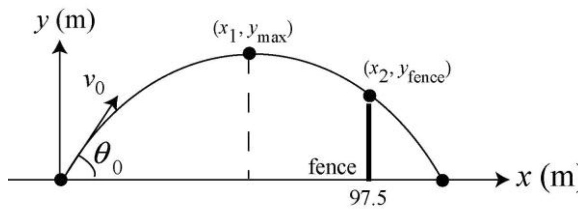
$$\Delta\vec{r} = \quad -755\text{ m} \;-\; 276\text{ m} \;\;\hat{\text{i}} \quad 231\text{ m} - 231\text{ m} \;\;\hat{} \;= -1031\text{ m}\;\;\hat{\text{i}}.$$

he magnitude of the displacement $\Delta\vec{r}$ is $\Delta\vec{r} \;\; = 1031\text{ m}.$

b he direction of $\Delta\vec{r}$ is $-\hat{\text{i}}$ or est ard.

87. **THINK** his problem deals ith the pro ectile motion of a baseball. iven the information on the position of the ball at t o instants e are asked to analy e its tra ectory.

EXPRESS he tra ectory of the baseball is sho n in the figure on the right. ccording to the problem statement at $t_1 = 3.0\text{ s}$ the ball reaches it ma imum height y_{ma} and at $t_2 = t_1 + 2.5\text{ s} = 5.5\text{ s}$ it barely clears a fence at $x_2 = 97.5\text{ m}$.



. 2 15 can be applied to the vertical y a is motion related to reaching the ma imum height hen t_1 = 3.0 s and v_y = 0

$$y_{ma} - y_0 = v_y t - \frac{1}{2} g t^2 \;.$$

ANALYZE a ith ground level chosen so $y_0 = 0$ this e uation gives the result

$$y_{ma} = \frac{1}{2} g t_1^2 = \frac{1}{2} \;\; 9.8\text{ m s}^2 \;\; 3.0\text{ s} \;^2 = 44.1\text{ m}$$

b fter the moment it reached ma imum height it is falling at $t_2 = t_1 + 2.5\text{ s} = 5.5\text{ s}$ it ill have fallen an amount given by . 2 18

$$y_{fence} - y_{ma} = 0 - \frac{1}{2} g \;\; t_2 - t_1 \;^2 .$$

hus the height of the fence is

$$y_{fence} = y_{ma} - \frac{1}{2} g \;\; t_2 - t_1 \;^2 = 44.1\text{ m} - \frac{1}{2} \;\; 9.8\text{ m s}^2 \;\; 2.5\text{ s} \;^2 = 13.48\text{ m} .$$

c ince the hori ontal component of velocity in a pro ectile motion problem is constant neglecting air friction e find from 97.5 m = v_{0x} 5.5 s that v_{0x} = 17.7 m s. he total flight time of the ball is $T = 2t_1 = 2 \;\; 3.0\text{ s} \;\; = 6.0\text{ s}$. hus the range of the baseball is

$$R = v_{0x} T = \;\; 17.7\text{ m s} \;\; 6.0\text{ s} \;\; = 106.4\text{ m}$$

hich means that the ball travels an additional distance

$$\Delta x = R - x_2 = 106.4 \text{ m} - 97.5 \text{ m} = 8.86 \text{ m}$$

beyond the fence before striking the ground.

LEARN art c can also be solved by noting that after passing the fence the ball ill strike the ground in 0.5 s so that the total fall time e uals the rise time . ith v_{0x} = 17.7 m s e have Δx = 17.7 m s 0.5 s = 8.86 m.

88. hen moving in the same direction as the et stream of speed v_s the time is

$$t_1 = \frac{d}{v_{ja} + v_s}$$

here d = 4000 km is the distance and v a is the speed of the et relative to the air 1000 km h . hen moving against the et stream the time is

$$t_2 = \frac{d}{v_{ja} - v_s}$$

here $t_2 - t_1 = \frac{70}{60}$ h . ombining these e uations and using the uadratic formula to solve gives v_s = 143 km h.

89. **THINK** e have a particle moving in a t o dimensional plane ith a constant acceleration. ince the x and y components of the acceleration are constants e can use able 2 1 for the motion along both a es.

EXPRESS sing vector notation ith $\vec{r}_0 = 0$ the position and velocity of the particle as a function of time are given by $\vec{r}\ \ t\ \ = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ and $\vec{v}\ \ t\ \ = \vec{v}_0 + \vec{a}t$ respectively. here units are not sho n units are to be understood.

ANALYZE a iven the initial velocity $\vec{v}_0 = \ 8.0$ m s $\hat{\ }$ and the acceleration $\vec{a} = \ 4.0$ m s^2 $\hat{\text{i}} + \ 2.0$ m s^2 $\hat{\ }$ the position vector of the particle is

$$\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = \left(8.0\hat{\ }\right)t + \frac{1}{2}\left(4.0\hat{\text{i}} + 2.0\hat{\ }\right)t^2 = \left(2.0t^2\right)\hat{\text{i}} \quad \left(8.0t \quad 1.0t^2\right)\hat{\ }.$$

herefore the time that corresponds to x = 29 m can be found by solving the e uation $2.0t^2$ = 29 hich leads to t = 3.8 s. he y coordinate at that time is

$$y = \ 8.0 \text{ m s} \ \ 3.8 \text{ s} \quad \ 1.0 \text{ m s}^2 \ \ 3.8 \text{ s}^{\ 2} = 45 \text{ m}.$$

b he velocity of the particle is given by $\vec{v} = \vec{v}_0 + \vec{a}t$. hus at t = 3.8 s the velocity is

$$\vec{v} = 8.0 \text{ m s } \hat{} + \left(4.0 \text{ m s}^2 \ \hat{\text{i}} + 2.0 \text{ m s}^2 \ \hat{} \right)(3.8 \text{ s}) = 15.2 \text{ m s } \hat{\text{i}} + 15.6 \text{ m s } \hat{}$$

hich has a magnitude of $v = \sqrt{v_x^2 + v_y^2} = \sqrt{15.2 \text{ m s }^2 + 15.6 \text{ m s }^2} = 22 \text{ m s}$.

LEARN nstead of using the vector notation e can also deal ith the x and the y components individually.

90. sing the same coordinate system assumed in . 4 25 e rearrange that e uation to solve for the initial speed

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2 \ \ x\tan\theta_0 - y}}$$

hich yields v_0 = 23 ft s for g = 32 ft s^2 x = 13 ft y = 3 ft and θ_0 = 55 .

91. e make use of . 4 25.

a y rearranging . 4 25 e obtain the initial speed

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2 \ x\tan\theta_0 - y}}$$

hich yields $v_0 = 255.5 \approx 2.6 \times 10^2$ m s for x = 9400 m y = –3300 m and θ_0 = 35 .

b rom . 4 21 e obtain the time of flight

$$t = \frac{x}{v_0\cos\theta_0} = \frac{9400 \text{ m}}{255.5 \text{ m s } \cos 35^\circ} = 45 \text{ s}.$$

c e e pect the air to provide resistance but no appreciable lift to the rock so e ould need a greater launching speed to reach the same target.

92. e apply . 4 34 to solve for speed v and . 4 35 to find the period T.

a e obtain

$$v = \sqrt{ra} = \sqrt{(5.0 \text{ m})(7.0)(9.8 \text{ m } \ \text{s}^2)} = 19 \text{ m } \ \text{s}.$$

b he time to go around once the period is $T = 2\pi r \ v$ = 1.7 s. herefore in one minute t = 60 s the astronaut e ecutes

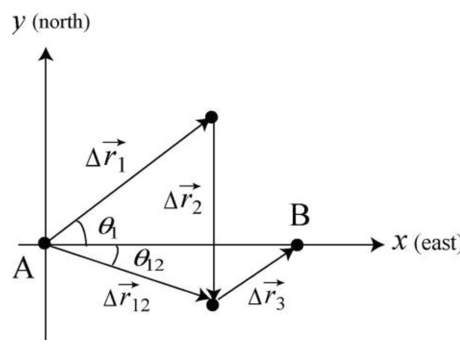
$$\frac{t}{T}=\frac{60\text{ s}}{1.7\text{ s}}=35$$

revolutions. hus 35 rev min is needed to produce a centripetal acceleration of 7g hen the radius is 5.0 m.

c s noted above T = 1.7 s.

93. **THINK** his problem deals ith the t o dimensional kinematics of a desert camel moving from oasis to oasis .

EXPRESS he ourney of the camel is illustrated in the figure on the right. e use a 'standard' coordinate system with $+x$ ast and y orth. engths are in kilometers and times are in hours. sing vector notation e rite the displacements for the first t o segments of the trip as



$$\Delta\vec{r}_1 = \;75\text{ km}\cos 37°\;\hat{\text{i}}+\;75\text{ km}\;\sin 37°\;\hat{}$$

$$\Delta\vec{r}_2 = \;-65\text{ km}\;\hat{}$$

he net displacement is $\Delta\vec{r}_{12}=\Delta\vec{r}_1+\Delta\vec{r}_2$. s can be seen from the figure to reach oasis re uires an additional displacement $\Delta\vec{r}_3$.

ANALYZE a e perform the vector addition of individual displacements to find the net displacement of the camel $\Delta\vec{r}_{12}=\Delta\vec{r}_1+\Delta\vec{r}_2=\;60\text{ km}\;\hat{\text{i}}-\;20\text{ km}\;\hat{}$. ts corresponding magnitude is

$$\Delta\vec{r}_{12}\;=\sqrt{\;60\text{ km}\;^2+\;-20\text{ km}\;^2}=63\text{ km}.$$

b he direction of $\Delta\vec{r}_{12}$ is $\theta_{12}=\tan^{-1}\;-20\text{ km}\quad 60\text{ km}\;=-18°$ or 18° south of east.

c o calculate the average velocity for the first t o segments of the ourney including rest e use the result from part a in . 4 8 along ith the fact that

$$\Delta t_{12}=\Delta t_1+\Delta t_2+\Delta t_{\text{rest}}=50\text{ h}+35\text{ h}+5.0\text{ h}=90\text{ h}.$$

n unit vector notation e have $\vec{v}_{12\text{ avg}}=\dfrac{60\,\hat{\text{i}}-20\,\hat{}\;\;\text{km}}{90\text{ h}}=\;0.67\,\hat{\text{i}}-0.22\,\hat{}\;\;\text{km h}.$

his leads to $\vec{v}_{12\text{ avg}}\;=0.70\text{ km h}.$

d he direction of $\vec{v}_{12\text{ avg}}$ is $\theta_{12}=\tan^{-1}\;-0.22\text{ km h}\quad 0.67\text{ km h}\;=-18°$ or 18° south of east.

e he average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. ince the camel travels 140 km e obtain 140 km 90 h = 1.56 km h $\approx$1.6 km h.

f he net displacement is re uired to be the 90 km ast from A to B. he displacement from the resting place to B is denoted $\Delta\vec{r}_3$. hus e must have

$$\Delta\vec{r}_1 \quad \Delta\vec{r}_2 \quad \Delta\vec{r}_3 = \ 90 \text{ km } \hat{\text{i}}$$

hich produces $\Delta\vec{r}_3 =$ 30 km $\hat{\text{i}}$ + 20 km $\hat{}$ in unit vector notation or 36 $\angle$ 33° in magnitude angle notation. herefore using . 4 8 e obtain

$$\vec{v}_{3\text{ avg}} = \frac{36\text{ km}}{120-90 \text{ h}} = 1.2 \text{ km h.}$$

g he direction of $\vec{v}_{3\text{ avg}}$ is the same as $\Delta\vec{r}_3$ that is 33 north of east .

LEARN ith a vector capable calculator in polar mode e could perform the vector addition of the displacements as 75 $\angle$ 37° + 65 $\angle$ − 90° = 63 $\angle$ − 18° . ote the distinction bet een average velocity and average speed.

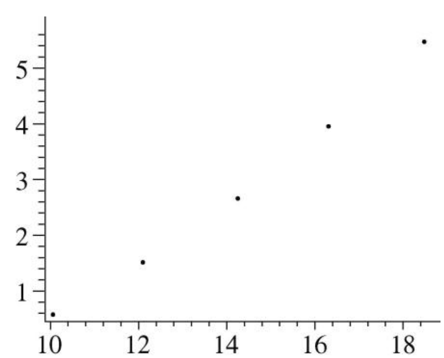
94. e compute the coordinate pairs x, y from $x =$ v_0 cos$\theta)t$ and $y = v_0$ sin $\theta t - \frac{1}{2}gt^2$ for t = 20 s and the speeds and angles given in the problem.

a e obtain

$$(x_A \ \ y_A) = (10.1 \text{ km} \ \ 0.556 \text{ km}) \qquad (x_B \ \ y_B) = (12.1 \text{ km} \ \ 1.51 \text{ km})$$
$$(x_C \ \ y_C) = (14.3 \text{ km} \ \ 2.68 \text{ km}) \qquad (x_D \ \ y_D) = (16.4 \text{ km} \ \ 3.99 \text{ km})$$

and x_E y_E = 18.5 km 5.53 km hich e plot in the ne t part.

b he vertical y and hori ontal x a es are in kilometers. he graph does not start at the origin. The curve to "fit" the data is not sho n but is easily imagined forming the "curtain of death").



95. a ith Δx = 8.0 m $t = \Delta t_1$ $a = a_x$ and v_{0x} = 0 . 2 15 gives

$$8.0\text{ m} = \tfrac{1}{2} a_x \ \Delta t_1 \ ^2$$

and the corresponding e pression for motion along the y a is leads to

$$\Delta y = 12\text{ m} = \tfrac{1}{2} a_y \ \Delta t_1 \ ^2 .$$

ividing the second e pression by the first leads to $a_y \quad a_x = 3 \quad 2 = 1.5$.

b etting $t = 2\Delta t_1$ then . 2 15 leads to $\Delta x =$ 8.0 m 2 ^{2} = 32 m hich implies that its x coordinate is no 4.0 32 m = 36 m. imilarly $\Delta y =$ 12 m 2 ^{2} = 48 m hich means its y coordinate has become 6.0 48 m = 54 m.

96. We assume the ball's initial velocity is perpendicular to the plane of the net. e choose coordinates so that $x_0 \ y_0$ = 0 3.0 m and v_x 0 note that $v_{0y} = 0$.

a o barely clear the net e have

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 \ \Rightarrow \ 2.24\text{ m} - 3.0\text{ m} = 0 - \frac{1}{2}\left(9.8\text{ m s}^2\right)t^2$$

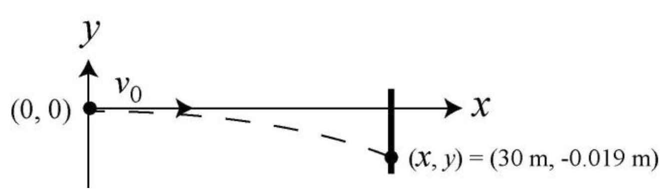
hich gives t = 0.39 s for the time it is passing over the net. his is plugged into the x e uation to yield the minimum initial velocity v_x = 8.0 m 0.39 s = 20.3 m s.

b e re uire y = 0 and find time t from the e uation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$. his value $t = \sqrt{2(3.0\text{ m}) \quad 9.8\text{ m s}^2} = 0.78\text{ s}$ is plugged into the x e uation to yield the ma imum initial velocity

$$v_x = \ 17.0\text{ m} \quad 0.78\text{ s} \ = 21.7\text{ m s}.$$

97. **THINK** bullet fired hori ontally from a rifle strikes the target at some distance below its aiming point. We're asked to find its total flight time and speed.

EXPRESS he tra ectory of the bullet is sho n in the figure on the right not to scale . ote that the origin is chosen to be at the firing point. ith this convention the y coordinate of the bullet is given by $y = -\frac{1}{2}gt^2$. no ing the coordinates $x \ y$ at the target allo s us to calculate the total flight time and speed of the bullet.



ANALYZE a f t is the time of flight and y = – 0.019 m indicates here the bullet hits the target then

$$t=\sqrt{\frac{-2y}{g}}=\sqrt{\frac{-2(-0.019\text{ m})}{9.8\text{ m s}^2}}=6.2\times10^{-2}\text{ s.}$$

b he mu le velocity is the initial hori ontal velocity of the bullet. ince x = 30 m is the hori ontal position of the target e have $x = v_0t$. hus

$$v_0=\frac{x}{t}=\frac{30\text{ m}}{6.3\times10^{-2}\text{ s}}=4.8\times10^{2}\text{ m s.}$$

LEARN lternatively e may use . 4 25 to solve for the initial velocity. ith $\theta_0=0$ and $y_0=0$ the e uation simplifies to $y=-\frac{gx^2}{2v_0^2}$ from hich e find

$$v_0=\sqrt{-\frac{gx^2}{2y}}=\sqrt{-\frac{9.8\text{ m s}^2\ \ 30\text{ m}\ ^2}{2\ -0.019\text{ m}}}=4.8\times10^{2}\text{ m s}$$

in agreement ith hat e calculated in part b .

98. or circular motion e must have $\vec{v}$ ith direction perpendicular to $\vec{r}$ and since the speed is constant magnitude $v=2\pi r\ \ T$ here $r=\sqrt{\ 2.00\text{ m}\ ^2+\ -3.00\text{ m}\ ^2}$ and $T=7.00\text{ s}$. he $\vec{r}$ given in the problem statement specifies a point in the fourth uadrant and since the motion is clock ise then the velocity must have both components negative. ur result satisfying these three conditions using unit vector notation hich makes it easy to double check that $\vec{r}\cdot\vec{v}=0$ for $\vec{v}$ = –2.69 m s i –1.80 m s .

99. et v_0 = 2π 0.200 m 0.00500 s $\approx$ 251 m s using . 4 35 be the speed it had in circular motion and θ_0 = 1 hr 360 12 hr for full rotation = 30.0 . hen . 4 25 leads to

$$y=\ 2.50\text{ m}\ \tan 30.0^\circ-\frac{9.8\text{ m s}^2\ \ 2.50\text{ m}\ ^2}{2\ 251\text{ m s}\ ^2\ \cos 30.0^\circ\ ^2}\approx 1.44\text{ m}$$

hich means its height above the floor is 1.44 m 1.20 m = 2.64 m.

100. oting that $\vec{v}_2=0$ then using . 4 15 the average acceleration is

$$\vec{a}_{\text{avg}}=\frac{\Delta\vec{v}}{\Delta t}=\frac{0-\left(6.30\,\hat{\text{i}}-8.42\,\hat{}\right)\text{m s}}{3\text{ s}}=\left(-2.1\,\hat{\text{i}}+2.8\,\hat{}\right)\text{m s}^2$$

101. sing . 2 16 e obtain $v^2=v_0^2-2gh$ or $h=\ v_0^2-v^2\ \ \ 2g$.

a ince $v = 0$ at the ma imum height of an up ard motion ith $v_0 = 7.00$ m s e have

$$h = \ 7.00 \text{ m s}\ ^2 \ \ 2\ 9.80 \text{ m s}^2 \ = 2.50 \text{ m.}$$

b he relative speed is $v_r = v_0 - v_c = 7.00 \text{ m s} - 3.00 \text{ m s} = 4.00 \text{ m s}$ ith respect to the floor. sing the above e uation e obtain $h = \ 4.00 \text{ m s}\ ^2 \ \ 2\ 9.80 \text{ m s}^2 \ = 0.82$ m.

c he acceleration or the rate of change of speed of the ball ith respect to the ground is 9.80 m s^2 do n ard .

d ince the elevator cab moves at constant velocity the rate of change of speed of the ball ith respect to the cab floor is also 9.80 m s^2 do n ard .

102. a ith r = 0.15 m and a = 3.0 × 10^{14} m s^2 . 4 34 gives

$$v = \sqrt{ra} = 6.7 \times 10^6 \text{ m s.}$$

b he period is given by . 4 35

$$T = \frac{2\pi r}{v} = 1.4 \times 10^{-7} \text{ s.}$$

103. a he magnitude of the displacement vector $\Delta\vec{r}$ is given by

$$\Delta\vec{r} \ = \sqrt{\ 21.5 \text{ km}\ ^2 + \ 9.7 \text{ km}\ ^2 + \ 2.88 \text{ km}\ ^2} = 23.8 \text{ km.}$$

hus

$$\vec{v}_{\text{avg}} \ = \frac{\Delta\vec{r}}{\Delta t} = \frac{23.8 \text{ km}}{3.50 \text{ h}} = 6.79 \text{ km h.}$$

b he angle θ in uestion is given by

$$\theta = \tan^{-1}\left(\frac{2.88 \text{ km}}{\sqrt{\ 21.5 \text{ km}\ ^2 + \ 9.7 \text{ km}\ ^2}}\right) = 6.96°.$$

104. he initial velocity has magnitude v_0 and because it is hori ontal it is e ual to v_x the hori ontal component of velocity at impact. hus the speed at impact is

$$\sqrt{v_0^2 + v_y^2} = 3v_0$$

here $v_y = \sqrt{2gh}$ and e have used . 2 16 ith Δx replaced ith h = 20 m. uaring both sides of the first e uality and substituting from the second e find

$$v_0^2 + 2gh = (3v_0)^2$$

hich leads to $gh = 4v_0^2$ and therefore to $v_0 = \sqrt{9.8\text{ m s}^2 \quad 20\text{ m}} \quad 2 = 7.0\text{ m s}$.

105. e choose hori ontal x and vertical y a es such that both components of $\vec{v}_0$ are positive. ositive angles are counterclock ise from x and negative angles are clock ise from it. n unit vector notation the velocity at each instant during the pro ectile motion is

$$\vec{v} = v_0 \cos\theta_0\,\hat{\text{i}} + (v_0 \sin\theta_0 - gt)\,\hat{}.$$

a ith v_0 = 30 m s and θ_0 = 60 e obtain $\vec{v} = 15\hat{\text{i}} \quad 6.4\hat{}$ m s for t = 2.0 s. he magnitude of $\vec{v}$ is $\vec{v} = \sqrt{15\text{ m s}^{\,2} + \;6.4\text{ m s}^{\,2}} = 16\text{ m s}$.

b he direction of $\vec{v}$ is

$$\theta = \tan^{-1}\;6.4\text{ m s} \quad 15\text{ m s} \; = 23°$$

measured counterclock ise from x.

c ince the angle is positive it is above the hori ontal.

d ith t = 5.0 s e find $\vec{v} = 15\hat{\text{i}} - 23\hat{}$ m s hich yields

$$\vec{v} = \sqrt{15\text{ m s}^{\,2} + \;-23\text{ m s}^{\,2}} = 27\text{ m s}.$$

e he direction of $\vec{v}$ is $\theta = \tan^{-1}\;-23\text{ m s} \quad 15\text{ m s} \; = -57°$ or 57° measured *clockwise* from x.

f ince the angle is negative it is belo the hori ontal.

106. e use . 4 2 and . 4 3.

a ith the initial position vector as $\vec{r}_1$ and the later vector as $\vec{r}_2$. 4 3 yields

$$\Delta r = \; -2.0\text{ m} \;-5.0\text{ m}\;\hat{\text{i}} + \;6.0\text{m} - \;-6.0\text{ m}\;\hat{} + \;2.0\text{ m} - 2.0\text{ m}\;\hat{\text{k}} = \;-7.0\text{ m}\;\hat{\text{i}} + \;12\text{ m}\;\hat{}$$

for the displacement vector in unit vector notation.

b ince there is no z component that is the coefficient of $\hat{\text{k}}$ is ero the displacement vector is in the xy plane.

107. e rite our magnitude angle results in the form $(R\angle\theta)$ ith units for the magnitude understood m for distances m s for speeds m s^2 for accelerations . ll angles θ are measured counterclock ise from x but e ill occasionally refer to angles ϕ hich are measured counterclock ise from the vertical line bet een the circle center and the coordinate origin and the line dra n from the circle center to the particle location see r in the figure . e note that the speed of the particle is $v = 2\pi r\ T$ here r = 3.00 m and T = 20.0 s thus v = 0.942 m s. he particle is moving counterclock ise in ig. 4 56.

a t t = 5.0 s the particle has traveled a fraction of

$$\frac{t}{T}=\frac{5.00\,\mathrm{s}}{20.0\,\mathrm{s}}=\frac{1}{4}$$

of a full revolution around the circle starting at the origin . hus relative to the circle center the particle is at

$$\phi=\frac{1}{4}\ 360° \ =90°$$

measured from vertical as e plained above . eferring to ig. 4 56 e see that this position (which is the “3 o’clock” position on the circle) corresponds to x = 3.0 m and y = 3.0 m relative to the coordinate origin. n our magnitude angle notation this is e pressed as $(R\angle\theta)=(4.2\angle 45°)$. lthough this position is easy to analy e ithout resorting to trigonometric relations it is useful for the computations belo to note that these values of x and y relative to coordinate origin can be gotten from the angle ϕ from the relations

$$x=r\sin\phi \qquad y=r-r\cos\phi\,.$$

f course $R=\sqrt{x^2+y^2}$ and θ comes from choosing the appropriate possibility from $\tan^{-1}\ y\ x$ or by using particular functions of vector capable calculators .

b t t = 7.5 s the particle has traveled a fraction of 7.5 20 = 3 8 of a revolution around the circle starting at the origin . elative to the circle center the particle is therefore at ϕ = 3 8 360 = 135 measured from vertical in the manner discussed above. eferring to ig. 4 56 e compute that this position corresponds to

$$x = \ 3.00\ \mathrm{m}\ \sin 135 \ = 2.1\ \mathrm{m}$$
$$y = \ 3.0\ \mathrm{m}\ - \ 3.0\ \mathrm{m}\ \cos 135 \ = 5.1\ \mathrm{m}$$

relative to the coordinate origin. n our magnitude angle notation this is e pressed as $R\ \angle\ \theta$ = 5.5 $\angle$ 68 .

c t t = 10.0 s the particle has traveled a fraction of 10 20 = 1 2 of a revolution around the circle. elative to the circle center the particle is at ϕ = 180 measured from vertical see e planation above . eferring to ig. 4 56 e see that this position corresponds to x

= 0 and y = 6.0 m relative to the coordinate origin. n our magnitude angle notation this is e pressed as $(R\angle\theta)=(6.0\angle 90°)$.

d e subtract the position vector in part a from the position vector in part c

$$(6.0\angle 90°)-(4.2\angle 45°)=(4.2\angle 135°)$$

using magnitude angle notation convenient hen using vector capable calculators . f e ish instead to use unit vector notation e rite

$$\Delta\vec{R}= 0-3.0 \text{ m} \ \hat{\text{i}}+ 6.0 \text{ m}-3.0 \text{ m} \ \hat{} = -3.0 \text{ m} \ \hat{\text{i}}+ 3.0 \text{ m} \ \hat{}$$

hich leads to $\Delta\vec{R} = 4.2$ m and $\theta = 135$.

e rom . 4 8 e have $\vec{v}_{\text{avg}} = \Delta\vec{R} \ \ \Delta t$. ith $\Delta t = 5.0$ s e have

$$\vec{v}_{\text{avg}} = -0.60 \text{ m s} \ \hat{\text{i}}+ 0.60 \text{ m s} \ \hat{}$$

in unit vector notation or 0.85 $\angle$ 135 in magnitude angle notation.

f he speed has already been noted v = 0.94 m s but its direction is best seen by referring again to ig. 4 56. The velocity vector is tangent to the circle at its "3 o'clock position" (see part (a)), which means $\vec{v}$ is vertical. hus our result is $(0.94 \angle 90°)$.

g gain the speed has been noted above v = 0.94 m s but its direction is best seen by referring to ig. 4 56. The velocity vector is tangent to the circle at its "12 o'clock position" (see part (c)), which means $\vec{v}$ is hori ontal. hus our result is $(0.94 \angle 180°)$.

h he acceleration has magnitude $a = v^2 \ r$ = 0.30 m s^2 and at this instant see part a it is hori ontal to ard the center of the circle . hus our result is $(0.30 \angle 180°)$.

i gain $a = v^2 \ r$ = 0.30 m s^2 but at this instant see part c it is vertical to ard the center of the circle . hus our result is $(0.30 \angle 270°)$.

108. uation 4 34 describes an inverse proportionality bet een r and a so that a large acceleration results from a small radius. hus an upper limit for a corresponds to a lo er limit for r.

a he minimum turning radius of the train is given by

$$r_{\text{min}} = \frac{v^2}{a_{\text{ma}}} = \frac{(216\text{ km}\quad\text{h})^2}{(0.050)(9.8\text{ m}\quad\text{s}^2)} = 7.3\times10^3\text{ m.}$$

b he speed of the train must be reduced to no more than

$$v = \sqrt{a_{\text{ma}}\,r} = \sqrt{0.050\left(9.8\text{ m s}^2\right)\left(1.00\times10^3\text{ m}\right)} = 22\text{ m s}$$

hich is roughly 80 km h.

109. a sing the same coordinate system assumed in . 4 25 e find

$$y = x\tan\theta_0 - \frac{gx^2}{2(v_0\cos\theta_0)^2} = -\frac{gx^2}{2v_0^2} \quad \text{if } \theta_0 = 0.$$

hus ith v_0 = 3.0 × 10^6 m s and x = 1.0 m e obtain y = –5.4 × 10^{-13} m hich is not practical to measure and suggests hy gravitational processes play such a small role in the fields of atomic and subatomic physics .

b t is clear from the above e pression that y decreases as v_0 is increased.

110. hen the escalator is stalled the speed of the person is $v_p = \ell/t$ here ℓ is the length of the escalator and t is the time the person takes to alk up it. his is v_p = 15 m 90 s = 0.167 m s. he escalator moves at v_e = 15 m 60 s = 0.250 m s. he speed of the person alking up the moving escalator is

$$v = v_p \quad v_e = 0.167\text{ m s} \quad 0.250\text{ m s} = 0.417\text{ m s}$$

and the time taken to move the length of the escalator is

$$t = \ell \quad v = 15\text{ m} \quad 0.417\text{ m}\quad\text{s} = 36\text{ s.}$$

f the various times given are independent of the escalator length then the ans er does not depend on that length either. n terms of ℓ in meters the speed in meters per second of the person alking on the stalled escalator is $\ell/90$ the speed of the moving escalator is $\ell/60$ and the speed of the person alking on the moving escalator is $v = (\ell/90) + (\ell/60) = 0.0278\ell$. he time taken is $t = \ell/v = \ell/0.0278\ell = 36\text{s}$ and is independent of ℓ.

111. he radius of arth may be found in ppendi .

(a) The speed of an object at Earth's equator is $v = 2\pi R\;T$ here R is the radius of arth 6.37 × 10^6 m and T is the length of a day 8.64 × 10^4 s

$$v = 2\pi\ 6.37\times10^{6}\ \text{m}\quad 8.64\times10^{4}\ \text{s} = 463\ \text{m s}.$$

he magnitude of the acceleration is given by

$$a=\frac{v^2}{R}=\frac{(463\ \text{m}\ \ \text{s})^2}{6.37\times10^6\ \text{m}}=0.034\ \text{m}\ \ \text{s}^2.$$

b f T is the period then $v = 2\pi R\ T$ is the speed and the magnitude of the acceleration is

$$a=\frac{v^2}{R}=\frac{2\pi R\ \ T^{\ 2}}{R}=\frac{4\pi^2 R}{T^2}.$$

hus

$$T=2\pi\sqrt{\frac{R}{a}}=2\pi\sqrt{\frac{6.37\times10^6\,\text{m}}{9.8\ \text{m}\ \ \text{s}^2}}=5.1\times10^3\ \text{s}=84\ \text{min}.$$

112. ith g_B = 9.8128 m s^2 and g_M = 9.7999 m s^2 e apply . 4 26

$$R_M - R_B=\frac{v_0^2\sin 2\theta_0}{g_M}-\frac{v_0^2\sin 2\theta_0}{g_B}=\frac{v_0^2\sin 2\theta_0}{g_B}\left(\frac{g_B}{g_M}-1\right)$$

hich becomes

$$R_M - R_B = R_B\left(\frac{9.8128\ \text{m s}^2}{9.7999\ \text{m s}^2}-1\right)$$

and yields upon substituting R_B = 8.09 m $R_M - R_B$ = 0.01 m = 1 cm.

113. rom the figure the three displacements can be ritten as

$$\vec d_1 = d_1\ \cos\theta_1\hat{\text{i}}+\sin\theta_1\hat{} \ = \ 5.00\ \text{m}\ \ \cos 30°\hat{\text{i}}+\sin 30°\hat{} \ = \ 4.33\ \text{m}\ \hat{\text{i}}+\ 2.50\ \text{m}\ \hat{}$$

$$\vec d_2 = d_2\ \cos\ 180°+\theta_1-\theta_2\ \hat{\text{i}}+\sin\ 180°+\theta_1-\theta_2\ \hat{} \ = \ 8.00\ \text{m}\ \ \cos 160°\hat{\text{i}}+\sin 160°\hat{}$$
$$= \ -7.52\ \text{m}\ \hat{\text{i}}+\ 2.74\ \text{m}\ \hat{}$$

$$\vec d_3 = d_3\ \cos\ 360°-\theta_3-\theta_2+\theta_1\ \hat{\text{i}}+\sin\ 360°-\theta_3-\theta_2+\theta_1\ \hat{} \ = \ 12.0\ \text{m}\ \ \cos 260°\hat{\text{i}}+\sin 260°\hat{}$$
$$= \ -2.08\ \text{m}\ \hat{\text{i}}-\ 11.8\ \text{m}\ \hat{}$$

here the angles are measured from the x a is. he net displacement is

$$\vec d=\vec d_1+\vec d_2+\vec d_3 = \ -5.27\ \text{m}\ \hat{\text{i}}-\ 6.58\ \text{m}\ \hat{}.$$

a he magnitude of the net displacement is

$$\vec{d} = \sqrt{\ -5.27\ \text{m}\ ^2 + \ -6.58\ \text{m}\ ^2} = 8.43\ \text{m}.$$

b he direction of $\vec{d}$ is $\theta = \tan^{-1}\left(\frac{d_y}{d_x}\right) = \tan^{-1}\left(\frac{-6.58\ \text{m}}{-5.27\ \text{m}}\right) = 51.3° \text{ or } 231°.$

 e choose $231°$ measured counterclock ise from x since the desired angle is in the third uadrant. n e uivalent ans er is $-129°$ measured clock ise from x.

114. aking derivatives of $\vec{r} = 2t\hat{\imath} + 2\sin\ \pi t\ 4\ \hat{}$ ith lengths in meters time in seconds and angles in radians provides e pressions for velocity and acceleration

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{\imath} + \frac{\pi}{2}\cos\left(\frac{\pi t}{4}\right)\hat{}$$
$$\vec{a} = \frac{d\vec{v}}{dt} = -\frac{\pi^2}{8}\sin\left(\frac{\pi t}{4}\right)\hat{}.$$

 hus e obtain

time t s			0.0	1.0	2.0	3.0	4.0
a	$\vec{r}$ position	x m	0.0	2.0	4.0	6.0	8.0
		y m	0.0	1.4	2.0	1.4	0.0
b	$\vec{v}$ velocity	v_x m s		2.0	2.0	2.0	
		v_y m s		1.1	0.0	–1.1	
c	$\vec{a}$ acceleration	a_x m s^2		0.0	0.0	0.0	
		a_y m s^2		–0.87	–1.2	–0.87	

115. ince this problem involves constant do n ard acceleration of magnitude a similar to the pro ectile motion situation   e use the e uations of 4 6 as long as e substitute a for g. e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he initial velocity is hori ontal so that $v_{0_y} = 0$ and

$$v_{0x} = v_0 = 1.00\times10^9\ \text{cm s}.$$

a f ℓ is the length of a plate and t is the time an electron is bet een the plates then $\ell = v_0 t$ here v_0 is the initial speed. hus

$$t = \frac{\ell}{v_0} = \frac{2.00\,\text{cm}}{1.00\times10^9\,\text{cm s}} = 2.00\times10^{-9}\ \text{s}.$$

b he vertical displacement of the electron is

$$y = -\frac{1}{2}at^2 = -\frac{1}{2}\left(1.00\times10^{17}\text{ cm s}^2\right)\left(2.00\times10^{-9}\text{ s}\right)^2 = -0.20\text{cm} = -2.00\text{ mm}$$

or $y = 2.00$ mm.

c he x component of velocity does not change

$$v_x = v_0 = 1.00 \times 10^9 \text{ cm s} = 1.00 \times 10^7 \text{ m s.}$$

d he y component of the velocity is

$$v_y = a_y t = \left(1.00\times10^{17}\text{ cm s}^2\right)\left(2.00\times10^{-9}\text{ s}\right) = 2.00\times10^8\text{ cm s}$$
$$= 2.00\times10^6\text{ m s.}$$

116. e neglect air resistance hich ustifies setting $a = -g = -9.8$ m s^2 taking *down* as the $-y$ direction for the duration of the motion of the shot ball. e are allo ed to use able 2 1 ith Δy replacing Δx because the ball has constant acceleration motion. e use primed variables e cept t ith the constant velocity elevator so $v = 10$ m s and unprimed variables ith the ball ith initial velocity $v_0 = v' + 20 = 30$ m s relative to the ground . units are used throughout.

a aking the time to be ero at the instant the ball is shot e compute its ma imum height y relative to the ground ith $v^2 = v_0^2 - 2g\ y - y_0$ here the highest point is characteri ed by $v = 0$. hus

$$y = y_o + \frac{v_0^2}{2g} = 76\text{ m}$$

here $y_o = y_o' + 2 = 30$ m here $y_o' = 28$ m is given in the problem and v_0 = 30 m s relative to the ground as noted above.

b here are a variety of approaches to this uestion. ne is to continue orking in the frame of reference adopted in part (a) (which treats the ground as motionless and "fixes" the coordinate origin to it in this case one describes the elevator motion ith $y' = y_o' + v't$ and the ball motion ith . 2 15 and solves them for the case here they reach the same point at the same time. nother is to ork in the frame of reference of the elevator the boy in the elevator might be oblivious to the fact the elevator is moving since it isn't accelerating), which is what we show here in detail

$$\Delta y_e = v_{0_e} t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{v_{0_e} + \sqrt{v_{0e}^2 - 2g\Delta y_e}}{g}$$

here v_{0e} = 20 m s is the initial velocity of the ball relative to the elevator and Δy_e = –2.0 m is the ball's displacement relative to the floor of the elevator. The positive root is chosen to yield a positive value for t the result is t = 4.2 s.

117. e adopt the positive direction choices used in the te tbook so that e uations such as . 4 22 are directly applicable. he coordinate origin is at the initial position for the football as it begins pro ectile motion in the sense of 4 5 and e let θ_0 be the angle of its initial velocity measured from the x a is.

a x = 46 m and y = –1.5 m are the coordinates for the landing point it lands at time t = 4.5 s. ince $x = v_{0x}t$

$$v_{0x} = \frac{x}{t} = \frac{46\ \text{m}}{4.5\ \text{s}} = 10.2\ \text{m s}.$$

ince $y = v_{0_y} t - \frac{1}{2} g t^2$

$$v_{0_y} = \frac{y + \frac{1}{2}gt^2}{t} = \frac{-1.5\ \text{m} + \frac{1}{2}\ 9.8\ \text{m}\ \ \text{s}^2\ \ 4.5\ \text{s}^{\ 2}}{4.5\ \text{s}} = 21.7\ \text{m}\ \ \text{s}.$$

he magnitude of the initial velocity is

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{\ 10.2\ \text{m}\ \ \text{s}^{\ 2} + \ 21.7\ \text{m}\ \ \text{s}^{\ 2}} = 24\ \text{m}\ \ \text{s}.$$

b he initial angle satisfies tan $\theta_0 = v_{0y}\ v_{0x}$. hus

$$\theta_0 = \tan^{-1}\ \ 21.7\ \text{m s}\ \ \ 10.2\ \text{m s}\ \ = 65\ .$$

118. he velocity of arry is v_1 and that of urly is v_2. lso e denote the length of the corridor by L. Now, Larry's time of passage is t_1 = 150 s hich must e ual $L\ v_1$ and Curly's time of passage is t_2 = 70 s hich must e ual $L\ v_2$. he time oe takes is therefore

$$t = \frac{L}{v_1 + v_2} = \frac{1}{v_1\ \ L + v_2\ \ L} = \frac{1}{\frac{1}{150\,\text{s}} + \frac{1}{70\,\text{s}}} = 48\text{s}.$$

119. he bo car has velocity $\vec{v}_{c\,g} = v_1\ \hat{\text{i}}$ relative to the ground and the bullet has velocity

$$\vec{v}_{0b\,g} = v_2 \cos\theta\,\hat{\text{i}} + v_2 \sin\theta\,\hat{}$$

relative to the ground before entering the car e are neglecting the effects of gravity on the bullet . hile in the car its velocity relative to the outside ground is

$$\vec{v}_{bg} = 0.8 v_2 \cos\theta\,\hat{\text{i}} + 0.8 v_2 \sin\theta\,\hat{}$$

due to the 20 reduction mentioned in the problem . he problem indicates that the velocity of the bullet in the car *relative to the car* is ith v_3 unspecified $\vec{v}_{b\,c} = v_3\,\hat{}$. o . 4 44 provides the condition

$$\vec{v}_{b\,g} = \vec{v}_{b\,c} + \vec{v}_{c\,g}$$

$$0.8v_2 \cos\theta\ \hat{\text{i}} + 0.8v_2 \sin\theta\ \hat{} \ = v_3\ \hat{} + v_1\ \hat{\text{i}}$$

so that e uating x components allo s us to find θ. f one ished to find v_3 one could also e uate the y components and from this if the car idth ere given one could find the time spent by the bullet in the car but this information is not asked for hich is hy the idth is irrelevant . herefore e amining the x components in units leads to

$$\theta = \cos^{-1}\left(\frac{v_1}{0.8v_2}\right) = \cos^{-1}\left(\frac{85\text{ km h}\left(\frac{1000\text{ m km}}{3600\text{ s h}}\right)}{0.8\ \ 650\text{ m s}}\right)$$

hich yields 87 for the direction of $\vec{v}_{b\,g}$ measured from $\hat{\text{i}}$ hich is the direction of motion of the car). The problem asks, "from what direction was it fired?" — hich means the ans er is not 87 but rather its supplement 93 measured from the direction of motion . tating this more carefully in the coordinate system e have adopted in our solution the bullet velocity vector is in the first uadrant at 87 measured counterclock ise from the x direction the direction of train motion hich means that the direction from hich the bullet came here the sniper is is in the third uadrant at –93 that is 93 measured clock ise from x .

120. a sing $a = v^2\ \ R$ the radius of the track is

$$R = \frac{v^2}{a} = \frac{9.20\text{ m s}\ \ ^2}{3.80\text{ m s}^2} = 22.3\text{ m}\,.$$

b sing $T = 2\pi R\ \ v$ the period of the circular motion is

$$T = \frac{2\pi R}{v} = \frac{2\pi\ \ 22.3\text{ m}}{9.20\text{ m s}} = 15.2\text{ s}$$

121. a ith $v = c\ \ 10 = 3\times10^7$ m s and $a = 20g = 196\text{ m s}^2$. 4 34 gives

$$r = v^2\ \ a = 4.6\times10^{12}\text{ m.}$$

b he period is given by . 4 35 $T = 2\pi r\ \ v = 9.6\times10^5$ s. hus the time to make a uarter turn is T 4 = 2.4 × 10^5 s or about 2.8 days.

122. ince $v_y^2 = v_{0y}^2 - 2g\Delta y$ and v_y=0 at the target e obtain

$$v_{0y} = \sqrt{2\left(9.80 \text{ m s}^2\right)\left(5.00 \text{ m}\right)} = 9.90 \text{ m s}$$

a ince $v_0 \sin \theta_0 = v_{0y}$ ith v_0 = 12.0 m s e find θ_0 = 55.6 .

b o $v_y = v_{0y} - gt$ gives t = 9.90 m s 9.80 m s^2 = 1.01 s. hus

$$\Delta x = v_0 \cos \theta_0 \, t = 6.85 \text{ m}.$$

c he velocity at the target has only the v_x component hich is e ual to $v_{0x} = v_0 \cos \theta_0$ = 6.78 m s.

123. ith v_0 = 30.0 m s and R = 20.0 m . 4 26 gives

$$\sin 2\theta_0 = \frac{gR}{v_0^2} = 0.218.$$

ecause sin ϕ = sin 180 $-\phi$ there are t o roots of the above e uation

$$2\theta_0 = \sin^{-1} \; 0.218 \; = 12.58° \text{ and } 167.4°.$$

hich correspond to the t o possible launch angles that ill hit the target in the absence of air friction and related effects .

a he smallest angle is θ_0 = 6.29 .

b he greatest angle is and θ_0 = 83.7 .

n alternative approach to this problem in terms of . 4 25 ith y = 0 and 1 cos^2 = 1 tan^2 is possible — and leads to a uadratic e uation for tanθ_0 ith the roots providing these t o possible θ_0 values.

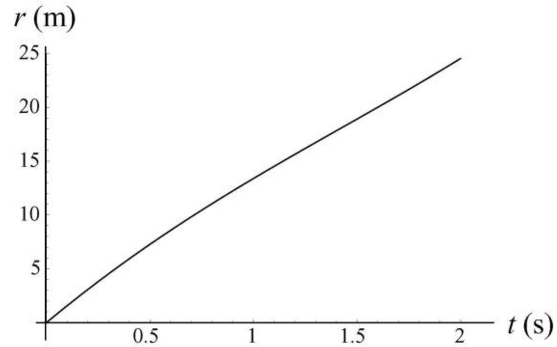
124. e make use of . 4 21 and .4 22.

a ith v_0 = 16 m s e s uare . 4 21 and . 4 22 and add them then using Pythagoras' theorem) take the square root to obtain r

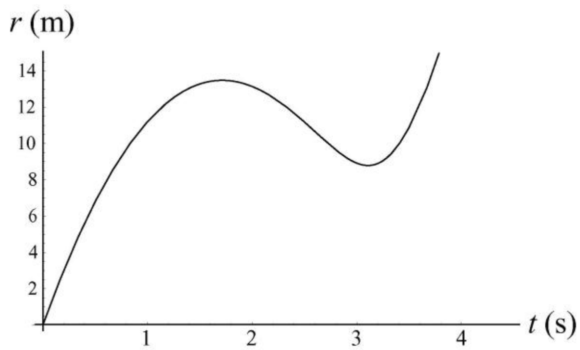
$$r = \sqrt{\; x - x_0 \;^2 + \; y - y_0 \;^2} = \sqrt{\; v_0 \cos\theta_0 t \;^2 + \; v_0 \sin\theta_0 t - gt^2 \;\; 2 \;^2}$$

$$= t\sqrt{v_0^2 - v_0 g \sin\theta_0 t + g^2 t^2 \;\; 4}$$

elo e plot r as a function of time for θ_0 = 40.0



b or this ne t graph for r versus t e set θ_0 = 80.0 .



c ifferentiating r ith respect to t e obtain

$$\frac{dr}{dt}=\frac{v_0^2-3v_0gt\sin\theta_0 \;\; 2+g^2t^2 \;\; 2}{\sqrt{v_0^2-v_0g\sin\theta_0 t+g^2t^2 \;\; 4}}$$

etting $dr \;\; dt=0$ ith $v_0=16.0$ m s and $\theta_0=40.0°$ e have $256-151t+48t^2=0$. he e uation has no real solution. his means that the ma imum is reached at the end of the flight ith

$$t_{total}=2v_0\sin\theta_0 \;\; g=2 \;\; 16.0 \text{ m s } \sin 40.0° \;\; 9.80 \text{ m s}^2 \;\; =2.10 \text{ s.}$$

d he value of r is given by

$$r= \;\; 2.10 \sqrt{\;16.0\;^2- \;16.0 \;\; 9.80 \;\sin 40.0° \;\; 2.10 \; + \; 9.80\;^2 \;\; 2.10\;^2 \;\; 4}=25.7 \text{ m.}$$

e he hori ontal distance is $r_x=v_0\cos\theta_0 t= \;\; 16.0 \text{ m s } \;\cos 40.0° \;\; 2.10 \text{ s } \;=25.7$ m.

f he vertical distance is $r_y=0$.

g or the θ_0 = 80 launch the condition for ma imum r is $256-232t+48t^2=0$ or $t=1.71$ s the other solution t = 3.13 s corresponds to a minimum.

h he distance traveled is

$$r = 1.71\sqrt{16.0^{\ 2} - 16.0 \;\; 9.80 \;\; \sin 80.0° \; 1.71 \; + \; 9.80^{\ 2} \; 1.71^{\ 2} \;\; 4} = 13.5 \text{ m.}$$

i he hori ontal distance is

$$r_x = v_0 \cos\theta_0 t = 16.0 \text{ m s } \cos 80.0° \; 1.71 \text{s} = 4.75 \text{ m.}$$

he vertical distance is

$$r_y = v_0 \sin\theta_0 t - \frac{gt^2}{2} = 16.0 \text{ m s } \sin 80° \; 1.71 \text{s} - \frac{9.80 \text{ m s}^2 \;\; 1.71 \text{s}^{\ 2}}{2} = 12.6 \text{ m.}$$

125. sing the same coordinate system assumed in . 4 25 e find x for the elevated cannon from

$$y = x\tan\theta_0 - \frac{gx^2}{2(v_0\cos\theta_0)^2} \qquad \text{here } y = -30 \text{ m.}$$

sing the uadratic formula choosing the positive root e find

$$x = v_0\cos\theta_0\left(\frac{v_0\sin\theta_0 + \sqrt{(v_0\sin\theta_0)^2 - 2gy}}{g}\right)$$

hich yields x = 715 m for v_0 = 82 m s and θ_0 = 45 . his is 29 m longer than the distance of 686 m.

126. t ma imum height the y component of a projectile's velocity vanishes, so the given 10 m s is the constant x component of velocity.

a sing v_{0y} to denote the y velocity 1.0 s before reaching the ma imum height then ith v_y = 0 the e uation $v_y = v_{0y} - gt$ leads to v_{0y} = 9.8 m s. he magnitude of the velocity vector or *speed* at that moment is therefore

$$\sqrt{v_x^2 + v_{0y}^{\ 2}} = \sqrt{10 \text{ m s}^{\ 2} + \; 9.8 \text{ m s}^{\ 2}} = 14 \text{ m s.}$$

b t is clear from the symmetry of the problem that the speed is the same 1.0 s after reaching the top as it as 1.0 s before 14 m s again . his may be verified by using $v_y = v_{0y} - gt$ again but now "starting the clock" at the highest point so that v_{0y} = 0 and $t = 1.0$ s . his leads to v_y = –9.8 m s and $\sqrt{10 \text{ m s}^{\ 2} + (-9.8 \text{ m s})^2} = 14 \text{ m s}$.

c he x_0 value may be obtained from $x = 0 = x_0$ 10 m s 1.0s hich yields $x_0 = -10$ m.

d ith v_{0y} = 9.8 m s denoting the y component of velocity one second before the top of the tra ectory then e have $y = 0 = y_0 + v_{0y}t - \frac{1}{2}gt^2$ here t = 1.0 s. his yields $y_0 = -4.9$ m.

e y using $x - x_0$ = 10 m s 1.0 s here x_0 = 0 e obtain x = 10 m.

f et t = 0 at the top ith $y_0 = v_{0y} = 0$. rom $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ e have for t = 1.0 s

$$y = - \; 9.8 \text{ m s}^2 \;\; 1.0 \text{ s} \;^2 \;\; 2 = -4.9 \text{ m}.$$

127. ith no acceleration in the x direction yet a constant acceleration of 1.40 m s^2 in the y direction the position in meters as a function of time in seconds must be

$$\vec{r} = \; 6.00t \;\hat{\text{i}} \quad \left(\frac{1}{2}\; 1.40 \;\; t^2\right)\hat{}$$

and $\vec{v}$ is its derivative ith respect to t.

a t t = 3.00 s therefore $\vec{v} = \; 6.00\hat{\text{i}} + 4.20\hat{}$ m s.

b t t = 3.00 s the position is $\vec{r} = \; 18.0\hat{\text{i}} + 6.30\hat{}$ m.

128. e note that

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$$

describes a right triangle ith one leg being $\vec{v}_{PG}$ east another leg being $\vec{v}_{AG}$ magnitude = 20 direction = south and the hypotenuse being $\vec{v}_{PA}$ magnitude = 70 . engths are in kilometers and time is in hours. sing the ythagorean theorem e have

$$|\vec{v}_{PA}| = \sqrt{\;\vec{v}_{PG}\;^2 + \;\vec{v}_{AG}\;^2} \;\Rightarrow\; 70 \text{ km h} = \sqrt{\;\vec{v}_{PG}\;^2 + \;20 \text{ km h}\;^2}$$

hich can be solved to give the ground speed $\vec{v}_{PG}$ = 67 km h.

129. he figure offers many interesting points to analy e and others are easily inferred such as the point of ma imum height . he focus here to begin ith ill be the final point sho n 1.25 s after the ball is released hich is hen the ball returns to its original height. n nglish units g = 32 ft s^2.

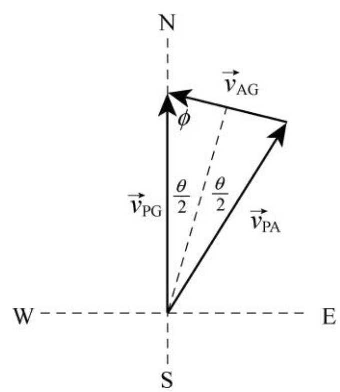
a sing $x - x_0 = v_x t$ e obtain v_x = 40 ft 1.25 s = 32 ft s. nd $y - y_0 = 0 = v_{0y} t - \frac{1}{2} g t^2$ yields $v_{0y} = \frac{1}{2}\left(32 \text{ ft s}^2\right)\left(1.25 \text{ s}\right) = 20 \text{ ft s}$. hus the initial speed is

$$v_0 = \vec{v}_0 = \sqrt{32 \text{ ft s}^2 + 20 \text{ ft s}^2} = 38 \text{ ft s}.$$

b ince v_y = 0 at the ma imum height and the hori ontal velocity stays constant then the speed at the top is the same as v_x = 32 ft s.

c e can infer from the figure or compute from $v_y = 0 = v_{0y} - gt$ that the time to reach the top is 0.625 s. ith this e can use $y - y_0 = v_{0y} t - \frac{1}{2} g t^2$ to obtain 9.3 ft here y_0 = 3 ft has been used . n alternative approach is to use $v_y^2 = v_{0y}^2 - 2g\left(y - y_0\right)$.

130. e denote $\vec{v}$ as the velocity of the plane relative to the ground $\vec{v}$ as the velocity of the air relative to the ground and $\vec{v}$ as the velocity of the plane relative to the air.



a he vector diagram is sho n on the right $\vec{v} = \vec{v} + \vec{v}$. ince the magnitudes v and v are e ual the triangle is isosceles ith t o sides of e ual length.

onsider either of the right triangles formed hen the bisector of θ is dra n the dashed line . t bisects $\vec{v}$ so

$$\sin\left(\theta \;\; 2\right) = \frac{v}{2v} = \frac{70.0 \text{ mi h}}{2\left(135 \text{ mi h}\right)}$$

hich leads to θ = 30.1 . o $\vec{v}$ makes the same angle ith the line as the dashed line does ith the line. he ind is blo ing in the direction 15.0 north of est. hus it is blo ing *from* 75.0 east of south.

b he plane is headed along $\vec{v}$ in the direction 30.0 east of north. here is another solution ith the plane headed 30.0 est of north and the ind blo ing 15 north of east that is from 75 est of south .

131. e make use of . 4 24 and . 4 25.

a ith x = 180 m θ_0 = 30 and v_0 = 43 m s e obtain

$$y = \tan 30^\circ \;\; 180 \text{ m} \;\; - \frac{9.8 \text{ m s}^2 \;\; 180 \text{ m}^{\;2}}{2 \;\; 43 \text{ m s}^{\;2} \;\; \cos 30^\circ \;\; ^2} = -11 \text{ m}$$

or $y = 11$ m. his implies the rise is roughly eleven meters above the fair ay.

b he hori ontal component in the absence of air friction is unchanged but the vertical component increases see . 4 24 . he ythagorean theorem then gives the magnitude of final velocity right before striking the ground 45 m s.

132. e let g_p denote the magnitude of the gravitational acceleration on the planet. number of the points on the graph (including some "inferred" points — such as the ma height point at x = 12.5 m and t = 1.25 s can be analy ed profitably for future reference e label ith subscripts the first x_0 y_0 = 0 2 at $t_0 = 0$) and last ("final") points ((x_f y_f = 25 2 at t_f = 2.5 ith lengths in meters and time in seconds.

a he x component of the initial velocity is found from $x_f - x_0 = v_{0x}\, t_f$. herefore $v_{0x} = 25 \;\; 2.5 = 10$ m s. e try to obtain the y component from

$$y_f - y_0 = 0 = v_{0y} t_f - \tfrac{1}{2} g_p t_f^2 .$$

his gives us v_{0y} = 1.25g_p and e see e need another e uation by analy ing another point say the ne t to last one $y - y_0 = v_{0y} t - \tfrac{1}{2} g_p t^2$ ith y = 6 and t = 2 this produces our second e uation v_{0y} = 2 g_p. imultaneous solution of these t o e uations produces results for v_{0y} and g_p relevant to part b . hus our complete ans er for the initial velocity is $\vec{v} =$ 10 m s $\hat{\text{i}}$ + 10 m s $\hat{}$.

b s a by product of the part a computations e have g_p = 8.0 m s^2.

c olving for t_g the time to reach the ground in $y_g = 0 = y_0 + v_{0y} t_g - \tfrac{1}{2} g_p t_g^2$ leads to a positive ans er t_g = 2.7 s.

d ith g = 9.8 m s^2 the method employed in part c ould produce the uadratic e uation $-4.9 t_g^2 + 10 t_g + 2 = 0$ and then the positive result t_g = 2.2 s.

133. (a) The helicopter's speed is v' = 6.2 m s hich implies that the speed of the package is $v_0 = 12 - v'$ = 5.8 m s relative to the ground.

b etting x be in the direction of $\vec{v}_0$ for the package and y be do n ard e have for the motion of the package

$$\Delta x = v_0 t \quad \text{and} \quad \Delta y = \frac{1}{2} g t^2$$

here Δy = 9.5 m. rom these e find t = 1.39 s and Δx = 8.08 m for the package hile $\Delta x'$ for the helicopter hich is moving in the opposite direction is $-v'\, t$ = –8.63 m. hus the hori ontal separation bet een them is 8.08 – –8.63 = 16.7 m ≈ 17 m.

c he components of $\vec{v}$ at the moment of impact are v_x v_y = 5.8 13.6 in units. he vertical component has been computed using . 2 11. he angle hich is belo hori ontal for this vector is $\tan^{-1}$ 13.6 5.8 = 67 .

134. he type of acceleration involved in steady speed circular motion is the centripetal acceleration $a = v^2 r$ hich is at each moment directed to ards the center of the circle. he radius of the circle is r = 12 ^{2} 3 = 48 m.

a hus if at the instant the car is traveling *clockwise* around the circle it is 48 m est of the center of its circular path.

b he same result holds here if at the instant the car is traveling *counterclockwise*. hat is it is 48 m est of the center of its circular path.

135. a sing the same coordinate system assumed in . 4 21 and . 4 22 so that θ_0 = –20.0 e use v_0 = 15.0 m s and find the hori ontal displacement of the ball at t = 2.30 s

$$\Delta x = (v_0 \cos\theta_0)t = 32.4 \text{ m}.$$

b he vertical displacement is $\Delta y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = -37.7$ m.

136. e take the initial x, y specification to be 0.000 0.762 m and the positive x direction to be towards the "green monster." The components of the initial velocity are 33.53 $\angle$ 55° $\rightarrow$ 19.23 27.47 m s.

a ith t = 5.00 s e have $x = x_0$ $v_x t$ = 96.2 m.

b t that time $y = y_0$ $v_{0y}t - \frac{1}{2}gt^2$ = 15.59 m hich is 4.31 m above the all.

c he moment in uestion is specified by t = 4.50 s. t that time $x - x_0$ = 19.23 4.50 = 86.5 m.

d he vertical displacement is $y = y_0$ $v_{0y}t - \frac{1}{2}gt^2$ = 25.1 m.

137. hen moving in the same direction as the et stream of speed v_s the time is $t = d \;\; v_{ja} + v_s$ here d = 4350 km is the distance and $v_{ja} = 966$ km h is the speed of the et relative to the air. hen moving against the et stream the time is $t' = d \;\; v_{ja} - v_s$ ith $t' - t = 50$ min = 5 6 h. ombining the e pressions gives

$$t' - t = \frac{d}{v_{ja} - v_s} - \frac{d}{v_{ja} + v_s} = \frac{2dv_s}{v_{ja}^2 - v_s^2} = \frac{5}{6} \text{ h}$$

pon rearranging and using the uadratic formula to solve for v_s e get v_s = 88.63 km h.

138. e establish coordinates ith $\hat{\text{i}}$ pointing to the far side of the river perpendicular to the current and $\hat{}$ pointing in the direction of the current. e are told that the magnitude presumed constant of the velocity of the boat relative to the ater is $\vec{v}_{bw}$ = 6.4 km h. ts angle relative to the x a is is θ. ith km and h as the understood units the velocity of the ater relative to the ground is $\vec{v}_{wg}$ = 3.2$\hat{}$.

(a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{v}_{bg} = v_{bg}\hat{\text{i}}$ here $v > 0$ is unkno n. hus all $\hat{}$ components must cancel in the vector sum

$$\vec{v}_{bw} \quad \vec{v}_{wg} = \vec{v}_{bg}$$

hich means the $u \sin \theta = -3.2$ so $\theta = \sin^{-1}$ -3.2 6.4 $= -30$.

b sing the result from part a e find $v_{bg} = v_{bw} \cos\theta$ = 5.5 km h. hus traveling a distance of ℓ = 6.4 km re uires a time of 6.4 5.5 = 1.15 h or 69 min.

c f her motion is completely along the y a is as the problem implies then ith v_{wg} = 3.2 km h the ater speed e have

$$t_{\text{total}} = \frac{D}{v_{bw} \quad v_{wg}} \quad \frac{D}{v_{bw} - v_{wg}} = 1.33 \text{ h}$$

here D = 3.2 km. his is e uivalent to 80 min.

d ince

$$\frac{D}{v_{bw} \quad v_{wg}} \quad \frac{D}{v_{bw} - v_{wg}} = \frac{D}{v_{bw} - v_{wg}} \quad \frac{D}{v_{bw} + v_{wg}}$$

the ans er is the same as in the previous part i.e. t_{total} = 80 min .

e he shortest time path should have θ = 0. his can also be sho n by noting that the case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} \quad \vec{v}_{wg} = v_{bw} \cos\theta \,\hat{\text{i}} \quad v_{bw} \sin\theta \quad v_{wg} \quad \hat{}$$

here the x component of $\vec{v}_{bg}$ must e ual l t. hus $t = \dfrac{l}{v_{bw}\cos\theta}$ hich can be minimi ed using the condition dt $d\theta$ = 0. he above e pression leads to t = 6.4 6.4 = 1.0 h or 60 min.

Chapter

1. e are only concerned ith hori ontal forces in this problem gravity plays no direct role . e take ast as the x direction and orth as y. his calculation is efficiently implemented on a vector capable calculator using magnitude angle notation ith units understood .

$$\vec{a} = \frac{\vec{F}}{m} = \frac{(9.0 \angle 0°) + (8.0 \angle 118°)}{3.0} = (2.9 \angle 53°)$$

herefore the acceleration has a magnitude of 2.9 m s^2.

2. We apply Newton's second law (Eq. 5 1 or e uivalently . 5 2 . he net force applied on the chopping block is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$ here the vector addition is done using unit vector notation. he acceleration of the block is given by $\vec{a} = (\vec{F}_1 + \vec{F}_2) \quad m$.

a n the first case

$$\vec{F}_1 + \vec{F}_2 = \left[(3.0 \quad)\hat{\text{i}} + (4.0 \quad)\hat{\ }\right] + \left[(-3.0 \quad)\hat{\text{i}} + (-4.0 \quad)\hat{\ }\right] = 0$$

so $\vec{a} = 0$.

b n the second case the acceleration $\vec{a}$ e uals

$$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0 \quad)\hat{\text{i}} + (4.0 \quad)\hat{\ }\right) + \left((-3.0 \quad)\hat{\text{i}} + (4.0 \quad)\hat{\ }\right)}{2.0\,\text{kg}} = \ 4.0\,\text{m s}^2\ \hat{\ }.$$

c n this final situation $\vec{a}$ is

$$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0 \quad)\hat{\text{i}} + (4.0 \quad)\hat{\ }\right) + \left((3.0 \quad)\hat{\text{i}} + (-4.0 \quad)\hat{\ }\right)}{2.0\,\text{kg}} = \ 3.0\,\text{m s}^2\ \hat{\text{i}}.$$

3. We apply Newton's second law (specifically, Eq. 5 2 .

a e find the x component of the force is

$$F_x = ma_x = ma \cos 20.0° = (1.00\,\text{kg})\left(2.00\,\text{m s}^2\right) \cos 20.0° = 1.88 \quad .$$

b he y component of the force is

$$F_y = ma_y = ma \sin 20.0° = (1.0\,\text{kg})\left(2.00\,\text{m s}^2\right) \sin 20.0° = 0.684 \quad .$$

c n unit vector notation the force vector is

$$\vec{F} = F_x \hat{\imath} + F_y \hat{} = 1.88 \quad \hat{\text{i}} + 0.684 \quad \hat{} .$$

4. ince $\vec{v}$ = constant e have $\vec{a}$ = 0 hich implies

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = m\vec{a} = 0 .$$

hus the other force must be

$$\vec{F}_2 = -\vec{F}_1 = -2 \quad \hat{\text{i}} + 6 \quad \hat{} .$$

5. he net force applied on the chopping block is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ here the vector addition is done using unit vector notation. he acceleration of the block is given by $\vec{a} = \left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right) \; m.$

a he forces e erted by the three astronauts can be e pressed in unit vector notation as follo s

$$\begin{aligned}
\vec{F}_1 &= 32 \quad \left(\cos 30°\hat{\text{i}} + \sin 30°\hat{}\right) = 27.7 \quad \hat{\text{i}} + 16 \quad \hat{} \\
\vec{F}_2 &= 55 \quad \left(\cos 0°\hat{\text{i}} + \sin 0°\hat{}\right) = 55 \quad \hat{\text{i}} \\
\vec{F}_3 &= 41 \quad \left(\cos(-60°)\hat{\text{i}} + \sin(-60°)\hat{}\right) = 20.5 \quad \hat{\text{i}} - 35.5 \quad \hat{} .
\end{aligned}$$

he resultant acceleration of the asteroid of mass m = 120 kg is therefore

$$\vec{a} = \frac{\left(27.7\hat{\text{i}} + 16\hat{}\right) \quad + \left(55\hat{\text{i}}\right) \quad + \left(20.5\hat{\text{i}} - 35.5\hat{}\right)}{120\,\text{kg}} = 0.86\,\text{m s}^2 \;\hat{\text{i}} - 0.16\,\text{m s}^2 \;\hat{} .$$

b he magnitude of the acceleration vector is

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{0.86\,\text{m s}^2 \;\; ^2 + \left(-0.16\,\text{m s}^2\right)^2} = 0.88\,\text{m s}^2 .$$

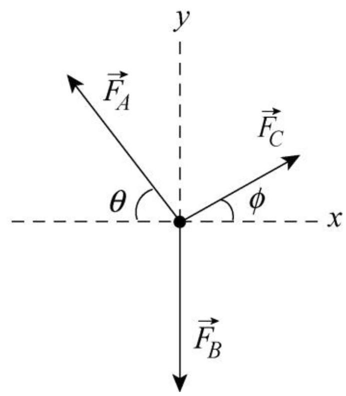
c he vector $\vec{a}$ makes an angle θ ith the x a is here

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-0.16\,\text{m s}^2}{0.86\,\text{m s}^2}\right) = -11° .$$

6. Since the tire remains stationary, by Newton's second law, the net force must be zero:

$$\vec{F}_{\text{net}} = \vec{F}_A + \vec{F}_B + \vec{F}_C = m\vec{a} = 0.$$

rom the free body diagram sho n on the right e have



$$0 = \sum F_{\text{net }x} = F_C \cos\phi - F_A \cos\theta$$
$$0 = \sum F_{\text{net }y} = F_A \sin\theta + F_C \sin\phi - F_B$$

o solve for F_B e first compute ϕ. ith $F_A = 220$ $F_C = 170$ and $\theta = 47°$ e get

$$\cos\phi = \frac{F_A \cos\theta}{F_C} = \frac{220 \quad \cos 47.0°}{170} = 0.883 \quad \Rightarrow \quad \phi = 28.0°$$

ubstituting the value into the second force e uation e find

$$F_B = F_A \sin\theta + F_C \sin\phi = \ 220 \quad \sin 47.0° + \ 170 \quad \sin 28.0 = 241 \quad .$$

7. **THINK** A box is under acceleration by two applied forces. We use Newton's second la to solve for the unkno n second force.

EXPRESS e denote the t o forces as $\vec{F}_1$ and $\vec{F}_2$. ccording to Newton's second law, $\vec{F}_1 + \vec{F}_2 = m\vec{a}$ so the second force is $\vec{F}_2 = m\vec{a} - \vec{F}_1$. ote that since the acceleration is in the third uadrant e e pect $\vec{F}_2$ to be in the third uadrant as ell.

ANALYZE a n unit vector notation $\vec{F}_1 = (20.0 \quad)\hat{\text{i}}$ and

$$\vec{a} = -\left(12.0 \sin 30.0° \text{m s}^2\right)\hat{\text{i}} - \left(12.0 \cos 30.0° \text{m s}^2\right)\hat{} = -\left(6.00 \text{ m s}^2\right)\hat{\text{i}} - \left(10.4 \text{ m s}^2\right)\hat{}.$$

herefore e find the second force to be

$$\begin{aligned}\vec{F}_2 &= m\vec{a} - \vec{F}_1 \\ &= (2.00 \text{ kg})\left(-6.00 \text{ m s}^2\right)\hat{\text{i}} + (2.00 \text{ kg})\left(-10.4 \text{ m s}^2\right)\hat{} - (20.0 \quad)\hat{\text{i}} \\ &= (-32.0 \quad)\hat{\text{i}} - (20.8 \quad)\hat{}.\end{aligned}$$

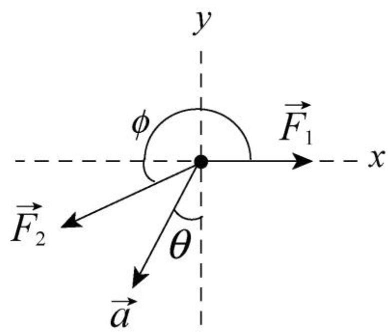
b he magnitude of $\vec{F}_2$ is $\vec{F}_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = \sqrt{\ -32.0 \quad ^2 + \ -20.8 \quad ^2} = 38.2 \quad .$

c he angle that $\vec{F}_2$ makes ith the positive x a is is found from

$$\tan\phi = \left(\frac{F_{2y}}{F_{2x}}\right) = \frac{-20.8}{-32.0} = 0.656.$$

onse uently the angle is either 33.0 or 33.0 180 = 213 . ince both the x and y components are negative the correct result is ϕ = 213 from the x a is. n alternative ans er is $213° - 360° = -147°$.

LEARN he result is sho n in the figure on the right. he calculation confirms our e pectation that $\vec{F}_2$ lies in the third uadrant same as $\vec{a}$. he net force is



$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = (20.0\quad)\hat{\text{i}} + \left[(-32.0\quad)\hat{\text{i}} - (20.8\quad)\hat{\ }\right]$$
$$= (-12.0\quad)\hat{\text{i}} - (20.8\quad)\hat{\ }$$

hich points in the same direction as $\vec{a}$.

8. e note that $m\vec{a}$ = –16 i 12 . ith the other forces as specified in the problem, then Newton's second law gives the third force as

$$\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2 = -34 \quad \text{i} - 12 \quad .$$

9. o solve the problem e note that acceleration is the second time derivative of the position function it is a vector and can be determined from its components. he net force is related to the acceleration via Newton's second law. Thus, differentiating $x\ t\ = -15.0 + 2.00t + 4.00t^3$ t ice ith respect to t e get

$$\frac{dx}{dt} = 2.00 - 12.0t^2 \qquad \frac{d^2x}{dt^2} = -24.0t$$

imilarly differentiating $y\ t\ = 25.0 + 7.00t - 9.00t^2$ t ice ith respect to t yields

$$\frac{dy}{dt} = 7.00 - 18.0t \qquad \frac{d^2y}{dt^2} = -18.0$$

a he acceleration is

$$\vec{a} = a_x\hat{\text{i}} + a_y\hat{\ } = \frac{d^2x}{dt^2}\hat{\text{i}} + \frac{d^2y}{dt^2}\hat{\ } = \ -24.0t\ \hat{\text{i}} + \ -18.0\ \hat{\ }.$$

t $t = 0.700$ s e have $\vec{a} = \ -16.8\ \hat{\text{i}} + \ -18.0\ \hat{\ }$ ith a magnitude of

$$a = \ \vec{a}\ = \sqrt{\ -16.8\ ^2 + \ -18.0\ ^2} = 24.6 \text{ m s}^2.$$

hus the magnitude of the force is $F = ma =$ 0.34 kg 24.6 m s^2 $= 8.37$.

b he angle $\vec{F}$ or $\vec{a} = \vec{F}$ m makes ith $+x$ is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-18.0 \text{ m s}^2}{-16.8 \text{ m s}^2}\right) = 47.0° \text{ or } -133°.$$

e choose the latter –133° since $\vec{F}$ is in the third uadrant.

c he direction of travel is the direction of a tangent to the path hich is the direction of the velocity vector

$$\vec{v}\ t \ = v_x\hat{\text{i}} + v_y\hat{\ } = \frac{dx}{dt}\hat{\text{i}} + \frac{dy}{dt}\hat{\ } = \ 2.00 - 12.0t^2\ \hat{\text{i}} + \ 7.00 - 18.0t\ \hat{\ }.$$

t $t = 0.700$ s e have $\vec{v}\ t = 0.700 \text{ s}\ =\ -3.88$ m s $\hat{\text{i}}$ + –5.60 m s $\hat{\ }$. herefore the angle $\vec{v}$ makes ith $+x$ is

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-5.60 \text{ m s}}{-3.88 \text{ m s}}\right) = 55.3° \text{ or } -125°.$$

e choose the latter –125° since $\vec{v}$ is in the third uadrant.

10. o solve the problem e note that acceleration is the second time derivative of the position function, and the net force is related to the acceleration via Newton's second la . hus differentiating

$$x\ t\ = -13.00 + 2.00t + 4.00t^2 - 3.00t^3$$

t ice ith respect to t e get

$$\frac{dx}{dt} = 2.00 + 8.00t - 9.00t^2 \qquad \frac{d^2x}{dt^2} = 8.00 - 18.0t$$

he net force acting on the particle at $t = 3.40$ s is

$$\vec{F} = m\frac{d^2x}{dt^2}\hat{\text{i}} = \ 0.150\ \left[8.00 - 18.0\ 3.40\ \right]\hat{\text{i}} = \ -7.98 \quad \hat{\text{i}}$$

11. he velocity is the derivative ith respect to time of given function x and the acceleration is the derivative of the velocity. hus a = 2c – 3 2.0 2.0 t hich e use in Newton's second law: F = 2.0 kg a = 4.0c – 24t ith units understood . t t = 3.0 s e are told that F = –36 . hus –36 = 4.0c – 24 3.0 can be used to solve for c. he result is c = 9.0 m s^2.

12. rom the slope of the graph e find a_x = 3.0 m s^2. Applying Newton's second law to the x a is and taking θ to be the angle bet een F_1 and F_2 e have

$$F_1 \quad F_2\cos\theta = ma_x \quad \Rightarrow \quad \theta = 56°.$$

13. a rom the fact that T_3 = 9.8 e conclude the mass of disk D is 1.0 kg. oth this and that of disk C cause the tension T_2 = 49 hich allo s us to conclude that disk C has a mass of 4.0 kg. he eights of these t o disks plus that of disk B determine the tension T_1 = 58.8 hich leads to the conclusion that m_B = 1.0 kg. he eights of all the disks must add to the 98 force described in the problem therefore disk A has mass 4.0 kg.

b m_B = 1.0 kg as found in part a .

c m_C = 4.0 kg as found in part a .

d m_D = 1.0 kg as found in part a .

14. hree vertical forces are acting on the block the earth pulls do n on the block ith gravitational force 3.0 a spring pulls up on the block ith elastic force 1.0 and the surface pushes up on the block ith normal force F_N. here is no acceleration so

$$\sum F_y = 0 = F_N + (1.0 \quad) + (-3.0 \quad)$$

yields F_N = 2.0 .

a By Newton's third law, the force exerted by the block on the surface has that same magnitude but opposite direction 2.0 .

b he direction is do n.

15. **THINK** e have a piece of salami hung to a spring scale in various ays. he problem is to e plore the concept of eight.

EXPRESS e first note that the reading on the spring scale is proportional to the eight of the salami. n all three cases a – c depicted in ig. 5 34 the scale is not accelerating hich means that the t o cords e ert forces of e ual magnitude on it. he scale reads the magnitude of either of these forces. n each case the tension force of the cord attached to the salami must be the same in magnitude as the eight of the salami because the salami is not accelerating. hus the scale reading is mg here m is the mass of the salami.

ANALYZE n all three cases a – c the reading on the scale is

$$w = mg = \quad 11.0 \text{ kg} \quad 9.8 \text{ m s}^2 \; = 108 \quad .$$

LEARN he eight of an ob ect is measured hen the ob ect is not accelerating vertically relative to the ground. f it is then the eight measured is called the apparent eight.

16. a here are si legs and the vertical component of the tension force in each leg is $T\sin\theta$ here $\theta = 40°$. or vertical e uilibrium ero acceleration in the y direction then Newton's second law leads to

$$6T\sin\theta = mg \Rightarrow T = \frac{mg}{6\sin\theta}$$

which (expressed as a multiple of the bug's weight mg gives roughly $T \quad mg \approx 0.26\,0$.

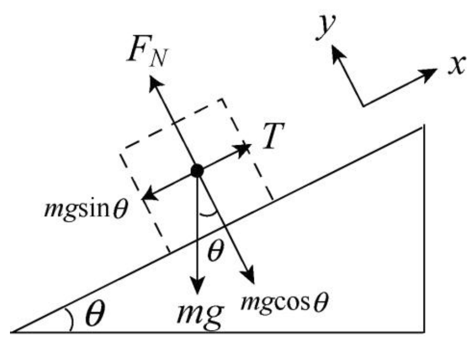
b he angle θ is measured from horizontal, so as the insect "straightens out the legs" θ ill increase getting closer to 90° hich causes $\sin\theta$ to increase getting closer to 1 and conse uently since $\sin\theta$ is in the denominator causes T to decrease.

17. **THINK** A block attached to a cord is resting on an incline plane. We apply Newton's second la to solve for the tension in the cord and the normal force on the block.

EXPRESS he free body diagram of the problem is sho n to the right. ince the acceleration of the block is zero, the components of Newton's second la e uation yield

$$T - mg\sin\theta = 0$$
$$F_N - mg\cos\theta = 0$$

here T is the tension in the cord and F_N is the normal force on the block.



ANALYZE a olving the first e uation for the tension in the string e find

$$T = mg\sin\theta = (8.5\text{ kg})(9.8\text{ m} \quad \text{s}^2)\sin 30° = 42 \quad .$$

b e solve the second e uation above for the normal force F_N

$$F_N = mg\cos\theta = (8.5\text{ kg})(9.8\text{ m s}^2)\cos 30° = 72 \quad .$$

c hen the cord is cut it no longer e erts a force on the block and the block accelerates. he x component of the second la becomes $-mg\sin\theta = ma$ so the acceleration becomes

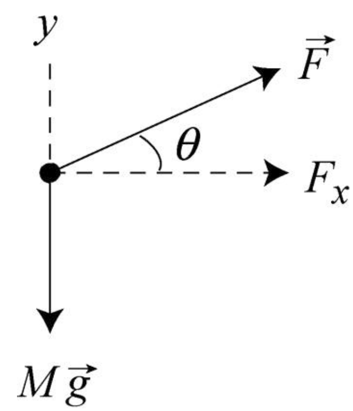
$$a = -g\sin\theta = - \; 9.8\text{ m s}^2 \;\; \sin 30° = -4.9\text{ m s}^2.$$

he negative sign indicates the acceleration is do n the plane. he magnitude of the acceleration is 4.9 m s^2.

LEARN he normal force F_N on the block must be e ual to $mg\cos\theta$ so that the block is in contact ith the surface of the incline at all time. hen the cord is cut the block has an acceleration $a=-g\sin\theta$ hich in the limit $\theta\to 90°$ becomes $-g$ as in the case of a free fall.

18. he free body diagram of the cars is sho n on the right. he force e erted by ohn assis is

$$F = 2.5mg = 2.5\ 80\ \text{kg}\ \ 9.8\ \text{m s}^2\ = 1960\ \ .$$



ince the motion is along the hori ontal x axis, using Newton's second la e have $Fx = F\cos\theta = Ma_x$ here M is the total mass of the railroad cars. hus the acceleration of the cars is

$$a_x = \frac{F\cos\theta}{M} = \frac{1960\ \ \ \cos 30°}{7.0\times 10^5\ \ \ 9.8\ \text{m s}^2} = 0.024\ \text{m s}^2.$$

sing . 2 16 the speed of the car at the end of the pull is

$$v_x = \sqrt{2a_x\Delta x} = \sqrt{2\ 0.024\ \text{m s}^2\ \ 1.0\ \text{m}\ } = 0.22\ \text{m s}.$$

19. **THINK** In this problem we're interested in the force applied to a rocket sled to accelerate it from rest to a given speed in a given time interval.

EXPRESS In terms of magnitudes, Newton's second law is $F = ma$ here $F = |\vec{F}_{\text{net}}|$ $a = \vec{a}$ and m is the al ays positive mass. he magnitude of the acceleration can be found using constant acceleration kinematics able 2 1 . olving $v = v_0\ \ at$ for the case here it starts from rest e have $a = v\ t$ hich e interpret in terms of magnitudes making specification of coordinate directions unnecessary . hus the re uired force is $F = ma = mv\ \ t$.

ANALYZE pressing the velocity in units as

$$v = \ 1600\ \text{km h}\ \ \ 1000\ \text{m km}\ \ \ 3600\ \text{s h}\ = 444\ \text{m s}$$

e find the force to be

$$F = m\frac{v}{t} = (500\,\text{kg})\,\frac{444\,\text{m/s}}{1.8\text{s}} = 1.2\times 10^5\ \ .$$

LEARN rom the e pression $F = mv\ t$ e see that the shorter the time to attain a given speed the greater the force re uired.

20. he stopping force $\vec{F}$ and the path of the passenger are hori ontal. ur x a is is in the direction of the passenger's motion, so that the passenger's acceleration (''deceleration'') is negative valued and the stopping force is in the $-x$ direction $\vec{F} = -F\,\hat{\imath}$. sing . 2 16 ith

$$v_0 = \ 53 \text{ km h} \ \ 1000 \text{ m km} \ \ 3600 \text{ s h} \ = 14.7 \text{ m s}$$

and $v = 0$ the acceleration is found to be

$$v^2 = v_0^2 + 2a\Delta x \ \Rightarrow \ a = -\frac{v_0^2}{2\Delta x} = -\frac{14.7 \text{ m s}\ ^2}{2(0.65 \text{ m})} = -167 \text{ m s}^2.$$

ssuming there are no significant hori ontal forces other than the stopping force . 5 1 leads to

$$\vec{F} = m\vec{a} \ \Rightarrow \ -F = (41 \text{ kg})\left(-167 \text{ m/s}^2\right)$$

hich results in $F = 6.8 \times 10^3$.

21. a he slope of each graph gives the corresponding component of acceleration. hus e find a_x = 3.00 m s^2 and a_y = –5.00 m s^2. he magnitude of the acceleration vector is therefore

$$a = \sqrt{\ 3.00 \text{ m s}^2\ ^2 + \ -5.00 \text{ m s}^2\ ^2} = 5.83 \text{ m s}^2$$

and the force is obtained from this by multiplying ith the mass m = 2.00 kg . he result is $F = ma$ =11.7 .

b he direction of the force is the same as that of the acceleration

$$\theta = \tan^{-1} \ \ -5.00 \text{ m s}^2 \ \ 3.00 \text{ m s}^2 \ \ = -59.0°.$$

22. a he coin undergoes free fall. herefore ith respect to ground its acceleration is

$$\vec{a}_{\text{coin}} = \vec{g} = \ -9.8 \text{ m s}^2 \ \hat{\ }.$$

b ince the customer is being pulled do n ith an acceleration of $\vec{a}\,'_{\text{customer}} = 1.24\vec{g} = \ -12.15 \text{ m s}^2 \ \hat{\ }$ the acceleration of the coin ith respect to the customer is

$$\vec{a}_{\text{rel}} = \vec{a}_{\text{coin}} - \vec{a}\,'_{\text{customer}} = \ -9.8 \text{ m s}^2 \ \hat{\ } - \ -12.15 \text{ m s}^2 \ \hat{\ } = \ +2.35 \text{ m s}^2 \ \hat{\ }.$$

c he time it takes for the coin to reach the ceiling is

$$t = \sqrt{\frac{2h}{a_{\text{rel}}}} = \sqrt{\frac{2\ 2.20\text{ m}}{2.35\text{ m s}^2}} = 1.37\text{ s.}$$

d ince gravity is the only force acting on the coin the actual force on the coin is

$$\vec{F}_{\text{coin}} = m\vec{a}_{\text{coin}} = m\vec{g} = \ 0.567\times10^{-3}\text{ kg}\ \ -9.8\text{ m s}^2\ \hat{} = \ -5.56\times10^{-3}\ \ \hat{}.$$

(e) In the customer's frame, the coin travels up ard at a constant acceleration. herefore the apparent force on the coin is

$$\vec{F}_{\text{app}} = m\vec{a}_{\text{rel}} = \ 0.567\times10^{-3}\text{ kg}\ \ +2.35\text{ m s}^2\ \hat{} = \ +1.33\times10^{-3}\ \ \hat{}.$$

23. e note that the rope is 22.0 from vertical and therefore 68.0 from hori ontal.

a ith T = 760 then its components are

$$\vec{T} = T\cos 68.0°\ \hat{\text{i}}\ \ T\sin 68.0°\ \hat{} = \ 285\ \ \hat{\text{i}}\ \ 705\ \ \hat{}.$$

(b) No longer in contact with the cliff, the only other force on Tarzan is due to earth's gravity his eight . hus

$$\vec{F}_{\text{net}} = \vec{T} + \vec{W} = \ 285\ \ \hat{\text{i}}\ \ 705\ \ \hat{} - \ 820\ \ \hat{} = \ 285\ \ \hat{\text{i}} - \ 115\ \ \hat{}.$$

c n a manner that is efficiently implemented on a vector capable calculator e convert from rectangular x y components to magnitude angle notation

$$\vec{F}_{\text{net}} = (285\ \ -115) \to (307\angle -22.0°)$$

so that the net force has a magnitude of 307 .

d he angle see part c has been found to be –22.0 or 22.0 belo hori ontal a ay from the cliff .

e ince $\vec{a} = \vec{F}_{\text{net}}/m$ here $m = W\ g$ = 83.7 kg e obtain $a = 3.67\ \text{m/s}^2$.

f . 5 1 re uires that $\vec{a} \parallel \vec{F}_{\text{net}}$ so that the angle is also –22.0 or 22.0 belo hori ontal a ay from the cliff .

24. e take right ard as the x direction. hus $\vec{F}_1 = \ 20\ \ \hat{\text{i}}$. n each case e use Newton's second law $\vec{F}_1 + \vec{F}_2 = m\vec{a}$ here m = 2.0 kg.

a f $\vec{a} = +10$ m s^2 î then the e uation above gives $\vec{F}_2 = 0$.

b f $\vec{a} = +20$ m s^2 î then that e uation gives $\vec{F}_2 = 20$ î.

c f $\vec{a} = 0$ then the e uation gives $\vec{F}_2 = -20$ î.

d f $\vec{a} = -10$ m s^2 î the e uation gives $\vec{F}_2 = -40$ î.

e f $\vec{a} = -20$ m s^2 î the e uation gives $\vec{F}_2 = -60$ î.

25. a he acceleration is

$$a = \frac{F}{m} = \frac{20}{900\,\text{kg}} = 0.022 \text{ m/s}^2 .$$

b he distance traveled in 1 day = 86400 s is

$$s = \frac{1}{2}at^2 = \frac{1}{2}\left(0.0222\,\text{m/s}^2\right)\left(86400\,\text{s}\right)^2 = 8.3 \times 10^7 \text{ m} .$$

c he speed it ill be traveling is given by

$$v = at = \left(0.0222 \text{ m/s}^2\right)\left(86400 \text{ s}\right) = 1.9 \times 10^3 \text{ m/s} .$$

26. ome assumptions not so much for realism but rather in the interest of using the given information efficiently are needed in this calculation e assume the fishing line and the path of the salmon are hori ontal. hus the eight of the fish contributes only via . 5 12 to information about its mass $m = W/g = 8.7$ kg . ur x a is is in the direction of the salmon's velocity (away from the fisherman), so that its acceleration (''deceleration'') is negative valued and the force of tension is in the $-x$ direction $\vec{T} = -T$. e use . 2 16 and units noting that $v = 0$.

$$v^2 = v_0^2 + 2a\Delta x \;\Rightarrow\; a = -\frac{v_0^2}{2\Delta x} = -\frac{2.8 \text{ m s}^2}{2\left(0.11\text{ m}\right)} = -36 \text{ m s}^2 .$$

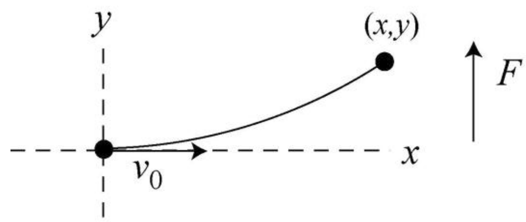
ssuming there are no significant hori ontal forces other than the tension . 5 1 leads to

$$\vec{T} = m\vec{a} \;\Rightarrow\; -T = \left(8.7\,\text{kg}\right)\left(-36\,\text{m/s}^2\right)$$

hich results in $T = 3.1 \times 10^2$.

27. **THINK** n electron moving hori ontally is under the influence of a vertical force. ts path ill be deflected to ard the direction of the applied force.

EXPRESS he setup is sho n in the figure belo . he acceleration of the electron is vertical and for all practical purposes the only force acting on it is the electric force. he force of gravity is negligible. e take the x a is to be in the direction of the initial velocity v_0 and the y a is to be in the direction of the electrical force and place the origin at the initial position of the electron.



ince the force and acceleration are constant e use the e uations from able 2 1 $x = v_0 t$ and

$$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)t^2.$$

ANALYZE he time taken by the electron to travel a distance x = 30 mm hori ontally is $t = x\ v_0$ and its deflection in the direction of the force is

$$y = \frac{1}{2}\frac{F}{m}\left(\frac{x}{v_0}\right)^2 = \frac{1}{2}\left(\frac{4.5\times10^{-16}}{9.11\times10^{-31}\ \text{kg}}\right)\left(\frac{30\times10^{-3}\ \text{m}}{1.2\times10^{7}\ \text{m s}}\right)^2 = 1.5\times10^{-3}\ \text{m}.$$

LEARN ince the applied force is constant the acceleration in the y direction is also constant and the path is parabolic ith $y \propto x^2$.

28. he stopping force $\vec{F}$ and the path of the car are hori ontal. hus the eight of the car contributes only via . 5 12 to information about its mass $m = W\ g$ = 1327 kg . ur x a is is in the direction of the car's velocity, so that its acceleration ("deceleration") is negative valued and the stopping force is in the $-x$ direction $\vec{F} = -F\,\hat{\imath}$.

a e use . 2 16 and units noting that v = 0 and v_0 = 40 1000 3600 = 11.1 m s .

$$v^2 = v_0^2 + 2a\Delta x \quad\Rightarrow\quad a = -\frac{v_0^2}{2\Delta x} = -\frac{11.1\,\text{m s}^{\ 2}}{2(15\ \text{m})}$$

hich yields a = – 4.12 m s^2. ssuming there are no significant hori ontal forces other than the stopping force . 5 1 leads to

$$\vec{F} = m\vec{a} \quad \Rightarrow \quad -F = (1327\ \text{kg})\left(-4.12\ \text{m/s}^2\right)$$

hich results in $F = 5.5 \times 10^3$.

b uation 2 11 readily yields $t = -v_0\ a$ = 2.7 s.

c eeping F the same means keeping a the same in hich case since $v = 0$. 2 16 e presses a direct proportionality bet een Δx and v_0^2. herefore doubling v_0 means uadrupling Δx. hat is the ne over the old stopping distances is a factor of 4.0.

d uation 2 11 illustrates a direct proportionality bet een t and v_0 so that doubling one means doubling the other. hat is the ne time of stopping is a factor of 2.0 greater than the one found in part b .

29. e choose up as the y direction so $\vec{a} = -3.00\ \text{m s}^2$ ˆ hich ithout the unit vector e denote as a since this is a 1 dimensional problem in hich able 2 1 applies . rom . 5 12, we obtain the firefighter's mass: $m = W\ g$ = 72.7 kg.

a e denote the force e erted by the pole on the firefighter $\vec{F}_{fp} = F_{fp}$ ˆ and apply . 5 1. ince $\vec{F}_{net} = m\vec{a}$ e have

$$F_{fp} - F_g = ma \quad \Rightarrow \quad F_{fp} - 712 \quad = \ 72.7\ \text{kg} \quad -3.00\ \text{m s}^2$$

hich yields F_{fp} = 494 .

b he fact that the result is positive means $\vec{F}_{fp}$ points up.

(c) Newton's third law indicates $\vec{F}_{fp} = -\vec{F}_{pf}$ hich leads to the conclusion that $\vec{F}_{pf} = 494$.

d he direction of $\vec{F}_{pf}$ is do n.

30. he stopping force $\vec{F}$ and the path of the toothpick are hori ontal. ur x a is is in the direction of the toothpick's motion, so that the toothpick's acceleration (''deceleration'') is negative valued and the stopping force is in the $-x$ direction $\vec{F} = -F\,\hat{\imath}$. sing . 2 16 ith v_0 = 220 m s and $v = 0$ the acceleration is found to be

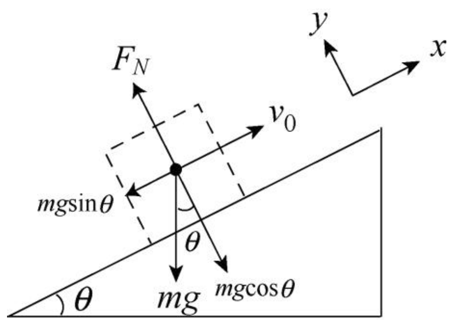
$$v^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad a = -\frac{v_0^2}{2\Delta x} = -\frac{220\ \text{m s}\ ^2}{2(0.015\ \text{m})} = -1.61\times10^6\ \text{m s}^2.$$

hus the magnitude of the force e erted by the branch on the toothpick is

$$F = m\ \ a = 1.3\times10^{-4}\ \text{kg}\ \ 1.61\times10^{6}\ \text{m s}^{2}\ = 2.1\times10^{2}\quad .$$

31. **THINK** n this problem e analy e the motion of a block sliding up an inclined plane and back do n.

EXPRESS he free body diagram is sho n belo . $\vec{F}_N$ is the normal force of the plane on the block and $m\vec{g}$ is the force of gravity on the block. e take the x direction to be up the incline and the y direction to be in the direction of the normal force e erted by the incline on the block.



he x component of Newton's second law is then $mg\sin\theta = -ma$ thus the acceleration is $a = -g\sin\theta$. lacing the origin at the bottom of the plane the kinematic e uations able 2 1 for motion along the x a is hich e ill use are $v^2 = v_0^2 + 2ax$ and $v = v_0 + at$. he block momentarily stops at its highest point here $v = 0$ according to the second e uation this occurs at time $t = -v_0/a$.

ANALYZE a he position here the block stops is

$$x = v_0 t + \frac{1}{2}at^2 = v_0\left(\frac{-v_0}{a}\right) + \frac{1}{2}a\left(\frac{-v_0}{a}\right)^2 = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\left(\frac{3.50\ \text{m s}\ ^2}{-\left(9.8\ \text{m s}^2\right)\sin 32.0^\circ}\right) = 1.18\ \text{m}.$$

b he time it takes for the block to get there is

$$t = \frac{v_0}{a} = -\frac{v_0}{-g\sin\theta} = -\frac{3.50\text{m s}}{-\ 9.8\text{m s}^2\ \ \sin 32.0^\circ} = 0.674\ \text{s}.$$

c hat the return speed is identical to the initial speed is to be e pected since there are no dissipative forces in this problem. n order to prove this one approach is to set $x = 0$ and solve $x = v_0 t + \frac{1}{2}at^2$ for the total time up and back do n t. he result is

$$t=-\frac{2v_0}{a}=-\frac{2v_0}{-g\sin\theta}=-\frac{2(3.50\text{ m s})}{-9.8\text{ m s}^2\ \sin 32.0°}=1.35\text{ s}.$$

he velocity hen it returns is therefore

$$v=v_0+at=v_0-gt\sin\theta=3.50\text{ m s}-\ 9.8\text{ m s}^2\ (1.35\text{ s})\sin 32°=-3.50\text{ m s}.$$

he negative sign indicates the direction is do n the plane.

LEARN s e pected the speed of the block hen it gets back to the bottom of the incline is the same as its initial speed. s e shall see in hapter 8 this is a conse uence of energy conservation. f friction is present then the return speed ill be smaller than the initial speed.

32. a sing notation suitable to a vector capable calculator the $\vec{F}_{\text{net}}$ = 0 condition becomes

$$\vec{F}_1\quad \vec{F}_2\quad \vec{F}_3 = \quad 6.00\angle 150 \qquad 7.00\angle -60.0 \qquad \vec{F}_3 = 0 .$$

hus $\vec{F}_3$ = 1.70 i 3.06 .

b constant velocity condition re uires ero acceleration so the ans er is the same.

c o the acceleration is

$$\vec{a}=\ 13.0\text{ m s}^2\ \hat{\text{i}}-\ 14.0\text{ m s}^2\ \hat{}.$$

sing $\vec{F}_{\text{net}}$ = $m\ \vec{a}$ ith m = 0.025 kg e no obtain

$$\vec{F}_3 = 2.02 \quad \text{i} \quad 2.71 \quad .$$

33. he free body diagram is sho n belo . et $\vec{T}$ be the tension of the cable and $m\vec{g}$ be the force of gravity. f the up ard direction is positive then e ton's second law is $T - mg = ma$ here a is the acceleration.

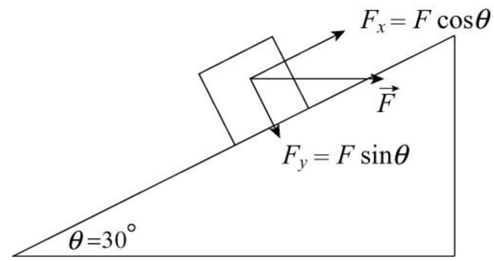
hus the tension is $T = m\ g \quad a$. e use constant acceleration kinematics able 2 1 to find the acceleration here v = 0 is the final velocity v_0 = – 12 m s is the initial velocity and $y = -42$ m is the coordinate at the stopping point . onse uently $v^2 = v_0^2 + 2ay$ leads to

$$a=-\frac{v_0^2}{2y}=-\frac{(-12\text{ m s})^2}{2(-42\text{ m})}=1.71\text{ m s}^2.$$

e no return to calculate the tension

$$T = m(g + a)$$
$$= (1600\text{ kg})\left(9.8\text{ m}\ \ \text{s}^2 + 1.71\text{ m}\ \ \text{s}^2\right)$$
$$= 1.8 \times 10^4\quad .$$

34. e resolve this hori ontal force into appropriate components.



(a) Newton's second law applied to the x a is produces

$$F\cos\theta - mg\sin\theta = ma.$$

or a = 0 this yields F = 566 .

(b) Applying Newton's second law to the y a is here there is no acceleration e have

$$F_N - F\sin\theta - mg\cos\theta = 0$$

hich yields the normal force F_N = 1.13 × 10^3 .

35. he acceleration vector as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(8.00t\,\hat{\mathrm{i}} + 3.00t^2\ \hat{}\right)\text{m s} = \ 8.00\,\hat{\mathrm{i}} + 6.00t\ \hat{}\ \ \text{m s}^2.$$

a he magnitude of the force acting on the particle is

$$F = ma = m\ \ \vec{a}\ \ =\ 3.00\ \sqrt{8.00\ ^2 +\ 6.00t\ ^2} =\ 3.00\ \sqrt{64.0 + 36.0\,t^2}\quad .$$

hus $F = 35.0$ corresponds to $t = 1.415\text{ s}$ and the acceleration vector at this instant is

$$\vec{a} =\ 8.00\,\hat{\mathrm{i}} + 6.00\ 1.415\ \ \hat{}\ \ \text{m s}^2 =\ 8.00\text{ m s}^2\ \ \hat{\mathrm{i}} +\ 8.49\text{ m s}^2\ \ \hat{}.$$

he angle $\vec{a}$ makes ith x is

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{8.49\text{ m s}^2}{8.00\text{ m s}^2}\right) = 46.7°.$$

b he velocity vector at $t = 1.415\text{ s}$ is

$$\vec{v} = \left[8.00\ 1.415\ \ \hat{\text{i}} + 3.00\ 1.415\ ^{2}\ \hat{} \right] \text{m s} = \ 11.3\,\text{m s}\ \ \hat{\text{i}} + \ 6.01\,\text{m s}\ \hat{}.$$

herefore the angle $\vec{v}$ makes ith x is

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{6.01\,\text{m s}}{11.3\,\text{m s}}\right) = 28.0°.$$

36. (a) Constant velocity implies zero acceleration, so the "uphill" force must equal (in magnitude) the "downhill" force: $T = mg \sin \theta$. hus ith m = 50 kg and $\theta = 8.0°$ the tension in the rope e uals 68 .

b ith an uphill acceleration of 0.10 m s^2, Newton's second law (applied to the x a is yields

$$T - mg \sin \theta = ma \ \ \Rightarrow \ \ T - (50\,\text{kg})(9.8\,\text{m s}^2) \sin 8.0° = (50\,\text{kg})(0.10\,\text{m s}^2)$$

hich leads to T = 73 .

37. a ince friction is negligible the force of the girl is the only hori ontal force on the sled. he vertical forces the force of gravity and the normal force of the ice sum to ero. he acceleration of the sled is

$$a_s = \frac{F}{m_s} = \frac{5.2}{8.4\,\text{kg}} = 0.62\ \text{m/s}^2\ .$$

(b) According to Newton's third la the force of the sled on the girl is also 5.2 . er acceleration is

$$a_g = \frac{F}{m_g} = \frac{5.2}{40\,\text{kg}} = 0.13\ \text{m/s}^2\ .$$

c he accelerations of the sled and girl are in opposite directions. ssuming the girl starts at the origin and moves in the x direction her coordinate is given by $x_g = \frac{1}{2} a_g t^2$. he sled starts at x_0 = 15 m and moves in the $-x$ direction. ts coordinate is given by $x_s = x_0 - \frac{1}{2} a_s t^2$. hey meet hen $x_g = x_s$ or

$$\frac{1}{2} a_g t^2 = x_0 - \frac{1}{2} a_s t^2.$$

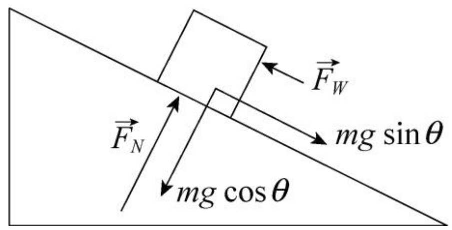
his occurs at time

$$t = \sqrt{\frac{2x_0}{a_g + a_s}}.$$

y then the girl has gone the distance

$$x_g = \frac{1}{2}a_g t^2 = \frac{x_0 a_g}{a_g + a_s} = \frac{(15\text{ m})(0.13\text{ m s}^2)}{0.13\text{ m s}^2 + 0.62\text{ m s}^2} = 2.6\text{ m}.$$

38. We label the 40 kg skier “m” which is represented as a block in the figure shown. he force of the ind is denoted $\vec{F}_w$ and might be either “uphill” or “downhill” (it is sho n uphill in our sketch . he incline angle θ is 10 . he $-x$ direction is do nhill.



(a) Constant velocity implies zero acceleration; thus, application of Newton’s second law along the x a is leads to $mg\sin\theta - F_w = 0$. his yields F_w = 68 uphill .

b iven our coordinate choice e have a = a = 1.0 m s^2. Newton’s second law

$$mg\sin\theta - F_w = ma$$

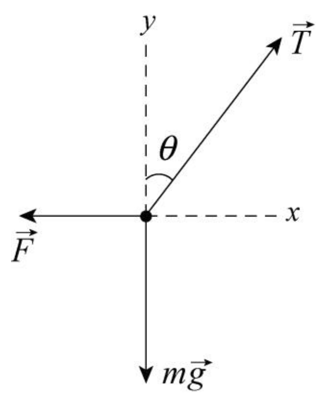
no leads to F_w = 28 uphill .

c ontinuing ith the forces as sho n in our figure the e uation

$$mg\sin\theta - F_w = ma$$

ill lead to F_w = – 12 hen a = 2.0 m s^2. his simply tells us that the ind is opposite to the direction sho n in our sketch in other ords $\vec{F}_w = 12$ *downhill*.

39. he solutions to parts a and b have been combined here. he free body diagram is sho n to the right ith the tension of the string $\vec{T}$ the force of gravity $m\vec{g}$ and the force of the air $\vec{F}$. ur coordinate system is sho n. ince the sphere is motionless the net force on it is ero and the x and the y components of the e uations are



$$T\sin\theta - F = 0$$
$$T\cos\theta - mg = 0$$

here θ = 37 . e ans er the uestions in the reverse order. olving $T\cos\theta - mg = 0$ for the tension e obtain

$$T = mg \cos\theta = (3.0 \times 10^{-4}\ \text{kg})(9.8\ \text{m s}^2)\cos 37 = 3.7 \times 10^{-3}\ .$$

olving $T \sin\theta - F = 0$ for the force of the air

$$F = T \sin\theta = (3.7 \times 10^{-3}\ )\sin 37 = 2.2 \times 10^{-3}\ .$$

40. he acceleration of an ob ect neither pushed nor pulled by any force other than gravity on a smooth inclined plane of angle θ is $a = -g \sin\theta$. he slope of the graph sho n ith the problem statement indicates $a = -2.50$ m s^2. herefore e find $\theta = 14.8°$. amining the forces perpendicular to the incline hich must sum to ero since there is no component of acceleration in this direction e find $F_N = mg\cos\theta$ here $m = 5.00$ kg. hus the normal perpendicular force e erted at the bo ramp interface is 47.4 .

41. he mass of the bundle is $m = 449 \quad 9.80$ m s^2 $= 45.8$ kg and e choose y up ard.

(a) Newton's second law, applied to the bundle, leads to

$$T - mg = ma \Rightarrow a = \frac{387 \quad - 449}{45.8\ \text{kg}}$$

hich yields $a = -1.4$ m s^2 or $a = 1.4$ m s^2 for the acceleration. he minus sign in the result indicates the acceleration vector points do n. ny do n ard acceleration of magnitude greater than this is also acceptable since that ould lead to even smaller values of tension .

b e use . 2 16 ith Δx replaced by $\Delta y = -6.1$ m . e assume $v_0 = 0$.

$$|v| = \sqrt{2a\Delta y} = \sqrt{2(-1.35\ \text{m s}^2)(-6.1\ \text{m})} = 4.1\ \text{m s}.$$

or do n ard accelerations greater than 1.4 m s^2 the speeds at impact ill be larger than 4.1 m s.

42. he direction of motion (the direction of the barge's acceleration) is $+\hat{\text{i}}$ and $+\vec{\ }$ is chosen so that the pull $\vec{F}_\text{h}$ from the horse is in the first uadrant. he components of the unkno n force of the ater are denoted simply F_x and F_y.

(a) Newton's second law applied to the barge, in the x and y directions leads to

$$(7900 \quad)\cos 18° + F_x = ma$$
$$(7900 \quad)\sin 18° + F_y = 0$$

respectively. lugging in a = 0.12 m s^2 and m = 9500 kg e obtain F_x = – 6.4 × 10^3 and F_y = – 2.4 × 10^3 . he magnitude of the force of the ater is therefore

$$F_{\text{ater}} = \sqrt{F_x^2 + F_y^2} = 6.8 \times 10^3 \quad .$$

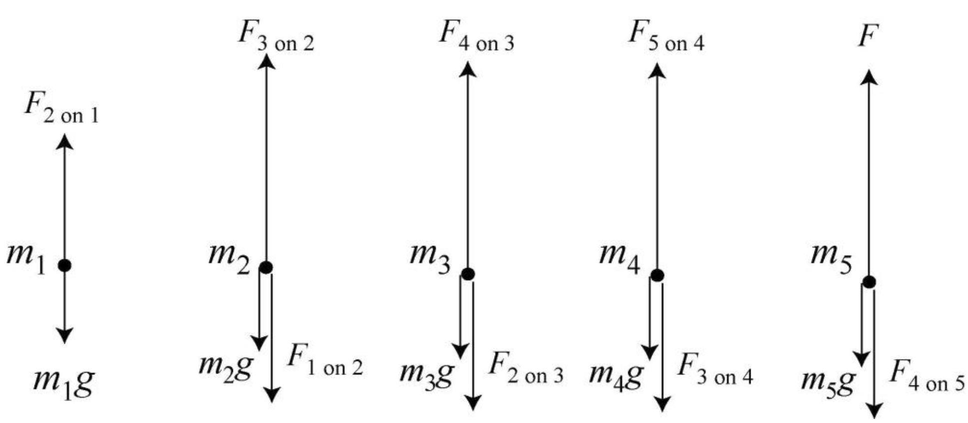
b ts angle measured from $+\hat{\text{i}}$ is either

$$\tan^{-1}\left(\frac{F_y}{F_x}\right) = +21° \text{ or } 201°.$$

he signs of the components indicate the latter is correct so $\vec{F}_{\text{ater}}$ is at 201° measured counterclock ise from the line of motion x a is .

43. **THINK** chain of five links is accelerated vertically up ard by an e ternal force. e are interested in the forces e erted by one link on its ad acent one.

EXPRESS he links are numbered from bottom to top. he forces on the first link are the force of gravity $m\vec{g}$ do n ard and the force $\vec{F}_{2\text{on}1}$ of link 2 up ard as sho n in the free body diagram belo not dra n to scale . ake the positive direction to be upward. Then Newton's second law for the first link is $F_{2\text{on}1} - m_1 g = m_1 a$. he e uations for the other links can be ritten in a similar manner see belo .



ANALYZE a iven that $a = 2.50$ m s^2 from $F_{2\text{on}1} - m_1 g = m_1 a$ the force e erted by link 2 on link 1 is

$$F_{2\text{on}1} = m_1\ a + g\ =\ 0.100\,\text{kg}\ \ 2.5\,\text{m s}^2 + 9.80\,\text{m s}^2\ = 1.23 \quad .$$

b rom the free body diagram above e see that the forces on the second link are the force of gravity $m_2\vec{g}$ do n ard the force $\vec{F}_{1\text{on}2}$ of link 1 do n ard and the force $\vec{F}_{3\text{on}2}$

of link 3, upward. According to Newton's third law $\vec{F}_{1on2}$ has the same magnitude as $\vec{F}_{2on1}$. Newton's second law for the second link is

$$F_{3on2} - F_{1on2} - m_2 g = m_2 a$$

so

$$F_{3on2} = m_2 \; a \quad g \quad F_{1on2} = \; 0.100 \text{ kg} \quad 2.50 \text{ m s}^2 \quad 9.80 \text{ m s}^2 \quad 1.23 \quad = 2.46 \quad .$$

(c) Newton's second law equation for link 3 is $F_{4on3} - F_{2on3} - m_3 g = m_3 a$ so

$$F_{4on3} = m_3 \; a \quad g \quad F_{2on3} = \; 0.100 \qquad 2.50 \text{ m s}^2 \quad 9.80 \text{ m s}^2 \quad 2.46 \quad = 3.69$$

where Newton's third law implies $F_{2on3} = F_{3on2}$ since these are magnitudes of the force vectors .

(d) Newton's second law for link 4 is

$$F_{5on4} - F_{3on4} - m_4 g = m_4 a$$

so

$$F_{5on4} = m_4 \; a \quad g \quad F_{3on4} = \; 0.100 \text{ kg} \quad 2.50 \text{ m s}^2 \quad 9.80 \text{ m s}^2 \quad 3.69 \quad = 4.92$$

where Newton's third law implies $F_{3on4} = F_{4on3}$.

(e) Newton's second law for the top link is $F - F_{4on5} - m_5 g = m_5 a$ so

$$F = m_5 \; a \quad g \quad F_{4on5} = \; 0.100 \text{ kg} \quad 2.50 \text{ m s}^2 \quad 9.80 \text{ m s}^2 \quad 4.92 \quad = 6.15$$

here $F_{4on5} = F_{5on4}$ by Newton's third law.

f ach link has the same mass $m_1 = m_2 = m_3 = m_4 = m_5 = m$ and the same acceleration so the same net force acts on each of them

$$F_{net} = ma = \; 0.100 \text{ kg} \quad 2.50 \text{ m s}^2 \; = 0.250 \quad .$$

LEARN n solving this problem e have used both Newton's second and third laws. ach pair of links constitutes a third la force pair ith $\vec{F}_{i\,on} = -\vec{F}_{on\,i}$.

44. (a) The term "deceleration" means the acceleration vector is in the direction opposite to the velocity vector hich the problem tells us is do n ard . hus ith y up ard the acceleration is $a =$ 2.4 m s^2. Newton's second law leads to

$$T - mg = ma \;\Rightarrow\; m = \frac{T}{g + a}$$

hich yields m = 7.3 kg for the mass.

b epeating the above computation no to solve for the tension ith a = 2.4 m s^2 ill of course lead us right back to T = 89 . ince the direction of the velocity did not enter our computation this is to be e pected.

45. a he mass of the elevator is m = 27800 9.80 = 2837 kg and ith y up ard the acceleration is a = 1.22 m s^2. Newton's second law leads to

$$T - mg = ma \;\Rightarrow\; T = m(g + a)$$

hich yields $T = 3.13 \times 10^4$ for the tension.

(b) The term "deceleration" means the acceleration vector is in the direction opposite to the velocity vector hich the problem tells us is up ard . hus ith y up ard the acceleration is no a = –1.22 m s^2 so that the tension is

$$T = m \;\; g \;\; a = 2.43 \times 10^4 \;\; .$$

46. ith a_{ce} meaning "the acceleration of the coin relative to the elevator" and a_{eg} meaning "the acceleration of the elevator relative to the ground," we have

$$a_{ce} \;\; a_{eg} = a_{cg} \;\; \Rightarrow \;\; –8.00 \text{ m s}^2 \;\; a_{eg} = –9.80 \text{ m s}^2$$

hich leads to a_{eg} = –1.80 m s^2. e have chosen up ard as the positive y direction. Then Newton's second law (in the "ground" reference frame) yields $T - m\,g = m\,a_{eg}$ or

$$T \;\; = m\,g \;\; m\,a_{eg} = m \;\; g \;\; a_{eg} \;\; = \;\; 2000 \text{ kg} \;\; 8.00 \text{ m s}^2 \;\; = 16.0 \text{ k} \;\; .$$

47. sing . 4 26 the launch speed of the pro ectile is

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta}} = \sqrt{\frac{9.8 \text{ m s}^2 \;\; 69 \text{ m}}{\sin 2 \;\; 53^\circ}} = 26.52 \text{ m s} .$$

he hori ontal and vertical components of the speed are

$$v_x = v_0 \cos\theta = \;\; 26.52 \text{ m s} \;\; \cos 53^\circ = 15.96 \text{ m s}$$
$$v_y = v_0 \sin\theta = \;\; 26.52 \text{ m s} \;\; \sin 53^\circ = 21.18 \text{ m s}.$$

ince the acceleration is constant e can use . 2 16 to analy e the motion. he component of the acceleration in the hori ontal direction is

$$a_x = \frac{v_x^2}{2x} = \frac{15.96 \text{ m s} \;\; ^2}{2 \;\; 5.2 \text{ m} \;\; \cos 53^\circ} = 40.7 \text{ m s}^2$$

and the force component is

$$F_x = ma_x = \ 85\text{ kg} \quad 40.7\text{ m s}^2 \ = 3460 \quad .$$

imilarly in the vertical direction e have

$$a_y = \frac{v_y^2}{2y} = \frac{21.18\text{ m s} \quad ^2}{2\ 5.2\text{ m} \ \sin 53°} = 54.0\text{ m s}^2.$$

and the force component is

$$F_y = ma_y + mg = \ 85\text{ kg} \quad 54.0\text{ m s}^2 + 9.80\text{ m s}^2 \ = 5424 \quad .$$

hus the magnitude of the force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\ 3460 \quad ^2 + \ 5424 \quad ^2} = 6434 \quad \approx 6.4 \times 10^3$$

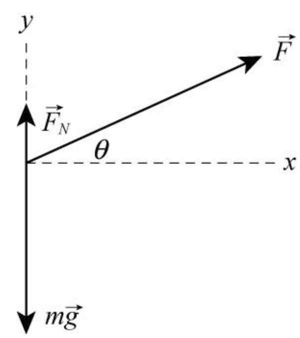
to t o significant figures.

48. Applying Newton's second law to cab B of mass m e have

$$a = \frac{T}{m} - g = 4.89\text{ m s}^2.$$

e t e apply it to the bo of mass m_b to find the normal force

$$F_N = m_b \ g \quad a \ = 176 \quad .$$



49. he free body diagram not to scale for the block is sho n to the right. $\vec{F}_N$ is the normal force e erted by the floor and $m\vec{g}$ is the force of gravity.

a he x component of Newton's second law is $F\cos\theta = ma$ here m is the mass of the block and a is the x component of its acceleration. e obtain

$$a = \frac{F\cos\theta}{m} = \frac{(12.0 \quad)\cos 25.0°}{5.00\text{ kg}} = 2.18\text{ m} \quad \text{s}^2.$$

his is its acceleration provided it remains in contact ith the floor. ssuming it does e find the value of F_N and if F_N is positive then the assumption is true but if F_N is negative then the block leaves the floor . he y component of Newton's second law becomes

$$F_N \quad F\sin\theta - mg = 0$$

so

$$F_N = mg - F\sin\theta = \ 5.00\text{ kg} \quad 9.80\text{ m s}^2 \ - \ 12.0 \quad \sin 25.0° = 43.9 \quad .$$

ence the block remains on the floor and its acceleration is a = 2.18 m s^2.

b f F is the minimum force for hich the block leaves the floor then F_N = 0 and the y component of the acceleration vanishes. he y component of the second la becomes

$$F\sin\theta - mg = 0 \quad \Rightarrow \quad F = \frac{mg}{\sin\theta} = \frac{(5.00\text{ kg})(9.80\text{ m s}^2)}{\sin 25.0°} = 116 \quad .$$

c he acceleration is still in the x direction and is still given by the e uation developed in part a

$$a = \frac{F\cos\theta}{m} = \frac{116 \quad \cos 25.0°}{5.00\text{ kg}} = 21.0\text{m s}^2.$$

50. a he net force on the *system* of total mass M = 80.0 kg is the force of gravity acting on the total overhanging mass m_{BC} = 50.0 kg . he magnitude of the acceleration is therefore a = $m_{BC}\,g$ M = 6.125 m s^2. Next we apply Newton's second law to block C itself choosing *down* as the y direction and obtain

$$m_C g - T_{BC} = m_C a.$$

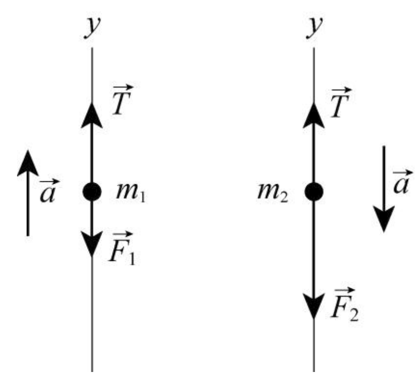
his leads to T_{BC} = 36.8 .

b e use . 2 15 choosing *rightward* as the x direction Δx = 0 $\frac{1}{2}at^2$ = 0.191 m.

51. he free body diagrams for m_1 and m_2 are sho n in the figures belo . he only forces on the blocks are the up ard tension $\vec{T}$ and the do n ard gravitational forces $\vec{F}_1 = m_1 g$ and $\vec{F}_2 = m_2 g$. pplying e ton's second la e obtain

$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$



hich can be solved to yield

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)g$$

ubstituting the result back e have

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right)g$$

a ith $m_1 = 1.3$ kg and $m_2 = 2.8$ kg the acceleration becomes

$$a=\left(\frac{2.80\text{ kg}-1.30\text{ kg}}{2.80\text{ kg}+1.30\text{ kg}}\right)\ 9.80\text{ m s}^2\ =3.59\text{ m s}^2\approx 3.6\text{ m s}^2.$$

b imilarly the tension in the cord is

$$T=\frac{2\ 1.30\text{ kg}\ \ 2.80\text{ kg}}{1.30\text{ kg}+2.80\text{ kg}}\ 9.80\text{ m s}^2\ =17.4\quad\approx 17\quad .$$

52. ie ing the man rope sandbag as a system means that e should be careful to choose a consistent positive direction of motion though there are other ays to proceed say, starting with individual application of Newton's law to each mass). We take *down* as positive for the man's motion and *up* as positive for the sandbag's motion and, without ambiguity denote their acceleration as a. he net force on the system is the different bet een the eight of the man and that of the sandbag. he system mass is m_{sys} = 85 kg 65 kg = 150 kg. hus . 5 1 leads to

$$85\text{ kg}\ \ 9.8\text{ m s}^2\ -\ 65\text{ kg}\ \ 9.8\text{ m s}^2\ =m_{sys}\,a$$

hich yields a = 1.3 m s^2. ince the system starts from rest . 2 16 determines the speed after traveling Δy = 10 m as follo s

$$v=\sqrt{2a\Delta y}=\sqrt{2\ 1.3\text{ m s}^2\ \ 10\text{ m}}\ =5.1\text{ m s}.$$

53. We apply Newton's second law first to the three blocks as a single system and then to the individual blocks. he x direction is to the right in ig. 5 48.

a ith $m_{sys} = m_1\ \ m_2\ \ m_3$ = 67.0 kg e apply . 5 2 to the x motion of the system in hich case there is only one force $\vec{T}_3=+\vec{T}_3\ \hat{\text{i}}$. herefore

$$T_3=m_{sys}a\ \Rightarrow\ 65.0\quad=\ 67.0\text{kg}\ a$$

hich yields a = 0.970 m s^2 for the system and for each of the blocks individually .

b pplying . 5 2 to block 1 e find

$$T_1=m_1a=(12.0\text{kg})(0.970\text{m s}^2)=11.6\quad .$$

c n order to find T_2 e can either analy e the forces on block 3 or e can treat blocks 1 and 2 as a system and e amine its forces. e choose the latter.

$$T_2=(m_1+m_2)a=(12.0\text{ kg}+24.0\text{ kg})(0.970\text{ m s}^2)=34.9\quad .$$

54. irst e consider all the penguins 1 through 4 counting left to right as one system to hich e apply e ton's second law:

$$T_4 = (m_1 + m_2 + m_3 + m_4)a \;\Rightarrow\; 222 \quad = (12\text{kg} + m_2 + 15\text{kg} + 20\text{kg})a.$$

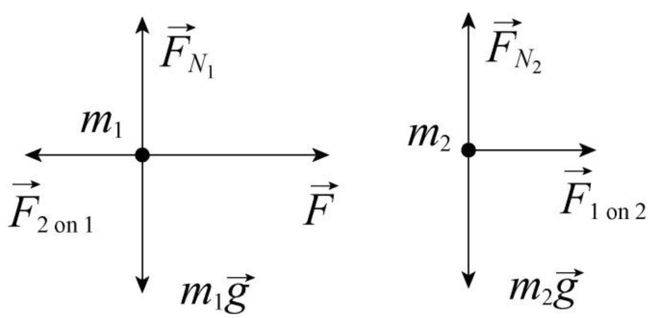
econd e consider penguins 3 and 4 as one system for hich e have

$$T_4 - T_2 = (m_3 + m_4)a$$
$$111 \quad = (15\text{ kg} + 20\text{kg})a \;\Rightarrow\; a = 3.2\text{ m s}^2.$$

ubstituting the value e obtain m_2 = 23 kg.

55. **THINK** n this problem a hori ontal force is applied to block 1 hich then pushes against block 2. oth blocks move together as a rigid connected system.

EXPRESS he free body diagrams for the t o blocks in a are sho n belo . $\vec{F}$ is the applied force and $\vec{F}_{1on2}$ is the force e erted by block 1 on block 2. e note that $\vec{F}$ is applied directly to block 1 and that block 2 e erts a force $\vec{F}_{2on1} = -\vec{F}_{1on2}$ on block 1 (taking Newton's third law into account).

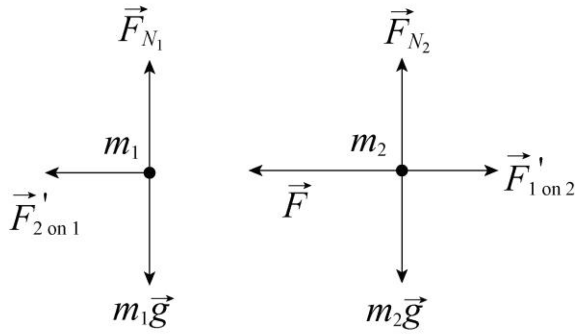


Newton's second law for block 1 is $F - F_{2on1} = m_1 a$ here a is the acceleration. he second la for block 2 is $F_{1on2} = m_2 a$. ince the blocks move together they have the same acceleration and the same symbol is used in both e uations.

ANALYZE a rom the second e uation e obtain the e pression $a = F_{1on2} \quad m_2$ hich e substitute into the first e uation to get $F - F_{2on1} = m_1 F_{1on2} \quad m_2$. ince $F_{2on1} = F_{1on2}$ same magnitude for third la force pair e obtain

$$F_{2on1} = F_{1on2} = \frac{m_2}{m_1 + m_2} F = \frac{1.2\text{ kg}}{2.3\text{ kg} + 1.2\text{ kg}}(3.2 \quad) = 1.1 \quad .$$

b f $\vec{F}$ is applied to block 2 instead of block 1 and in the opposite direction the free body diagrams ould look like the follo ing



he corresponding force of contact bet een the blocks ould be

$$F'_{2\text{on}1} = F'_{1\text{on}2} = \frac{m_1}{m_1 + m_2} F = \frac{2.3\text{ kg}}{2.3\text{ kg} + 1.2\text{ kg}}(3.2 \quad) = 2.1 \quad .$$

c e note that the acceleration of the blocks is the same in the t o cases. n part a the force $F_{1\text{on}2}$ is the only hori ontal force on the block of mass m_2 and in part b $F'_{2\text{on}1}$ is the only hori ontal force on the block ith m_1 m_2. ince $F_{1\text{on}2} = m_2 a$ in part a and $F'_{2\text{on}1} = m_1 a$ in part b then for the accelerations to be the same $F'_{2\text{on}1} > F_{1\text{on}2}$ i.e. force bet een blocks must be larger in part b .

LEARN his problem demonstrates that hen t o blocks are being accelerated together under an e ternal force the contact force bet een the t o blocks is greater if the smaller mass is pushing against the bigger one as in part b . n the special case here the t o masses are e ual $m_1 = m_2 = m$ $F'_{2\text{on}1} = F_{2\text{on}1} = F$ 2.

56. oth situations involve the same applied force and the same total mass so the accelerations must be the same in both figures.

a he direct force causing B to have this acceleration in the first figure is t ice as big as the direct force causing A to have that acceleration. herefore B has the t ice the mass of A. ince their total is given as 12.0 kg then B has a mass of m_B = 8.00 kg and A has mass m_A = 4.00 kg. onsidering the first figure 20.0 8.00 kg = 2.50 m s^2. f course the same result comes from considering the second figure 10.0 4.00 kg = 2.50 m s^2 .

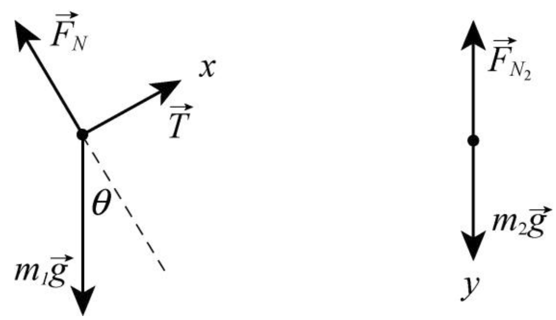
b F_a = 12.0 kg 2.50 m s^2 = 30.0

57. he free body diagram for each block is sho n belo . T is the tension in the cord and θ = 30 is the angle of the incline. or block 1 e take the x direction to be up the incline and the y direction to be in the direction of the normal force $\vec{F}_N$ that the plane e erts on the block. or block 2 e take the y direction to be do n. n this ay the accelerations of the t o blocks can be represented by the same symbol a ithout

ambiguity. Applying Newton's second law to the x and y a es for block 1 and to the y a is of block 2 e obtain

$$T - m_1 g \sin\theta = m_1 a$$
$$F_N - m_1 g \cos\theta = 0$$
$$m_2 g - T = m_2 a$$

respectively. he first and third of these e uations provide a simultaneous set for obtaining values of a and T. he second e uation is not needed in this problem since the normal force is neither asked for nor is it needed as part of some further computation such as can occur in formulas for friction .



a e add the first and third e uations above

$$m_2 g - m_1 g \sin\theta = m_1 a \quad m_2 a.$$

onse uently e find

$$a = \frac{(m_2 - m_1 \sin\theta)g}{m_1 + m_2} = \frac{2.30\text{ kg} - 3.70\text{ kg} \sin 30.0^\circ \ (9.80\text{ m s}^2)}{3.70\text{ kg} + 2.30\text{ kg}} = 0.735\text{ m s}^2.$$

b he result for a is positive indicating that the acceleration of block 1 is indeed up the incline and that the acceleration of block 2 is vertically do n.

c he tension in the cord is

$$T = m_1 a + m_1 g \sin\theta = (3.70\text{ kg})(0.735\text{ m s}^2) + (3.70\text{ kg})(9.80\text{ m s}^2)\sin 30.0^\circ = 20.8 \quad .$$

58. he motion of the man and chair is positive if up ard.

a hen the man is grasping the rope pulling ith a force e ual to the tension T in the rope the total up ard force on the man and chair due its t o contact points ith the rope is $2T$. Thus, Newton's second law leads to

$$2T - mg = ma$$

so that hen $a = 0$ the tension is $T = 466$.

b hen $a =$ 1.30 m s^2 the e uation in part a predicts that the tension ill be $T = 527$.

c hen the man is not holding the rope instead the co orker attached to the ground is pulling on the rope ith a force e ual to the tension T in it there is only one contact point bet een the rope and the man and chair, and Newton's second law now leads to

$$T - mg = ma$$

so that hen $a = 0$ the tension is $T = 931$.

d hen $a =$ 1.30 m s^2 the e uation in c yields $T = 1.05 \times 10^3$.

e he rope comes into contact pulling do n in each case at the left edge and the right edge of the pulley producing a total do n ard force of magnitude $2T$ on the ceiling. hus in part a this gives $2T = 931$.

f n part b the do n ard force on the ceiling has magnitude $2T = 1.05 \times 10^3$.

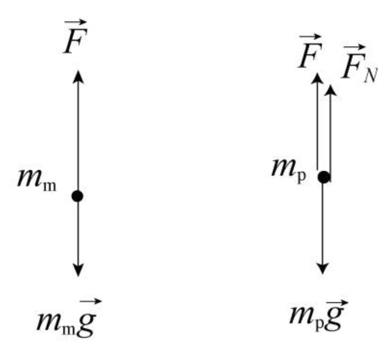
g n part c the do n ard force on the ceiling has magnitude $2T = 1.86 \times 10^3$.

h n part d the do n ard force on the ceiling has magnitude $2T = 2.11 \times 10^3$.

59. **THINK** his problem involves the application of Newton's third law. As the monkey climbs up a tree it pulls do n ard on the rope but the rope pulls up ard on the monkey.

EXPRESS e take y to be up for both the monkey and the package. he force the monkey pulls do n ard on the rope has magnitude F.

he free body diagrams for the monkey and the package are sho n to the right not to scale . According to Newton's third law, the rope pulls up ard on the monkey ith a force of the same magnitude, so Newton's second law for forces acting on the monkey leads to



$$F - m_m g = m_m a_m$$

here m_m is the mass of the monkey and a_m is its acceleration.

ince the rope is massless $F = T$ is the tension in the rope. he rope pulls up ard on the package ith a force of magnitude F, so Newton's second la for the package is

$$F \quad F_N - m_p g = m_p a_p$$

here m_p is the mass of the package a_p is its acceleration and F_N is the normal force e erted by the ground on it. o if F is the minimum force re uired to lift the package then $F_N = 0$ and $a_p = 0$. ccording to the second la e uation for the package this means $F = m_p g$.

ANALYZE a ubstituting $m_p g$ for F in the e uation for the monkey e solve for a_m

$$a_m = \frac{F - m_m g}{m_m} = \frac{(m_p - m_m)g}{m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m s}^2)}{10\text{ kg}} = 4.9\text{ m s}^2.$$

(b) As discussed, Newton's second law leads to $F - m_p g = m_p a'_p$ for the package and $F - m_m g = m_m a'_m$ for the monkey. f the acceleration of the package is do n ard then the acceleration of the monkey is up ard so $a'_m = -a'_p$. olving the first e uation for F

$$F = m_p\left(g + a'_p\right) = m_p\left(g - a'_m\right)$$

and substituting this result into the second e uation

$$m_p \;\; g - a'_m \;\; - m_m g = m_m a'_m$$

e solve for a'_m

$$a'_m = \frac{(m_p - m_m)g}{m_p + m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m s}^2)}{15\text{ kg} + 10\text{ kg}} = 2.0\text{ m s}^2.$$

c he result is positive indicating that the acceleration of the monkey is up ard.

d olving the second la e uation for the package the tension in the rope is

$$F = m_p\left(g - a'_m\right) = (15\text{ kg})\left(9.8\text{ m s}^2 - 2.0\text{ m s}^2\right) = 120 \quad .$$

LEARN he situations described in b d are similar to that of an t ood machine. ith $m_p > m_m$ the package accelerates do n ard hile the monkey accelerates up ard.

60. he hori ontal component of the acceleration is determined by the net hori ontal force.

a f the rate of change of the angle is

$$\frac{d\theta}{dt} = \;\; 2.00\times10^{-2} \;\circ\; \text{s} = \;\; 2.00\times10^{-2} \;\circ\; \text{s}\cdot\left(\frac{\pi\text{ rad}}{180^\circ}\right) = 3.49\times10^{-4}\text{rad s}$$

then using $F_x = F\cos\theta$ e find the rate of change of acceleration to be

$$\frac{da_x}{dt} = \frac{d}{dt}\left(\frac{F\cos\theta}{m}\right) = -\frac{F\sin\theta}{m}\frac{d\theta}{dt} = -\frac{20.0 \quad \sin 25.0°}{5.00\text{ kg}}\left(3.49\times10^{-4}\text{ rad s}\right)$$
$$= -5.90\times10^{-4}\text{ m s}^3.$$

b f the rate of change of the angle is

$$\frac{d\theta}{dt} = - \ 2.00\times10^{-2} \ ° \ \text{ s} = - \ 2.00\times10^{-2} \ ° \ \text{ s}\cdot\left(\frac{\pi\text{ rad}}{180°}\right) = -3.49\times10^{-4}\text{ rad s}$$

then the rate of change of acceleration ould be

$$\frac{da_x}{dt} = \frac{d}{dt}\left(\frac{F\cos\theta}{m}\right) = -\frac{F\sin\theta}{m}\frac{d\theta}{dt} = -\frac{20.0 \quad \sin 25.0°}{5.00\text{ kg}}\left(-3.49\times10^{-4}\text{ rad s}\right)$$
$$= +5.90\times10^{-4}\text{ m s}^3.$$

61. **THINK** s more mass is thro n out of the hot air balloon its up ard acceleration increases.

EXPRESS he forces on the balloon are the force of gravity $m\vec{g}$ do n and the force of the air $\vec{F}_a$ up . e take the y to be up and use a to mean the *magnitude* of the acceleration. hen the mass is M before the ballast is thro n out the acceleration is downward and Newton's second law is

$$Mg - F_a = Ma$$

fter the ballast is thro n out the mass is $M - m$ here m is the mass of the ballast and the acceleration is now upward. Newton's second law leads to

$$F_a - \ M - m \ g = \ M - m \ a.$$

ombing the t o e uations allo s us to solve for m.

ANALYZE he first e uation gives $F_a = M \ g - a$ and this plugs into the ne e uation to give

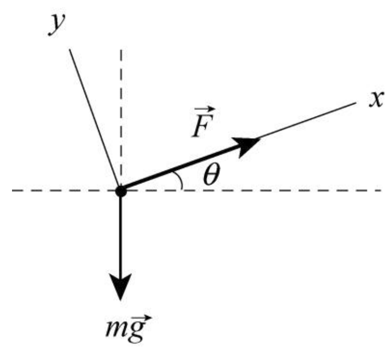
$$M(g-a) - (M-m)g = (M-m)a \ \Rightarrow \ m = \frac{2Ma}{g+a}.$$

LEARN ore generally if a ballast mass m' is tossed the resulting acceleration is a' hich is related to m' via

$$m' = M\frac{a'+a}{g+a}$$

sho ing that the more mass thro n out the greater is the up ard acceleration. or $a'=a$ e get $m' = 2Ma \quad g+a$ hich agrees ith hat as found above.

62. o solve the problem e note that the acceleration along the slanted path depends on only the force components along the path not the components perpendicular to the path.



a rom the free body diagram sho n e see that the net force on the putting shot along the x a is is

$$F_{\text{net } x} = F - mg\sin\theta = 380.0 \quad - \ 7.260\text{ kg} \ \ 9.80\text{ m s}^2 \ \ \sin 30^\circ = 344.4$$

hich in turn gives

$$a_x = F_{\text{net } x} \quad m = \ 344.4 \qquad 7.260\text{ kg} \ = 47.44\text{ m s}^2.$$

sing . 2 16 for constant acceleration motion the speed of the shot at the end of the acceleration phase is

$$v = \sqrt{v_0^2 + 2a_x\Delta x} = \sqrt{\ 2.500\text{ m s}\ ^2 + 2\ \ 47.44\text{ m s}^2 \ \ 1.650\text{ m}\ } = 12.76\text{ m s}.$$

b f $\theta = 42^\circ$ then

$$a_x = \frac{F_{\text{net } x}}{m} = \frac{F - mg\sin\theta}{m} = \frac{380.0 \quad - \ 7.260\text{ kg} \ \ 9.80\text{ m s}^2 \ \ \sin 42.00^\circ}{7.260\text{ kg}} = 45.78\text{ m s}^2$$

and the final launch speed is

$$v = \sqrt{v_0^2 + 2a_x\Delta x} = \sqrt{\ 2.500\text{ m s}\ ^2 + 2\ \ 45.78\text{ m s}^2 \ \ 1.650\text{ m}\ } = 12.54\text{ m s}.$$

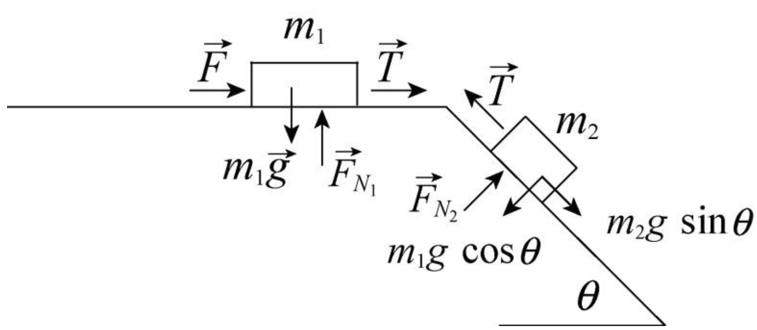
c he decrease in launch speed hen changing the angle from 30.00° to 42.00° is

$$\frac{12.76 \text{ m s} - 12.54 \text{ m s}}{12.76 \text{ m s}} = 0.0169 = 1.69 \quad .$$

63. a he acceleration hich e uals F/m in this problem is the derivative of the velocity. hus the velocity is the integral of F/m, so we find the "area" in the graph (15 units and divide by the mass 3 to obtain $v - v_0$ = 15 3 = 5. ince v_0 = 3.0 m s then $v = 8.0$ m s.

b ur positive ans er in part a implies $\vec{v}$ points in the x direction.

64. he x direction for m_2 = 1.0 kg is "downhill" and the $+x$ direction for m_1 = 3.0 kg is right ard thus they accelerate ith the same sign.



(a) We apply Newton's second law to the x a is of each bo

$$m_2 g \sin\theta - T = m_2 a$$
$$F + T = m_1 a$$

dding the t o e uations allo s us to solve for the acceleration

$$a = \frac{m_2 g \sin\theta + F}{m_1 + m_2}$$

ith F = 2.3 and $\theta = 30°$ e have a = 1.8 m s^2. e plug back in and find T = 3.1 .

b e consider the "critical" case where the F has reached the *max* value causing the tension to vanish. he first of the e uations in part a sho s that $a = g\sin 30°$ in this case thus a = 4.9 m s^2. his implies along ith T = 0 in the second e uation in part a that

$$F = \ 3.0 \text{ kg} \ \ 4.9 \text{ m s}^2 \ = 14.7 \quad \approx 15$$

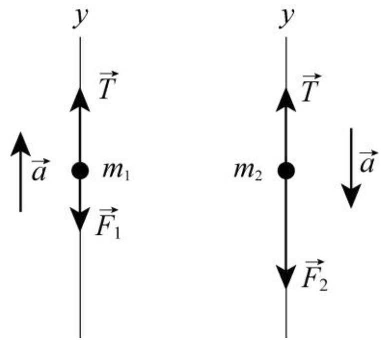
in the critical case.

65. he free body diagrams for m_1 and m_2 are sho n in the figures belo . he only forces on the blocks are the up ard tension $\vec{T}$ and the do n ard gravitational forces $\vec{F}_1 = m_1 g$ and $\vec{F}_2 = m_2 g$. pplying e ton's second la e obtain

$$T - m_1 g = m_1 a$$
$$m_2 g - T = m_2 a$$

hich can be solved to give

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$



a t $t = 0$ $m_{10} = 1.30\ \text{kg}$. ith $dm_1\ dt = -0.200\ \text{kg s}$ e find the rate of change of acceleration to be

$$\frac{da}{dt} = \frac{da}{dm_1}\frac{dm_1}{dt} = -\frac{2m_2 g}{m_2 + m_{10}\ {}^2}\frac{dm_1}{dt} = -\frac{2\ 2.80\ \text{kg}\ \ 9.80\ \text{m s}^2}{2.80\ \text{kg} + 1.30\ \text{kg}\ {}^2}\left(-0.200\ \text{kg s}\right) = 0.653\ \text{m s}^3.$$

b t $t = 3.00\ \text{s}$ $m_1 = m_{10} +\ dm_1\ \ dt\ t = 1.30\ \text{kg} +\ -0.200\ \text{kg s}\ \ 3.00\ \text{s}\ = 0.700\ \text{kg}$ and the rate of change of acceleration is

$$\frac{da}{dt} = \frac{da}{dm_1}\frac{dm_1}{dt} = -\frac{2m_2 g}{m_2 + m_1\ {}^2}\frac{dm_1}{dt} = -\frac{2\ 2.80\ \text{kg}\ \ 9.80\ \text{m s}^2}{2.80\ \text{kg} + 0.700\ \text{kg}\ {}^2}\left(-0.200\ \text{kg s}\right) = 0.896\ \text{m s}^3.$$

c he acceleration reaches its ma imum value hen

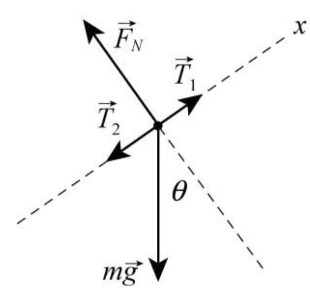
$$0 = m_1 = m_{10} +\ dm_1\ \ dt\ t = 1.30\ \text{kg} +\ -0.200\ \text{kg s}\ t$$

or $t = 6.50\ \text{s}$.

66. he free body diagram is shown to the right. Newton's second la for the mass m for the x direction leads to

$$T_1 - T_2 - mg\sin\theta = ma$$

hich gives the difference in the tension in the pull cable



$$T_1 - T_2 = m(g\sin\theta + a) = (2800\ \text{kg})\left[\ 9.8\ \text{m s}^2\ \ \sin 35^\circ + 0.81\ \text{m s}^2\right] = 1.8\times 10^4\quad .$$

67. irst e analy e the entire *system* with "clockwise" motion considered positive (that is do n ard is positive for block C right ard is positive for block B and up ard is positive for block A $m_C g - m_A g = Ma$ here M = mass of the *system* = 24.0 kg . his yields an acceleration of

$$a = g\ m_C - m_A\ \ M = 1.63 \text{ m s}^2.$$

e t e analy e the forces ust on block C $m_C g\ -\ T\ = m_C a$. hus the tension is

$$T = m_C g\ 2m_A\ \ m_B\ \ M = 81.7\ \ .$$

68. e first use . 4 26 to solve for the launch speed of the shot

$$y - y_0 = \ \tan\theta\ x - \frac{gx^2}{2\ v'\cos\theta\ ^2}.$$

ith $\theta = 34.10°$ $y_0 = 2.11$ m and $x\ y\ = 15.90$ m 0 e find the launch speed to be $v' = 11.85$ m s. uring this phase the acceleration is

$$a = \frac{v'^2 - v_0^2}{2L} = \frac{11.85 \text{ m s}\ ^2 - \ 2.50 \text{ m s}\ ^2}{2\ 1.65 \text{ m}} = 40.63 \text{ m s}^2.$$

ince the acceleration along the slanted path depends on only the force components along the path not the components perpendicular to the path the average force on the shot during the acceleration phase is

$$F = m\ a + g\sin\theta\ = \ 7.260 \text{ kg}\ \left[40.63 \text{ m s}^2 + \ 9.80 \text{ m s}^2\ \sin 34.10°\right] = 334.8\ \ .$$

69. e begin by e amining a slightly different problem similar to this figure but ithout the string. he motivation is that if ithout the string block A is found to accelerate faster or e actly as fast as block B then returning to the original problem the tension in the string is trivially ero. n the absence of the string

$$a_A = F_A\ \ m_A = 3.0 \text{ m s}^2$$
$$a_B = F_B\ \ m_B = 4.0 \text{ m s}^2$$

so the trivial case does not occur. e no ith the string consider the net force on the *system* $Ma = F_A\ \ F_B = 36$. ince $M = 10$ kg the total mass of the system e obtain a = 3.6 m s^2. he t o forces on block A are F_A and T in the same direction so e have

$$m_A a = F_A\ \ T\ \ \Rightarrow\ \ T = 2.4\ \ .$$

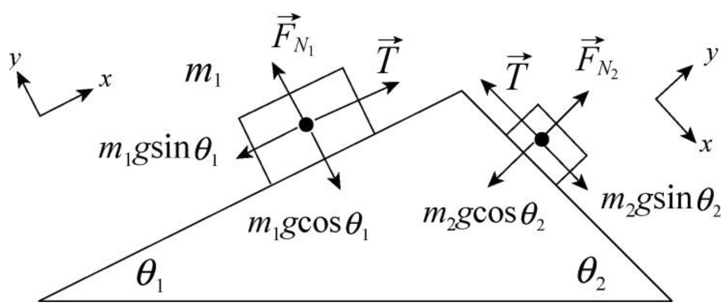
70. a For the 0.50 meter drop in "free fall," Eq. 2 16 yields a speed of 3.13 m s. sing this as the "initial speed" for the final motion over 0.02 meter during hich his motion slows at rate "a" we find the magnitude of his average acceleration from when his feet first touch the patio until the moment his body stops moving is a = 245 m s^2.

b We apply Newton's second law: $F_{\text{stop}} - \ mg = ma\ \Rightarrow\ F_{\text{stop}} = 20.4$ k .

71. **THINK** e have t o bo es connected together by a cord and placed on a edge. The system accelerates together and we'd like to know the tension in the cord.

EXPRESS he x axis is "uphill" for m_1 = 3.0 kg and "downhill" for m_2 = 2.0 kg so they both accelerate ith the same sign . he x components of the t o masses along the x a is are given by $m_1 g\sin\theta_1$ and $m_2 g\sin\theta_2$ respectively. he free body diagram is sho n belo . pplying Newton's second la e obtain

$$T - m_1 g\sin\theta_1 = m_1 a$$
$$m_2 g\sin\theta_2 - T = m_2 a$$



dding the t o e uations allo s us to solve for the acceleration

$$a = \left(\frac{m_2\sin\theta_2 - m_1\sin\theta_1}{m_2 + m_1}\right) g$$

ANALYZE ith $\theta_1 = 30°$ and $\theta_2 = 60°$ e have a = 0.45 m s^2. his value is plugged back into either of the t o e uations to yield the tension

$$T = \frac{m_1 m_2 g}{m_2 + m_1}\ \sin\theta_2 + \sin\theta_1\ = 16.1$$

LEARN n this problem e find $m_2\sin\theta_2 > m_1\sin\theta_1$ so that $a > 0$ indicating that m_2 slides do n and m_1 slides up. he situation ould reverse if $m_2\sin\theta_2 < m_1\sin\theta_1$. hen $m_2\sin\theta_2 = m_1\sin\theta_1$ the acceleration is a = 0 and the t o masses hang in balance. otice also the symmetry bet een the t o masses in the e pression for T.

72. ince the velocity of the particle does not change it undergoes no acceleration and must therefore be sub ect to ero net force. herefore

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0\,.$$

hus the third force $\vec{F}_3$ is given by

$$\vec{F}_3 = -\vec{F}_1 - \vec{F}_2 = -\left(2\hat{\text{i}} + 3\hat{\ } - 2\hat{\text{k}}\right) \ \ - \left(-5\hat{\text{i}} + 8\hat{\ } - 2\hat{\text{k}}\right) \ \ = \left(3\hat{\text{i}} - 11\hat{\ } + 4\hat{\text{k}}\right) \ \ .$$

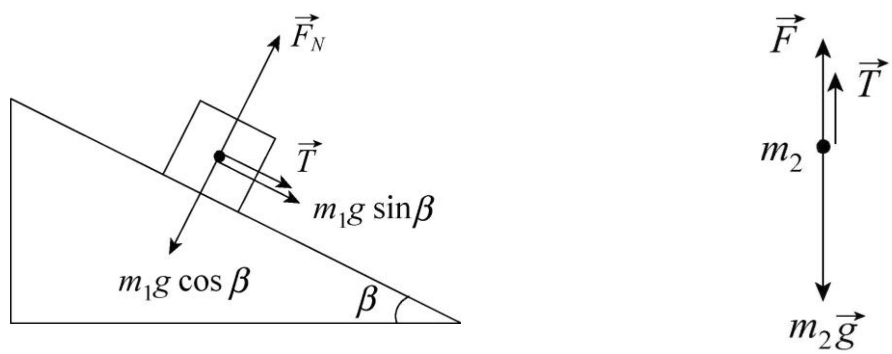
he specific value of the velocity is not used in the computation.

73. **THINK** e have t o masses connected together by a cord. force is applied to the second mass and the system accelerates together. We apply Newton's second law to solve the problem.

EXPRESS he free body diagrams for the t o masses are sho n belo not to scale . e first analy e the forces on m_1=1.0 kg. he x direction is "downhill" (parallel to $\vec{T}$. ith an acceleration a = 5.5 m s^2 in the positive x direction for m_1, Newton's second law applied to the x a is gives

$$T + m_1 g \sin\beta = m_1 a .$$

n the other hand for the second mass m_2=2.0 kg e have $m_2 g - F - T = m_2 a$ here the tension comes in as an up ard force the cord can pull not push . he t o e uations can be combined to solve for T and β.



ANALYZE e solve b first. y combining the t o e uations above e obtain

$$\sin\beta = \frac{m_1 + m_2 \;\; a + F - m_2 g}{m_1 g} = \frac{1.0\text{ kg} + 2.0\text{ kg} \;\; 5.5\text{ m s}^2 \;\; + 6.0 \quad - \;\; 2.0\text{ kg} \;\; 9.8\text{ m s}^2}{1.0\text{ kg} \;\; 9.8\text{ m s}^2}$$
$$= 0.296$$

hich gives $\beta = 17.2°$.

a ubstituting the value for β found in a into the first e uation e have

$$T = m_1 \;\; a - g \sin\beta \;\; = \;\; 1.0\text{ kg} \left[5.5\text{ m s}^2 - \;\; 9.8\text{ m s}^2 \;\; \sin 17.2° \right] = 2.60 \quad .$$

LEARN or $\beta = 0$ the problem becomes the same as that discussed in ample roblem "Block on table, block hanging." In this case, our results reduce to the familiar e pressions $a = m_2 g \quad m_1 + m_2$ and $T = m_1 m_2 g \quad m_1 + m_2$.

74. e are only concerned ith hori ontal forces in this problem gravity plays no direct role . ithout loss of generality e take one of the forces along the x direction and the other at 80° measured counterclock ise from the x a is . his calculation is efficiently implemented on a vector capable calculator in polar mode as follo s using magnitude angle notation ith angles understood to be in degrees

$$\vec{F}_{\text{net}} = \ 20 \angle 0 \quad 35 \angle 80 \ = \ 43 \angle 53 \ \Rightarrow \ \vec{F}_{\text{net}} \ = \ 43 \quad .$$

herefore the mass is $m = \ 43 \quad 20 \text{ m s}^2 \ = 2.2$ kg.

75. he goal is to arrive at the least magnitude of $\vec{F}_{\text{net}}$ and as long as the magnitudes of $\vec{F}_2$ and $\vec{F}_3$ are in total less than or e ual to $|\vec{F}_1|$ then e should orient them opposite to the direction of $\vec{F}_1$ hich is the x direction .

a e orient both $\vec{F}_2$ and $\vec{F}_3$ in the $-x$ direction. hen the magnitude of the net force is $50 - 30 - 20 = 0$ resulting in ero acceleration for the tire.

b e again orient $\vec{F}_2$ and $\vec{F}_3$ in the negative x direction. e obtain an acceleration along the x a is ith magnitude

$$a = \frac{F_1 - F_2 - F_3}{m} = \frac{50 \quad - 30 \quad - 10}{12\,\text{kg}} = 0.83 \text{ m } \ \text{s}^2 \ .$$

c he least value is $a = 0$. n this case the forces $\vec{F}_2$ and $\vec{F}_3$ are collectively strong enough to have y components one positive and one negative that cancel each other and still have enough x contributions in the $-x$ direction to cancel $\vec{F}_1$. ince $|\vec{F}_2| = |\vec{F}_3|$ e see that the angle above the $-x$ a is to one of them should e ual the angle belo the $-x$ a is to the other one e denote this angle θ. e re uire

$$-50 \quad = F_{2x} + F_{3x} = -(30 \quad)\cos\theta - (30 \quad)\cos\theta$$

hich leads to

$$\theta = \cos^{-1}\left(\frac{50}{60}\right) = 34° \ .$$

76. a small segment of the rope has mass and is pulled do n by the gravitational force of the arth. uilibrium is reached because neighboring portions of the rope pull up sufficiently on it. ince tension is a force *along* the rope at least one of the neighboring portions must slope up a ay from the segment e are considering. hen the tension has an up ard component hich means the rope sags.

b he only force acting ith a hori ontal component is the applied force $\vec{F}$. reating the block and rope as a single object, we write Newton's second law for it: $F = M \quad m\ a$ here a is the acceleration and the positive direction is taken to be to the right. he acceleration is given by $a = F \quad M \quad m$.

c he force of the rope F_r is the only force ith a hori ontal component acting on the block. Then Newton's second law for the block gives

$$F_r = Ma = \frac{MF}{M + m}$$

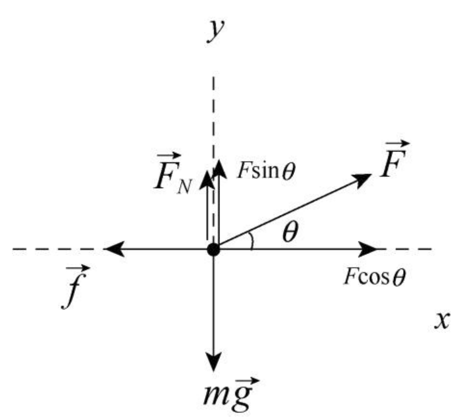
here the e pression found above for a has been used.

d reating the block and half the rope as a single ob ect ith mass $M + \frac{1}{2}m$ here the hori ontal force on it is the tension T_m at the midpoint of the rope, we use Newton's second la

$$T_m = \left(M + \frac{1}{2}m\right)a = \frac{(M + m \quad 2)F}{(M + m)} = \frac{(2M + m)F}{2(M + m)}.$$

77. **THINK** e have a crate that is being pulled at an angle. We apply Newton's second la to analy e the motion.

EXPRESS lthough the full specification of $\vec{F}_{\text{net}} = m\vec{a}$ in this situation involves both x and y a es only the x application is needed to find hat this particular problem asks for. e note that $a_y = 0$ so that there is no ambiguity denoting a_x simply as a. e choose x to the right and y up. he free body diagram not to scale is sho n to the right. he x component of the rope's tension acting on the crate is



$$F_x = F\cos\theta = \quad 450 \qquad \cos 38 \quad = 355$$

and the resistive force pointing in the $-x$ direction has magnitude $f = 125$.

ANALYZE (a) Newton's second law leads to

$$F_x - f = ma \Rightarrow a = \frac{F\cos\theta - f}{m} = \frac{355 \quad - 125}{310\text{ kg}} = 0.74\text{m s}^2.$$

b n this case e use . 5 12 to find the mass $m' = W \quad g = 31.6\text{ kg}$. Newton's second la then leads to

$$F_x - f = m'a' \;\Rightarrow\; a' = \frac{F_x - f}{m'} = \frac{355 \quad - 125}{31.6\ \text{kg}} = 7.3\ \text{m s}^2.$$

LEARN he resistive force opposing the motion is due to the friction bet een the crate and the floor. his topic is discussed in greater detail in hapter 6.

78. e take x uphill for the m_2 = 1.0 kg bo and x right ard for the m_1 = 3.0 kg bo so the accelerations of the t o bo es have the same magnitude and the same sign . he uphill force on m_2 is F and the do nhill forces on it are T and $m_2 g$ sin θ here θ = 37 . he only hori ontal force on m_1 is the right ard pointed tension. Applying Newton's second la to each bo e find

$$F - T - m_2 g \sin\theta = m_2 a$$
$$T = m_1 a$$

hich can be added to obtain

$$F - m_2 g \sin\theta = m_1 \quad m_2\ a.$$

his yields the acceleration

$$a = \frac{12 \quad - \ 1.0\ \text{kg} \ \ 9.8\ \text{m s}^2 \ \sin 37°}{1.0\ \text{kg} + 3.0\ \text{kg}} = 1.53\ \text{m s}^2.$$

hus the tension is $T = m_1 a =$ 3.0 kg 1.53 m s^2 = 4.6 .

79. e apply . 5 12.

a he mass is

$$m = W\ g = 22 \qquad 9.8\ \text{m s}^2 = 2.2\ \text{kg}.$$

t a place here g = 4.9 m s^2 the mass is still 2.2 kg but the gravitational force is

$$F_g = mg = 2.2\ \text{kg} \quad 4.0\ \text{m s}^2 = 11 \quad .$$

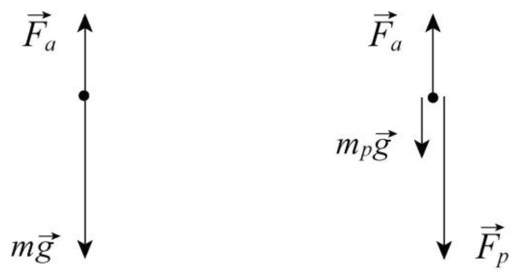
b s noted m = 2.2 kg.

c t a place here g = 0 the gravitational force is ero.

d he mass is still 2.2 kg.

80. e take do n to be the y direction.

a he first diagram sho n belo left is the free body diagram for the person and parachute considered as a single ob ect ith a mass of 80 kg 5.0 kg = 85 kg.



$\vec{F}_a$ is the force of the air on the parachute and $m\vec{g}$ is the force of gravity. pplication of Newton's second law produces $mg - F_a = ma$ here a is the acceleration. olving for F_a e find

$$F_a = m(g - a) = (85\text{ kg})\left(9.8\text{ m s}^2 - 2.5\text{ m s}^2\right) = 620 \quad .$$

b he second diagram above right is the free body diagram for the parachute alone. $\vec{F}_a$ is the force of the air $m_p\vec{g}$ is the force of gravity and $\vec{F}_p$ is the force of the person. Now, Newton's second law leads to

$$m_p g \quad F_p - F_a = m_p a.$$

olving for F_p e obtain

$$F_p = m_p(a - g) + F_a = (5.0\text{ kg})\left(2.5\text{ m s}^2 - 9.8\text{ m s}^2\right) + 620 \quad = 580 \quad .$$

81. he mass of the pilot is m = 735 9.8 = 75 kg. enoting the up ard force e erted by the spaceship his seat presumably on the pilot as $\vec{F}$ and choosing up ard as the y direction, then Newton's second law leads to

$$F - mg_{\text{moon}} = ma \quad \Rightarrow \quad F = (75\text{ kg})\left(1.6\text{ m s}^2 + 1.0\text{ m s}^2\right) = 195 \quad .$$

82. ith units understood the net force on the bo is

$$\vec{F}_{\text{net}} = (3.0 + 14\cos 30^\circ - 11)\,\hat{\text{i}} + (14\sin 30^\circ + 5.0 - 17)\,\hat{}$$

hich yields $\vec{F}_{\text{net}} = \; 4.1 \quad \hat{\text{i}} - \; 5.0 \quad \hat{}$.

(a) Newton's second law applied to the m = 4.0 kg bo leads to

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \; 1.0\text{m s}^2 \; \hat{\text{i}} - \; 1.3\text{m s}^2 \; \hat{} \; .$$

b he magnitude of $\vec{a}$ is $a = \sqrt{1.0\text{ m s}^2 \;{}^2 + \left(-1.3\text{ m s}^2\right)^2} = 1.6\ \text{m/s}^2$.

c ts angle is $\tan^{-1}$ –1.3 m s^2 1.0 m s^2 = –50 that is 50 measured clock ise from the right ard a is .

83. **THINK** his problem deals ith the relationship bet een the three uantities force mass and acceleration in Newton's second law $F = ma$.

EXPRESS The "certain force," denoted as F is assumed to be the net force on the ob ect hen it gives m_1 an acceleration a_1 = 12 m s^2 and hen it gives m_2 an acceleration a_2 = 3.3 m s^2 i.e. $F = m_1 a_1 = m_2 a_2$. he accelerations for $m_2 + m_1$ and $m_2 - m_1$ can be solved by substituting $m_1 = F\ a_1$ and $m_2 = F\ \ a_2$.

ANALYZE a o e seek the acceleration a of an ob ect of mass $m_2 - m_1$ hen F is the net force on it. he result is

$$a = \frac{F}{m_2 - m_1} = \frac{F}{F\ \ a_2\ - F\ \ a_1} = \frac{a_1 a_2}{a_1 - a_2} = \frac{12.0\text{ m s}^2\ \ 3.30\text{ m s}^2}{12.0\text{ m s}^2 - 3.30\text{ m s}^2} = 4.55\text{ m s}^2 .$$

b imilarly for an ob ect of mass $m_2\ \ m_1$ e have

$$a' = \frac{F}{m_2 + m_1} = \frac{F}{F\ \ a_2\ + F\ \ a_1} = \frac{a_1 a_2}{a_1 + a_2} = \frac{12.0\text{ m s}^2\ \ 3.30\text{ m s}^2}{12.0\text{ m s}^2 + 3.30\text{ m s}^2} = 2.59\text{ m s}^2 .$$

LEARN ith the same applied force the greater the mass the smaller the acceleration. n this problem e have $a_1 > a > a_2 > a'$. his implies $m_1 < m_2 - m_1 < m_2 < m_2 + m_1$.

84. e assume the direction of motion is x and assume the refrigerator starts from rest so that the speed being discussed is the velocity $\vec{v}$ that results from the process . he only force along the x a is is the x component of the applied force $\vec{F}$.

a ince v_0 = 0 the combination of . 2 11 and . 5 2 leads simply to

$$F_x = m\left(\frac{v}{t}\right) \Rightarrow v_i = \left(\frac{F\ \cos\theta_i}{m}\right)t$$

for i = 1 or 2 here e denote θ_1 = 0 and θ_2 = θ for the t o cases . ence e see that the ratio v_2 over v_1 is e ual to cos θ.

b ince v_0 = 0 the combination of . 2 16 and . 5 2 leads to

$$F_x = m\left(\frac{v^2}{2\Delta x}\right) \Rightarrow v_i = \sqrt{2\left(\frac{F\ \cos\theta_i}{m}\right)\Delta x}$$

for i = 1 or 2 again θ_1 = 0 and θ_2 = θ is used for the t o cases . n this scenario e see that the ratio v_2 over v_1 is e ual to $\sqrt{\cos\theta}$.

85. a ince the performer's weight is (52 kg)(9.8 m/s^2 = 510 the rope breaks.

b etting T = 425 N in Newton's second law (with $+y$ up ard leads to

$$T - mg = ma \;\Rightarrow\; a = \frac{T}{m} - g$$

hich yields a = 1.6 m s^2.

86. e use $W_p = mg_p$ here W_p is the eight of an ob ect of mass m on the surface of a certain planet p and g_p is the acceleration of gravity on that planet.

a he eight of the space ranger on arth is

$$W_e = mg_e = \ 75\text{ kg}\ \ 9.8\text{ m s}^2 \ = 7.4 \times 10^2 \ .$$

b he eight of the space ranger on ars is

$$W_m = mg_m = \ 75\text{ kg}\ \ 3.7\text{ m s}^2 \ = 2.8 \times 10^2 \ .$$

c he eight of the space ranger in interplanetary space is ero here the effects of gravity are negligible.

d he mass of the space ranger remains the same m = 75 kg at all the locations.

87. rom the reading hen the elevator as at rest e kno the mass of the ob ect is m = 65 9.8 m s^2 = 6.6 kg. e choose y up ard and note there are t o forces on the ob ect mg do n ard and T up ard in the cord that connects it to the balance T is the reading on the scale by Newton's third law).

(a) "Upward at constant speed" means constant velocity, which means no acceleration. hus the situation is ust as it as at rest T = 65 .

b he term "deceleration" is used when the acceleration vector points in the direction opposite to the velocity vector. We're told the velocity is upward, so the acceleration vector points do n ard a = –2.4 m s^2). Newton's second law gives

$$T - mg = ma \;\Rightarrow\; T = \ 6.6\text{ kg}\ \ 9.8\text{ m s}^2 - 2.4\text{ m s}^2 \ = 49 \ .$$

88. e use the notation g as the acceleration due to gravity near the surface of allisto m as the mass of the landing craft a as the acceleration of the landing craft and F as the rocket thrust. We take down to be the positive direction. Thus, Newton's second la takes the form $mg - F = ma$. f the thrust is F_1 = 3260 then the acceleration is ero

so $mg - F_1 = 0$. f the thrust is F_2 = 2200 then the acceleration is a_2 = 0.39 m s^2 so $mg - F_2 = ma_2$.

a he first e uation gives the eight of the landing craft $mg = F_1 = 3260$.

b he second e uation gives the mass

$$m = \frac{mg - F_2}{a_2} = \frac{3260 \quad - 2200}{0.39 \text{ m s}^2} = 2.7 \times 10^3 \text{ kg} .$$

c he eight divided by the mass gives the acceleration due to gravity

$$g = 3260 \qquad 2.7 \times 10^3 \text{ kg} = 1.2 \text{ m s}^2.$$

89. a hen $\vec{F}_{\text{net}} = 3F - mg = 0$ e have

$$F = \frac{1}{3}mg = \frac{1}{3}(1400 \text{ kg})\left(9.8 \text{ m s}^2\right) = 4.6 \times 10^3$$

for the force e erted by each bolt on the engine.

b he force on each bolt no satisfies $3F - mg = ma$ hich yields

$$F = \frac{1}{3}m(g + a) = \frac{1}{3}(1400 \text{ kg})\left(9.8 \text{ m s}^2 + 2.6 \text{ m s}^2\right) = 5.8 \times 10^3 \quad .$$

90. e rite the length unit light month the distance traveled by light in one month as c month in this solution.

a he magnitude of the re uired acceleration is given by

$$a = \frac{\Delta v}{\Delta t} = \frac{(0.10)(3.0 \times 10^8 \text{ m s})}{(3.0 \text{ days})(86400 \text{ s day})} = 1.2 \times 10^2 \text{ m s}^2 .$$

b he acceleration in terms of g is $a = \left(\frac{a}{g}\right)g = \left(\frac{1.2 \times 10^2 \text{ m s}^2}{9.8 \text{ m s}^2}\right)g = 12g$.

c he force needed is

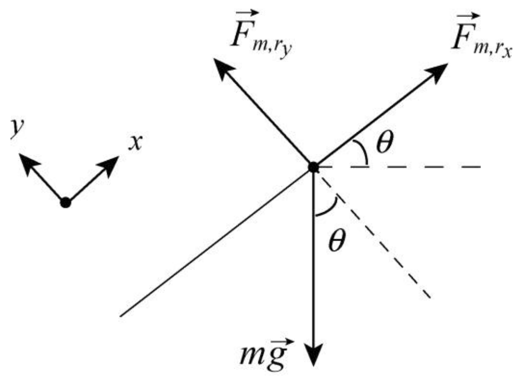
$$F = ma = \left(1.20 \times 10^6 \text{ kg}\right)\left(1.2 \times 10^2 \text{ m s}^2\right) = 1.4 \times 10^8 \quad .$$

d he spaceship ill travel a distance $d = 0.1\ c$ month during one month. he time it takes for the spaceship to travel at constant speed for 5.0 light months is

$$t=\frac{d}{v}=\frac{5.0\ \text{c}\cdot\text{months}}{0.1c}=50\ \text{months}\ \approx\ 4.2\ \text{years.}$$

91. **THINK** e have a motorcycle going up a ramp at a constant acceleration. e apply Newton's second law to calculate the net force on the rider and the force on the rider from the motorcycle.

EXPRESS he free body diagram is sho n to the right not to scale . ote that $F_{m\ r_y}$ and $F_{m\ r_x}$ respectively denote the y and x components of the force $\vec{F}_{m\ r}$ e erted by the motorcycle on the rider. he net force on the rider is



$$F_{\text{net}}=ma.$$

ANALYZE a ince the net force e uals ma then the magnitude of the net force on the rider is

$$F_{\text{net}}=ma=\ 60.0\ \text{kg}\quad 3.0\ \text{m s}^2\ =1.8\times10^2\quad .$$

(b) To calculate the force by the motorcycle on the rider, we apply Newton's second law to the x- and the y a es separately. or the x a is e have

$$F_{m\ r_x}-mg\sin\theta=ma$$

here m = 60.0 kg a = 3.0 m s^2 and θ = 10 . hus $F_{m\ r_x}=282$. pplying it to the y-a is here there is no acceleration e have

$$F_{m\ r_y}-mg\cos\theta=0$$

hich gives $F_{m\ r_y}=579$. sing the ythagorean theorem e find

$$F_{m\ r}=\sqrt{F^2_{m\ r_x}+F^2_{m\ r_y}}=\sqrt{\ 282\quad ^2+\ 579\quad ^2}=644\quad .$$

o the magnitude of the force e erted on the rider by the motorcycle is the same magnitude of force e erted by the rider on the motorcycle so the ans er is 644 .

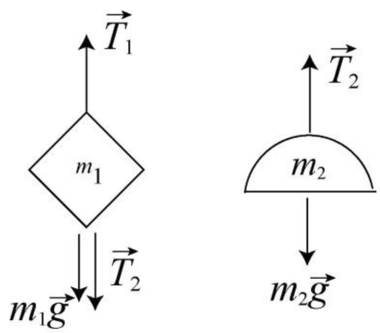
LEARN he force e erted by the motorcycle on the rider keeps the rider accelerating in the x direction hile maintaining contact ith the inclines surface $a_y=0$.

92. e denote the thrust as T and choose y up ard. Newton's second law leads to

$$T - Mg = Ma \;\Rightarrow\; a = \frac{2.6\times10^5}{1.3\times10^4\text{ kg}} - 9.8\text{ m s}^2 = 10\text{ m s}^2.$$

93. **THINK** n this problem e have mobiles consisting of masses connected by cords. We apply Newton's second law to calculate the tensions in the cords.

EXPRESS he free body diagrams for m_1 and m_2 for part a are sho n to the right.



he bottom cord is only supporting m_2 = 4.5 kg against gravity so its tension is $T_2 = m_2 g$. n the other hand the top cord is supporting a total mass of m_1 m_2 = 3.5 kg 4.5 kg = 8.0 kg against gravity. Applying Newton's second law gives

$$T_1 - T_2 - m_1 g = 0$$

so the tension is

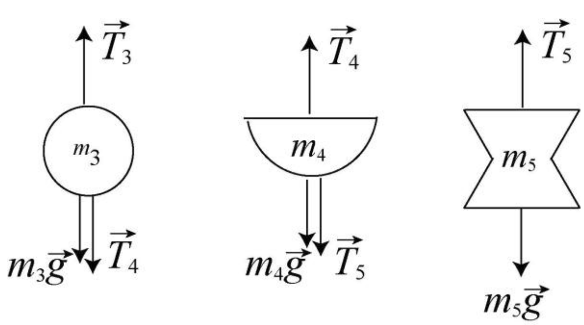
$$T_1 = m_1 g + T_2 = \; m_1 + m_2 \; g.$$

ANALYZE a rom the e uations above e find the tension in the bottom cord to be

$$T_2 = m_2 g = \; 4.5\text{ kg} \;\; 9.8\text{ m s}^2 \; = 44 \quad .$$

b imilarly the tension in the top cord is $T_1 = \; m_1 \;\; m_2 \; g = \; 8.0\text{ kg} \;\; 9.8\text{ m s}^2 \; = 78$.

c he free body diagrams for m_3 m_4 and m_5 for part b are sho n belo not to scale .



rom the diagram e see that the lo est cord supports a mass of m_5 = 5.5 kg against gravity and conse uently has a tension of

$$T_5 = m_5 g = 5.5 \text{ kg} \quad 9.8 \text{ m s}^2 = 54 \quad .$$

d he top cord as e are told has a tension T_3 =199 hich supports a total of 199 9.80 m s^2 = 20.3 kg 10.3 kg of hich is already accounted for in the figure. hus the unkno n mass in the middle must be m_4 = 20.3 kg – 10.3 kg = 10.0 kg and the tension in the cord above it must be enough to support

$$m_4 \quad m_5 = 10.0 \text{ kg} \quad 5.50 \text{ kg} = 15.5 \text{ kg}$$

so T_4 = 15.5 kg 9.80 m s^2 = 152 .

LEARN nother ay to calculate T_4 is to e amine the forces on m_3 – one of the do n ard forces on it is T_4. From Newton's second law, we have $T_3 - m_3 g - T_4 = 0$ hich can be solved to give

$$T_4 = T_3 - m_3 g = 199 \quad - 4.8 \text{ kg} \quad 9.8 \text{ m s}^2 = 152 \quad .$$

94. he coordinate choices are made in the problem statement.

a e rite the velocity of the armadillo as $\vec{v} = v_x \hat{\imath} + v_y \hat{\ }$. ince there is no net force e erted on it in the x direction the x component of the velocity of the armadillo is a constant v_x = 5.0 m s. n the y direction at t = 3.0 s e have using . 2 11 ith $v_{0y} = 0$

$$v_y = v_{0y} + a_y t = v_{0y} + \left(\frac{F_y}{m}\right) t = \left(\frac{17}{12 \text{ kg}}\right)(3.0 \text{ s}) = 4.3 \text{ m s}.$$

hus $\vec{v} =$ 5.0 m s $\hat{\text{i}} +$ 4.3 m s $\hat{\ }$.

b e rite the position vector of the armadillo as $\vec{r} = r_x \hat{\text{i}} + r_y \hat{\ }$. t t = 3.0 s e have r_x = 5.0 m s 3.0 s = 15 m and using . 2 15 ith v_{0y} = 0

$$r_y = v_{0y} t + \frac{1}{2} a_y t^2 = \frac{1}{2}\left(\frac{F_y}{m}\right) t^2 = \frac{1}{2}\left(\frac{17}{12 \text{ kg}}\right)(3.0 \text{ s})^2 = 6.4 \text{ m}.$$

he position vector at t = 3.0 s is therefore $\vec{r} =$ 15 m $\hat{\text{i}} +$ 6.4 m $\hat{\ }$.

95. a ntuition readily leads to the conclusion that the heavier block should be the hanging one, for largest acceleration. The force that "drives" the system into motion is the eight of the hanging block gravity acting on the block on the table has no effect on the dynamics so long as e ignore friction . hus m = 4.0 kg.

he acceleration of the system and the tension in the cord can be readily obtained by solving

$$mg - T = ma \qquad T = Ma.$$

b he acceleration is given by $a = \left(\frac{m}{m+M}\right) g = 6.5\text{ m s}^2$.

c he tension is

$$T = Ma = \left(\frac{Mm}{m+M}\right) g = 13 \quad .$$

96. According to Newton's second law, the magnitude of the force is given by $F = ma$ here a is the magnitude of the acceleration of the neutron. e use kinematics able 2 1 to find the acceleration that brings the neutron to rest in a distance d. ssuming the acceleration is constant then $v^2 = v_0^2 + 2ad$ produces the value of a

$$a = \frac{(v^2 - v_0^2)}{2d} = \frac{-(1.4 \times 10^7 \text{ m s})^2}{2(1.0 \times 10^{-14}\text{ m})} = -9.8 \times 10^{27}\text{ m s}^2.$$

he magnitude of the force is conse uently

$$F = ma = (1.67 \times 10^{-27}\text{ kg})\,(9.8 \times 10^{27}\text{ m s}^2) = 16 \quad .$$

97. a ith units understood the net force is

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = (3.0 \quad + (-2.0 \quad))\hat{\text{i}} + (4.0 \quad + (-6.0 \quad))\hat{}$$

hich yields $\vec{F}_{\text{net}} = 1.0 \quad \hat{\text{i}} - 2.0 \quad \hat{}$.

b he magnitude of $\vec{F}_{\text{net}}$ is $F_{\text{net}} = \sqrt{1.0 \quad ^2 + -2.0 \quad ^2} = 2.2 \quad$.

c he angle of $\vec{F}_{\text{net}}$ is $\theta = \tan^{-1}\left(\frac{-2.0}{1.0}\right) = -63°$.

d he magnitude of $\vec{a}$ is $a = F_{\text{net}} \quad m = 2.2 \qquad 1.0\text{ kg} = 2.2\text{ m s}^2$.

e ince $\vec{F}_{\text{net}}$ is e ual to $\vec{a}$ multiplied by mass m hich is a positive scalar that cannot affect the direction of the vector it multiplies $\vec{a}$ has the same angle as the net force i.e $\theta = -63°$. n magnitude angle notation e may rite $\vec{a} = (2.2\text{ m s}^2 \angle -63°)$.

Chapter

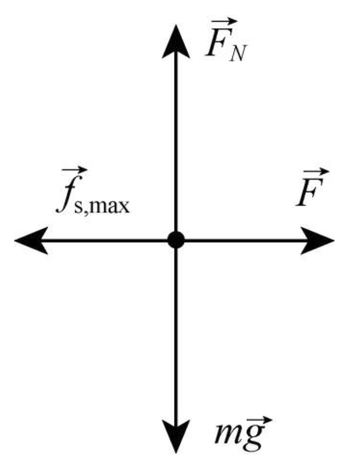
1. he greatest deceleration of magnitude a is provided by the ma imum friction force . 6 1 ith F_N = mg in this case). Using Newton's second law, we find

$$a = f_{s\ ma} \quad m = \mu_s g.$$

. 2 16 then gives the shortest distance to stop Δx = v^2 2a = 36 m. n this calculation it is important to first convert v to 13 m s.

2. Applying Newton's second law to the horizontal motion, we have $F - \mu_k m g = ma$ here e have used . 6 2 assuming that F_N = mg hich is e uivalent to assuming that the vertical force from the broom is negligible . . 2 16 relates the distance traveled and the final speed to the acceleration $v^2 = 2a\Delta x$. his gives a = 1.4 m s^2. eturning to the force e uation e find ith F = 25 and m = 3.5 kg that μ_k = 0.58.

3. **THINK** n the presence of friction bet een the floor and the bureau a minimum hori ontal force must be applied before the bureau ould begin to move.



EXPRESS he free body diagram for the bureau is sho n to the right. e denote $\vec{F}$ as the hori ontal force of the person $\vec{f}_s$ is the force of static friction in the $-x$ direction F_N is the vertical normal force e erted by the floor in the y direction and $m\vec{g}$ is the force of gravity. e do not consider the possibility that the bureau might tip and treat this as a purely hori ontal motion problem ith the person's push $\vec{F}$ in the x direction). Applying Newton's second la to the x and y a es e obtain

$$F - f_{s\ ma} = ma$$
$$F_N - mg = 0$$

respectively.

he second e uation yields the normal force F_N = mg hereupon the ma imum static friction is found to be from . 6 1 $f_{s\ ma} = \mu_s mg$. hus the first e uation becomes

$$F - \mu_s mg = ma = 0$$

here e have set a = 0 to be consistent ith the fact that the static friction is still ust barely able to prevent the bureau from moving.

ANALYZE a ith $\mu_s = 0.45$ and m = 45 kg the e uation above leads to

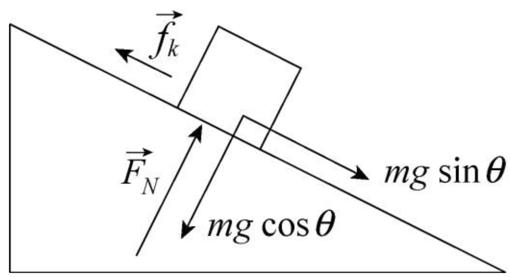
$$F = \mu_s mg = \ 0.45 \quad 45\,\text{kg} \quad 9.8\,\text{m s}^2 \ = 198 \quad .$$

o bring the bureau into a state of motion the person should push ith any force greater than this value. ounding to t o significant figures e can therefore say the minimum re uired push is $F = 2.0 \times 10^2$.

b eplacing m = 45 kg ith m = 28 kg the reasoning above leads to roughly $F = 1.2\times10^2$.

LEARN he values found above represent the minimum force re uired to overcome the friction. pplying a force greater than $\mu_s mg$ results in a net force in the x direction and hence non ero acceleration.

4. e first analy e the forces on the pig of mass m. he incline angle is θ.



he x direction is "downhill.'' Application of Newton's second law to the x and y a es leads to

$$mg\sin\theta - f_k = ma$$
$$F_N - mg\cos\theta = 0.$$

olving these along ith . 6 2 $f_k = \mu_k F_N$) produces the following result for the pig's do nhill acceleration

$$a = g(\sin\theta - \mu_k\cos\theta) .$$

o compute the time to slide from rest through a do nhill distance ℓ e use . 2 15

$$\ell = v_0 t + \frac{1}{2}at^2 \;\Rightarrow\; t = \sqrt{\frac{2\ell}{a}}.$$

e denote the frictionless $\mu_k = 0$ case ith a prime and set up a ratio

$$\frac{t}{t'} = \frac{\sqrt{2\ell \;\; a}}{\sqrt{2\ell \;\; a'}} = \sqrt{\frac{a'}{a}}$$

hich leads us to conclude that if $t\ t' = 2$ then $a' = 4a$. utting in hat e found out above about the accelerations e have

$$g\sin\theta = 4g\left(\sin\theta - \mu_k\cos\theta\right).$$

sing θ = 35 e obtain μ_k = 0.53.

5. n addition to the forces already sho n in ig. 6 17 a free body diagram ould include an up ard normal force $\vec{F}_N$ e erted by the floor on the block a do n ard $m\vec{g}$ representing the gravitational pull e erted by arth and an assumed left ard $\vec{f}$ for the kinetic or static friction. e choose x right ards and y upwards. We apply Newton's second la to these a es

$$F - f = ma$$
$$P + F_N - mg = 0$$

here F = 6.0 and m = 2.5 kg is the mass of the block.

a n this case P = 8.0 leads to

$$F_N = \ 2.5\text{ kg}\ \ 9.8\text{ m s}^2\ - 8.0\ \ = 16.5\ \ .$$

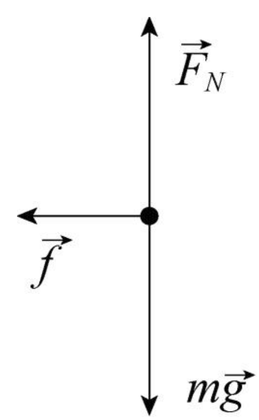
sing . 6 1 this implies $f_{s\,\text{ma}} = \mu_s F_N = 6.6$ hich is larger than the 6.0 right ard force – so the block hich as initially at rest does not move. utting a = 0 into the first of our e uations above yields a static friction force of $f = P$ = 6.0 .

b n this case P = 10 the normal force is

$$F_N = \ 2.5\text{ kg}\ \ 9.8\text{ m s}^2\ - 10\ \ = 14.5\ \ .$$

sing . 6 1 this implies $f_{s\,\text{ma}} = \mu_s F_N = 5.8$ hich is less than the 6.0 right ard force – so the block does move. ence e are dealing not ith static but ith kinetic friction hich . 6 2 reveals to be $f_k = \mu_k F_N = 3.6$.

c n this last case P = 12 leads to F_N = 12.5 and thus to $f_{s\,\text{ma}} = \mu_s F_N = 5.0$ hich as e pected is less than the 6.0 right ard force – so the block moves. he kinetic friction force then is $f_k = \mu_k F_N = 3.1$.

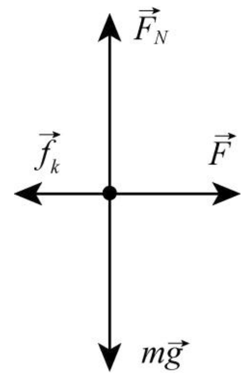


6. he free body diagram for the player is sho n to the right. $\vec{F}_N$ is the normal force of the ground on the player $m\vec{g}$ is the force of gravity and $\vec{f}$ is the force of friction. he force of friction is related to the normal force by $f = \mu_k F_N$. We use Newton's second la applied to the vertical a is to find the normal force. he vertical component of the acceleration is ero so e obtain $F_N - mg$ = 0 thus $F_N = mg$. onse uently

$$\mu_k = \frac{f}{F_N} = \frac{470}{(79\,\text{kg})\,(9.8\,\text{m s}^2)} = 0.61.$$

7. **THINK** force is being applied to accelerate a crate in the presence of friction. e apply Newton's second law to solve for the acceleration.

EXPRESS he free body diagram for the crate is sho n to the right. e denote $\vec{F}$ as the hori ontal force of the person e erted on the crate in the x direction $\vec{f}_k$ is the force of kinetic friction in the $-x$ direction F_N is the vertical normal force e erted by the floor in the y direction and $m\vec{g}$ is the force of gravity. he magnitude of the force of friction is given by . 6 2 $f_k = \mu_k F_N$. Applying Newton's second la to the x and y a es e obtain



$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively.

ANALYZE a he second e uation above yields the normal force $F_N = mg$ so that the friction is

$$f_k = \mu_k F_N = \mu_k mg = (0.35)(55\text{ kg})\;\; 9.8\text{ m s}^2 \;\; = 1.9 \times 10^2 \quad .$$

b he first e uation becomes

$$F - \mu_k mg = ma$$

hich ith F = 220 e solve to find

$$a = \frac{F}{m} - \mu_k g = \frac{220}{55\,\text{kg}} - \;\; 0.35 \;\; 9.8\,\text{m s}^2 \;\; = 0.56\text{ m s}^2.$$

LEARN or the crate to accelerate the condition $F > f_k = \mu_k mg$ must be met. s can be seen from the e uation above the greater the value of μ_k the smaller the acceleration under the same applied force.

8. To maintain the stone's motion, a horizontal force (in the $+x$ direction is needed that cancels the retarding effect due to kinetic friction. Applying Newton's second to the x and y a es e obtain

$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively. he second e uation yields the normal force $F_N = mg$ so that using . 6 2 the kinetic friction becomes $f_k = \mu_k\, mg$. hus the first e uation becomes

$$F - \mu_k mg = ma = 0$$

here e have set a = 0 to be consistent ith the idea that the hori ontal velocity of the stone should remain constant. ith m = 20 kg and μ_k = 0.80 e find $F = 1.6 \times 10^2$.

9. e choose x hori ontally right ards and y up ards and observe that the 15 force has components $F_x = F\cos\theta$ and $F_y = -F\sin\theta$.

(a) We apply Newton's second law to the y a is

$$F_N - F\sin\theta - mg = 0 \;\Rightarrow\; F_N = \;15 \quad \sin 40° + \;3.5\text{ kg}\;\; 9.8\text{ m s}^2 \;\; = 44 \quad .$$

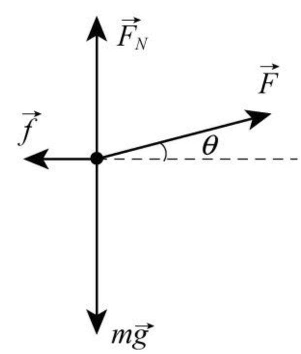
ith μ_k = 0.25 . 6 2 leads to f_k = 11 .

(b) We apply Newton's second law to the x a is

$$F\cos\theta - f_k = ma \;\Rightarrow\; a = \frac{(15 \quad)\cos 40° - 11}{3.5\text{ kg}} = 0.14\text{ m s}^2 .$$

ince the result is positive valued then the block is accelerating in the x right ard direction.

10. a he free body diagram for the block is sho n belo ith $\vec{F}$ being the force applied to the block $\vec{F}_N$ the normal force of the floor on the block $m\vec{g}$ the force of gravity and $\vec{f}$ the force of friction.



e take the x direction to be hori ontal to the right and the y direction to be up. he e uations for the x and the y components of the force according to Newton's second law are

$$F_x = F\cos\theta - f = ma$$
$$F_y = F\sin\theta + F_N - mg = 0$$

o $f = \mu_k F_N$ and the second e uation gives $F_N = mg - F\sin\theta$ hich yields $f = \mu_k \;\; mg - F\sin\theta$. his e pression is substituted for f in the first e uation to obtain

$$F\cos\theta - \mu_k \;\; mg - F\sin\theta \;\; = ma$$

so the acceleration is

$$a = \frac{F}{m}(\cos\theta + \mu_k \sin\theta) - \mu_k g .$$

a f $\mu_s = 0.600$ and $\mu_k = 0.500$ then the magnitude of $\vec{f}$ has a ma imum value of

$$f_{s\,\mathrm{ma}} = \mu_s F_N = 0.600\ \ mg - 0.500mg\sin 20°\ \ = 0.497mg.$$

n the other hand $F\cos\theta = 0.500mg\cos 20° = 0.470mg$. herefore $F\cos\theta < f_{s\,\mathrm{ma}}$ and the block remains stationary ith $a = 0$.

b f $\mu_s = 0.400$ and $\mu_k = 0.300$ then the magnitude of $\vec{f}$ has a ma imum value of

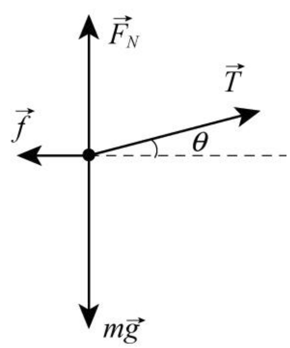
$$f_{s\,\mathrm{ma}} = \mu_s F_N = 0.400\ \ mg - 0.500mg\sin 20°\ \ = 0.332mg.$$

n this case $F\cos\theta = 0.500mg\cos 20° = 0.470mg > f_{s\,\mathrm{ma}}$. herefore the acceleration of the block is

$$\begin{aligned} a &= \frac{F}{m}(\cos\theta + \mu_k \sin\theta) - \mu_k g \\ &= \ 0.500\ \ 9.80\ \mathrm{m\ s}^2\ [\cos 20° + \ 0.300\ \sin 20°] - \ 0.300\ \ 9.80\ \mathrm{m\ s}^2 \\ &= 2.17\ \mathrm{m\ s}^2. \end{aligned}$$

11. **THINK** ince the crate is being pulled by a rope at an angle ith the hori ontal e need to analy e the force components in both the x and y directions.

EXPRESS he free body diagram for the crate is sho n to the right. ere $\vec{T}$ is the tension force of the rope on the crate $\vec{F}_N$ is the normal force of the floor on the crate $m\vec{g}$ is the force of gravity and $\vec{f}$ is the force of friction. e take the x direction to be hori ontal to the right and the y direction to be up. e assume the crate is motionless.



he e uations for the x and the y components of the force according to Newton's second la are

$$T\cos\theta - f = 0 \qquad T\sin\theta + F_N - mg = 0$$

here θ = 15 is the angle bet een the rope and the hori ontal. he first e uation gives $f = T\cos\theta$ and the second gives $F_N = mg - T\sin\theta$. f the crate is to remain at rest f must be less than $\mu_s F_N$ or $T\cos\theta \quad \mu_s\ \ mg - T\sin\theta$. hen the tension force is sufficient to ust start the crate moving e must have $T\cos\theta = \mu_s\ \ mg - T\sin\theta$.

ANALYZE a e solve for the tension

$$T = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} = \frac{(0.50)\,(68\text{ kg})\,\left(9.8\text{ m s}^2\right)}{\cos 15° + 0.50\sin 15°} = 304 \quad \approx 3.0\times 10^2 \quad .$$

b he second la e uations for the moving crate are

$$T\cos\theta - f = ma \qquad T\sin\theta + F_N - mg = 0\,.$$

o $f = \mu_k F_N$ and the second e uation above gives $F_N = mg - T\sin\theta$ hich then yields $f = \mu_k\; mg - T\sin\theta$. his e pression is substituted for f in the first e uation to obtain

$$T\cos\theta - \mu_k\; mg - T\sin\theta\; = ma$$

so the acceleration is

$$a = \frac{T\left(\cos\theta + \mu_k\sin\theta\right)}{m} - \mu_k g$$

$$= \frac{304 \quad \cos 15° + 0.35\sin 15°}{68\text{ kg}} - \;0.35\;\; 9.8\text{ m s}^2 \;\; = 1.3\text{ m s}^2.$$

LEARN Let's check the limit where $\theta = 0$. n this case e recover the familiar e pressions $T = \mu_s mg$ and $a = \; T - \mu_k mg \quad m$.

12. here is no acceleration so the up ard static friction forces there are four of them one for each thumb and one for each set of opposing fingers e uals the magnitude of the do n ard pull of gravity. sing . 6 1 e have

$$4\mu_s F_N = mg = \; 79\text{ kg} \;\; 9.8\text{ m s}^2$$

hich ith μ_s = 0.70 yields $F_N = 2.8\times 10^2$.

13. e denote the magnitude of 110 force e erted by the orker on the crate as F. he magnitude of the static frictional force can vary bet een ero and $f_{s\text{ ma}} = \mu_s F_N$.

(a) In this case, application of Newton's second la in the vertical direction yields $F_N = mg$. hus

$$f_{s\text{ ma}} = \mu_s F_N = \mu_s mg = (0.37)(35\text{kg})\;\; 9.8\text{m s}^2 \;\; = 1.3\times 10^2$$

hich is greater than F.

b he block hich is initially at rest stays at rest since $F \quad f_{s\text{ ma}}$. hus it does not move.

c By applying Newton's second la to the hori ontal direction that the magnitude of the frictional force e erted on the crate is $f_s = 1.1\times 10^2$.

d enoting the up ard force e erted by the second orker as F_2 then application of Newton's second law in the vertical direction yields $F_N = mg - F_2$ hich leads to

$$f_{s\,\text{ma}} = \mu_s F_N = \mu_s\ mg - F_2\ .$$

n order to move the crate F must satisfy the condition $F\ \ f_{s,\text{ma}} = \mu_s\ \ mg - F_2)$, or

$$110\ \ > (0.37)\left[\ 35\,\text{kg}\ \ 9.8\,\text{m s}^2\ - F_2\right].$$

he minimum value of F_2 that satisfies this ine uality is a value slightly bigger than 45.7 so e e press our ans er as $F_{2\ \text{min}} = 46$.

e n this final case moving the crate re uires a greater hori ontal push from the orker than static friction (as computed in part (a)) can resist. Thus, Newton's law in the hori ontal direction leads to

$$F + F_2 > f_{s\ \text{ma}} \quad \Rightarrow \quad 110\ \ + F_2 > 126.9$$

hich leads after appropriate rounding to $F_{2\ \text{min}} = 17$.

14. a sing the result obtained in ample roblem – "Friction, applied force at an angle," the maximum angle for which static friction applies is

$$\theta_{\text{ma}} = \tan^{-1}\mu_s = \tan^{-1}0.63 \approx 32^\circ\ .$$

his is greater than the dip angle in the problem so the block does not slide.

(b) Applying Newton's second law, we have

$$F + mg\sin\theta - f_{s\ \text{ma}} = ma = 0$$
$$F_N - mg\cos\theta = 0.$$

long ith . 6 1 $f_{s\ \text{ma}} = \mu_s F_N$ e have enough information to solve for F. ith $\theta = 24^\circ$ and $m = 1.8 \times 10^7$ kg e find

$$F = mg\left(\mu_s\cos\theta - \sin\theta\right) = 3.0\times 10^7\ \ .$$

15. n e cellent discussion and e uation development related to this problem is given in ample roblem – "Friction, applied force at an angle." We merely quote (and apply) their main result

$$\theta = \tan^{-1}\mu_s = \tan^{-1}0.04 \approx 2^\circ\ .$$

16. a n this situation e take $\vec{f}_s$ to point uphill and to be e ual to its ma imum value in hich case $f_{s\ \text{ma}} = \mu_s F_N$ applies here $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W\ g$ = 8.2 kg in the x and y directions produces

$$F_{\text{min 1}} - mg\sin\theta + f_{s\ \text{ma}} = ma = 0$$
$$F_N - mg\cos\theta = 0$$

hich ith θ = 20 leads to

$$F_{\text{min 1}} - mg\left(\sin\theta + \mu_s\cos\theta\right) = 8.6 \quad .$$

b o e take $\vec{f}_s$ to point do nhill and to be e ual to its ma imum value in hich case $f_{s\ \text{ma}} = \mu_s F_N$ applies here $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W\ g$ = 8.2 kg in the x and y directions produces

$$F_{\text{min 2}} = mg\sin\theta - f_{s\ \text{ma}} = ma = 0$$
$$F_N - mg\cos\theta = 0$$

hich ith θ = 20 leads to

$$F_{\text{min 2}} = mg\left(\sin\theta + \mu_s\cos\theta\right) = 46 \quad .$$

A value slightly larger than the "exact" result of this calculation is re uired to make it accelerate uphill but since e uote our results here to t o significant figures 46 is a "good enough" answer.

c inally e are dealing ith kinetic friction pointing do nhill so that

$$0 = F - mg\sin\theta - f_k = ma$$
$$0 = F_N - mg\cos\theta$$

along ith $f_k = \mu_k F_N$ here μ_k = 0.15 brings us to

$$F = mg\left(\sin\theta + \mu_k\cos\theta\right) = 39 \quad .$$

17. f the block is sliding then e compute the kinetic friction from . 6 2 if it is not sliding, then we determine the extent of static friction from applying Newton's law, with ero acceleration to the x a is hich is parallel to the incline surface . he uestion of hether or not it is sliding is therefore crucial and depends on the ma imum static friction force as calculated from . 6 1. he forces are resolved in the incline plane coordinate system in igure 6 5 in the te tbook. he acceleration if there is any is along the x a is and e are taking uphill as x. he net force along the y a is then is certainly ero hich provides the follo ing relationship

$$\sum \vec{F}_y = 0 \Rightarrow F_N = W\cos\theta$$

here $W = mg = 45$ is the eight of the block and $\theta = 15$ is the incline angle. hus $F_N = 43.5$ hich implies that the ma imum static friction force should be

$$f_{s\,\text{ma}} = 0.50 \quad 43.5 \quad = 21.7 \quad .$$

a or $\vec{P} = -5.0 \quad \hat{\text{i}}$, Newton's second law, applied to the x a is becomes

$$f - P \quad - mg\sin\theta = ma .$$

ere e are assuming $\vec{f}$ is pointing uphill as sho n in igure 6 5 and if it turns out that it points do nhill hich *is* a possibility then the result for f_s ill be negative. f $f = f_s$ then $a = 0$ e obtain

$$f_s = P \quad mg\sin\theta = 5.0 \quad 43.5 \quad \sin 15 \quad = 17$$

or $\vec{f}_s = 17 \quad \hat{\text{i}}$. his is clearly allo ed since f_s is less than $f_{s\,\text{ma}}$.

b or $\vec{P} = -8.0 \quad \hat{\text{i}}$ e obtain from the same e uation $\vec{f}_s = 20 \quad \hat{\text{i}}$ hich is still allo ed since it is less than $f_{s\,\text{ma}}$.

c ut for $\vec{P} = -15 \quad \hat{\text{i}}$ e obtain from the same e uation $f_s = 27$ hich is not allo ed since it is larger than $f_{s\,\text{ma}}$. hus e conclude that it is the kinetic friction instead of the static friction that is relevant in this case. he result is

$$\vec{f}_k = \mu_k F_N \hat{\text{i}} = 0.34 \quad 43.5 \quad \hat{\text{i}} = 15 \quad \hat{\text{i}} .$$

18. a We apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma$$

here using . 6 11

$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta .$$

hus ith $\mu_k = 0.600$ e have

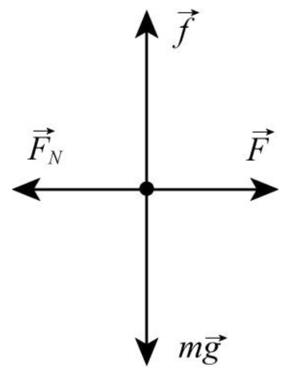
$$a = g\sin\theta - \mu_k\cos\theta = -3.72 \text{ m s}^2$$

hich means since e have chosen the positive direction in the direction of motion do n the slope then the acceleration vector points "uphill"; it is decelerating. With $v_0 = 18.0$ m s and $\Delta x = d = 24.0$ m . 2 16 leads to

$$v=\sqrt{v_0^2+2ad}=12.1 \text{ m s}.$$

b n this case e find a = 1.1 m s^2 and the speed hen impact occurs is 19.4 m s.

19. a he free body diagram for the block is sho n belo . $\vec{F}$ is the applied force $\vec{F}_N$ is the normal force of the all on the block $\vec{f}$ is the force of friction and $m\vec{g}$ is the force of gravity. o determine if the block falls e find the magnitude f of the force of friction re uired to hold it ithout accelerating and also find the normal force of the all on the block. e compare f and $\mu_s F_N$. f f $\mu_s F_N$ the block does not slide on the all but if f $\mu_s F_N$ the block does slide. he horizontal component of Newton's second law is $F - F_N = 0$ so $F_N = F = 12$ and



$$\mu_s F_N = 0.60 \quad 12 \quad = 7.2 \quad .$$

he vertical component is $f - mg = 0$ so $f = mg = 5.0$. ince f $\mu_s F_N$ the block does not slide.

b ince the block does not move $f = 5.0$ and $F_N = 12$. he force of the all on the block is

$$\vec{F}_w = -F_N\,\hat{\mathrm{i}} + f\,\hat{} = -(12 \quad)\,\hat{\mathrm{i}} + (5.0 \quad)\,\hat{}$$

here the a es are as sho n on ig. 6 26 of the te t.

20. reating the t o bo es as a single system of total mass m m =1.0 3.0 = 4.0 kg sub ect to a total left ard friction of magnitude 2.0 4.0 = 6.0 e apply Newton's second law (with $+x$ right ard

$$F - f_{\text{total}} = m_{\text{total}}\, a \;\Rightarrow\; 12.0 \quad -6.0 \quad = \; 4.0 \text{ kg}\; a$$

hich yields the acceleration a = 1.5 m s^2. e have treated F as if it ere kno n to the nearest tenth of a Newton so that our acceleration is "good" to two significant figures. urning our attention to the larger bo the heaties bo of mass m = 3.0 kg e apply Newton's second law to find the contact force F' e erted by the heerios bo on it.

$$F' - f \quad = m \quad a \quad \Rightarrow \quad F' - 4.0 \quad = \; 3.0 \text{ kg} \;\; 1.5 \text{ m s}^2 \;.$$

rom the above e uation e find the contact force to be F' = 8.5 .

21. ig. 6 4 in the te tbook sho s a similar situation using ϕ for the unkno n angle along ith a free body diagram. e use the same coordinate system as in that figure.

(a) Thus, Newton’s second law leads to

$$x \qquad T\cos\phi - f = ma$$
$$y \quad T\sin\phi + F_N - mg = 0$$

etting a = 0 and f = $f_{s\,\text{ma}}$ = $\mu_s F_N$ e solve for the mass of the bo and sand as a function of angle

$$m = \frac{T}{g}\left(\sin\phi + \frac{\cos\phi}{\mu_s}\right)$$

hich e ill solve ith calculus techni ues to find the angle ϕ_m corresponding to the ma imum mass that can be pulled .

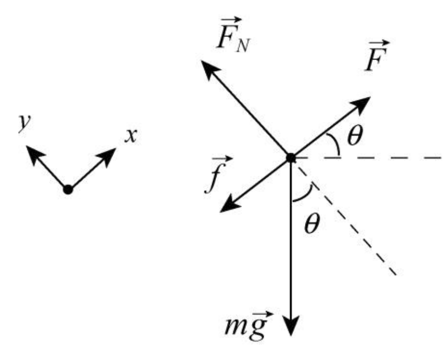
$$\frac{dm}{dt} = \frac{T}{g}\left(\cos\phi_m - \frac{\sin\phi_m}{\mu_s}\right) = 0$$

his leads to tan $\phi_m = \mu_s$ hich for $\mu_s = 0.35$ yields $\phi_m = 19°$.

b lugging our value for ϕ_m into the e uation e found for the mass of the bo and sand yields m = 340 kg. his corresponds to a eight of mg = 3.3 × 10^3 .

22. he free body diagram for the sled is sho n belo ith $\vec{F}$ being the force applied to the sled $\vec{F}_N$ the normal force of the inclined plane on the sled $m\vec{g}$ the force of gravity and $\vec{f}$ the force of friction.

e take the x direction to be along the inclined plane and the y direction to be in its normal direction. he e uations for the x and the y components of the force according to Newton’s second law are



$$F_x = F - f - mg\sin\theta = ma = 0$$
$$F_y = F_N - mg\cos\theta = 0$$

o $f = \mu F_N$ and the second e uation gives F_N = $mg\cos\theta$ hich yields $f = \mu mg\cos\theta$. his e pression is substituted for f in the first e uation to obtain

$$F = mg\ \sin\theta + \mu\cos\theta$$

rom the figure e see that $F = 2.0$ hen $\mu = 0$. his implies $mg\sin\theta = 2.0$. imilarly e also find $F = 5.0$ hen $\mu = 0.5$

$$5.0 \quad = mg\ \sin\theta + 0.50\cos\theta\ = 2.0 \quad + 0.50mg\cos\theta$$

hich yields $mg\cos\theta = 6.0$. ombining the t o results e get

$$\tan\theta = \frac{2}{6} = \frac{1}{3} \;\;\Rightarrow\;\; \theta = 18°.$$

23. et the tensions on the strings connecting m_2 and m_3 be T_{23} and that connecting m_2 and m_1 be T_{12} respectively. Applying Newton's second law (and Eq. 6 2 ith $F_N = m_2 g$ in this case to the *system* e have

$$\begin{aligned} m_3 g - T_{23} &= m_3 a \\ T_{23} - \mu_k m_2 g - T_{12} &= m_2 a \\ T_{12} - m_1 g &= m_1 a \end{aligned}$$

dding up the three e uations and using $m_1 = M \;\; m_2 = m_3 = 2M$ e obtain

$$2Mg - 2\mu_k Mg - Mg = 5Ma .$$

ith a = 0.500 m s^2 this yields μ_k = 0.372. hus the coefficient of kinetic friction is roughly μ_k = 0.37.

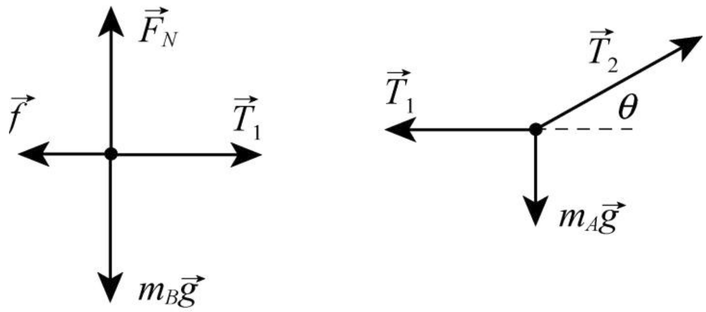
24. e find the acceleration from the slope of the graph recall . 2 11 a = 4.5 m s^2. Thus, Newton's second law leads to

$$F - \mu_k mg = ma$$

here F = 40.0 is the constant hori ontal force applied. ith m = 4.1 kg e arrive at μ_k =0.54.

25. **THINK** n order that the t o blocks remain in e uilibrium friction must be present bet een block B and the surface.

EXPRESS he free body diagrams for block B and for the knot ust above block A are sho n belo . $\vec{T}_1$ is the tension force of the rope pulling on block B or pulling on the knot as the case may be $\vec{T}_2$ is the tension force e erted by the second rope at angle θ = 30 on the knot $\vec{f}$ is the force of static friction e erted by the hori ontal surface on block B $\vec{F}_N$ is normal force e erted by the surface on block B W_A is the eight of block A W_A is the magnitude of $m_A\vec{g}$ and W_B is the eight of block B W_B = 711 is the magnitude of $m_B\vec{g}$.



or each ob ect e take x hori ontally right ard and y upward. Applying Newton's second la in the x and y directions for block B and then doing the same for the knot results in four e uations

$$\begin{aligned} T_1 - f_{s\,\text{ma}} &= 0 \\ F_N - W_B &= 0 \\ T_2 \cos\theta - T_1 &= 0 \\ T_2 \sin\theta - W_A &= 0 \end{aligned}$$

here e assume the static friction to be at its ma imum value permitting us to use . 6 1 . he above e uations yield $T_1 = \mu_s F_N$ $F_N = W_B$ and $T_1 = T_2 \cos\theta$.

ANALYZE olving these e uations ith $\mu_s = 0.25$ e obtain

$$\begin{aligned} W_A &= T_2 \sin\theta = T_1 \tan\theta = \mu_s F_N \tan\theta = \mu_s W_B \tan\theta \\ &= 0.25 \quad 711 \quad \tan 30^\circ = 1.0\times 10^2 \end{aligned}$$

LEARN s e pected the ma imum eight of A is proportional to the eight of B as ell as the coefficient of static friction. n addition e see that W_A is proportional to $\tan\theta$ the larger the angle the greater the vertical component of T_2 that supports its eight .

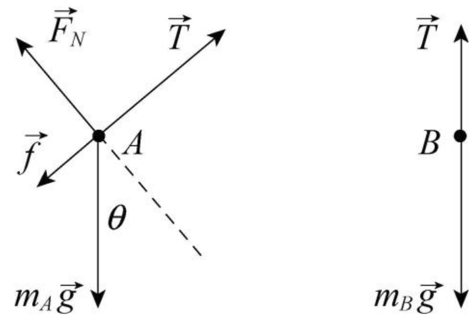
26. a Applying Newton's second law to the *system* of total mass M = 60.0 kg and using . 6 2 ith $F_N = Mg$ in this case e obtain

$$F - \mu_k Mg = Ma \;\Rightarrow\; a = 0.473 \text{ m s}^2.$$

e t e e amine the forces ust on m_3 and find $F_{32} = m_3\; a \quad \mu_k g$ = 147 . f the algebra steps are done more systematically one ends up ith the interesting relationship $F_{32} = \; m_3 \;\; M \;\; F$ hich is independent of the friction .

b s remarked at the end of our solution to part a the result does not depend on the frictional parameters. he ans er here is the same as in part a .

27. irst e check to see if the bodies start to move. e assume they remain at rest and compute the force of static friction hich holds them there and compare its magnitude ith the ma imum value $\mu_s F_N$. he free body diagrams are sho n belo .



T is the magnitude of the tension force of the string f is the magnitude of the force of friction on body A F_N is the magnitude of the normal force of the plane on body A $m_A\vec{g}$ is the force of gravity on body A ith magnitude W_A = 102 and $m_B\vec{g}$ is the force of gravity on body B ith magnitude W_B = 32 . θ = 40 is the angle of incline. e are told the direction of $\vec{f}$ but e assume it is do nhill. f e obtain a negative result for f then e kno the force is actually up the plane.

a or A e take the x to be uphill and y to be in the direction of the normal force. he x and y components of Newton's second law become

$$T - f - W_A \sin\theta = 0$$
$$F_N - W_A \cos\theta = 0.$$

aking the positive direction to be *downward* for body B, Newton's second law leads to $W_B - T = 0$. olving these three e uations leads to

$$f = W_B - W_A \sin\theta = 32 \quad - 102 \quad \sin 40° = -34$$

indicating that the force of friction is *uphill* and to

$$F_N = W_A \cos\theta = 102 \quad \cos 40° = 78$$

hich means that

$$f_{s\,\mathrm{ma}} = \mu_s F_N = 0.56 \quad 78 \quad = 44 \quad .$$

ince the magnitude f of the force of friction that holds the bodies motionless is less than $f_{s\,\mathrm{ma}}$ the bodies remain at rest. he acceleration is ero.

b ince A is moving up the incline the force of friction is do nhill ith magnitude $f_k = \mu_k F_N$. Newton's second law, using the same coordinates as in part (a), leads to

$$T - f_k - W_A \sin\theta = m_A a$$
$$F_N - W_A \cos\theta = 0$$
$$W_B - T = m_B a$$

for the t o bodies. e solve for the acceleration

$$a = \frac{W_B - W_A \sin\theta - \mu_k W_A \cos\theta}{m_B + m_A} = \frac{32 \quad -(102 \quad)\sin 40° - (0.25)(102 \quad)\cos 40°}{(32 \quad 102 \quad)/(9.8\ \text{m/s}^2)}$$
$$= -3.9\ \text{m/s}^2.$$

he acceleration is do n the plane i.e. $\vec{a} = -3.9$ m s^2 $\hat{\text{i}}$ hich is to say since the initial velocity as uphill that the ob ects are slo ing do n. e note that $m = W\ g$ has been used to calculate the masses in the calculation above.

c o body A is initially moving do n the plane so the force of friction is uphill ith magnitude $f_k = \mu_k F_N$. he force e uations become

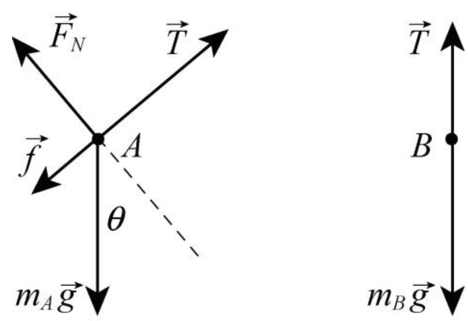
$$T + f_k - W_A \sin\theta = m_A a$$
$$F_N - W_A \cos\theta = 0$$
$$W_B - T = m_B a$$

hich e solve to obtain

$$a = \frac{W_B - W_A \sin\theta + \mu_k W_A \cos\theta}{m_B + m_A} = \frac{32 \quad -(102 \quad)\sin 40° + (0.25)(102 \quad)\cos 40°}{(32 \quad 102 \quad)/(9.8\ \text{m/s}^2)}$$
$$= -1.0\ \text{m/s}^2.$$

he acceleration is again do nhill the plane i.e. $\vec{a} = -1.0$ m s^2 $\hat{\text{i}}$. n this case the ob ects are speeding up.

28. he free body diagrams are sho n to the right here T is the magnitude of the tension force of the string f is the magnitude of the force of friction on block A F_N is the magnitude of the normal force of the plane on block A $m_A\vec{g}$ is the force of gravity on body A here m_A = 10 kg and $m_B\vec{g}$ is the force of gravity on block B. θ = 30 is the angle of incline. or A e take the x to be uphill and y to be in the direction of the normal force the positive direction is chosen *downward* for block B.



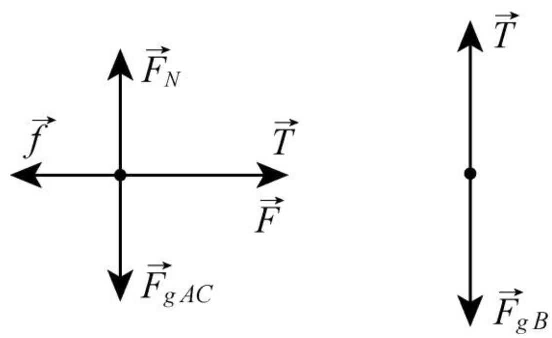
ince A is moving do n the incline the force of friction is uphill ith magnitude $f_k = \mu_k F_N$ here $\mu_k = 0.20$). Newton's second law leads to

$$T - f_k + m_A g \sin\theta = m_A a = 0$$
$$F_N - m_A g \cos\theta = 0$$
$$m_B g - T = m_B a = 0$$

for the t o bodies here a = 0 is a conse uence of the velocity being constant . e solve these for the mass of block B.

$$m_B = m_A\left(\sin\theta - \mu_k \cos\theta\right) = 3.3 \text{ kg.}$$

29. a ree body diagrams for the blocks A and C considered as a single ob ect and for the block B are sho n belo .



T is the magnitude of the tension force of the rope F_N is the magnitude of the normal force of the table on block A f is the magnitude of the force of friction W_{AC} is the combined eight of blocks A and C the magnitude of force $\vec{F}_{g\,AC}$ sho n in the figure and W_B is the eight of block B the magnitude of force $\vec{F}_{g\,B}$ sho n . ssume the blocks are not moving. or the blocks on the table e take the x a is to be to the right and the y a is to be up ard. rom Newton's second law, we have

$$x \text{ component} \qquad T - f = 0$$
$$y \text{ component} \qquad F_N - W_{AC} = 0.$$

or block B take the downward direction to be positive. Then Newton's second law for that block is $W_B - T = 0$. he third e uation gives $T = W_B$ and the first gives $f = T = W_B$. he second e uation gives $F_N = W_{AC}$. f sliding is not to occur f must be less than $\mu_s F_N$ or W_B $\mu_s W_{AC}$. he smallest that W_{AC} can be ith the blocks still at rest is

$$W_{AC} = W_B\ \mu_s = 22 \quad 0.20 = 110 \quad .$$

ince the eight of block A is 44 the least eight for C is 110 – 44 = 66 .

b he second la e uations become

$$T - f = W_A\ g\ a$$

$$F_N - W_A \ = 0$$
$$W_B - T = \ W_B \ g \ a.$$

n addition $f = \mu_k F_N$. he second e uation gives $F_N = W_A$ so $f = \mu_k W_A$. he third gives $T = W_B - \ W_B \ g \ a$. ubstituting these t o e pressions into the first e uation e obtain

$$W_B - \ W_B \ g \ a - \mu_k W_A = \ W_A \ g \ a.$$

herefore

$$a = \frac{g\left(W_B - \mu_k W_A\right)}{W_A + W_B} = \frac{9.8 \text{ m s}^2 \ \left(22 \quad - (0.15)(44 \quad)\right)}{44 \qquad 22} = 2.3 \text{ m s}^2.$$

30. e use the familiar hori ontal and vertical a es for x and y directions ith right ard and up ard positive respectively. he rope is assumed massless so that the force e erted by the child $\vec{F}$ is identical to the tension uniformly through the rope. he x and y components of $\vec{F}$ are $F\cos\theta$ and $F\sin\theta$ respectively. he static friction force points left ard.

(a) Newton's Law applied to the y a is here there is presumed to be no acceleration leads to

$$F_N + F\sin\theta - mg = 0$$

hich implies that the ma imum static friction is $\mu_s \ mg - F \sin \theta$. f $f_s = f_{s \ \text{ma}}$ is assumed then e ton's second law applied to the x a is hich also has $a = 0$ even though it is "verging" on moving) yields

$$F\cos\theta - f_s = ma \quad \Rightarrow \quad F\cos \ \theta - \mu_s \ mg - F\sin\theta \ = 0$$

hich e solve for $\theta = 42$ and $\mu_s = 0.42$ to obtain $F = 74$.

b olving the above e uation algebraically for F ith W denoting the eight e obtain

$$F = \frac{\mu_s W}{\cos\theta + \mu_s \sin\theta} = \frac{0.42 \ \ 180}{\cos\theta + \ 0.42 \ \ \sin\theta} = \frac{76}{\cos\theta + \ 0.42 \ \ \sin\theta}.$$

c e minimi e the above e pression for F by orking through the condition

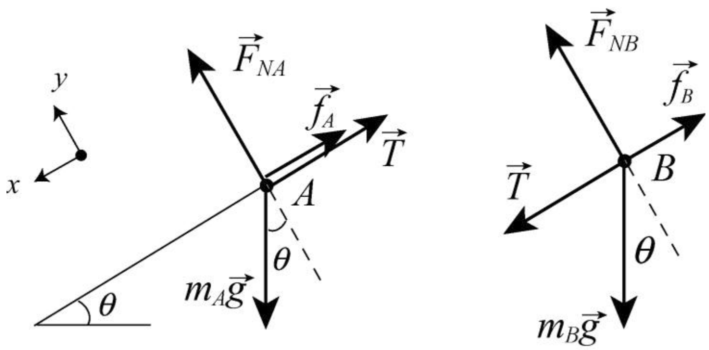
$$\frac{dF}{d\theta} = \frac{\mu_s W \ \sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \ \sin \ \theta \ ^2} = 0$$

hich leads to the result $\theta = \tan^{-1} \mu_s = 23$.

d lugging $\theta = 23$ into the above result for F ith $\mu_s = 0.42$ and $W = 180$ yields $F = 70$.

31. **THINK** n this problem e have t o blocks connected by a string sliding do n an inclined plane the blocks have different coefficient of kinetic friction.

EXPRESS he free body diagrams for the t o blocks are sho n belo . T is the magnitude of the tension force of the string $\vec{F}_{NA}$ is the normal force on block A the leading block $\vec{F}_{NB}$ is the normal force on block B $\vec{f}_A$ is kinetic friction force on block A $\vec{f}_B$ is kinetic friction force on block B. lso m_A is the mass of block A here m_A = W_A g and W_A = 3.6 and m_B is the mass of block B here m_B = W_B g and W_B = 7.2 . he angle of the incline is θ = 30 .



or each block e take x do nhill hich is to ard the lo er left in these diagrams and y in the direction of the normal force. Applying Newton's second law to the x and y directions of both blocks A and B e arrive at four e uations

$$W_A \sin\theta - f_A - T = m_A a$$
$$F_{NA} - W_A \cos\theta = 0$$
$$W_B \sin\theta - f_B + T = m_B a$$
$$F_{NB} - W_B \cos\theta = 0$$

hich hen combined ith . 6 2 $f_A = \mu_{kA} F_{NA}$ here $\mu_{k\,A}$ = 0.10 and $f_B = \mu_{kB} F_{NB} f_B$ here $\mu_{k\,B}$ = 0.20 fully describe the dynamics of the system so long as the blocks have the same acceleration and T 0.

ANALYZE a rom these e uations e find the acceleration to be

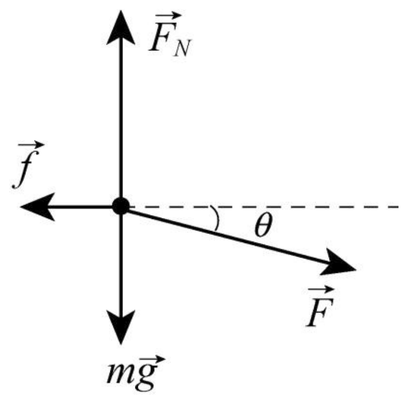
$$a = g\left(\sin\theta - \left(\frac{\mu_{k\,A} W_A + \mu_{k\,B} W_B}{W_A + W_B}\right)\cos\theta\right) = 3.5 \text{ m s}^2.$$

b e solve the above e uations for the tension and obtain

$$T = \left(\frac{W_A W_B}{W_A + W_B}\right)\,(\mu_{k\,B} - \mu_{k\,A})\cos\theta = \frac{3.6 \quad 7.2}{3.6 \quad +7.2}(0.20 - 0.10)\cos 30^\circ = 0.21 \quad .$$

LEARN he tension in the string is proportional to $\mu_{kB} - \mu_{kA}$ the difference in coefficients of kinetic friction for the t o blocks. hen the coefficients are e ual $\mu_{kB} = \mu_{kA}$, the t o blocks can be vie ed as moving independent of one another and the tension is ero. imilarly hen $\mu_{kB} < \mu_{kA}$ the leading block A has larger coefficient than the B the string is slack so the tension is also ero.

32. he free body diagram for the block is sho n belo ith $\vec{F}$ being the force applied to the block $\vec{F}_N$ the normal force of the floor on the block $m\vec{g}$ the force of gravity and $\vec{f}$ the force of friction. e take the x direction to be hori ontal to the right and the y direction to be up. he e uations for the x and the y components of the force according to Newton's second la are



$$F_x = F\cos\theta - f = ma$$
$$F_y = F_N - F\sin\theta - mg = 0$$

o $f = \mu_k F_N$ and the second e uation gives $F_N = mg$ $F\sin\theta$ hich yields

$$f = \mu_k \; mg + F\sin\theta \; .$$

his e pression is substituted for f in the first e uation to obtain

$$F\cos\theta - \mu_k \; mg \quad F\sin\theta \; = ma$$

so the acceleration is

$$a = \frac{F}{m}\left(\cos\theta - \mu_k \sin\theta\right) - \mu_k g \, .$$

rom the figure e see that $a = 3.0 \text{ m s}^2$ hen $\mu_k = 0$. his implies

$$3.0 \text{ m s}^2 = \frac{F}{m}\cos\theta.$$

e also find $a = 0$ hen $\mu_k = 0.20$

$$0 = \frac{F}{m}\left(\cos\theta - \;\; 0.20 \;\; \sin\theta\right) - \;\; 0.20 \;\; 9.8 \text{ m s}^2 \;\; = 3.00 \text{ m s}^2 - 0.20\frac{F}{m}\sin\theta - 1.96 \text{ m s}^2$$
$$= 1.04 \text{ m s}^2 - 0.20\frac{F}{m}\sin\theta$$

hich yields $5.2 \text{ m s}^2 = \frac{F}{m}\sin\theta.$ ombining the t o results e get

$$\tan\theta = \left(\frac{5.2\ \text{m s}^2}{3.0\ \text{m s}^2}\right) = 1.73 \quad \Rightarrow \quad \theta = 60°.$$

33. **THINK** n this problem the frictional force is not a constant but instead proportional to the speed of the boat. ntegration is re uired to solve for the speed.

EXPRESS e denote the magnitude of the frictional force as αv here $\alpha = 70\ \ \cdot \text{s/m}$. We take the direction of the boat's motion to be positive. Newton's second law gives

$$-\alpha v = m\frac{dv}{dt} \quad \Rightarrow \quad \frac{dv}{v} = -\frac{\alpha}{m}dt.$$

ntegrating the e uation gives

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{\alpha}{m}\int_0^t dt$$

here v_0 is the velocity at time ero and v is the velocity at time t. olving the integral allo s us to calculate the time it takes for the boat to slo do n to 45 km h or $v = v_0$ 2 here $v_0 = 90$ km h .

ANALYZE he integrals are evaluated ith the result

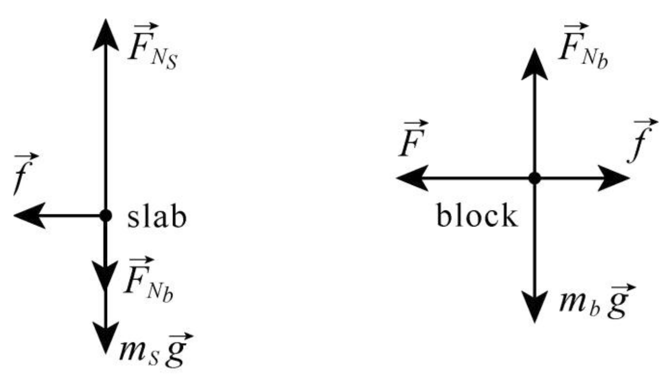
$$\ln\left(\frac{v}{v_0}\right) = -\frac{\alpha t}{m}$$

ith $v = v_0$ 2 and $m = 1000$ kg e find the time to be

$$t = -\frac{m}{\alpha}\ln\left(\frac{v}{v_0}\right) = -\frac{m}{\alpha}\ln\left(\frac{1}{2}\right) = -\frac{1000\ \text{kg}}{70\ \ \cdot \text{s m}}\ln\left(\frac{1}{2}\right) = 9.9\ \text{s}.$$

LEARN he speed of the boat is given by $v\ t = v_0 e^{-\alpha t\ m}$ sho ing e ponential decay ith time. he greater the value of α the more rapidly the boat slo s do n.

34. he free body diagrams for the slab and block are sho n belo .



$\vec{F}$ is the 100 force applied to the block $\vec{F}_{Ns}$ is the normal force of the floor on the slab F_{Nb} is the magnitude of the normal force bet een the slab and the block $\vec{f}$ is the force of friction bet een the slab and the block m_s is the mass of the slab and m_b is the mass of the block. or both ob ects e take the x direction to be to the right and the y direction to be up.

Applying Newton's second law for the x and y a es for first the slab and second the block results in four e uations

$$\begin{aligned} -f &= m_s a_s \\ F_{Ns} - F_{Ns} - m_s g &= 0 \\ f - F &= m_b a_b \\ F_{Nb} - m_b g &= 0 \end{aligned}$$

from hich e note that the ma imum possible static friction magnitude ould be

$$\mu_s F_{Nb} = \mu_s m_b g = 0.60 \quad 10\text{ kg} \quad 9.8\text{ m s}^2 = 59 \quad .$$

e check to see if the block slides on the slab. ssuming it does not then $a_s = a_b$ hich e denote simply as a and e solve for f

$$f = \frac{m_s F}{m_s + m_b} = \frac{40\text{ kg} \quad 100}{40\text{ kg} + 10\text{ kg}} = 80$$

hich is greater than $f_{s\,ma}$ so that e conclude the block is sliding across the slab their accelerations are different .

a sing $f = \mu_k F_{Nb}$ the above e uations yield

$$a_b = \frac{\mu_k m_b g - F}{m_b} = \frac{0.40 \quad 10\text{ kg} \quad 9.8\text{ m s}^2 \quad -100}{10\text{ kg}} = -6.1\text{ m s}^2.$$

he negative sign means that the acceleration is left ard. hat is $\vec{a}_b = -6.1\text{ m s}^2\ \hat{\text{i}}$

b e also obtain

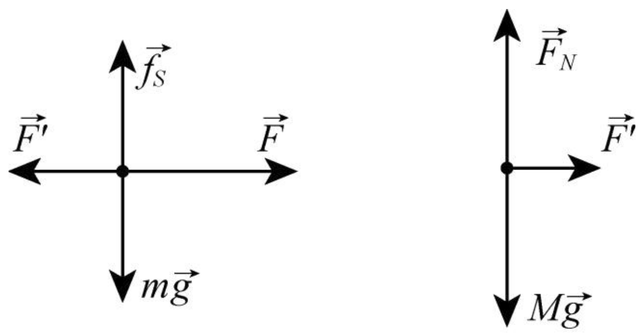
$$a_s = -\frac{\mu_k m_b g}{m_s} = -\frac{0.40 \quad 10\text{ kg} \quad 9.8\text{ m s}^2}{40\text{ kg}} = -0.98\text{ m s}^2.$$

s mentioned above this means it accelerates to the left. hat is $\vec{a}_s = -0.98\text{ m s}^2\ \hat{\text{i}}$

35. he free body diagrams for the t o blocks treated individually are sho n belo first m and then M . F' is the contact force bet een the t o blocks and the static friction force $\vec{f}_s$ is at its ma imum value so . 6 1 leads to $f_s = f_{s\,\text{ma}} = \mu_s F'$ here $\mu_s = 0.38$.

reating the t o blocks together as a single system sliding across a frictionless floor we apply Newton's second law (with $+x$ right ard to find an e pression for the acceleration

$$F = m_{\text{total}}\, a \quad \Rightarrow a = \frac{F}{m + M}$$



his is e uivalent to having analy ed the t o blocks individually and then combined their equations. Now, when we analyze the small block individually, we apply Newton's second la to the x and y a es substitute in the above e pression for a and use . 6 1.

$$F - F = ma \quad \Rightarrow \quad F = F - m\left(\frac{F}{m + M}\right)$$
$$f_s - mg = 0 \quad \Rightarrow \quad \mu_s F - mg = 0$$

hese e pressions are combined to eliminate F' and e arrive at

$$F = \frac{mg}{\mu_s\left(1 - \dfrac{m}{m + M}\right)} = 4.9\times10^2 \quad .$$

36. sing . 6 16 e solve for the area $A \dfrac{2m\,g}{C\rho\, v_t^2}$ hich illustrates the inverse proportionality bet een the area and the speed s uared. hus hen e set up a ratio of areas – of the slo er case to the faster case – e obtain

$$\frac{A_{\text{slo}}}{A_{\text{fast}}} = \left(\frac{310 \text{ km} \quad \text{h}}{160 \text{ km} \quad \text{h}}\right)^2 = 3.75.$$

37. n the solution to e ercise 4 e found that the force provided by the ind needed to e ual F = 157 here that last figure is not "significant'').

a etting $F = D$ for rag force e use . 6 14 to find the ind speed v along the ground hich actually is relative to the moving stone but e assume the stone is moving slo ly enough that this does not invalidate the result

$$v = \sqrt{\frac{2F}{C\rho A}} = \sqrt{\frac{2\ 157}{0.80\ \ 1.21\,\text{kg m}^3\ \ 0.040\ \text{m}^2}} = 90\ \text{m s} = 3.2\times 10^2\ \text{km h}.$$

b oubling our previous result e find the reported speed to be 6.5 × 10^2 km h.

c he result is not reasonable for a terrestrial storm. category 5 hurricane has speeds on the order of 2.6 × 10^2 m s.

38. a rom able 6 1 and . 6 16 e have

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} \Rightarrow C\rho A = 2\frac{mg}{v_t^2}$$

here v_t = 60 m/s. We estimate the pilot's mass at about m = 70 kg. o e convert v = 1300 1000 3600 ≈ 360 m s and plug into . 6 14

$$D = \frac{1}{2}C\rho A v^2 = \frac{1}{2}\left(2\frac{mg}{v_t^2}\right)v^2 = mg\left(\frac{v}{v_t}\right)^2$$

hich yields D = 70 kg 9.8 m s^2 360 60 ^{2} ≈ 2 × 10^4 .

b e assume the mass of the e ection seat is roughly e ual to the mass of the pilot. Thus, Newton's second law (in the horizontal direction) applied to this system of mass 2m gives the magnitude of acceleration

$$|a| = \frac{D}{2m} = \frac{g}{2}\left(\frac{v}{v_t}\right)^2 = 18g\ .$$

39. or the passenger et $D_j = \frac{1}{2}C\rho_1 A v_j^2$ and for the prop driven transport $D_t = \frac{1}{2}C\rho_2 A v_t^2$ here ρ_1 and ρ_2 represent the air density at 10 km and 5.0 km respectively. hus the ratio in uestion is

$$\frac{D_j}{D_t} = \frac{\rho_1 v_j^2}{\rho_2 v_t^2} = \frac{(0.38\ \text{kg m}^3)(1000\ \text{km h})^2}{(0.67\ \text{kg m}^3)(500\ \text{km h})^2} = 2.3.$$

40. This problem involves Newton's second law for motion along the slope.

a he force along the slope is given by

$$F_g = mg\sin\theta - \mu F_N = mg\sin\theta - \mu mg\cos\theta = mg\ \sin\theta - \mu\cos\theta$$
$$= 85.0\text{ kg}\ \ 9.80\text{ m s}^2\ \left[\sin 40.0° - 0.04000\ \cos 40.0°\right]$$
$$= 510\ \ .$$

hus the terminal speed of the skier is

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} = \sqrt{\frac{2\ 510}{0.150\ \ 1.20\text{ kg m}^3\ \ 1.30\text{ m}^2}} = 66.0\text{ m s.}$$

b ifferentiating v_t ith respect to C e obtain

$$dv_t = -\frac{1}{2}\sqrt{\frac{2F_g}{\rho A}}\,C^{-3\ 2}dC = -\frac{1}{2}\sqrt{\frac{2\ 510}{1.20\text{ kg m}^3\ \ 1.30\text{ m}^2}}\ \ 0.150\ ^{-3\ 2}dC$$
$$= -\ 2.20\times10^2\ \text{ m s } dC.$$

41. erhaps surprisingly the e uations pertaining to this situation are e actly those in ample roblem – "Car in flat circular turn," although the logic is a little different. In the ample roblem the car moves along a stationary road hereas in this problem the cat is stationary relative to the merry go around platform. ut the static friction plays the same role in both cases since the bottom most point of the car tire is instantaneously at rest ith respect to the race track ust as static friction applies to the contact surface bet een cat and platform. sing . 6 23 ith . 4 35 e find

$$\mu_s = \ 2\pi R/T\ ^2\ gR = 4\pi^2 R/gT^2.$$

ith T = 6.0 s and R = 5.4 m e obtain μ_s = 0.60.

42. he magnitude of the acceleration of the car as it rounds the curve is given by $v^2\ R$ here v is the speed of the car and R is the radius of the curve. ince the road is hori ontal only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2\ R$. f F_N is the normal force of the road on the car and m is the mass of the car the vertical component of Newton's second law leads to $F_N = mg$. hus using . 6 1 the ma imum value of static friction is

$$f_{s\ \text{ma}} = \mu_s F_N = \mu_s mg.$$

f the car does not slip $f \le \mu_s mg$. his means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad v \le \sqrt{\mu_s R g}.$$

onse uently the ma imum speed ith hich the car can round the curve ithout slipping is

$$v_{\text{ma}} = \sqrt{\mu_s R g} = \sqrt{0.60\ \ 30.5\text{ m}\ \ 9.8\text{ m s}^2} = 13\text{ m s} \approx 48\text{ km h}.$$

43. he magnitude of the acceleration of the cyclist as it rounds the curve is given by v^2 R here v is the speed of the cyclist and R is the radius of the curve. ince the road is hori ontal only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second la is $f = mv^2$ R. f F_N is the normal force of the road on the bicycle and m is the mass of the bicycle and rider the vertical component of Newton's second law leads to $F_N = mg$. hus using . 6 1 the ma imum value of static friction is

$$f_{s\,\text{ma}} = \mu_s F_N = \mu_s mg.$$

f the bicycle does not slip $f \le \mu_s mg$. his means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad R \ge \frac{v^2}{\mu_s g}.$$

onse uently the minimum radius ith hich a cyclist moving at 29 km h = 8.1 m s can round the curve ithout slipping is

$$R_{\text{min}} = \frac{v^2}{\mu_s g} = \frac{8.1\text{ m s}\ ^2}{0.32\ \ 9.8\text{ m s}^2} = 21\text{ m}.$$

44. ith v = 96.6 km h = 26.8 m s . 6 17 readily yields

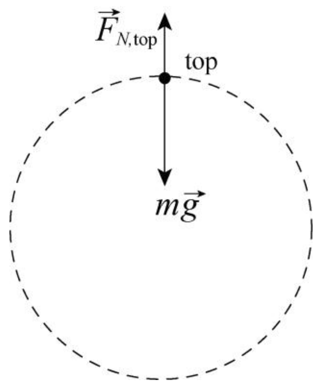
$$a = \frac{v^2}{R} = \frac{26.8\text{ m s}\ ^2}{7.6\text{ m}} = 94.7\text{ m s}^2$$

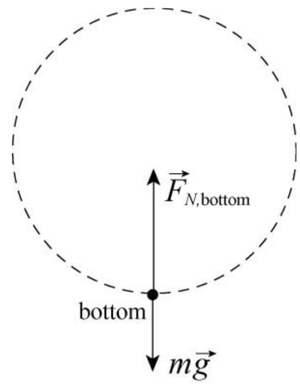
hich e e press as a multiple of g

$$a = \left(\frac{a}{g}\right) g = \left(\frac{94.7\text{ m s}^2}{9.80\text{ m s}^2}\right) g = 9.7g.$$

45. **THINK** erris heel ride is a vertical circular motion. he apparent eight of the rider varies ith his position.

EXPRESS he free body diagrams of the student at the top and bottom of the erris heel are sho n ne t





t the top the highest point in the circular motion the seat pushes up on the student ith a force of magnitude $F_{N\text{ top}}$ hile the arth pulls do n ith a force of magnitude mg. Newton's second law for the radial direction gives

$$mg - F_{N\text{ top}} = \frac{mv^2}{R}.$$

t the bottom of the ride $F_{N\text{ bottom}}$ is the magnitude of the up ard force e erted by the seat. he net force to ard the center of the circle is choosing up ard as the positive direction

$$F_{N\text{ bottom}} - mg = \frac{mv^2}{R}.$$

The Ferris wheel is "steadily rotating" so the value $F_c = mv^2 \quad R$ is the same every here. he apparent eight of the student is given by F_N.

ANALYZE a t the top e are told that $F_{N\text{ top}} = 556$ and $mg = 667$. his means that the seat is pushing up with a force that is smaller than the student's weight, and we say the student experiences a decrease in his "apparent weight" at the highest point. hus he feels "light."

b rom a e find the centripetal force to be

$$F_c = \frac{mv^2}{R} = mg - F_{N\text{ top}} = 667 \quad - 556 \quad = 111 \quad .$$

hus the normal force at the bottom is

$$F_{N\text{ bottom}} = \frac{mv^2}{R} + mg = F_c + mg = 111 \quad + 667 \quad = 778 \quad .$$

c f the speed is doubled

$$F_c' = \frac{m \ 2v \ ^2}{R} = 4 \ 111 \quad = 444 \quad .$$

herefore at the highest point e have

$$F'_{N\,\text{top}} = mg - F'_c = 667 \quad - 444 \quad = 223 \quad .$$

d imilarly the normal force at the lo est point is no found to be

$$F'_{N\,\text{bottom}} = F'_c + mg = 444 \quad + 667 \quad = 1111 \quad .$$

LEARN he apparent eight of the student is the greatest at the bottom and smallest at the top of the ride. he speed $v = \sqrt{gR}$ ould result in $F_{N\,\text{top}} = 0$ giving the student a sudden sensation of "weightlessness" at the top of the ride.

46. a e note that the speed 80.0 km h in units is roughly 22.2 m s. he hori ontal force that keeps her from sliding must e ual the centripetal force . 6 18 and the up ard force on her must e ual mg. hus

$$F_{\text{net}} = \sqrt{\ mg\ ^2 \quad m\text{v}^2\ R\ ^2} = 547 \quad .$$

b he angle is

$$\tan^{-1}[\ mv^2\ R\ /\ mg\ ] = \tan^{-1}(v^2\ gR) = 9.53$$

as measured from a vertical a is.

47. a . 4 35 gives $T = 2\pi R\ v = 2\pi\ 10\text{ m} \quad 6.1\text{ m s} = 10\text{ s}$.

b he situation is similar to that of ample roblem – "Vertical circular loop, Diavolo," but ith the normal force direction reversed. dapting . 6 19 e find

$$F_N = m\ g - v^2/R\ = 486 \quad \approx 4.9 \times 10^2 \quad .$$

c o e reverse both the normal force direction and the acceleration direction from hat is sho n in ample roblem – "Vertical circular loop, Diavolo") and adapt Eq. 6 19 accordingly. hus e obtain

$$F_N = m\ g + v^2/R\ = 1081 \quad \approx 1.1\text{ k} \quad .$$

48. e ill start by assuming that the normal force on the car from the rail points up. ote that gravity points do n and the y a is is chosen positive up ards. lso the direction to the center of the circle the direction of centripetal acceleration is do n. Thus, Newton's second law leads to

$$F_N - mg = m\left(-\frac{v^2}{r}\right).$$

a hen $v = 11$ m s e obtain $F_N = 3.7 \times 10^3 \quad .$

b $\vec{F}_N$ points up ard.

c   hen v = 14 m s e obtain F_N = –1.3 × 10^3 or F_N = 1.3 × 10^3 .

d he fact that this ans er is negative means that $\vec{F}_N$ points opposite to hat e had assumed. hus the magnitude of $\vec{F}_N$ is $\vec{F}_N$ $=$1.3 k and its direction is *down*.

49. t the top of the hill the situation is similar to that of ample roblem – "Vertical circular loop, Diavolo," but with the normal force direction reversed. Adapting Eq. 6 19 e find

$$F_N = m\ g - v^2/R\ .$$

 ince F_N = 0 there as stated in the problem then $v^2 = gR$. ater at the bottom of the valley e reverse both the normal force direction and the acceleration direction from hat is sho n in the ample roblem and adapt . 6 19 accordingly. hus e obtain

$$F_N = m\ g + v^2/R\ = 2mg = 1372 \quad \approx 1.37 \times 10^3 \quad .$$

50. he centripetal force on the passenger is $F = mv^2\ \ r$.

a he slope of the plot at $v = 8.30$ m s is

$$\left.\frac{dF}{dv}\right|_{v=8.30\text{ m s}} = \left.\frac{2mv}{r}\right|_{v=8.30\text{ m s}} = \frac{2\ 85.0\text{ kg}\ \ 8.30\text{ m s}}{3.50\text{ m}} = 403 \quad \cdot\text{s m}.$$

b he period of the circular ride is $T = 2\pi r\ \ v$. hus

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2}$$

and the variation of F ith respect to T hile holding r constant is

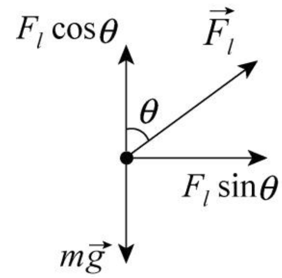
$$dF = -\frac{8\pi^2 mr}{T^3}\,dT.$$

 he slope of the plot at $T = 2.50$ s is

$$\left.\frac{dF}{dT}\right|_{T=2.50\text{ s}} = -\left.\frac{8\pi^2 mr}{T^3}\right|_{T=2.50\text{ s}} = \frac{8\pi^2\ 85.0\text{ kg}\ \ 3.50\text{ m}}{2.50\text{ s}\ ^3} = -1.50\times 10^3 \quad \text{s}.$$

51. **THINK** n airplane ith its ings tilted at an angle is in a circular motion. entripetal force is involved in this problem.

EXPRESS he free body diagram for the airplane of mass m is sho n to the right. e note that $\vec{F}_l$ is the force of aerodynamic lift and $\vec{a}$ points right ards in the figure. e also note that $|\vec{a}| = v^2 \ R$. Applying Newton's law to the axes of the problem x right ard and y up ard e obtain



$$F_l \sin\theta = m\frac{v^2}{R}$$
$$F_l \cos\theta = mg$$

liminating mass from these e uations leads to $\tan\theta = \dfrac{v^2}{gR}$. he e uation allo s us to solve for the radius R.

ANALYZE ith v = 480 km h = 133 m s and θ = 40 e find

$$R = \frac{v^2}{g\tan\theta} = \frac{133\text{ m s }^2}{9.8\text{ m s}^2 \tan 40°} = 2151\text{ m} \approx 2.2\times 10^3\text{ m}.$$

LEARN ur approach to solving this problem is identical to that discussed in the ample roblem – "Car in banked circular turn." Do you see the similarities?

52. he situation is some hat similar to that sho n in the "loop the loop" example done in the te tbook see igure 6 10 e cept that instead of a do n ard normal force e are dealing ith the force of the boom $\vec{F}_B$ on the car – hich is capable of pointing any direction. e ill assume it to be upward as we apply Newton's second law to the car (of total eight 5000 $F_B - W = ma$ here $m = W \ g$ and $a = -v^2 \ r$. ote that the centripetal acceleration is do n ard our choice for negative direction for a body at the top of its circular tra ectory.

a f r = 10 m and v = 5.0 m s e obtain F_B = 3.7 × 10^3 = 3.7 k .

b he direction of $\vec{F}_B$ is up.

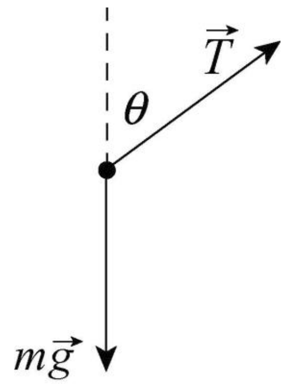
c f r = 10 m and v = 12 m s e obtain F_B = – 2.3 × 10^3 = – 2.3 k or F_B = 2.3 k .

d he minus sign indicates that $\vec{F}_B$ points do n ard.

53. he free body diagram for the hand straps of mass m is the vie that a passenger might see if she as looking for ard and the streetcar as curving to ards the right so $\vec{a}$ points right ards in the figure . e note that $|\vec{a}| = v^2 \ R$ here v = 16 km h = 4.4 m s.

Applying Newton's law to the axes of the problem (+x is right ard and y is up ard e obtain

$$T\sin\theta = m\frac{v^2}{R}$$
$$T\cos\theta = mg.$$



e solve these e uations for the angle

$$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$$

hich yields θ = 12 .

54. he centripetal force on the passenger is $F = mv^2 \;\; r$.

a he variation of F ith respect to r hile holding v constant is $dF = -\frac{mv^2}{r^2}dr$.

b he variation of F ith respect to v hile holding r constant is $dF = \frac{2mv}{r}dv$.

c he period of the circular ride is $T = 2\pi r \;\; v$. hus

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2}$$

and the variation of F ith respect to T hile holding r constant is

$$dF = -\frac{8\pi^2 mr}{T^3}dT = -8\pi^2 mr\left(\frac{v}{2\pi r}\right)^3 dT = -\left(\frac{mv^3}{\pi r^2}\right)dT.$$

55. e note that the period T is eight times the time bet een flashes $\frac{1}{2000}$ s so T = 0.0040 s. ombining . 6 18 ith . 4 35 leads to

$$F = \frac{4m\pi^2 R}{T^2} = \frac{4 \; 0.030 \text{ kg} \; \pi^2 \; 0.035 \text{ m}}{0.0040 \text{ s} \;\; ^2} = 2.6 \times 10^3 \quad .$$

56. e refer the reader to ample roblem – "Car in banked circular turn," and use the result . 6 26

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)$$

ith v = 60 1000 3600 = 17 m s and R = 200 m. he banking angle is therefore θ = 8.1 . o e consider a vehicle taking this banked curve at v' = 40 1000 3600 = 11 m s. ts hori ontal acceleration is $a' = v'^2 R$ hich has components parallel the incline and perpendicular to it

$$a = a'\cos\theta = \frac{v'^2\cos\theta}{R}$$
$$a_\perp = a'\sin\theta = \frac{v'^2\sin\theta}{R}.$$

These enter Newton's second law as follows (choosing downhill as the x direction and a ay from incline as y

$$mg\sin\theta - f_s = ma$$
$$F_N - mg\cos\theta = ma_\perp$$

and e are led to

$$\frac{f_s}{F_N} = \frac{mg\sin\theta - mv'^2\cos\theta \;\; R}{mg\cos\theta + mv'^2\sin\theta \;\; R}.$$

e cancel the mass and plug in obtaining f_s F_N = 0.078. he problem implies e should set f_s = $f_{s\,\mathrm{ma}}$ so that by . 6 1 e have μ_s = 0.078.

57. or the puck to remain at rest the magnitude of the tension force T of the cord must e ual the gravitational force Mg on the cylinder. he tension force supplies the centripetal force that keeps the puck in its circular orbit so $T = mv^2$ r. hus $Mg = mv^2$ r. e solve for the speed

$$v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{2.50\text{ kg}\;\; 9.80\text{ m s}^2\;\; 0.200\text{ m}}{1.50\text{ kg}}} = 1.81\text{ m s}.$$

58. a sing the kinematic e uation given in able 2 1 the deceleration of the car is

$$v^2 = v_0^2 + 2ad \quad\Rightarrow\quad 0 = \;35\text{ m s}\;^2 + 2a\;107\text{ m}$$

hich gives $a = –5.72$ m s^2. hus the force of friction re uired to stop by car is

$$f = m\;\; a\; = \;1400\text{ kg}\;\; 5.72\text{ m s}^2\;\; \approx 8.0\times10^3\;\; .$$

b he ma imum possible static friction is

$$f_{s\,\mathrm{ma}} = \mu_s mg = \;0.50\;\; 1400\text{ kg}\;\; 9.80\text{ m s}^2\;\; \approx 6.9\times10^3\;\; .$$

c f $\mu_k = 0.40$ then $f_k = \mu_k mg$ and the deceleration is $a = -\mu_k g$. herefore the speed of the car hen it hits the all is

$$v=\sqrt{v_0^2+2ad}=\sqrt{35\text{ m s}^{\ 2}-2\ 0.40\ \ 9.8\text{ m s}^2\ \ 107\text{ m}}\approx 20\text{ m s.}$$

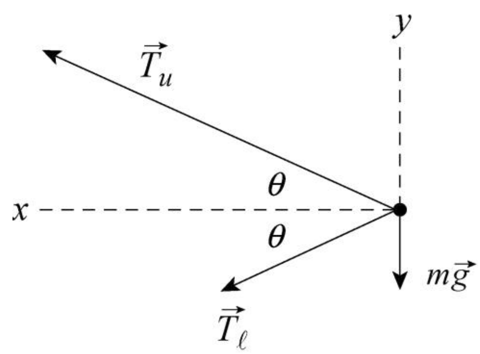
d he force re uired to keep the motion circular is

$$F_r=\frac{mv_0^2}{r}=\frac{1400\text{ kg}\ \ 35.0\text{ m s}^{\ 2}}{107\text{ m}}=1.6\times10^4\quad .$$

e ince $F_r > f_{s\,\text{ma}}$ no circular path is possible.

59. **THINK** s illustrated in ig. 6 45 our system consists of a ball connected by t o strings to a rotating rod. he tensions in the strings provide the source of centripetal force.

EXPRESS he free body diagram for the ball is sho n belo . $\vec{T}_u$ is the tension e erted by the upper string on the ball $\vec{T}_\ell$ is the tension in the lo er string and m is the mass of the ball. ote that the tension in the upper string is greater than the tension in the lo er string. t must balance the do n ard pull of gravity and the force of the lo er string.



 e take the x direction to be left ard to ard the center of the circular orbit and y up ard. ince the magnitude of the acceleration is $a = v^2\ R$ the x component of Newton's second law is

$$T_u\cos\theta+T_\ell\cos\theta=\frac{mv^2}{R}$$

 here v is the speed of the ball and R is the radius of its orbit. he y component is

$$T_u\sin\theta-T_\ell\sin\theta-mg=0.$$

 he second e uation gives the tension in the lo er string $T_\ell = T_u - mg\ \ \sin\theta$.

ANALYZE a ince the triangle is e uilateral the angle is θ = 30.0 . hus

$$T_\ell=T_u-\frac{mg}{\sin\theta}=35.0\quad -\frac{1.34\text{ kg}\ \ 9.80\text{ m s}^2}{\sin 30.0^\circ}=8.74\quad .$$

b he net force in the y direction is ero. n the x direction the net force has magnitude

$$F_{\text{net str}} = (T_u + T_\ell)\cos\theta = 35.0 \quad + 8.74 \quad \cos 30.0° = 37.9 \quad .$$

c he radius of the path is

$$R = L\cos\theta = 1.70 \text{ m} \cos 30° = 1.47 \text{ m}.$$

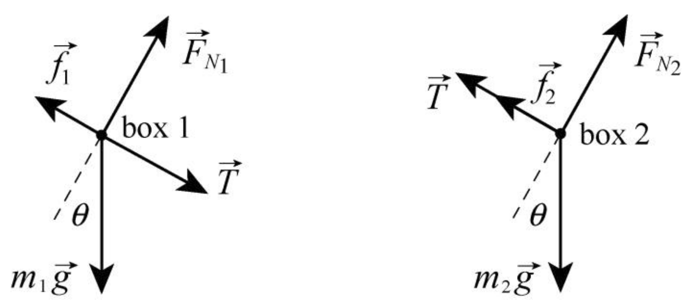
sing $F_{\text{net str}} = mv^2 \; R$ e find the speed of the ball to be

$$v = \sqrt{\frac{RF_{\text{net str}}}{m}} = \sqrt{\frac{1.47 \text{ m} \quad 37.9}{1.34 \text{ kg}}} = 6.45 \text{ m s}.$$

d he direction of $\vec{F}_{\text{net str}}$ is leftward (“radially inward” .

LEARN he upper string ith a tension about 4 times that in the lo er string $T_u \approx 4T_\ell$ ill break more easily than the lo er one.

60. he free body diagrams for the t o bo es are sho n belo . T is the magnitude of the force in the rod hen T 0 the rod is said to be in tension and hen T 0 the rod is under compression $\vec{F}_{N2}$ is the normal force on bo 2 the uncle bo $\vec{F}_{N1}$ is the the normal force on the aunt bo bo 1 $\vec{f}_1$ is kinetic friction force on the aunt bo and $\vec{f}_2$ is kinetic friction force on the uncle bo . lso m_1 = 1.65 kg is the mass of the aunt bo and m_2 = 3.30 kg is the mass of the uncle bo hich is a lot of ants .



or each block e take x do nhill hich is to ard the lo er right in these diagrams and y in the direction of the normal force. Applying Newton’s second law to the x and y directions of first bo 2 and ne t bo 1 e arrive at four e uations

$$m_2 g \sin\theta - f_2 - T = m_2 a$$
$$F_{N2} - m_2 g \cos\theta = 0$$
$$m_1 g \sin\theta - f_1 + T = m_1 a$$
$$F_{N1} - m_1 g \cos\theta = 0$$

hich hen combined ith . 6 2 $f_1 = \mu_1 F_{N1}$ here μ_1 = 0.226 and $f_2 = \mu_2 F_{N2}$ here μ_2 = 0.113 fully describe the dynamics of the system.

a e solve the above e uations for the tension and obtain

$$T = \left(\frac{m_2 m_1 g}{m_2 + m_1}\right) \mu_1 - \mu_2 \cos\theta = 1.05 \quad .$$

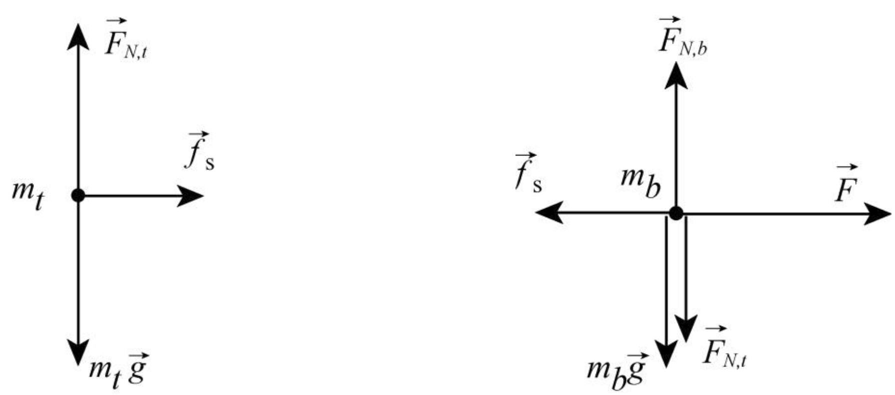
b hese e uations lead to an acceleration e ual to

$$a = g\left(\sin\theta - \left(\frac{\mu_2 m_2 + \mu_1 m_1}{m_2 + m_1}\right)\cos\theta\right) = 3.62 \text{ m } \text{s}^2.$$

c eversing the blocks is e uivalent to s itching the labels. e see from our algebraic result in part a that this gives a negative value for T e ual in magnitude to the result e got before . hus the situation is as it as before e cept that the rod is no in a state of compression.

61. **THINK** ur system consists of t o blocks one on top of the other. f e pull the bottom block too hard, the top block will slip on the bottom one. We're interested in the ma imum force that can be applied such that the t o ill move together.

EXPRESS he free body diagrams for the t o blocks are sho n belo .



e first calculate the coefficient of static friction for the surface bet een the t o blocks. hen the force applied is at a ma imum the frictional force bet een the t o blocks must also be a ma imum. ince $F_t = 12$ of force has to be applied to the top block for slipping to take place using $F_t = f_{s\,\text{ma}} = \mu_s F_{N\,t} = \mu_s m_t g$ e have

$$\mu_s = \frac{F_t}{m_t g} = \frac{12}{4.0\text{ kg} \quad 9.8\text{ m } \text{s}^2} = 0.31.$$

sing the same reasoning for the t o masses to move together the ma imum applied force ould be

$$F = \mu_s \;\; m_t + m_b \;\; g$$

ANALYZE a ubstituting the value of μ_s found above the ma imum hori ontal force has a magnitude

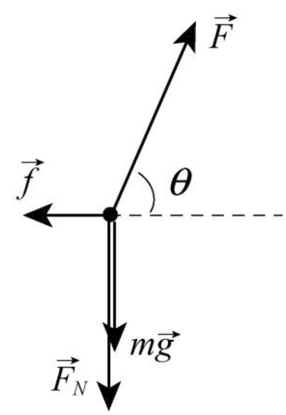
$$F = \mu_s \;\; m_t + m_b \;\; g = \;\; 0.31 \;\; 4.0\,\text{kg} + 5.0\,\text{kg} \;\; 9.8\,\text{m s}^2 \;\; = 27$$

b he ma imum acceleration is

$$a_{\text{ma}} \;\; = \frac{F}{m_t + m_b} = \mu_s g = \;\; 0.31 \;\; 9.8\,\text{m s}^2 \;\; = 3.0\,\text{m s}^2 .$$

LEARN lipping ill occur if the applied force e ceeds 27.3 . n the absence of friction $\mu_s = 0$ bet een the t o blocks any amount of force ould cause the top block to slip.

62. he free body diagram for the stone is sho n to the right ith $\vec{F}$ being the force applied to the stone $\vec{F}_N$ the *downward* normal force of the ceiling on the stone $m\vec{g}$ the force of gravity and $\vec{f}$ the force of friction. e take the x direction to be hori ontal to the right and the y direction to be up. he e uations for the x and the y components of the force according to Newton's second law are



$$F_x = F\cos\theta - f = ma$$
$$F_y = F\sin\theta - F_N - mg = 0$$

o $f = \mu_k F_N$ and the second e uation gives $F_N = F\sin\theta - mg$ hich yields $f = \mu_k \;\; F\sin\theta - mg$. his e pression is substituted for f in the first e uation to obtain

$$F\cos\theta - \mu_k \;\; F\sin\theta - mg \;\; = ma.$$

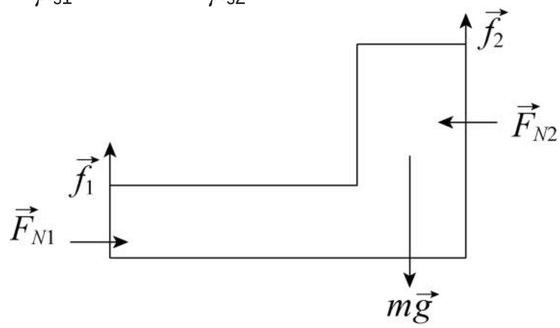
or $a = 0$ the force is

$$F = \frac{-\mu_k mg}{\cos\theta - \mu_k \sin\theta}.$$

ith μ_k = 0.65 m =5.0 kg and θ = 70 e obtain F = 118 .

63. a he free body diagram for the person sho n as an shaped block is sho n belo . he force that she e erts on the rock slabs is not directly sho n since the diagram should only show forces exerted on her), but it is related by Newton's third law) to the normal forces $\vec{F}_{N1}$ and $\vec{F}_{N2}$ e erted hori ontally by the slabs onto her shoes and

back respectively. e ill sho in part b that F_{N1} = F_{N2} so that e there is no ambiguity in saying that the magnitude of her push is F_{N2}. he total up ard force due to ma imum static friction is $\vec{f} = \vec{f}_1 + \vec{f}_2$ here $f_1 = \mu_{s1}F_{N1}$ and $f_2 = \mu_{s2}F_{N2}$. he problem gives the values μ_{s1} = 1.2 and μ_{s2} = 0.8.



(b) We apply Newton's second law to the x and y a es ith x right ard and y up ard and there is no acceleration in either direction .

$$F_{N1} - F_{N2} = 0$$
$$f_1 + f_2 - mg = 0$$

he first e uation tells us that the normal forces are e ual F_{N1} = F_{N2} = F_N. onse uently from . 6 1

$$f_1 = \mu_{s1}F_N$$
$$f_2 = \mu_{s2}F_N$$

e conclude that

$$f_1 = \left(\frac{\mu_{s1}}{\mu_{s2}}\right) f_2 \, .$$

herefore f_1 $f_2 - mg$ = 0 leads to

$$\left(\frac{\mu_{s1}}{\mu_{s2}} + 1\right) f_2 = mg$$

hich ith m = 49 kg yields f_2 = 192 . rom this e find $F_N = f_2 \ \mu_{s2} = 240$. his is e ual to the magnitude of the push e erted by the rock climber.

c rom the above calculation e find $f_1 = \mu_{s1}F_N = 288$ hich amounts to a fraction

$$\frac{f_1}{W} = \frac{288}{(49)\,(9.8)} = 0.60$$

or 60 of her eight.

64. a he up ard force e erted by the car on the passenger is e ual to the do n ard force of gravity W = 500 on the passenger. o the *net* force does not have a vertical contribution it only has the contribution from the hori ontal force hich is necessary for maintaining the circular motion . hus $\left|\vec{F}_{\text{net}}\right| = F = 210$.

b sing . 6 18 e have

$$v=\sqrt{\frac{FR}{m}}=\sqrt{\frac{210 \quad 470\text{ m}}{51.0\text{ kg}}}=44.0\text{ m s.}$$

65. he layer of ice has a mass of

$$m_{\text{ice}}=\left(917\text{ kg m}^3\right)\quad 400\text{ m}\times 500\text{ m}\times 0.0040\text{ m} \;=7.34\times 10^5\text{ kg.}$$

his added to the mass of the hundred stones at 20 kg each comes to m = 7.36 × 10^5 kg.

a etting $F = D$ for rag force e use . 6 14 to find the ind speed v along the ground hich actually is relative to the moving stone but e assume the stone is moving slo ly enough that this does not invalidate the result

$$v=\sqrt{\frac{\mu_k mg}{4C_{\text{ice}}\rho A_{\text{ice}}}}=\sqrt{\frac{(0.10)\left(7.36\times 10^5\text{ kg}\right)\left(9.8\text{ m s}^2\right)}{4(0.002)\left(1.21\text{ kg m}^3\right)\left(400\times 500\text{ m}^2\right)}}=19\text{ m s}\;\approx 69\text{ km h.}$$

b oubling our previous result e find the reported speed to be 139 km h.

c he result is reasonable for storm inds. category 5 hurricane has speeds on the order of 2.6 × 10^2 m s.

66. ote that since no static friction coefficient is mentioned e assume f_s is not relevant to this computation. e apply e ton s second la to each block s x a is hich for m_1 is positive right ard and for m_2 is positive do nhill

$$\begin{aligned} T - f_k &= m_1 a \\ m_2 g\sin\theta - T &= m_2 a \end{aligned}$$

dding the e uations e obtain the acceleration

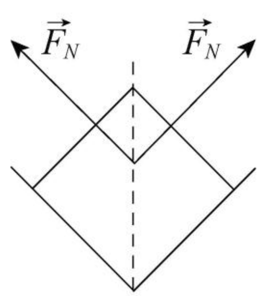
$$a=\frac{m_2 g\sin\theta - f_k}{m_1+m_2}$$

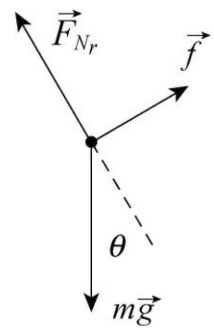
or $f_k = \mu_k F_N = \mu_k m_1 g$ e obtain

$$a = \frac{3.0\ \text{kg}\quad 9.8\ \text{m s}^2\quad \sin 30° - \ 0.25\quad 2.0\ \text{kg}\quad 9.8\ \text{m s}^2}{3.0\ \text{kg} + 2.0\ \text{kg}} = 1.96\ \text{m s}^2 .$$

eturning this value to either of the above t o e uations e find T = 8.8 .

67. ach side of the trough e erts a normal force on the crate. he first diagram sho s the vie looking in to ard a cross section.





he net force is along the dashed line. ince each of the normal forces makes an angle of 45 ith the dashed line the magnitude of the resultant normal force is given by

$$F_{Nr} = 2F_N \cos 45° = \sqrt{2}F_N .$$

he second diagram is the free body diagram for the crate (from a "side" view, similar to that sho n in the first picture in ig. 6 51 . he force of gravity has magnitude mg here m is the mass of the crate and the magnitude of the force of friction is denoted by f. e take the x direction to be do n the incline and y to be in the direction of $\vec{F}_{Nr}$. hen the x and the y components of Newton's second law are

$$x \qquad mg \sin\,\theta - f = ma$$
$$y \qquad F_{Nr} - mg \cos\,\theta = 0.$$

ince the crate is moving each side of the trough e erts a force of kinetic friction so the total frictional force has magnitude

$$f = 2\mu_k F_N = 2\mu_k F_{Nr}\quad \sqrt{2} = \sqrt{2}\mu_k F_{Nr}$$

ombining this e pression ith F_{Nr} = mg cos θ and substituting into the x component e uation e obtain

$$mg \sin\theta - \sqrt{2}mg\cos\theta = ma .$$

herefore $a = g\ \sin\theta - \sqrt{2}\mu_k \cos\theta$.

68. a o be on the verge of sliding out means that the force of static friction is acting "down the bank" (in the sense explained in the problem statement ith ma imum

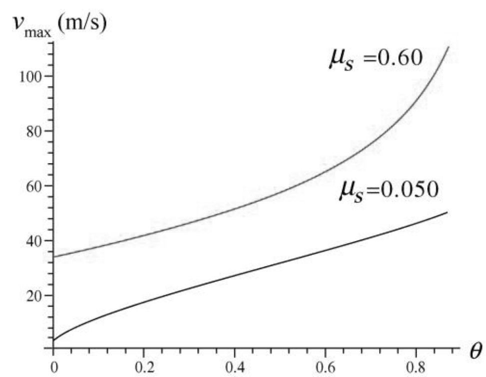
possible magnitude. e first consider the vector sum $\vec{F}$ of the ma imum static friction force and the normal force. ue to the facts that they are perpendicular and their magnitudes are simply proportional . 6 1 e find $\vec{F}$ is at angle measured from the *vertical* a is $\phi = \theta \quad \theta_s$ here $\tan\theta_s = \mu_s$ compare ith . 6 13 and θ is the bank angle as stated in the problem . o the vector sum of $\vec{F}$ and the vertically do n ard pull mg of gravity must be e ual to the hori ontal centripetal force mv^2/R hich leads to a surprisingly simple relationship

$$\tan\phi = \frac{mv^2 \quad R}{mg} = \frac{v^2}{Rg} \quad .$$

riting this as an e pression for the ma imum speed e have

$$v_{\text{ma}} = \sqrt{Rg\tan\ \theta + \tan^{-1}\mu_s} = \sqrt{\frac{Rg\ \tan\theta + \mu_s}{1-\mu_s\tan\theta}}$$

b he graph is sho n belo ith θ in radians



c ither estimating from the graph $\mu_s = 0.60$ upper curve or calculated it more carefully leads to v = 41.3 m s = 149 km h hen θ = 10 = 0.175 radian.

d imilarly for $\mu_s = 0.050$ the lo er curve e find v = 21.2 m s = 76.2 km h hen θ = 10 = 0.175 radian.

69. or simplicity e denote the 70 angle as θ and the magnitude of the push 80 as P. he vertical forces on the block are the do n ard normal force e erted on it by the ceiling the do n ard pull of gravity of magnitude mg and the vertical component of $\vec{P}$ hich is up ard ith magnitude $P\sin\theta$. ince there is no acceleration in the vertical direction e must have

$$F_N = P\sin\theta - mg$$

in hich case the left ard pointed kinetic friction has magnitude

$$f_k = \mu_k \ \ P\sin\theta - mg \ .$$

hoosing x rightward, Newton's second law leads to

$$P\cos\ \theta - f_k = ma \Rightarrow a = \frac{P\cos\theta - u_k \ \ P\sin\theta - mg}{m}$$

hich yields a = 3.4 m s^2 hen μ_k = 0.40 and m = 5.0 kg.

70. a e note that R the hori ontal distance from the bob to the a is of rotation is the circumference of the circular path divided by 2π therefore R = 0.94 2π = 0.15 m. he angle that the cord makes ith the hori ontal is no easily found

$$\theta = \cos^{-1}\ R/L \ = \cos^{-1}\ 0.15\text{ m}/0.90\text{ m} \ = 80\ .$$

he vertical component of the force of tension in the string is $T\sin\theta$ and must e ual the do n ard pull of gravity mg . hus

$$T = \frac{mg}{\sin\theta} = 0.40 \quad .$$

ote that e are using T for tension not for the period .

b he hori ontal component of that tension must supply the centripetal force . 6 18 so e have $T\cos\theta$ = mv^2 R. his gives speed v = 0.49 m s. his divided into the circumference gives the time for one revolution 0.94 0.49 = 1.9 s.

71. a To be "on the verge of sliding" means the applied force is e ual to the ma imum possible force of static friction . 6 1 ith $F_N = mg$ in this case

$$f_{s\ ma} = \mu_s mg = 35.3 \quad .$$

b n this case the applied force $\vec{F}$ indirectly decreases the ma imum possible value of friction since its y component causes a reduction in the normal force as ell as directly opposing the friction force itself because of its x component . he normal force turns out to be

$$F_N = mg - F\sin\theta$$

here θ = 60 so that the hori ontal e uation the x application of Newton's second law) becomes

$$F\cos\theta - f_{s\ ma} = F\cos\theta - \mu_s\ mg - F\sin\theta \ = 0 \quad \Rightarrow F = 39.7 \quad .$$

c o the applied force $\vec{F}$ indirectly increases the ma imum possible value of friction since its y component causes a reduction in the normal force as ell as directly opposing the friction force itself because of its x component . he normal force in this case turns out to be

$$F_N = mg \quad F\sin\theta$$

here $\theta = 60$ so that the hori ontal e uation becomes

$$F\cos\theta - f_{s\,ma} = F\cos\theta - \mu_s \; mg \quad F\sin\theta \;\; = 0 \quad \Rightarrow F = 320 \quad .$$

72. ith $\theta = 40^{o}$, we apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma$$

$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta$$

using . 6 12. hus

$$a = 0.75 \text{ m s}^2 = g \;\sin\theta - \mu_k\cos\theta$$

determines the coefficient of kinetic friction $\mu_k = 0.74$.

73. a ith $\theta = 60^{o}$, we apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma$$
$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta.$$

hus

$$a = g \;\sin\theta - \mu_k\cos\theta \;\; = 7.5 \text{ m s}^2.$$

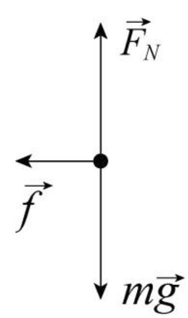
b he direction of the acceleration $\vec{a}$ is do n the slope.

c Now the friction force is in the "downhill" direction (which is our positive direction) so that e obtain

$$a = g \;\sin\theta \quad \mu_k\cos\theta \;\; = 9.5 \text{ m s}^2.$$

d he direction is do n the slope.

74. he free body diagram for the puck is sho n on the right. $\vec{F}_N$ is the normal force of the ice on the puck $\vec{f}$ is the force of friction in the $-x$ direction and $m\vec{g}$ is the force of gravity.



a he hori ontal component of Newton's second law gives $-f = ma$ and constant acceleration kinematics able 2 1 can be used to find the acceleration.

ince the final velocity is ero $v^2 = v_0^2 + 2ax$ leads to $a = -v_0^2 \;\; 2x$. his is substituted into the e ton's law equation to obtain

$$f = \frac{mv_0^2}{2x} = \frac{(0.110 \text{ kg})(6.0 \text{ m s})^2}{2(15 \text{ m})} = 0.13 \quad .$$

(b) The vertical component of Newton's second law gives $F_N - mg = 0$ so $F_N = mg$ hich implies using . 6 2 $f = \mu_k mg$. e solve for the coefficient

$$\mu_k = \frac{f}{mg} = \frac{0.13}{(0.110 \text{ kg})\ 9.8 \text{ m s}^2} = 0.12 \; .$$

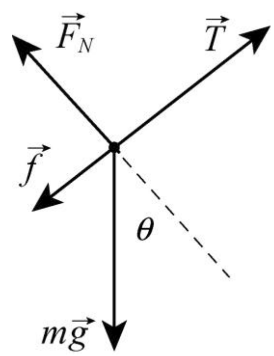
75. e may treat all 25 cars as a single ob ect of mass $m = 25 \times 5.0 \times 10^4$ kg and hen the speed is 30 km h = 8.3 m s sub ect to a friction force e ual to

$$f = 25 \times 250 \times 8.3 = 5.2 \times 10^4 \quad .$$

(a) Along the level track, this object experiences a "forward" force T e erted by the locomotive so that Newton's second law leads to

$$T - f = ma \quad \Rightarrow \quad T = 5.2\times10^4 + 1.25\times10^6 \quad 0.20 \; = 3.0\times10^5 \quad .$$

b he free body diagram is sho n ne t ith θ as the angle of the incline. he x direction hich is the only direction to hich e ill be applying Newton's second law) is uphill (to the upper right in our sketch . hus e obtain



$$T - f - mg\sin\theta = ma$$

here e set $a = 0$ implied by the problem statement and solve for the angle. e obtain $\theta = 1.2$.

76. n e cellent discussion and e uation development related to this problem is given in ample roblem – "Friction, applied force at an angle." Using the result e obtain

$$\theta = \tan^{-1}\mu_s = \tan^{-1} 0.50 = 27°$$

hich implies that the angle through hich the slope should be *reduced* is

$$\phi = 45 \;\; - 27 \;\; \approx 20 \; .$$

77. e make use of . 6 16 hich yields

$$\sqrt{\frac{2mg}{C\rho\pi R^2}} = \sqrt{\frac{2\;\; 6\;\; 9.8}{1.6\;\; 1.2\;\; \pi\;\; 0.03\;\; ^2}} = 147 \text{ m s}.$$

78. a he coefficient of static friction is $\mu_s = \tan\,\theta_{\text{slip}} = 0.577 \approx 0.58$.

b sing

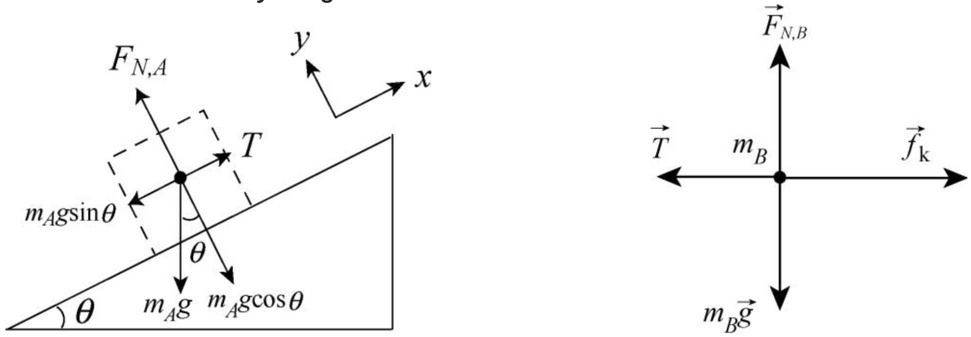
$$mg\sin\theta - f = ma$$

$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta$$

and $a = 2d/t^2$ ith d = 2.5 m and t = 4.0 s e obtain μ_k = 0.54.

79. **THINK** e have t o blocks connected by a cord as sho n in ig. 6 56. s block A slides do n the frictionless inclined plane it pulls block B, so there's a tension in the cord.

EXPRESS he free body diagrams for blocks A and B are sho n belo



Newton's law gives

$$m_A g\sin\theta - T = m_A a$$

for block A here θ = 30 . or block B e have

$$T - f_k = m_B a$$

o the frictional force is given by $f_k = \mu_k F_{N\ B} = \mu_k m_B g$. he e uations allo us to solve for the tension T and the acceleration a.

ANALYZE a ombining the above e uations to solve for T e obtain

$$T = \frac{m_A m_B}{m_A + m_B}(\sin\theta + \mu_k)g = \frac{4.0\,\text{kg}\ \ 2.0\,\text{kg}}{4.0\,\text{kg} + 2.0\,\text{kg}}(\sin 30° + 0.50)\ 9.80\,\text{m}\ \text{s}^2\ = 13\ \ .$$

b imilarly the acceleration of the t o block system is

$$a = \left(\frac{m_A\sin\theta - \mu_k m_B}{m_A + m_B}\right)g = \frac{4.0\,\text{kg}\ \sin 30° -\ 0.50\ \ 2.0\,\text{kg}}{4.0\,\text{kg} + 2.0\,\text{kg}}\ 9.80\,\text{m}\ \text{s}^2\ = 1.6\,\text{m}\ \text{s}^2.$$

LEARN n the case here $\theta = 90°$ and $\mu_k = 0$ e have

$$T = \frac{m_A m_B}{m_A + m_B} g \quad a = \frac{m_A}{m_A + m_B} g$$

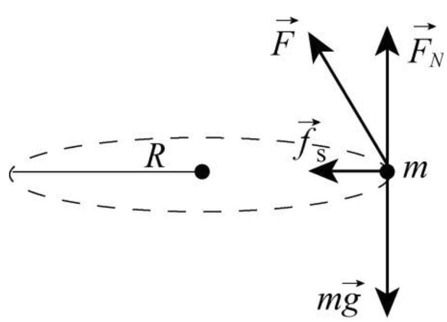
hich correspond to the ample roblem – “Block on table, block hanging,” discussed in hapter 5.

80. e use . 6 14 $D = \frac{1}{2} C\rho A v^2$ here ρ is the air density A is the cross sectional area of the missile v is the speed of the missile and C is the drag coefficient. he area is given by $A = \pi R^2$ here R = 0.265 m is the radius of the missile. hus

$$D = \frac{1}{2}\ 0.75\ (1.2 \text{ kg} \quad \text{m}^3)\pi(0.265 \text{ m})^2(250 \text{ m} \quad \text{s})^2 = 6.2 \times 10^3 \quad .$$

81. **THINK** o can a cyclist move in a circle t is the force of friction that provides the centripetal force re uired for the circular motion.

EXPRESS he free body diagram is sho n belo . he magnitude of the acceleration of the cyclist as it moves along the hori ontal circular path is given by $v^2\ R$ here v is the speed of the cyclist and R is the radius of the curve.



The horizontal component of Newton’s second law is $f_s = mv^2\ R$ here f_s is the static friction e erted hori ontally by the ground on the tires. imilarly if F_N is the vertical force of the ground on the bicycle and m is the mass of the bicycle and rider the vertical component of Newton’s second law leads to $F_N = mg = 833$.

ANALYZE a he frictional force is $f_s = \frac{mv^2}{R} = \frac{(85.0 \text{ kg})(9.00 \text{ m s})^2}{25.0 \text{ m}} = 275$.

b ince the frictional force $\vec{f}_s$ and $\vec{F}_N$ the normal force e erted by the road are perpendicular to each other the magnitude of the force e erted by the ground on the bicycle is

$$F = \sqrt{f_s^2 + F_N^2} = \sqrt{275 \quad ^2 + 833 \quad ^2} = 877 \quad .$$

LEARN he force e erted by the ground on the bicycle is at an angle $\theta = \tan^{-1}$ 275 833 $= 18.3°$ ith respect to the *vertical* a is.

82. t the top of the hill the vertical forces on the car are the up ard normal force e erted by the ground and the do n ard pull of gravity. esignating y do n ard e have

$$mg - F_N = \frac{mv^2}{R}$$

from Newton's second law. To find the greatest speed without leaving the hill, we set F_N = 0 and solve for v

$$v = \sqrt{gR} = \sqrt{9.8\text{ m s}^2 \quad 250\text{ m}} = 49.5\text{ m s} = 49.5 \; 3600 \; 1000 \;\text{ km h} = 178\text{ km h}.$$

83. a he push to get it moving must be at least as big as $f_{s\,ma} = \mu_s F_N$. 6 1 ith $F_N = mg$ in this case hich e uals 0.51 165 = 84.2 .

b hile in motion constant velocity ero acceleration is maintained if the push is e ual to the kinetic friction force $f_k = \mu_k F_N = \mu_k mg$ = 52.8 .

c e note that the mass of the crate is 165 9.8 = 16.8 kg. he acceleration using the push from part a is

$$a = 84.2 \quad - 52.8 \qquad 16.8\text{ kg} \approx 1.87\text{ m s}^2.$$

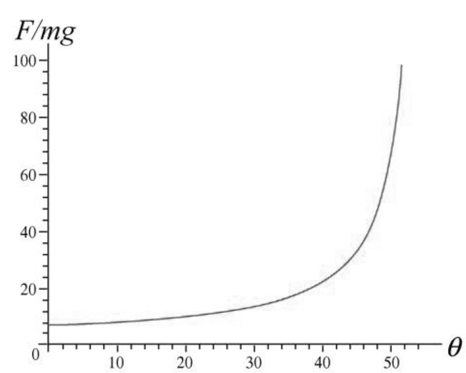
84. a he x component of $\vec{F}$ tries to move the crate hile its y component indirectly contributes to the inhibiting effects of friction by increasing the normal force . e ton's second la implies

$$x\text{ direction} \quad F\cos\theta - f_s = 0$$

$$y\text{ direction} \quad F_N - F\sin\theta - mg = 0.$$

To be "on the verge of sliding" means $f_s = f_{s\,ma} = \mu_s F_N$. 6 1 . olving these e uations for F actually for the ratio of F to mg yields

$$\frac{F}{mg} = \frac{\mu_s}{\cos\theta - \mu_s \sin\theta} \; .$$



his is plotted on the right θ in degrees .

b he denominator of our e pression for F/mg vanishes hen

$$\cos\theta - \mu_s \sin\theta = 0 \quad \Rightarrow \quad \theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right)$$

or $\mu_s = 0.70$ e obtain $\theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right) = 55°$.

c educing the coefficient means increasing the angle by the condition in part b .

d or $\mu_s = 0.60$ e have $\theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right) = 59°$.

85. The car is in "danger of sliding" down when

$$\mu_s = \tan\theta = \tan 35.0° = 0.700.$$

his value represents a 3.4 decrease from the given 0.725 value.

86. a he tension ill be the greatest at the lo est point of the s ing. ote that there is no substantive difference bet een the tension T in this problem and the normal force F_N in ample roblem – "Vertical circular loop, Diavolo." Eq. 6 19 of that ample roblem e amines the situation at the top of the circular path here F_N is the least and re riting that for the bottom of the path leads to

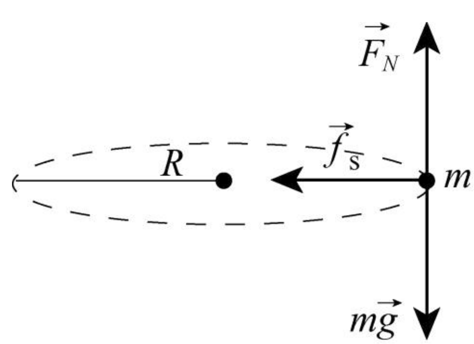
$$T = mg \quad mv^2 \; r$$

here F_N is at its greatest value.

b t the breaking point T = 33 = m g v^2 r here m = 0.26 kg and r = 0.65 m. olving for the speed e find that the cord should break hen the speed at the lo est point reaches 8.73 m s.

87. **THINK** car is making a turn on an unbanked curve. riction is hat provides the centripetal force needed for this circular motion.

EXPRESS he free body diagram is sho n to the right. he mass of the car is m = 10700 9.80 kg = 1.09 × 10^3 kg. We choose "inward" (horizontally to ard the center of the circular path as the positive direction. he normal force is F_N = mg in this situation and the re uired frictional force is $f_s = mv^2 \; R$.



ANALYZE a ith a speed of v = 13.4 m s and a radius R = 61 m, Newton's second law (using Eq. 6 18 leads to

$$f_s = \frac{mv^2}{R} = \frac{1.09\times10^3 \text{ kg} \;\; 13.4 \text{ m s}^{\;2}}{61.0 \text{ m}} = 3.21\times10^3 \quad .$$

b he ma imum possible static friction is found to be

$$f_{s\,\text{ma}} = \mu_s mg = (0.35)(10700 \quad) = 3.75\times10^3$$

using . 6 1. e see that the static friction found in part a is less than this so the car rolls no skidding and successfully negotiates the curve.

LEARN rom the above e pressions e see that ith a coefficient of friction μ_s the ma imum speed of the car negotiating a curve of radius R is $v_{\text{ma}} = \sqrt{\mu_s gR}$. o in this case the car can go up to a ma imum speed of

$$v_{\text{ma}} = \sqrt{0.35 \;\; 9.8 \text{ m s}^2 \;\; 61 \text{ m}} = 14.5 \text{ m s}$$

ithout skidding.

88. or the m_2 = 1.0 kg block application of e ton s la s result in

$$F\cos\theta - T - f_k = m_2 a \qquad x \text{ a is}$$
$$F_N - F\sin\theta - m_2 g = 0 \qquad y \text{ a is}$$

ince $f_k = \mu_k F_N$ these e uations can be combined into an e uation to solve for a

$$F\;\cos\theta - \mu_k \sin\theta \;\; - T - \mu_k m_2 g = m_2 a$$

imilarly but ithout the applied push e analy e the m_1= 2.0 kg block

$$T - f_k' = m_1 a \qquad x \text{ a is}$$
$$F_N' \;\; - m_1 g = 0 \qquad y \text{ a is}$$

sing $f_k = \mu_k F_N'$, the e uations can be combined

$$T - \mu_k m_1 g = m_1 a$$

ubtracting the t o e uations for a and solving for the tension e obtain

$$T = \frac{m_1 \;\cos\theta - \mu_k \sin\theta}{m_1 + m_2} F = \frac{2.0 \text{ kg} \;\; \cos 35^\circ - \;0.20\; \sin 35^\circ}{2.0 \text{ kg} + 1.0 \text{ kg}} \;\; 20 \quad = 9.4 \quad .$$

89. **THINK** n order to move a filing cabinet the force applied must be able to overcome the frictional force.

EXPRESS We apply Newton's second law (as $F_{\text{push}} - f = ma$. f e find the applied force F_{push} to be less than $f_{s\ \text{ma}}$ the ma imum static frictional force our conclusion would then be "no, the cabinet does not move" (which means a is actually 0 and the frictional force is simply $f = F_{\text{push}}$. n the other hand if e obtain a 0 then the cabinet moves so $f = f_k$. or $f_{s\ \text{ma}}$ and f_k e use . 6 1 and . 6 2 respectively and in those formulas e set the magnitude of the normal force to the eight of the cabinet $F_N = mg = 556$. hus the ma imum static frictional force is

$$f_{s\ \text{ma}} = \mu_s F_N = \mu_s mg = 0.68 \quad 556 \qquad = 378 \quad .$$

and the kinetic frictional force is

$$f_k = \mu_k F_N = \mu_k mg = 0.56 \quad 556 \qquad = 311 \quad .$$

ANALYZE a ere e find F_{push} $f_{s\ \text{ma}}$ hich leads to $f = F_{\text{push}} = 222$. he cabinet does not move.

b gain e find F_{push} $f_{s\ \text{ma}}$ hich leads to $f = F_{\text{push}} = 334$. he cabinet does not move.

c o e have F_{push} $f_{s\ \text{ma}}$ hich means the cabinet moves and $f = f_k = 311$.

d gain e have F_{push} $f_{s\ \text{ma}}$ hich means the cabinet moves and $f = f_k = 311$.

e he cabinet moves in c and d .

LEARN n summary in order to make the cabinet move the minimum applied force is e ual to the ma imum static frictional force $f_{s\ \text{ma}}$.

90. nalysis of forces in the hori ontal direction here there can be no acceleration leads to the conclusion that $F = F_N$ the magnitude of the normal force is 60 . he ma imum possible static friction force is therefore $\mu_s F_N = 33$ and the kinetic friction force hen applicable is $\mu_k F_N = 23$.

a n this case $\vec{P} = 34$ up ard. ssuming $\vec{f}$ points do n then e ton s second la for the y leads to

$$P - mg - f = ma .$$

if e assume $f = f_s$ and $a = 0$ e obtain $f = 34 - 22 \qquad = 12$. his is less than $f_{s,\ \text{ma}}$ hich sho s the consistency of our assumption. he ans er is $\vec{f_s} = 12$ do n.

b n this case $\vec{P}$ = 12 up ard. he above e uation ith the same assumptions as in part a leads to f = 12 – 22 = –10 . hus f_s $f_{s,\ \text{ma}}$ ustifying our assumption that the block is stationary but its negative value tells us that our initial assumption about the direction of $\vec{f}$ is incorrect in this case. hus the ans er is $\vec{f_s}$ = 10 up.

c n this case $\vec{P}$ = 48 up ard. he above e uation ith the same assumptions as in part a leads to f = 48 – 22 = 26 . hus e again have f_s $f_{s,\ \text{ma}}$ and our ans er is $\vec{f_s}$ = 26 do n.

d n this case $\vec{P}$ = 62 up ard. he above e uation ith the same assumptions as in part a leads to f = 62 – 22 = 40 hich is larger than f_s ma invalidating our assumptions. herefore e take $f = f_k$ and $a \neq 0$ in the above e uation if e ished to find the value of a e ould find it to be positive as e should e pect. he ans er is $\vec{f_k}$ = 23 do n.

e n this case $\vec{P}$ = 10 do n ard. he above e uation but ith P replaced ith -P ith the same assumptions as in part a leads to f = –10 – 22 = –32 . hus e have f_s $f_{s,\ \text{ma}}$ ustifying our assumption that the block is stationary but its negative value tells us that our initial assumption about the direction of $\vec{f}$ is incorrect in this case. hus the ans er is $\vec{f_s}$ = 32 up.

f n this case $\vec{P}$ = 18 do n ard. he above e uation but ith P replaced ith –P ith the same assumptions as in part a leads to f = –18 – 22 = –40 hich is larger in absolute value than f_s ma invalidating our assumptions. herefore e take $f = f_k$ and $a \neq 0$ in the above e uation if e ished to find the value of a e ould find it to be negative as e should e pect. he ans er is $\vec{f_k}$ = 23 up.

g he block moves up the all in case d here a 0.

h he block moves do n the all in case f here a 0.

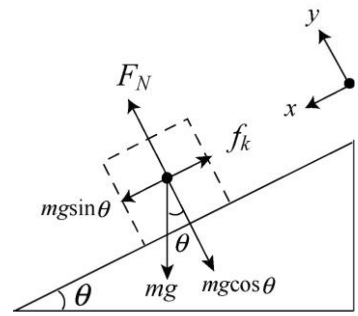
i he frictional force $\vec{f_s}$ is directed do n in cases a c and d .

91. **THINK** hether the block is sliding do n or up the incline there is a frictional force in the opposite direction of the motion.

EXPRESS he free body diagram for the first part of this problem hen the block is sliding do nhill ith ero acceleration is sho n ne t.

Newton's second law gives

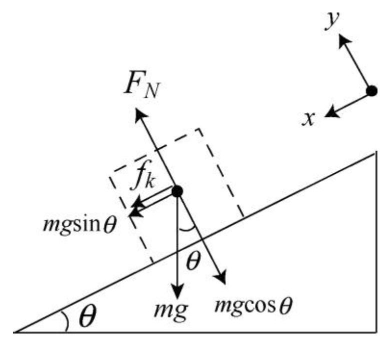
$$mg\sin\theta - f_k = mg\sin\theta - \mu_k F_N = ma_x = 0$$
$$mg\cos\theta - F_N = ma_y = 0$$



he t o e uations can be combined to give

$$\mu_k = \tan\theta.$$

o for the second part of the problem ith the block pro ected uphill the friction direction is reversed (see figure to the right). Newton's second la for the uphill motion and . 6 12 leads to



$$mg\sin\theta + f_k = mg\sin\theta + \mu_k F_N = ma_x$$
$$mg\cos\theta - F_N = ma_y = 0$$

ote that by our convention $a_x > 0$ means that the acceleration is do nhill and therefore the speed of the block ill decrease as it moves up the incline.

ANALYZE a sing $\mu_k = \tan\theta$ and $F_N = mg\cos\theta$ e find the x component of the acceleration to be

$$a_x = g\sin\theta + \frac{\mu_k F_N}{m} = g\sin\theta + \frac{\tan\theta \quad mg\cos\theta}{m} = 2g\sin\theta .$$

he distance the block travels before coming to a stop can be found by using . 2 16 $v_f^2 = v_0^2 - 2a_x\Delta x$ hich yields

$$\Delta x = \frac{v_0^2}{2a_x} = \frac{v_0^2}{2 \ 2g\sin\theta} = \frac{v_0^2}{4g\sin\theta} .$$

b e usually e pect μ_s μ_k see the discussion in ection 6 1). The "angle of repose" the minimum angle necessary for a stationary block to start sliding do nhill is μ_s = tan θ_{repose} . herefore e e pect θ_{repose} θ found in part a . onse uently hen the block comes to rest the incline is not steep enough to cause it to start slipping do n the incline again.

LEARN n alternative ay to see that the block ill not slide do n again is to note that the do n ard force of gravitation is not large enough to overcome the force of friction i.e. $mg\sin\theta = f_k < f_{s\,\text{ma}}$.

92. Consider that the car is “on the verge of sliding out” – meaning that the force of static friction is acting “down the bank” (or “downhill” from the point of view of an ant on the banked curve ith ma imum possible magnitude. e first consider the vector sum $\vec{F}$ of the ma imum static friction force and the normal force. ue to the facts that they are perpendicular and their magnitudes are simply proportional . 6 1 e find $\vec{F}$ is at angle measured from the <u>vertical</u> a is $\phi = \theta \quad \theta_s$ here tan $\theta_s = \mu_s$ compare ith . 6 13 and θ is the bank angle. o the vector sum of $\vec{F}$ and the vertically do n ard pull mg of gravity must be e ual to the hori ontal centripetal force $m\text{v}^2/R$ hich leads to a surprisingly simple relationship

$$\tan\phi = \frac{m\text{v}^2/R}{mg} = \frac{\text{v}^2}{Rg}\ .$$

riting this as an e pression for the ma imum speed e have

$$v_{\text{ma}} = \sqrt{Rg\tan\ \theta+\tan^{-1}\mu_s} = \sqrt{\frac{Rg\ \tan\theta+\mu_s}{1-\mu_s\tan\theta}}\ .$$

a e note that the given speed is in units roughly 17 m s. f e do not ant the cars to “depend” on the static friction to keep from sliding out (that is, if we want the component “down the back” of gravity to be sufficient), then we can set μ_s = 0 in the above e pression and obtain $v = \sqrt{Rg\tan\theta}$. ith R = 150 m this leads to θ = 11°.

b f ho ever the curve is not banked so θ = 0 then the above e pression becomes

$$v = \sqrt{Rg\tan\ \tan^{-1}\mu_s} = \sqrt{Rg\mu_s}$$

olving this for the coefficient of static friction μ_s = 0.19.

93. (a) The box doesn't move until t = 2.8 s hich is hen the applied force $\vec{F}$ reaches a magnitude of F = 1.8 2.8 = 5.0 implying therefore that $f_{s\ \text{ma}}$ = 5.0 . nalysis of the vertical forces on the block leads to the observation that the normal force magnitude e uals the eight $F_N = mg$ = 15 . hus

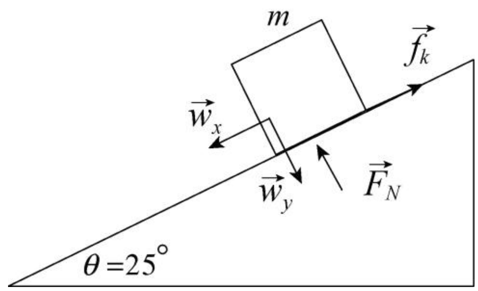
$$\mu_s = f_{s\ \text{ma}} \quad F_N = 0.34.$$

(b) We apply Newton's second law to the horizontal x a is positive in the direction of motion

$$F - f_k = ma \;\Rightarrow\; 1.8t - f_k = (1.5)(1.2t - 2.4)$$

hus e find f_k = 3.6 . herefore $\mu_k = f_k \quad F_N$ = 0.24.

94. n the figure belo m = 140 9.8 = 14.3 kg is the mass of the child. e use $\vec{w}_x$ and $\vec{w}_y$ as the components of the gravitational pull of arth on the block their magnitudes are $w_x = mg \sin\theta$ and $w_y = mg \cos\theta$.



a ith the x a is directed up along the incline so that a = –0.86 m s^2), Newton's second la leads to

$$f_k - 140\sin 25° = m \quad -0.86$$

hich yields f_k = 47 N. We also apply Newton's second law to the y a is perpendicular to the incline surface here the acceleration component is ero

$$F_N - 140\cos 25° = 0 \quad \Rightarrow \quad F_N = 127 \quad .$$

herefore $\mu_k = f_k \ F_N$ = 0.37.

b eturning to our first e uation in part a e see that if the do nhill component of the eight force ere insufficient to overcome static friction the child ould not slide at all. herefore e re uire 140 sin 25 $f_{s\,\mathrm{ma}} = \mu_s F_N$ hich leads to tan 25 = 0.47 μ_s. he minimum value of μ_s e uals μ_k and is more subtle reference to 6 1 is recommended. f μ_k e ceeded μ_s then hen static friction ere overcome as the incline is raised then it should start to move – hich is impossible if f_k is large enough to cause deceleration he bounds on μ_s are therefore given by 0.47 μ_s 0.37.

95. a he x component of $\vec{F}$ contributes to the motion of the crate hile its y component indirectly contributes to the inhibiting effects of friction by increasing the normal force . long the y direction e have $F_N - F\cos\theta - mg = 0$ and along the x direction e have $F\sin\theta - f_k = 0$ since it is not accelerating according to the problem . lso . 6 2 gives $f_k = \mu_k F_N$. olving these e uations for F yields

$$F = \frac{\mu_k mg}{\sin\theta - \mu_k \cos\theta} \quad .$$

b hen $\theta < \theta_0 = \tan^{-1}\mu_s$ F ill not be able to move the mop head.

96. a he distance traveled in one revolution is $2\pi R$ = 2π 4.6 m = 29 m. he constant speed is conse uently v = 29 m 30 s = 0.96 m s.

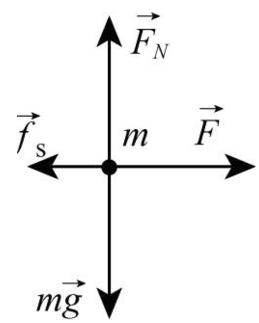
(b) Newton's second law (using Eq. 6 17 for the magnitude of the acceleration leads to

$$f_s = m\left(\frac{v^2}{R}\right) = m\ 0.20$$

in units. oting that F_N = mg in this situation the ma imum possible static friction is $f_{s\,\text{ma}}$ = $\mu_s mg$ using . 6 1. uating this ith f_s = m 0.20 e find the mass m cancels and e obtain μ_s = 0.20 9.8 = 0.021.

97. **THINK** n this problem a force is applied to accelerate a bo . rom the distance traveled and the speed at that instant e can calculate the coefficient of kinetic friction bet een the bo and the floor.

EXPRESS he free body diagram is sho n to the right. e adopt the familiar a es ith x right ard and y up ard and refer to the 85 hori ontal push of the orker as F and assume it to be right ard). Applying Newton's second law to the x a is and y a is respectively produces



$$F - f_k = ma_x \qquad F_N - mg = 0.$$

n the other hand using . 2 16 $v^2 = v_0^2 + 2a_x\Delta x$ e find the acceleration to be

$$a_x = \frac{v^2 - v_0^2}{2\Delta x} = \frac{1.0\ \text{m s}\ ^2 - 0}{2\ \ 1.4\ \text{m}} = 0.357\ \text{m s}^2.$$

he above e uations can be combined to give μ_k .

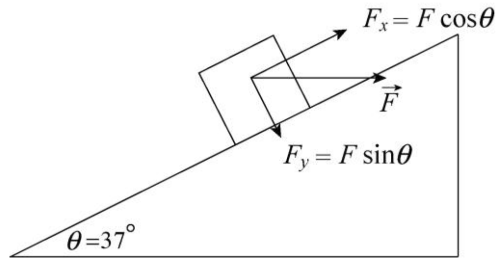
ANALYZE sing $f_k = \mu_k F_N$ e find the coefficient of kinetic friction bet een the bo and the floor to be

$$\mu_k = \frac{f_k}{F_N} = \frac{F - ma_x}{mg} = \frac{85\quad - \ 40\,\text{kg}\ \ 0.357\ \text{m s}^2}{40\,\text{kg}\ \ 9.8\ \text{m s}^2} = 0.18.$$

LEARN n general the acceleration can be ritten as $a_x = \ F\ \ m - \mu_k g$. e see that the smaller the value of μ_k the greater the acceleration. n the limit $\mu_k = 0$ e simply have $a_x = F\ \ m$.

98. e resolve this hori ontal force into appropriate components.

(a) Applying Newton's second law to the x directed uphill and y directed a ay from the incline surface a es e obtain



$$F\cos\theta - f_k - mg\sin\theta = ma$$
$$F_N - F\sin\theta - mg\cos\theta = 0.$$

sing $f_k = \mu_k F_N$ these e uations lead to

$$a = \frac{F}{m}\ \cos\theta - \mu_k\sin\theta\ \ - g\ \sin\theta + \mu_k\cos\theta$$

hich yields $a = -2.1$ m s^2 or a = 2.1 m s^2 for μ_k = 0.30 F = 50 and m = 5.0 kg.

b he direction of $\vec{a}$ is do n the plane.

c ith v_0 = 4.0 m s and v = 0 . 2 16 gives $\Delta x\ = -\dfrac{4.0\text{ m s}\ ^2}{2\ -2.1\text{ m s}^2} = 3.9$ m.

d e e pect $\mu_s \geq \mu_k$ other ise an ob ect started into motion ould immediately start decelerating before it gained any speed n the minimal e pectation case here μ_s = 0.30 the ma imum possible do nhill static friction is using . 6 1

$$f_{s\text{ ma}}\ = \mu_s F_N = \mu_s\ \ F\sin\theta + mg\cos\theta$$

hich turns out to be 21 . ut in order to have no acceleration along the x a is e must have

$$f_s = F\cos\theta - mg\sin\theta = 10$$

the fact that this is positive reinforces our suspicion that $\vec{f}_s$ points do nhill . ince the f_s needed to remain at rest is less than $f_{s\text{ ma}}$ then it stays at that location.

99. a e note that $F_N = mg$ in this situation so

$$f_{s\text{ ma}}\ = \mu_s mg = \ 0.52\ \ 11\text{ kg}\ \ 9.8\text{ m s}^2\ = 56\ \ .$$

onse uently the hori ontal force $\vec{F}$ needed to initiate motion must be at minimum slightly more than 56 .

b naly ing vertical forces hen $\vec{F}$ is at non ero θ yields

$$F\sin\ \theta + F_N = mg\ \Rightarrow f_{s\text{ ma}}\ = \mu_s\ \ mg - F\ \sin\ \theta\ .$$

o the hori ontal component of $\vec{F}$ needed to initiate motion must be at minimum slightly more than this so

$$F\cos\theta = \mu_s \ mg - F\sin \ \theta \ \Rightarrow F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin \ \theta}$$

hich yields F = 59 hen θ = 60 .

c e no set θ = –60 and obtain

$$F = \frac{0.52 \ \ 11\,\text{kg} \ \ 9.8\ \text{m s}^2}{\cos \ -60° \ + \ 0.52 \ \sin \ -60°} = 1.1\times10^3 \quad .$$

100. a f the skier covers a distance L during time t ith ero initial speed and a constant acceleration a then $L = at^2$ 2 hich gives the acceleration a_1 for the first old pair of skis

$$a_1 = \frac{2L}{t_1^2} = \frac{2(200\,\text{m})}{(61\ \text{s})^2} = 0.11\ \text{m} \ \ \text{s}^2.$$

b he acceleration a_2 for the second ne pair is

$$a_2 = \frac{2L}{t_2^2} = \frac{2(200\,\text{m})}{(42\,\text{s})^2} = 0.23\ \text{m} \ \ \text{s}^2 \ .$$

c he net force along the slope acting on the skier of mass m is

$$F_{\text{net}} = mg\sin\theta - f_k = mg(\sin\theta - \mu_k \cos\theta) = ma$$

hich e solve for μ_{k1} for the first pair of skis

$$\mu_{k1} = \tan\theta - \frac{a_1}{g\cos\theta} = \tan 3.0° - \frac{0.11\,\text{m s}^2}{9.8\ \text{m s}^2 \ \ \cos 3.0°} = 0.041$$

d or the second pair e have

$$\mu_{k2} = \tan\theta - \frac{a_2}{g\cos\theta} = \tan 3.0° - \frac{0.23\,\text{m s}^2}{9.8\ \text{m s}^2 \ \ \cos 3.0°} = 0.029 \ .$$

101. If we choose “downhill” positive, then Newton’s law gives

$$mg\sin\theta - f_k = ma$$

for the sliding child. o using . 6 12

$$f_k = \mu_k F_N = \mu_k m g$$

so e obtain $a = g \sin\theta - \mu_k \cos\theta = -0.5$ m s^2 note that the problem gives the direction of the acceleration vector as uphill even though the child is sliding do nhill so it is a deceleration . ith $\theta = 35$ e solve for the coefficient and find $\mu_k = 0.76$.

102. a ur x direction is hori ontal and is chosen as e also do ith y so that the components of the 100 force $\vec{F}$ are non negative. hus $F_x = F \cos\theta = 100$ hich the te tbook denotes F_h in this problem.

b ince there is no vertical acceleration, application of Newton's second law in the y direction gives

$$F_N + F_y = mg \Rightarrow F_N = mg - F\sin\theta$$

here $m = 25.0$ kg. his yields $F_N = 245$ in this case $\theta = 0$.

c o $F_x = F_h = F\cos\theta = 86.6$ for $\theta = 30.0$.

d nd $F_N = mg - F\sin\theta = 195$.

e e find $F_x = F_h = F\cos\theta = 50.0$ for $\theta = 60.0$.

f nd $F_N = mg - F\sin\theta = 158$.

g he condition for the chair to slide is

$$F_x > f_{s\,\mathrm{ma}} = \mu_s F_N \quad \text{here } \mu_s = 0.42.$$

or $\theta = 0$ e have

$$F_x = 100 \quad < f_{s\,\mathrm{ma}} = 0.42 \quad 245 \quad = 103$$

so the crate remains at rest.

h or $\theta = 30.0$ e find $F_x = 86.6 \quad > f_{s\,\mathrm{ma}} = 0.42 \quad 195 \quad = 81.9$ so the crate slides.

i or $\theta = 60$ e get $F_x = 50.0 \quad < f_{s\,\mathrm{ma}} = 0.42 \quad 158 \quad = 66.4$ hich means the crate must remain at rest.

103. a he intuitive conclusion that the tension is greatest at the bottom of the s ing is certainly supported by application of Newton's second law there:

$$T - mg = \frac{mv^2}{R} \Rightarrow T = m\left(g + \frac{v^2}{R}\right)$$

here . 6 18 has been used. ncreasing the speed eventually leads to the tension at the bottom of the circle reaching that breaking value of 40 .

b olving the above e uation for the speed e find

$$v = \sqrt{R\left(\frac{T}{m} - g\right)} = \sqrt{0.91\text{ m}\left(\frac{40}{0.37\text{ kg}} - 9.8\text{ m s}^2\right)}$$

hich yields v = 9.5 m s.

104. a he component of the eight along the incline ith do nhill understood as the positive direction is $mg\sin\theta$ here m = 630 kg and θ = 10.2 . ith f = 62.0 N, Newton's second la leads to $mg\sin\theta - f = ma$ hich yields a = 1.64 m s^2. sing . 2 15 e have

$$80.0\text{ m} = \left(6.20\ \frac{\text{m}}{\text{s}}\right) t + \frac{1}{2}\left(1.64\ \frac{\text{m}}{\text{s}^2}\right) t^2 .$$

his is solved using the uadratic formula. he positive root is t = 6.80 s.

b unning through the calculation of part a ith f = 42.0 instead of f = 62 results in t = 6.76 s.

105. cept for replacing f_s ith f_k ig 6 5 in the te tbook is appropriate. ith that figure in mind e choose uphill as the x direction. pplying e ton's second law to the x a is e have

$$f_k - W\sin\theta = ma \quad \text{here } m = \frac{W}{g}$$

and here W = 40 a = 0.80 m s^2 and θ = 25 . hus e find f_k = 20 . long the y a is e have

$$\sum \vec{F}_y = 0 \Rightarrow F_N = W\cos\theta$$

so that $\mu_k = f_k\ F_N$ = 0.56.

Chapter

1. **THINK** s the proton is being accelerated its speed increases and so does its kinetic energy.

EXPRESS o calculate the speed of the proton at a later time e use the e uation $v^2 = v_0^2 + 2a\Delta x$ from able 2 1. he change in kinetic energy is then e ual to

$$\Delta K = \frac{1}{2} m \;\, v_f^2 - v_i^2 \;\; .$$

ANALYZE a ith $\Delta x = 3.5 \text{ cm} = 0.035 \text{ m}$ and $a = 3.6 \times 10^{15} \text{ m s}^2$ e find the proton speed to be

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{\left(2.4 \times 10^7 \text{ m s}\right)^2 + 2\;\left(3.6 \times 10^{15} \text{ m s}^2\right)\left(0.035 \text{ m}\right)} = 2.9 \times 10^7 \text{ m s.}$$

b he initial kinetic energy is

$$K_i = \frac{1}{2} m v_0^2 = \frac{1}{2}\;\left(1.67 \times 10^{-27} \text{ kg}\right)\left(2.4 \times 10^7 \text{ m s}\right)^2 = 4.8 \times 10^{-13}$$

and the final kinetic energy is

$$K_f = \frac{1}{2} m v^2 = \frac{1}{2}\;\left(1.67 \times 10^{-27} \text{ kg}\right)\left(2.9 \times 10^7 \text{ m s}\right)^2 = 6.9 \times 10^{-13} \;\; .$$

hus the change in kinetic energy is

$$\Delta K = K_f - K_i = 6.9 \times 10^{-13} \;\; - 4.8 \times 10^{-13} \;\; = 2.1 \times 10^{-13} \;\; .$$

LEARN he change in kinetic energy can be re ritten as

$$\Delta K = \frac{1}{2} m \;\, v_f^2 - v_i^2 \;\; = \frac{1}{2} m \;\, 2a\Delta x \;\; = ma\Delta x = F\Delta x = W$$

hich according to the ork kinetic energy theorem is simply the ork done on the particle.

2. ith speed v = 11200 m s e find

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\ 2.9\times10^5\ \text{kg}\quad 11200\ \text{m s}\ ^2 = 1.8\times10^{13}\quad .$$

3. a he change in kinetic energy for the meteorite ould be

$$\Delta K = K_f - K_i = -K_i = -\frac{1}{2}m_i v_i^2 = -\frac{1}{2}\left(4\times10^6\ \text{kg}\right)\left(15\times10^3\ \text{m s}\right)^2 = -5\times10^{14}$$

or $\Delta K = 5\times10^{14}$. he negative sign indicates that kinetic energy is lost.

b he energy loss in units of megatons of ould be

$$-\Delta K = \left(5\times10^{14}\quad\right)\left(\frac{1\ \text{megaton}}{4.2\times10^{15}}\right) = 0.1\,\text{megaton}\qquad .$$

c he number of bombs N that the meteorite impact ould correspond to is found by noting that megaton = 1000 kilotons and setting up the ratio

$$N = \frac{0.1\times1000\,\text{kiloton}}{13\,\text{kiloton}} = 8.$$

4. a e set up the ratio

$$\frac{50\ \text{km}}{1\ \text{km}} = \left(\frac{E}{1\ \text{megaton}}\right)^{1\ 3}$$

and find $E = 50^3 \approx 1\times10^5$ megatons of .

b e note that 15 kilotons is e uivalent to 0.015 megatons. ividing the result from part a by 0.013 yields about ten million 10^7 bombs.

5. e denote the mass of the father as m and his initial speed v_i. he initial kinetic energy of the father is

$$K_i = \frac{1}{2}K_{\text{son}}$$

and his final kinetic energy hen his speed is $v_f = v_i$ 1.0 m s is $K_f = K_{\text{son}}$. e use these relations along ith . 7 1 in our solution.

a e see from the above that $K_i = \frac{1}{2}K_f$ hich ith units understood leads to

$$\frac{1}{2}mv_i^2 = \frac{1}{2}\left[\frac{1}{2}m\ \left(v_i + 1.0 \text{ m s}\right)^2\right].$$

he mass cancels and e find a second degree e uation for v_i $\frac{1}{2}v_i^2 - v_i - \frac{1}{2} = 0$. he positive root from the uadratic formula yields v_i = 2.4 m s.

b rom the first relation above $\left(K_i = \frac{1}{2}K_{\text{son}}\right)$ e have

$$\frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{1}{2}\ \ m\ 2\ \ v_{\text{son}}^2\right)$$

and after canceling m and one factor of 1 2 are led to $v_{\text{son}} = 2v_i = 4.8$ m/s.

6. e apply the e uation $x\ t\ = x_0 + v_0 t + \frac{1}{2}at^2$ found in able 2 1. ince at t = 0 s x_0 = 0 and $v_0 = 12$ m s the e uation becomes in unit of meters

$$x\ t\ = 12t + \tfrac{1}{2}at^2.$$

ith $x = 10$ m hen $t = 1.0$ s the acceleration is found to be $a = -4.0$ m s^2. he fact that $a < 0$ implies that the bead is decelerating. hus the position is described by $x\ t\ = 12t - 2.0t^2$. ifferentiating x ith respect to t then yields

$$v\ t\ = \frac{dx}{dt} = 12 - 4.0t.$$

ndeed at t =3.0 s $v\ t = 3.0\ = 0$ and the bead stops momentarily. he speed at $t = 10$ s is $v\ t = 10\ = -28$ m s and the corresponding kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\ 1.8\times10^{-2}\text{kg}\ \ -28\text{ m s}\ ^2 = 7.1\ \ .$$

7. ince this involves constant acceleration motion e can apply the e uations of able 2 1 such as $x = v_0 t + \frac{1}{2}at^2$ here $x_0 = 0$. e choose to analy e the third and fifth points obtaining

$$0.2\text{m} = v_0\ 1.0\text{ s}\ + \frac{1}{2}a\ \ 1.0\text{ s}\ ^2$$
$$0.8\text{m} = v_0\ 2.0\text{ s}\ + \frac{1}{2}a\ \ 2.0\text{ s}\ ^2.$$

imultaneous solution of the e uations leads to $v_0 = 0$ and $a = 0.40\,\mathrm{m/s^2}$. e no have t o ays to finish the problem. ne is to compute force from $F = ma$ and then obtain the ork from . 7 7. he other is to find ΔK as a ay of computing W in accordance ith . 7 10 . n this latter approach e find the velocity at $t = 2.0\,\mathrm{s}$ from $v = v_0 + at$ so $v = 0.80\,\mathrm{m/s}$. hus

$$W = \Delta K = \frac{1}{2}\ 3.0\,\mathrm{kg}\quad 0.80\,\mathrm{m\ s}\ ^2 = 0.96\ \ .$$

8. sing . 7 8 and . 3 23 e find the ork done by the ater on the ice block

$$W = \vec{F}\cdot\vec{d} = \left[\ 210\quad \hat{\mathrm{i}} - \ 150\quad \hat{}\ \right]\cdot\left[\ 15\ \mathrm{m}\ \hat{\mathrm{i}} - \ 12\ \mathrm{m}\ \hat{}\ \right] = \ 210\quad 15\ \mathrm{m}\ + \ -150\quad -12\ \mathrm{m}$$
$$= 5.0\times 10^3\ \ .$$

9. y the ork kinetic energy theorem

$$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}\ 2.0\,\mathrm{kg}\ \left(\ 6.0\,\mathrm{m\ s}\ ^2 - \ 4.0\,\mathrm{m\ s}\ ^2\right) = 20\ \ .$$

e note that the *directions* of $\vec{v}_f$ and $\vec{v}_i$ play no role in the calculation.

10. uation 7 8 readily yields

$$W = \ F_x\Delta x\quad F_y\Delta y\ = 2.0\quad \cos 100\quad 3.0\ \mathrm{m}\quad 2.0\quad \sin 100\quad 4.0\ \mathrm{m}\ = 6.8\ \ .$$

11. sing the ork kinetic energy theorem e have

$$\Delta K = W = \vec{F}\cdot\vec{d} = Fd\cos\phi .$$

n addition $F = 12$ and $d = \sqrt{\ 2.00\ \mathrm{m}\ ^2 + \ -4.00\ \mathrm{m}\ ^2 + \ 3.00\ \mathrm{m}\ ^2} = 5.39\ \mathrm{m}$.

a f $\Delta K = +30.0$ then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{30.0}{12.0\quad 5.39\ \mathrm{m}}\right) = 62.3^\circ .$$

b $\Delta K = -30.0$ then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{-30.0}{12.0\quad 5.39\ \mathrm{m}}\right) = 118^\circ .$$

12. a rom . 7 6 $F = W/x = 3.00$ this is the slope of the graph .

b uation 7 10 yields $K = K_i$ W = 3.00 6.00 = 9.00 .

13. e choose x as the direction of motion so $\vec{a}$ and $\vec{F}$ are negative valued .

(a) Newton's second law readily yields $\vec{F} =$ 85 ky –2.0 m s^2 so that

$$F = \vec{F} = 1.7\times 10^2$$.

b rom . 2 16 ith v = 0 e have

$$0 = v_0^2 + 2a\Delta x \;\Rightarrow\; \Delta x = -\frac{(37\,\text{m s})^2}{2(-2.0\,\text{m s}^2)} = 3.4\times 10^2\ \text{m}.$$

lternatively this can be orked using the ork energy theorem.

c ince $\vec{F}$ is opposite to the direction of motion so the angle ϕ bet een $\vec{F}$ and $\vec{d} = \Delta x$ is 180 then . 7 7 gives the ork done as $W = -F\Delta x = -5.8\times 10^4$.

(d) In this case, Newton's second law yields $\vec{F} = (85\,\text{ky})(-4.0\,\text{m s}^2)$ so that $F = \vec{F} = 3.4\times 10^2$.

e rom . 2 16 e no have

$$\Delta x = -\frac{(37\,\text{m s})^2}{2(-4.0\,\text{m s}^2)} = 1.7\times 10^2\ \text{m}.$$

f he force $\vec{F}$ is again opposite to the direction of motion so the angle ϕ is again 180 so that . 7 7 leads to $W = -F\Delta x = -5.8\times 10^4$. he fact that this agrees ith the result of part c provides insight into the concept of ork.

14. he forces are all constant so the total ork done by them is given by $W = F_{\text{net}}\Delta x$ here F_{net} is the magnitude of the net force and Δx is the magnitude of the displacement. e add the three vectors finding the x and y components of the net force

$$F_{\text{net}\,x} = -F_1 - F_2\sin 50.0° + F_3\cos 35.0° = -3.00 \; - \; 4.00 \quad \sin 35.0° + \; 10.0 \quad \cos 35.0°$$
$$= 2.13$$

$$F_{\text{net}\,y} = -F_2\cos 50.0° + F_3\sin 35.0° = - \; 4.00 \quad \cos 50.0° + \; 10.0 \quad \sin 35.0°$$
$$= 3.17$$.

he magnitude of the net force is

$$F_{\text{net}} = \sqrt{F^2_{\text{net}\,x} + F^2_{\text{net}\,y}} = \sqrt{\;2.13\quad^2 + \;3.17\quad^2} = 3.82 \quad .$$

he ork done by the net force is

$$W = F_{\text{net}}d = \;3.82 \qquad 4.00\,\text{m} \;= 15.3$$

here e have used the fact that $\vec{d}\;\;\vec{F}_{\text{net}}$ hich follo s from the fact that the canister started from rest and moved hori ontally under the action of hori ontal forces — the resultant effect of hich is e pressed by $\vec{F}_{\text{net}}$.

15. a he forces are constant so the ork done by any one of them is given by $W = \vec{F}\cdot\vec{d}$ here $\vec{d}$ is the displacement. orce $\vec{F}_1$ is in the direction of the displacement so

$$W_1 = F_1 d\cos\phi_1 = \;5.00 \qquad 3.00\,\text{m}\;\cos 0° = 15.0 \quad .$$

orce $\vec{F}_2$ makes an angle of 120 ith the displacement so

$$W_2 = F_2 d\cos\phi_2 = \;9.00 \qquad 3.00\,\text{m}\;\cos 120° = -13.5 \quad .$$

orce $\vec{F}_3$ is perpendicular to the displacement so

$$W_3 = F_3 d\cos\phi_3 = 0 \text{ since } \cos 90 \;\; = 0.$$

he net ork done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0 \quad -13.5 \quad + 0 = +1.50 \quad .$$

b f no other forces do ork on the bo its kinetic energy increases by 1.50 during the displacement.

16. he change in kinetic energy can be ritten as

$$\Delta K = \frac{1}{2}m\;v_f^2 - v_i^2\;\; = \frac{1}{2}m\;2a\Delta x\;\; = ma\Delta x$$

here e have used $v_f^2 = v_i^2 + 2a\Delta x$ from able 2 1. rom the figure e see that $\Delta K = \;0 - 30 \quad = -30$ hen $\Delta x = +5$ m. he acceleration can then be obtained as

$$a = \frac{\Delta K}{m\Delta x} = \frac{-30}{8.0\text{ kg}\;\;5.0\text{ m}} = -0.75\text{ m s}^2.$$

he negative sign indicates that the mass is decelerating. rom the figure e also see that hen $x = 5$ m the kinetic energy becomes ero implying that the mass comes to rest momentarily. hus

$$v_0^2 = v^2 - 2a\Delta x = 0 - 2\ \ -0.75\text{ m s}^2\ \ 5.0\text{ m}\ = 7.5\text{ m}^2\text{ s}^2$$

or $v_0 = 2.7\text{ m s}$. he speed of the ob ect hen x = –3.0 m is

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{7.5\text{ m}^2\text{ s}^2 + 2\ \ -0.75\text{ m s}^2\ \ -3.0\text{ m}\ } = \sqrt{12}\text{ m s} = 3.5\text{ m s}.$$

17. **THINK** he helicopter does ork to lift the astronaut up ard against gravity. he ork done on the astronaut is converted to the kinetic energy of the astronaut.

EXPRESS e use $\vec{F}$ to denote the up ard force e erted by the cable on the astronaut. he force of the cable is up ard and the force of gravity is mg do n ard. urthermore the acceleration of the astronaut is $a = g/10$ upward. According to Newton's second la the force is given by

$$F - mg = ma \;\Rightarrow\; F = m\ \ g + a\ = \frac{11}{10}mg$$

in the same direction as the displacement. n the other hand the force of gravity has magnitude $F_g = mg$ and is opposite in direction to the displacement.

ANALYZE a ince the force of the cable $\vec{F}$ and the displacement $\vec{d}$ are in the same direction the ork done by $\vec{F}$ is

$$W_F = Fd = \frac{11mgd}{10} = \frac{11\ \ 72\text{ kg}\ \ 9.8\text{ m s}^2\ \ 15\text{ m}}{10} = 1.164\times10^4\ \ \approx 1.2\times10^4\ \ .$$

b sing . 7 7 the ork done by gravity is

$$W_g = -F_g d = -mgd = -\ \ 72\text{ kg}\ \ 9.8\text{ m s}^2\ \ 15\text{ m}\ = -1.058\times10^4\ \ \approx -1.1\times10^4\ \ .$$

c he total ork done is the sum of the t o orks

$$W_{\text{net}} = W_F + W_g = 1.164\times10^4\ \ -1.058\times10^4\ \ = 1.06\times10^3\ \ \approx 1.1\times10^3\ \ .$$

ince the astronaut started from rest the ork kinetic energy theorem tells us that this is her final kinetic energy.

d ince $K = \frac{1}{2}mv^2$ her final speed is $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2\ 1.06\times10^3}{72\text{ kg}}} = 5.4\text{ m s}.$

LEARN or a general up ard acceleration a the net ork done is

$$W_{\text{net}} = W_F + W_g = Fd - F_g d = m\ g + a\ d - mgd = mad.$$

ince $W_{\text{net}} = \Delta K = mv^2\ \ 2$ by the ork kinetic energy theorem the speed of the astronaut ould be $v = \sqrt{2ad}$ hich is independent of the mass of the astronaut. n our case $v = \sqrt{2\ 9.8\,\text{m}\ \text{s}^2\ \ 10\ \ 15\ \text{m}} = 5.4\ \text{m}\ \text{s}$ hich agrees ith that calculated in d .

18. n both cases there is no acceleration so the lifting force is e ual to the eight of the ob ect.

a uation 7 8 leads to $W = \vec{F} \cdot \vec{d} =\ 360\,\text{k}\ \ \ 0.10\,\text{m}\ = 36\ \text{k}$.

b n this case e find $W =\ 4000\ \ \ 0.050\ \text{m}\ = 2.0 \times 10^2$.

19. uation 7 15 applies but the ording of the problem suggests that it is only necessary to examine the contribution from the rope (which would be the "W_a" term in . 7 15

$$W_a = -\ 50\ \ \ 0.50\ \text{m}\ = -25$$

the minus sign arises from the fact that the pull from the rope is anti parallel to the direction of motion of the block . hus the kinetic energy ould have been 25 greater if the rope had not been attached given the same displacement .

20. rom the figure one may rite the kinetic energy in units of as a function of x as

$$K = K_s - 20x = 40 - 20x.$$

ince $W = \Delta K = \vec{F}_x \cdot \Delta x$ the component of the force along the force along x is $F_x = dK\ \ dx = -20$. he normal force on the block is $F_N = F_y$ hich is related to the gravitational force by

$$mg = \sqrt{F_x^2 +\ -F_y\ ^2}.$$

ote that F_N points in the opposite direction of the component of the gravitational force. ith an initial kinetic energy $K_s = 40.0$ and $v_0 = 4.00\ \text{m s}$ the mass of the block is

$$m = \frac{2K_s}{v_0^2} = \frac{2\ 40.0}{4.00\ \text{m s}\ ^2} = 5.00\ \text{kg}.$$

hus the normal force is

$$F_y = \sqrt{\ mg\ ^2 - F_x^2} = \sqrt{\ 5.0\ \text{kg}\ ^2\ 9.8\ \text{m s}^2\ ^2 -\ 20\ \ ^2} = 44.7\ \ \approx 45\ \ .$$

21. **THINK** n this problem the cord is doing ork on the block so that it does not undergo free fall.

EXPRESS e use F to denote the magnitude of the force of the cord on the block. his force is up ard opposite to the force of gravity hich has magnitude $F_g = Mg$ to prevent the block from undergoing free fall. he acceleration is $\vec{a} = g$ 4 do n ard. Taking the downward direction to be positive, then Newton's second law yields

$$\vec{F}_{\text{net}} = m\vec{a} \Rightarrow Mg - F = M\left(\frac{g}{4}\right)$$

so F = 3Mg 4 in the opposite direction of the displacement. n the other hand the force of gravity $F_g = mg$ is in the same direction to the displacement.

ANALYZE (a) Since the displacement is downward, the work done by the cord's force is using . 7 7

$$W_F = -Fd = -\frac{3}{4}Mgd.$$

b imilarly the ork done by the force of gravity is $W_g = F_g d = Mgd$.

c he total ork done on the block is simply the sum of the t o orks

$$W_{\text{net}} = W_F + W_g = -\frac{3}{4}Mgd + Mgd = \frac{1}{4}Mgd\ .$$

ince the block starts from rest e use . 7 15 to conclude that this $(Mgd/4)$ is the block's kinetic energy K at the moment it has descended the distance d.

d ith $K = \frac{1}{2}Mv^2$ the speed is

$$v = \sqrt{\frac{2K}{M}} = \sqrt{\frac{2\ \ Mgd\ \ 4}{M}} = \sqrt{\frac{gd}{2}}$$

at the moment the block has descended the distance d.

LEARN or a general do n ard acceleration a the force e erted by the cord is $F = m\ \ g - a$ so that the net ork done on the block is $W_{\text{net}} = F_{\text{net}}d = mad$. he speed of the block after falling a distance d is $v = \sqrt{2ad}$. n the special case here the block hangs still $a = 0$ $F = mg$ and $v = 0$. n our case $a = g$ 4 and $v = \sqrt{2\ g\ \ 4\ d} = \sqrt{gd\ \ 2}$ hich agrees ith that calculated in d .

22. e use d to denote the magnitude of the spelunker's displacement during each stage. he mass of the spelunker is m = 80.0 kg. he ork done by the lifting force is denoted W_i here i = 1 2 3 for the three stages. e apply the ork energy theorem . 17 15.

a or stage 1 $W_1 - mgd = \Delta K_1 = \frac{1}{2} m v_1^2$ here $v_1 = 5.00$ m s. his gives

$$W_1 = mgd + \frac{1}{2} m v_1^2 = \ 80.0\text{ kg} \ \ 9.80\text{ m s}^2 \ \ 10.0\text{ m} \ + \frac{1}{2} \ 80.0\text{ kg} \ \ 5.00\text{ m s} \ ^2 = 8.84 \times 10^3 \ \ .$$

b or stage 2 $W_2 - mgd = \Delta K_2 = 0$ hich leads to

$$W_2 = mgd = \ 80.0\text{ kg} \ \ 9.80\text{ m s}^2 \ \ 10.0\text{ m} \ = 7.84 \times 10^3 \ \ .$$

c or stage 3 $W_3 - mgd = \Delta K_3 = -\frac{1}{2} m v_1^2$. e obtain

$$W_3 = mgd - \frac{1}{2} m v_1^2 = \ 80.0\text{ kg} \ \ 9.80\text{ m s}^2 \ \ 10.0\text{ m} \ - \frac{1}{2} \ 80.0\text{ kg} \ \ 5.00\text{ m s} \ ^2 = 6.84 \times 10^3 \ \ .$$

23. he fact that the applied force $\vec{F}_a$ causes the bo to move up a frictionless ramp at a constant speed implies that there is no net change in the kinetic energy $\Delta K = 0$. hus the ork done by $\vec{F}_a$ must be e ual to the negative ork done by gravity $W_a = -W_g$. ince the bo is displaced vertically up ard by $h = 0.150$ m e have

$$W_a = +mgh = \ 3.00\text{ kg} \ \ 9.80\text{ m s}^2 \ \ 0.150\text{ m} \ = 4.41$$

24. a sing notation common to many vector capable calculators e have from . 7 8 W = dot 20.0 0 0 – 3.00 9.8 0.500 ∠ 30.0 = 1.31 J , where "dot" stands for dot product.

b . 7 10 along ith . 7 1 then leads to $v = \sqrt{2\ 1.31 \ \ / 3.00\text{ kg}}$ = 0.935 m s.

25. a he net up ard force is given by

$$F + F_N - \ m + M \ g = \ m + M \ a$$

here m = 0.250 kg is the mass of the cheese M = 900 kg is the mass of the elevator cab F is the force from the cable and $F_N = 3.00$ is the normal force on the cheese. n the cheese alone e have

$$F_N - mg = ma \ \Rightarrow \ a = \frac{3.00 \ \ - \ 0.250\text{ kg} \ \ 9.80\text{ m s}^2}{0.250\text{ kg}} = 2.20\text{ m s}^2 .$$

hus the force from the cable is $F = \; m+M \quad a+g \quad -F_N = 1.08\times10^4$ and the ork done by the cable on the cab is

$$W = Fd_1 = \; 1.80\times10^4 \qquad 2.40\text{ m} \; = 2.59\times10^4 \quad .$$

b f $W = 92.61$ k and $d_2 = 10.5$ m the magnitude of the normal force is

$$F_N = \; m+M \; g - \frac{W}{d_2} = \; 0.250\text{ kg} + 900\text{ kg} \;\; 9.80\text{ m s}^2 \; - \frac{9.261\times10^4}{10.5\text{ m}} = 2.45 \quad .$$

26. e make use of . 7 25 and . 7 28 since the block is stationary before and after the displacement. he ork done by the applied force can be ritten as

$$W_a = -W_s = \frac{1}{2}k \; x_f^2 - x_i^2 \quad .$$

he spring constant is $k = \; 80 \qquad 2.0\text{ cm} \; = 4.0\times10^3 \quad \text{m}.$ ith $W_a = 4.0$ and $x_i = -2.0\text{ cm}$ e have

$$x_f = \pm\sqrt{\frac{2W_a}{k} + x_i^2} = \pm\sqrt{\frac{2 \;\, 4.0}{4.0\times10^3 \qquad \text{m}} + \; -0.020\text{ m} \; ^2} = \pm0.049\text{ m} = \pm4.9\text{ cm}.$$

27. rom . 7 25 e see that the ork done by the spring force is given by

$$W_s = \frac{1}{2}k \; x_i^2 - x_f^2 \quad .$$

he fact that 360 of force must be applied to pull the block to $x =$ 4.0 cm implies that the spring constant is

$$k = \frac{360}{4.0\text{ cm}} = 90 \qquad \text{cm} = 9.0\times10^3 \qquad \text{m}\,.$$

a hen the block moves from $x_i = +5.0\text{ cm}$ to $x = +3.0\text{ cm}$ e have

$$W_s = \frac{1}{2} \; 9.0\times10^3 \qquad \text{m} \quad 0.050\text{ m} \; ^2 - \; 0.030\text{ m} \; ^2 \; = 7.2 \quad .$$

b oving from $x_i = +5.0\text{ cm}$ to $x = -3.0\text{ cm}$ e have

$$W_s = \frac{1}{2} \; 9.0\times10^3 \qquad \text{m} \quad 0.050\text{ m} \; ^2 - \; -0.030\text{ m} \; ^2 \; = 7.2 \quad .$$

c oving from $x_i = +5.0$ cm to $x = -5.0$ cm e have

$$W_s = \frac{1}{2}\ 9.0\times10^3 \quad \text{m} \quad 0.050 \text{ m}^{\ 2} - \ -0.050 \text{ m}^{\ 2} = 0 \ .$$

d oving from $x_i = +5.0$ cm to $x = -9.0$ cm e have

$$W_s = \frac{1}{2}\ 9.0\times10^3 \quad \text{m} \quad 0.050 \text{ m}^{\ 2} - \ -0.090 \text{ m}^{\ 2} = -25 \ .$$

28. he spring constant is k = 100 m and the ma imum elongation is x_i = 5.00 m. sing . 7 25 ith x_f = 0 the ork is found to be

$$W = \frac{1}{2}kx_i^2 = \frac{1}{2}\ 100 \quad \text{m} \ \ 5.00 \text{ m}^{\ 2} = 1.25\times10^3 \ .$$

29. he ork done by the spring force is given by . 7 25 $W_s = \frac{1}{2}k\ x_i^2 - x_f^2$. he spring constant k can be deduced from the figure hich sho s the amount of ork done to pull the block from 0 to x = 3.0 cm. he parabola $W_a = kx^2$ 2 contains 0 0 2.0 cm 0.40 and 3.0 cm 0.90 . hus e may infer from the data that $k = 2.0\times10^3$ m .

a hen the block moves from $x_i = +5.0$ cm to $x = +4.0$ cm e have

$$W_s = \frac{1}{2}\ 2.0\times10^3 \quad \text{m} \quad 0.050 \text{ m}^{\ 2} - \ 0.040 \text{ m}^{\ 2} = 0.90 \ .$$

b oving from $x_i = +5.0$ cm to $x = -2.0$ cm e have

$$W_s = \frac{1}{2}\ 2.0\times10^3 \quad \text{m} \quad 0.050 \text{ m}^{\ 2} - \ -0.020 \text{ m}^{\ 2} = 2.1 \ .$$

c oving from $x_i = +5.0$ cm to $x = -5.0$ cm e have

$$W_s = \frac{1}{2}\ 2.0\times10^3 \quad \text{m} \quad 0.050 \text{ m}^{\ 2} - \ -0.050 \text{ m}^{\ 2} = 0 \ .$$

30. Hooke's law and the work done by a spring is discussed in the chapter. We apply the ork kinetic energy theorem in the form of $\Delta K = W_a + W_s$ to the points in the figure at x = 1.0 m and x = 2.0 m, respectively. The "applied" work W_a is that due to the constant force $\vec{P}$.

$$4 \quad = P \; 1.0 \text{ m} \; - \frac{1}{2} k \; 1.0 \text{ m} \;^2$$
$$0 = P \; 2.0 \text{ m} \; - \frac{1}{2} k \; 2.0 \text{ m} \;^2.$$

a imultaneous solution leads to P = 8.0 .

b imilarly e find k = 8.0 m.

31. **THINK** he applied force varies ith x so an integration is re uired to calculate the ork done on the body.

EXPRESS s the body moves along the x a is from x_i = 3.0 m to x_f = 4.0 m the ork done by the force is

$$W = \int_{x_i}^{x_f} F_x \; dx = \int_{x_i}^{x_f} -6x \; dx = -3 \; x_f^2 - x_i^2 \; = -3 \; 4.0^2 - 3.0^2 \; = -21 \; .$$

ccording to the ork kinetic energy theorem this gives the change in the kinetic energy

$$W = \Delta K = \frac{1}{2} m\left(v_f^2 - v_i^2\right)$$

here v_i is the initial velocity at x_i and v_f is the final velocity at x_f . iven v_i e can readily calculate v_f .

ANALYZE a he ork kinetic theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2 \; -21}{2.0 \text{ kg}} + \; 8.0 \text{ m s} \;^2} = 6.6 \text{ m s}.$$

b he velocity of the particle is v_f = 5.0 m s hen it is at $x = x_f$. he ork kinetic energy theorem is used to solve for x_f. he net ork done on the particle is $W = -3\left(x_f^2 - x_i^2\right)$ so the theorem leads to

$$W = \Delta K \quad \Rightarrow \quad -3\left(x_f^2 - x_i^2\right) = \frac{1}{2} m \; \left(v_f^2 - v_i^2\right).$$

hus

$$x_f = \sqrt{-\frac{m}{6}\left(v_f^2 - v_i^2\right) + x_i^2} = \sqrt{-\frac{2.0 \text{ kg}}{6 \quad \text{m}}\left(\; 5.0 \text{ m s} \;^2 - \; 8.0 \text{ m s} \;^2\right) + \; 3.0 \text{ m} \;^2} = 4.7 \text{ m}.$$

LEARN ince $x_f > x_i$ $W = -3 \; x_f^2 - x_i^2 \; < 0$ i.e. the ork done by the force is negative. rom the ork kinetic energy theorem this implies $\Delta K < 0$. ence the speed of the particle ill continue to decrease as it moves in the x direction.

32. he ork done by the spring force is given by . 7 25 $W_s = \frac{1}{2}k\ x_i^2 - x_f^2$. ince $F_x = -kx$ the slope in ig. 7 37 corresponds to the spring constant k. ts value is given by $k = 80$ cm=8.0×10^3 m.

a hen the block moves from $x_i = +8.0$ cm to $x = +5.0$ cm e have

$$W_s = \frac{1}{2}\ 8.0\times10^3 \quad \text{m} \quad 0.080\ \text{m}\ ^2 - \ 0.050\ \text{m}\ ^2 = 15.6 \quad \approx 16\ .$$

b oving from $x_i = +8.0$ cm to $x = -5.0$ cm e have

$$W_s = \frac{1}{2}\ 8.0\times10^3 \quad \text{m} \quad 0.080\ \text{m}\ ^2 - \ -0.050\ \text{m}\ ^2 = 15.6 \quad \approx 16\ .$$

c oving from $x_i = +8.0$ cm to $x = -8.0$ cm e have

$$W_s = \frac{1}{2}\ 8.0\times10^3 \quad \text{m} \quad 0.080\ \text{m}\ ^2 - \ -0.080\ \text{m}\ ^2 = 0\ .$$

d oving from $x_i = +8.0$ cm to $x = -10.0$ cm e have

$$W_s = \frac{1}{2}\ 8.0\times10^3 \quad \text{m} \quad 0.080\ \text{m}\ ^2 - \ -0.10\ \text{m}\ ^2 = -14.4 \quad \approx -14\ .$$

33. a his is a situation here . 7 28 applies so e have

$$Fx = \tfrac{1}{2}kx^2 \Rightarrow 3.0 \quad x = \tfrac{1}{2}\ 50 \quad \text{m}\ x^2$$

hich other than the trivial root gives $x =$ 3.0 25 m = 0.12 m.

b he ork done by the applied force is $W_{\text{a}} = Fx =$ 3.0 0.12 m = 0.36 .

c . 7 28 immediately gives $W_s = -W_{\text{a}} = -0.36$.

d ith $K_f = K$ considered variable and $K_i = 0$. 7 27 gives $K = Fx - \tfrac{1}{2}kx^2$. e take the derivative of K ith respect to x and set the resulting e pression e ual to ero in order to find the position x_{c} taht corresponds to a ma imum value of K

$$x_{\text{c}} = \frac{F}{k} = \ 3.0\ 50\ \text{m} = 0.060\ \text{m}.$$

e note that x_{c} is also the point here the applied and spring forces "balance."

e t x_c e find $K = K_{ma}$ = 0.090 .

34. ccording to the graph the acceleration a varies linearly ith the coordinate x. e may rite $a = \alpha x$ here α is the slope of the graph. umerically

$$\alpha = \frac{20 \text{ m } \text{s}^2}{8.0 \text{ m}} = 2.5 \text{ s}^{-2}.$$

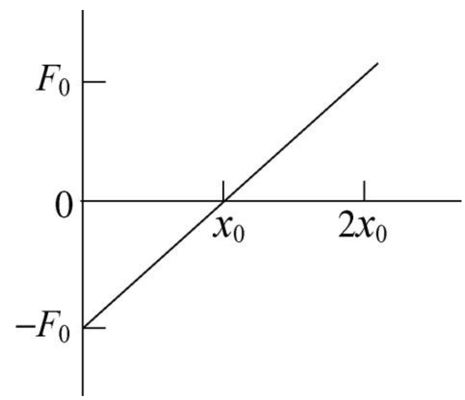
he force on the brick is in the positive x direction and, according to Newton's second la its magnitude is given by $F = ma = m\alpha x$. f x_f is the final coordinate the ork done by the force is

$$W = \int_0^{x_f} F \ dx = m\alpha \int_0^{x_f} x \ dx = \frac{m\alpha}{2} x_f^2 = \frac{10 \text{ kg} \ \ 2.5 \text{ s}^{-2}}{2} \ 8.0 \text{ m}^{\ 2} = 8.0 \times 10^2 \ \ .$$

35. **THINK** e have an applied force that varies ith x. n integration is re uired to calculate the ork done on the particle.

EXPRESS iven a one dimensional force $F \ x$ the ork done is simply e ual to the area under the curve $W = \int_{x_i}^{x_f} F \ x \ \ dx$.

ANALYZE a he plot of $F \ x$ is sho n to the right. ere e take x_0 to be positive. he ork is negative as the ob ect moves from $x = 0$ to $x = x_0$ and positive as it moves from $x = x_0$ to $x = 2x_0$.



ince the area of a triangle is base altitude 2 the ork done from $x = 0$ to $x = x_0$ is $W_1 = - \ x_0 \ \ F_0 \ \ 2$ and the ork done from $x = x_0$ to $x = 2x_0$ is

$$W_2 = \ 2x_0 - x_0 \ \ F_0 \ \ 2 = \ x_0 \ \ F_0 \ \ 2$$

he total ork is the sum of the t o

$$W = W_1 + W_2 = -\frac{1}{2} F_0 x_0 + \frac{1}{2} F_0 x_0 = 0.$$

b he integral for the ork is

$$W = \int_0^{2x_0} F_0 \left(\frac{x}{x_0} - 1 \right) dx = F_0 \left(\frac{x^2}{2x_0} - x \right) \Bigg|_0^{2x_0} = 0.$$

LEARN f the particle starts out at x = 0 ith an initial speed v_i ith a negative ork $W_1 = -F_0 x_0 \ \ 2 < 0$ its speed at $x = x_0$ ill decrease to

$$v=\sqrt{v_i^2+\frac{2W_1}{m}}=\sqrt{v_i^2-\frac{F_0x_0}{m}}<v_i$$

but return to v_i again at $x=2x_0$ ith a positive ork $W_2=F_0x_0$ $2>0$.

36. rom . 7 32, we see that the "area" in the graph is equivalent to the work done. inding that area in terms of rectangular length $\times$ idth and triangular $\frac{1}{2}$ base $\times$ height areas e obtain

$$W=W_{0<x<2}+W_{2<x<4}+W_{4<x<6}+W_{6<x<8}= 20+10+0-5 \quad =25 \quad .$$

37. a e first multiply the vertical a is by the mass so that it becomes a graph of the applied force. Now, adding the triangular and rectangular "areas" in the graph (for $0 \le x \le 4$ gives 42 for the ork done.

b Counting the "areas" under the axis as negative contributions, we find (for $0 \le x \le 7$ the ork to be 30 at x = 7.0 m.

c nd at x = 9.0 m the ork is 12 .

d uation 7 10 along ith . 7 1 leads to speed v = 6.5 m s at x = 4.0 m. eturning to the original graph here a as plotted e note that since it started from rest it has received acceleration s up to this point only in the $+x$ direction and conse uently must have a velocity vector pointing in the x direction at x = 4.0 m.

e o using the result of part b and . 7 10 along ith . 7 1 e find the speed is 5.5 m s at x = 7.0 m. lthough it has e perienced some deceleration during the $0 \le x \le 7$ interval its velocity vector still points in the x direction.

f inally using the result of part c and . 7 10 along ith . 7 1 e find its speed v = 3.5 m s at x = 9.0 m. t certainly has e perienced a significant amount of deceleration during the $0 \le x \le 9$ interval nonetheless its velocity vector *still* points in the x direction.

38. a sing the ork kinetic energy theorem

$$K_f=K_i+\int_0^{2.0} 2.5-x^2 \quad dx=0+ 2.5 \quad 2.0 \; -\frac{1}{3} \; 2.0 \;^3=2.3 \quad .$$

b or a variable end point e have K_f as a function of x hich could be differentiated to find the e tremum value but e recogni e that this is e uivalent to solving F = 0 for x

$$F=0 \Rightarrow 2.5- x^2=0.$$

hus K is e tremi ed at $x = \sqrt{2.5} \approx 1.6$ m and e obtain

$$K_f = K_i + \int_0^{\sqrt{2.5}} 2.5 - x^2 \; dx = 0 + 2.5 \quad \sqrt{2.5} \; - \frac{1}{3} \quad \sqrt{2.5}^{\;3} = 2.6 \quad .$$

ecalling our ans er for part a it is clear that this e treme value is a ma imum.

39. s the body moves along the x a is from x_i = 0 m to x_f = 3.00 m the ork done by the force is

$$W = \int_{x_i}^{x_f} F_x \; dx = \int_{x_i}^{x_f} cx - 3.00x^2 \; dx = \left(\frac{c}{2}x^2 - x^3 \right)_0^3 = \frac{c}{2} \; 3.00^{\;2} - \; 3.00^{\;3}$$
$$= 4.50c - 27.0.$$

o ever $W = \Delta K = \; 11.0 - 20.0 \; = -9.00$ from the ork kinetic energy theorem. hus

$$4.50c - 27.0 = -9.00$$

or $c = 4.00$ m.

40. sing . 7 32 e find

$$W = \int_{0.25}^{1.25} e^{-4x^2} \; dx = 0.21$$

here the result has been obtained numerically. any modern calculators have that capability as ell as most math soft are packages that a great many students have access to.

41. e choose to ork this using . 7 10 the ork kinetic energy theorem . o find the initial and final kinetic energies e need the speeds so

$$v = \frac{dx}{dt} = 3.0 - 8.0t + 3.0t^2$$

in units. hus the initial speed is v_i = 3.0 m s and the speed at t = 4 s is v_f = 19 m s. he change in kinetic energy for the ob ect of mass m = 3.0 kg is therefore

$$\Delta K = \frac{1}{2}m \; \left(v_f^2 - v_i^2 \right) = 528$$

hich e round off to t o figures and using the ork kinetic energy theorem conclude that the ork done is $W = 5.3 \times 10^2$.

42. e solve the problem using the ork kinetic energy theorem hich states that the change in kinetic energy is e ual to the ork done by the applied force $\Delta K = W$. n our

problem the ork done is $W = Fd$ here F is the tension in the cord and d is the length of the cord pulled as the cart slides from x_1 to x_2. rom the figure e have

$$d = \sqrt{x_1^2 + h^2} - \sqrt{x_2^2 + h^2} = \sqrt{\ 3.00\text{ m}\ ^2 + \ 1.20\text{ m}\ ^2} - \sqrt{\ 1.00\text{ m}\ ^2 + \ 1.20\text{ m}\ ^2}$$
$$= 3.23\text{ m} - 1.56\text{ m} = 1.67\text{ m}$$

hich yields $\Delta K = Fd = \ 25.0\ \ \ \ 1.67\text{ m}\ = 41.7$.

43. **THINK** his problem deals ith the po er and ork done by a constant force.

EXPRESS he po er done by a constant force F is given by $P = Fv$ and the ork done by F from time t_1 to time t_2 is

$$W = \int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2} Fv\,dt$$

ince F is the magnitude of the net force the magnitude of the acceleration is $a = F\ m$. hus if the initial velocity is $v_0 = 0$ then the velocity of the body as a function of time is given by $v = v_0 + at = \ F/m\ t$. ubstituting the e pression for v into the e uation above the ork done during the time interval $t_1\ t_2$ becomes

$$W = \int_{t_1}^{t_2} \ F^2\ \ m\ t\,dt = \frac{F^2}{2m}\left(t_2^2 - t_1^2\right).$$

ANALYZE a or $t_1 = 0$ and $t_2 = 1.0$ s $W = \frac{1}{2}\left(\frac{\ 5.0\ \ ^2}{15\text{ kg}}\right)\ 1.0\text{ s}\ ^2 - 0\ = 0.83$.

b or $t_1 = 1.0$s and $t_2 = 2.0$ s $W = \frac{1}{2}\left(\frac{\ 5.0\ \ ^2}{15\text{ kg}}\right)\ 2.0\text{ s}\ ^2 - \ 1.0\text{ s}\ ^2\ = 2.5$.

c or $t_1 = 2.0$ s and $t_2 = 3.0$ s $W = \frac{1}{2}\left(\frac{\ 5.0\ \ ^2}{15\text{ kg}}\right)\ 3.0\text{ s}\ ^2 - \ 2.0\text{ s}\ ^2\ = 4.2$.

d ubstituting $v = \ F\ m\ t$ into $P = Fv$ e obtain $P = F^2\,t/m$ for the po er at any time t. t the end of the third second the instantaneous po er is

$$P = \left(\frac{\ 5.0\ \ ^2\ \ 3.0\text{ s}}{15\text{ kg}}\right) = 5.0\ \ .$$

LEARN he ork done here is uadratic in t. herefore from the definition $P = dW\ \ dt$ for the instantaneous po er e see that P increases linearly ith t.

44. a ince constant speed implies $\Delta K = 0$ e re uire $W_a = -W_g$ by . 7 15. ince W_g is the same in both cases same eight and same path then $W_a = 9.0\times10^2$ ust as it as in the first case.

b ince the speed of 1.0 m s is constant then 8.0 meters is traveled in 8.0 seconds. sing . 7 42 and noting that average po er is *the* po er hen the ork is being done at a steady rate e have

$$P = \frac{W}{\Delta t} = \frac{900}{8.0 \text{ s}} = 1.1\times10^2 \quad .$$

c ince the speed of 2.0 m s is constant 8.0 meters is traveled in 4.0 seconds. sing . 7 42 ith *average power* replaced by *power* e have

$$P = \frac{W}{\Delta t} = \frac{900}{4.0 \text{ s}} = 225 \quad \approx 2.3\times10^2 \quad .$$

45. **THINK** block is pulled at a constant speed by a force directed at some angle ith respect to the direction of motion. The quantity we're interested in is the power, or the time rate at hich ork is done by the applied force.

EXPRESS he po er associated ith force $\vec{F}$ is given by $P = \vec{F}\cdot\vec{v} = Fv\cos\phi$ here $\vec{v}$ is the velocity of the ob ect on hich the force acts and ϕ is the angle bet een $\vec{F}$ and $\vec{v}$.

ANALYZE ith $F = 122$ $v = 5.0$ m s and $\phi = 37.0°$ e find the po er to be

$$P = Fv\cos\phi = \quad 122 \qquad 5.0 \text{ m s} \quad \cos 37.0° = 4.9\times10^2 \quad .$$

LEARN rom the e pression $P = Fv\cos\phi$ e see that the po er is at a ma imum hen $\vec{F}$ and $\vec{v}$ are in the same direction $\phi = 0$ and is ero hen they are perpendicular of each other. In addition, we're told that the block moves at a constant speed so $\Delta K = 0$ and the net ork done on it must also be ero by the ork kinetic energy theorem. hus the applied force here must be compensating another force e.g. friction for the net rate to be ero.

46. ecogni ing that the force in the cable must e ual the total eight since there is no acceleration e employ . 7 47

$$P = Fv\cos\theta = mg\left(\frac{\Delta x}{\Delta t}\right)$$

here e have used the fact that $\theta = 0°$ (both the force of the cable and the elevator's motion are up ard . hus

$$P = \ 3.0\times10^3 \text{ kg} \ \ 9.8 \text{ m s}^2 \ \left(\frac{210 \text{ m}}{23 \text{ s}}\right) = 2.7\times10^5 \quad .$$

47. a uation 7 8 yields

$W =$ $F_x\,\Delta x$ $F_y\,\Delta y$ $F_z\,\Delta z$
 = 2.00 7.5 m – 0.50 m 4.00 12.0 m – 0.75 m 6.00 7.2m – 0.20 m
 =101 ≈ 1.0× 10^2 .

b ividing this result by 12 s see . 7 42 yields P = 8.4 .

48. a ince the force e erted by the spring on the mass is ero hen the mass passes through the e uilibrium position of the spring the rate at hich the spring is doing ork on the mass at this instant is also ero.

b he rate is given by $P = \vec{F}\cdot\vec{v} = -Fv$ here the minus sign corresponds to the fact that $\vec{F}$ and $\vec{v}$ are anti parallel to each other. he magnitude of the force is given by

$$F = kx = \ 500 \quad \text{m} \ \ 0.10 \text{ m} \ = 50$$

 hile v is obtained from conservation of energy for the spring mass system

$$E = K + U = 10 \quad = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2} \ 0.30 \text{ kg} \ v^2 + \frac{1}{2} \ 500 \quad \text{m} \ \ 0.10 \text{ m} \ ^2$$

 hich gives v = 7.1 m s. hus

$$P = -Fv = - \ 50 \quad 7.1 \text{ m s} \ = -3.5\times10^2 \quad .$$

49. **THINK** e have a loaded elevator moving up ard at a constant speed. he forces involved are gravitational force on the elevator gravitational force on the counter eight and the force by the motor via cable.

EXPRESS he total ork is the sum of the ork done by gravity on the elevator the ork done by gravity on the counter eight and the ork done by the motor on the system

$$W = W_e + W_c + W_m.$$

 ince the elevator moves at constant velocity its kinetic energy does not change and according to the ork kinetic energy theorem the total ork done is ero i.e. $W = \Delta K = 0$.

ANALYZE he elevator moves *upward* through 54 m so the ork done by gravity on it is

$$W_e = -m_e g d = -\ 1200\text{ kg}\ \ 9.80\text{ m s}^2\ \ 54\text{ m}\ = -6.35\times 10^5\ \ .$$

he counter eight moves *downward* the same distance so the ork done by gravity on it is

$$W_c = m_c g d = \ 950\text{ kg}\ \ 9.80\text{ m s}^2\ \ 54\text{ m}\ = 5.03\times 10^5\ \ .$$

ince $W = 0$ the ork done by the motor on the system is

$$W_m = -W_e - W_c = 6.35\times 10^5\ \ -\ 5.03\times 10^5\ \ =\ 1.32\times 10^5\ \ .$$

his ork is done in a time interval of $\Delta t = 3.0\text{ min} = 180\text{ s}$ so the po er supplied by the motor to lift the elevator is

$$P = \frac{W_m}{\Delta t} = \frac{1.32\times 10^5}{180\text{ s}} = 7.4\times 10^2\ \ .$$

LEARN n general the ork done by the motor is $W_m = \ m_e - m_c\ gd$. o hen the counter eight mass balances the total mass $m_c = m_e$ no ork is re uired by the motor.

50. a sing . 7 48 and . 3 23 e obtain

$$P = \vec{F}\cdot\vec{v} = \ 4.0\ \ \ -2.0\text{ m s}\ +\ 9.0\ \ \ 4.0\text{ m s}\ = 28\ \ .$$

b e again use . 7 48 and . 3 23 but ith a one component velocity $\vec{v} = v\hat{\ }$.

$$P = \vec{F}\cdot\vec{v}\ \Rightarrow -12\ \ =\ -2.0\ \ v.$$

hich yields $v = 6$ m s.

51. (a) The object's displacement is

$$\vec{d} = \vec{d}_f - \vec{d}_i = \ -8.00\text{ m}\ \hat{\text{i}} +\ 6.00\text{ m}\ \hat{\ } +\ 2.00\text{ m}\ \hat{\text{k}}\,.$$

hus . 7 8 gives

$$W = \vec{F}\cdot\vec{d} = \ 3.00\ \ \ -8.00\text{ m}\ +\ 7.00\ \ \ 6.00\text{ m}\ +\ 7.00\ \ \ 2.00\text{ m}\ = 32.0\ \ .$$

b he average po er is given by . 7 42

$$P_{\text{avg}} = \frac{W}{t} = \frac{32.0}{4.00} = 8.00\ \ .$$

c he distance from the coordinate origin to the initial position is

$$d_i = \sqrt{3.00\text{ m}^2 + -2.00\text{ m}^2 + 5.00\text{ m}^2} = 6.16\text{ m}$$

and the magnitude of the distance from the coordinate origin to the final position is

$$d_f = \sqrt{-5.00\text{ m}^2 + 4.00\text{ m}^2 + 7.00\text{ m}^2} = 9.49\text{ m}.$$

heir scalar dot product is

$$\vec{d}_i \cdot \vec{d}_f = 3.00\text{ m} \;\; -5.00\text{ m} + -2.00\text{ m} \;\; 4.00\text{ m} + 5.00\text{ m} \;\; 7.00\text{ m} = 12.0\text{ m}^2.$$

hus the angle bet een the t o vectors is

$$\phi = \cos^{-1}\left(\frac{\vec{d}_i \cdot \vec{d}_f}{d_i d_f}\right) = \cos^{-1}\left(\frac{12.0}{6.16 \;\; 9.49}\right) = 78.2°.$$

52. ccording to the problem statement the po er of the car is

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\frac{dv}{dt} = \text{constant.}$$

he condition implies $dt = mvdv \;\; P$ hich can be integrated to give

$$\int_0^T dt = \int_0^{v_T} \frac{mvdv}{P} \;\; \Rightarrow \; T = \frac{mv_T^2}{2P}$$

here v_T is the speed of the car at $t = T$. n the other hand the total distance traveled can be ritten as

$$L = \int_0^T vdt = \int_0^{v_T} v\frac{mvdv}{P} = \frac{m}{P}\int_0^{v_T} v^2 dv = \frac{mv_T^3}{3P}.$$

y s uaring the e pression for L and substituting the e pression for T e obtain

$$L^2 = \left(\frac{mv_T^3}{3P}\right)^2 = \frac{8P}{9m}\left(\frac{mv_T^2}{2P}\right)^3 = \frac{8PT^3}{9m}$$

hich implies that

$$PT^3 = \frac{9}{8}mL^2 = \text{constant.}$$

ifferentiating the above e uation gives $dPT^3 + 3PT^2 dT = 0$ or $dT = -\frac{T}{3P}dP.$

53. a oting that the x component of the third force is F_{3x} = 4.00 cos 60 e apply . 7 8 to the problem

$$W = 5.00 \quad -1.00 \qquad 4.00 \quad \cos 60 \quad 0.20 \text{ m} \ = 1.20 \ .$$

b uation 7 10 along ith . 7 1 then yields $v = \sqrt{2W/m}$ = 1.10 m s.

54. rom . 7 32, we see that the "area" in the graph is equivalent to the work done. We find the area in terms of rectangular length × idth and triangular $\frac{1}{2}$ base × height areas and use the ork kinetic energy theorem appropriately. he initial point is taken to be x = 0 here v_0 = 4.0 m s.

a ith $K_i = \frac{1}{2} m v_0^2 = 16$ e have

$$K_3 - K_0 = W_{0<x<1} + W_{1<x<2} + W_{2<x<3} = -4.0$$

so that K_3 the kinetic energy hen x = 3.0 m is found to e ual 12 .

b ith units understood e rite $W_{3<x<x_f}$ as $F_x \Delta x = \ -4.0 \qquad x_f - 3.0$ m and apply the ork kinetic energy theorem

$$K_{x_f} - K_3 = W_{3<x<x_f}$$
$$K_{x_f} - 12 = \ -4 \quad x_f - 3.0$$

so that the re uirement $K_{xf} = 8.0$ leads to $x_f = 4.0$ m.

c s long as the ork is positive the kinetic energy gro s. he graph sho s this situation to hold until x = 1.0 m. t that location the kinetic energy is

$$K_1 = K_0 + W_{0<x<1} = 16 \quad + 2.0 \quad = 18 \ .$$

55. **THINK** A horse is doing work to pull a cart at a constant speed. We'd like to know the ork done during a time interval and the corresponding average po er.

EXPRESS he horse pulls ith a force $\vec{F}$. s the cart moves through a displacement $\vec{d}$ the ork done by $\vec{F}$ is $W = \vec{F} \cdot \vec{d} = Fd \cos\phi$ here ϕ is the angle bet een $\vec{F}$ and $\vec{d}$.

ANALYZE a n 10 min the cart moves a distance

$$d = v\Delta t = \left(6.0 \ \frac{\text{mi}}{\text{h}}\right)\left(\frac{5280 \text{ ft mi}}{60 \text{ min h}}\right) \ 10 \text{ min} \ = 5280 \text{ ft}$$

so that . 7 7 yields

$$W = Fd\cos\phi = \ 40\ \text{lb}\ \ 5280\ \text{ft}\ \ \cos 30° = 1.8\times10^5\ \text{ft}\cdot\text{lb}.$$

b he average po er is given by . 7 42. ith $\Delta t = 10\ \text{min} = 600\,\text{s}$ e obtain

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{1.8\times10^5\ \text{ft}\cdot\text{lb}}{600\ \text{s}} = 305\ \text{ft}\cdot\text{lb s}$$

hich using the conversion factor $1\,\text{hp} = 550\ \text{ft}\cdot\text{lb s}$ found on the inside back cover converts to P_{avg} = 0.55 hp.

LEARN he average po er can also be calculate by using . 7 48 $P_{\text{avg}} = Fv\cos\phi$.
onverting the speed to $v = \ 6.0\ \text{mi h}\left(\frac{5280\ \text{ft mi}}{3600\ \text{s h}}\right) = 8.8\ \text{ft s}$ e get

$$P_{\text{avg}} = Fv\cos\phi = \ 40\ \text{lb}\ \ 8.8\ \text{ft s}\ \cos30° = 305\ \text{ft}\cdot\text{lb} = 0.55\ \text{hp}$$

hich agrees ith that found in b .

56. he acceleration is constant so e may use the e uations in able 2 1. e choose the direction of motion as x and note that the displacement is the same as the distance traveled in this problem. e designate the force assumed singular along the x direction acting on the m = 2.0 kg ob ect as F.

a ith $v_0 = 0$. 2 11 leads to $a = v\ t$. nd . 2 17 gives $\Delta x = \frac{1}{2}vt$. Newton's second la yields the force $F = ma$. uation 7 8 then gives the ork

$$W = F\Delta x = m\left(\frac{v}{t}\right)\left(\frac{1}{2}vt\right) = \frac{1}{2}mv^2$$

as e e pect from the ork kinetic energy theorem. ith v = 10 m s this yields $W = 1.0\times10^2$.

b nstantaneous po er is defined in . 7 48. ith t = 3.0 s e find

$$P = Fv = m\left(\frac{v}{t}\right)v = 67\ \ .$$

c he velocity at $t' = 1.5\text{s}$ is $v' = at' = 5.0\ \text{m/s}$. hus $P' = Fv' = 33$.

57. a o hold the crate at e uilibrium in the final situation $\vec{F}$ must have the same magnitude as the horizontal component of the rope's tension T sin θ here θ is the angle bet een the rope in the final position and vertical

$$\theta = \sin^{-1}\left(\frac{4.00}{12.0}\right) = 19.5°.$$

ut the vertical component of the tension supports against the eight $T \cos\theta = mg$. hus the tension is

$$T = 230 \text{ kg} \;\; 9.80 \text{ m s}^2 \;\; \cos 19.5 \;\; = 2391$$

and F = 2391 sin 19.5 = 797 .

n alternative approach based on dra ing a vector triangle of forces in the final situation provides a uick solution.

b ince there is no change in kinetic energy the net ork on it is ero.

c he ork done by gravity is $W_g = \vec{F}_g \cdot \vec{d} = -mgh$ here h = L 1 – cos θ is the vertical component of the displacement. ith L = 12.0 m e obtain W_g = –1547 hich should be rounded to three significant figures –1.55 k .

d he tension vector is every here perpendicular to the direction of motion so its ork is ero since cos 90 = 0 .

e he implication of the previous three parts is that the ork due to $\vec{F}$ is $-W_g$ so the net ork turns out to be ero . hus $W_F = -W_g$ = 1.55 k .

f ince $\vec{F}$ does not have constant magnitude e cannot e pect . 7 8 to apply.

58. a he force of the orker on the crate is constant so the ork it does is given by $W_F = \vec{F} \cdot \vec{d} = Fd\cos\phi$ here $\vec{F}$ is the force $\vec{d}$ is the displacement of the crate and ϕ is the angle bet een the force and the displacement. ere F = 210 d = 3.0 m and ϕ = 20 . hus

$$W_F = 210 \quad 3.0 \text{ m} \;\; \cos 20 \;\; = 590 \;.$$

b he force of gravity is do n ard perpendicular to the displacement of the crate. he angle bet een this force and the displacement is 90 and cos 90 = 0 so the ork done by the force of gravity is ero.

c he normal force of the floor on the crate is also perpendicular to the displacement so the ork done by this force is also ero.

d hese are the only forces acting on the crate so the total ork done on it is 590 .

59. he ork done by the applied force $\vec{F}_a$ is given by $W = \vec{F}_a \cdot \vec{d} = F_a d \cos\phi$. rom the figure e see that $W = 25$ hen $\phi = 0$ and $d = 5.0$ cm. his yields the magnitude of $\vec{F}_a$

$$F_a = \frac{W}{d} = \frac{25}{0.050\text{ m}} = 5.0\times10^2 \quad .$$

a or $\phi = 64°$ e have $W = F_a d \cos\phi = 5.0\times10^2 \quad 0.050\text{ m} \cos 64° = 11$.

b or $\phi = 147°$ e have $W = F_a d \cos\phi = 5.0\times10^2 \quad 0.050\text{ m} \cos 147° = -21$.

60. a n the ork kinetic energy theorem e include both the ork due to an applied force W_a and ork done by gravity W_g in order to find the latter uantity.

$$\Delta K = W_a + W_g \quad \Rightarrow \quad 30 \quad = \quad 100 \qquad 1.8\text{ m} \cos 180° + W_g$$

leading to $W_g = 2.1\times10^2$.

b he value of W_g obtained in part a still applies since the eight and the path of the child remain the same so $\Delta K = W_g = 2.1\times10^2$.

61. One approach is to assume a "path" from $\vec{r}_i$ to $\vec{r}_f$ and do the line integral accordingly. nother approach is to simply use . 7 36 hich e demonstrate

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy = \int_2^{-4} 2x\ dx + \int_3^{-3} 3\ dy$$

ith units understood. hus e obtain $W = 12 - 18 = -6$.

62. a he compression of the spring is d = 0.12 m. he ork done by the force of gravity acting on the block is by . 7 12

$$W_1 = mgd = 0.25\text{ kg} \left(9.8\text{ m} \quad \text{s}^2\right) 0.12\text{ m} = 0.29 \quad .$$

b he ork done by the spring is by . 7 26

$$W_2 = -\frac{1}{2}kd^2 = -\frac{1}{2} \quad 250 \qquad \text{m} \quad 0.12\text{ m}^{\ 2} = -1.8 \quad .$$

c he speed v_i of the block ust before it hits the spring is found from the ork kinetic energy theorem . 7 15

$$\Delta K = 0 - \frac{1}{2} m v_i^2 = W_1 + W_2$$

hich yields

$$v_i = \sqrt{\frac{-2\ \ W_1 + W_2}{m}} = \sqrt{\frac{-2\ \ 0.29\ \ -1.8}{0.25\ \text{kg}}} = 3.5\ \text{m s}.$$

d f e instead had $v_i' = 7\ \text{m s}$ e reverse the above steps and solve for d'. ecalling the theorem used in part c e have

$$0 - \frac{1}{2} m v_i'^2 = W_1' + W_2' = mgd' - \frac{1}{2} k d'^2$$

hich choosing the positive root leads to

$$d' = \frac{mg + \sqrt{m^2 g^2 + mk v_i'^2}}{k}$$

hich yields d = 0.23 m. n order to obtain this result e have used more digits in our intermediate results than are sho n above so $v_i = \sqrt{12.048}\ \text{m s} = 3.471\ \text{m s}$ and v_i' = 6.942 m s .

63. **THINK** crate is being pushed up a frictionless inclined plane. he forces involved are gravitational force on the crate normal force on the crate and the force applied by the orker.

EXPRESS he ork done by a force $\vec{F}$ on an ob ect as it moves through a displacement $\vec{d}$ is $W = \vec{F} \cdot \vec{d} = Fd\cos\phi$ here ϕ is the angle bet een $\vec{F}$ and $\vec{d}$.

ANALYZE a he applied force is parallel to the inclined plane. hus using . 7 6 the ork done on the crate by the worker's applied force is

$$W_a = Fd\cos 0° = 209\ \ \ \ 1.50\ \text{m} \approx 314\ \ .$$

b sing . 7 12 e find the ork done by the gravitational force to be

$$\begin{aligned} W_g &= F_g d\cos\ 90° + 25°\ = mgd\cos 115° \\ &= 25.0\ \text{kg}\ \ 9.8\ \text{m s}^2\ \ 1.50\ \text{m}\ \cos 115° \\ &\approx -155\ \ . \end{aligned}$$

c he angle bet een the normal force and the direction of motion remains 90 at all times so the ork it does is ero

$$W_N = F_N d\cos 90° = 0.$$

d he total ork done on the crate is the sum of all three orks

$$W = W_a + W_g + W_N = 314 \quad + \; -155 \quad +0 \quad = 158 \quad .$$

LEARN y ork kinetic energy theorem if the crate is initially at rest $K_i = 0$ then its kinetic energy after having moved 1.50 m up the incline ould be $K_f = W = 158$ and the speed of the crate at that instant is

$$v = \sqrt{2K_f \quad m} = \sqrt{2\;158 \qquad 25.0\,\text{kg}} = 3.56\,\text{m s}.$$

64. a he force $\vec{F}$ of the incline is a combination of normal and friction force hich is serving to “cancel” the tendency of the box to fall downward (due to its 19.6 N weight). hus $\vec{F} = mg$ up ard. n this part of the problem the angle ϕ bet een the belt and $\vec{F}$ is 80 . rom . 7 47 e have

$$P = Fv\cos\phi = 19.6 \qquad 0.50\,\text{m s} \;\cos 80^\circ = 1.7 \quad .$$

b o the angle bet een the belt and $\vec{F}$ is 90 so that P = 0.

c n this part the angle bet een the belt and $\vec{F}$ is 100 so that

$$P = 19.6 \qquad 0.50\,\text{m s} \;\cos 100 = -1.7 \quad .$$

65. here is no acceleration so the lifting force is e ual to the eight of the ob ect. e note that the person’s pull $\vec{F}$ is e ual in magnitude to the tension in the cord.

a s indicated in the *hint* tension contributes t ice to the lifting of the canister $2T = mg$. ince $|\vec{F}| = T$ e find $|\vec{F}| = 98$.

b o rise 0.020 m t o segments of the cord see ig. 7 47 must shorten by that amount. hus the amount of string pulled do n at the left end this is the magnitude of $\vec{d}$ the do n ard displacement of the hand is d = 0.040 m.

c ince at the left end both $\vec{F}$ and $\vec{d}$ are do n ard then . 7 7 leads to

$$W = \vec{F} \cdot \vec{d} = 98 \qquad 0.040\,\text{m} = 3.9 \quad .$$

d ince the force of gravity $\vec{F}_g$ ith magnitude mg is opposite to the displacement $\vec{d}_c = 0.020$ m up of the canister . 7 7 leads to

$$W = \vec{F}_g \cdot \vec{d}_c = - \quad 196 \qquad 0.020 \text{ m} \quad = -3.9 \quad .$$

his is consistent ith . 7 15 since there is no change in kinetic energy.

66. fter converting the speed $v = 120 \text{ km h} = 33.33 \text{ m s}$ e find

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1200\,\text{kg})(33.33\,\text{m s})^2 = 6.67 \times 10^5 \quad .$$

67. **THINK** n this problem e have packages hung from the spring. he e tent of stretching can be determined from Hooke's law.

EXPRESS According to Hooke's law, the spring force is given by

$$F_x = -k \quad x - x_0 \quad = -k\Delta x$$

here Δx is the displacement from the e uilibrium position. hus the first t o situations in ig. 7 48 can be ritten as

$$\begin{aligned} -110 \quad &= -k \quad 40 \text{ mm} - x_0 \\ -240 \quad &= -k \quad 60 \text{ mm} - x_0 \end{aligned}$$

he t o e uations allo us to solve for k the spring constant as ell as x_0 the rela ed position hen no mass is hung.

ANALYZE a he t o e uations can be added to give

$$240 \quad -110 \quad = k \quad 60 \text{ mm} - 40 \text{ mm}$$

hich yields k = 6.5 mm. ubstituting the result into the first e uation e find

$$x_0 = 40 \text{ mm} - \frac{110}{k} = 40 \text{ mm} - \frac{110}{6.5 \qquad \text{mm}} = 23 \text{ mm}.$$

b sing the results from part a to analy e that last picture e find the eight to be

$$W = k \quad 30\,\text{mm} - x_0 \quad = \quad 6.5 \qquad \text{mm} \quad 30 \text{ mm} - 23 \text{ mm} \quad = 45 \qquad .$$

LEARN n alternative method to calculate W in the third picture is to note that since the amount of stretching is proportional to the eight hung e have $\frac{W}{W'} = \frac{\Delta x}{\Delta x'}$. pplying this relation to the second and the third pictures the eight W is

$$W = \left(\frac{\Delta x_3}{\Delta x_2}\right) W_2 = \left(\frac{30\text{ mm} - 23\text{ mm}}{60\text{ mm} - 23\text{ mm}}\right) 240 \qquad = 45$$

in agreement ith the result sho n in b .

68. sing . 7 7 e have $W = Fd\cos\phi = 1504$. hen by the ork kinetic energy theorem e find the kinetic energy $K_f = K_i$ $W = 0$ 1504 . he ans er is therefore 1.5 k .

69. he total eight is 100 660 = 6.60×10^4 and the words "raises … at constant speed" imply zero acceleration, so the lift force is e ual to the total eight. hus

$$P = Fv = \ 6.60 \times 10^4 \ \ 150\text{ m } 60.0\text{ s} \ = 1.65 \times 10^5 \quad .$$

70. ith units understood . 7 8 leads to $W =$ 4.0 3.0 $- c$ 2.0 = $12 - 2c$.

a f $W = 0$ then $c = 6.0$.

b f $W = 17$ then $c = -2.5$.

c f $W = -18$ then $c = 15$.

71. sing . 7 8 e find

$$W = \vec{F}\cdot\vec{d} = \ F\cos\theta\,\hat{\mathrm{i}} + F\sin\theta\,\hat{} \ \cdot\ x\hat{\mathrm{i}} + y\hat{} \ = Fx\cos\theta + Fy\sin\theta$$

here $x = 2.0$ m $y = -4.0$ m $F = 10$ and $\theta = 150°$. hus e obtain $W = -37$. ote that the given mass value 2.0 kg is not used in the computation.

72. a . 7 10 along ith . 7 1 and . 7 7 leads to

$$v_f = \ 2\frac{d}{m}F\cos\theta \ ^{1\ 2} = \ \cos\theta \ ^{1\ 2}$$

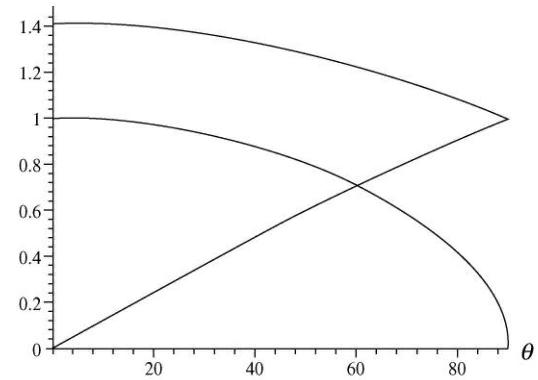
here e have substituted $F = 2.0$ $m = 4.0$ kg and $d = 1.0$ m.

b ith $v_i = 1$ those same steps lead to $v_f = \ 1 \quad \cos\theta \ ^{1\ 2}$.

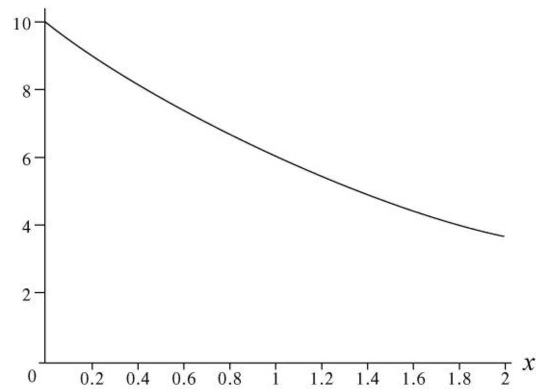
c eplacing θ ith 180 $- \theta$ and still using $v_i = 1$ e find

$$v_f = \ 1 \quad \cos\ 180 \ - \theta \ \ ^{1\ 2} = \ 1 - \cos\theta \ ^{1\ 2}.$$

d he graphs are sho n on the right. ote that as θ is increased in parts a and b the force provides less and less of a positive acceleration hereas in part c the force provides less and less of a deceleration as its θ value increases . he highest curve hich slo ly decreases from 1.4 to 1 is the curve for part b the other decreasing curve starting at 1 and ending at 0 is for part a . he rising curve is for part c it is e ual to 1 here θ = 90 .



73. a he plot of the function ith units understood is sho n belo .



stimating the area under the curve allo s for a range of ans ers. stimates from 11 to 14 are typical.

b valuating the ork analytically using . 7 32 e have

$$W = \int_0^2 10e^{-x\ 2}dx = -20e^{-x\ 2}\Big|_0^2 = 12.6 \quad \approx 13 \quad .$$

74. a sing . 7 8 and units e find

$$W = \vec{F}\cdot\vec{d} = \ 2\hat{\imath} - 4\hat{\ } \ \cdot\ 8\hat{\imath} + c\hat{\ } \ = 16 - 4c$$

hich if e ual ero implies c = 16 4 = 4 m.

b f W 0 then 16 4c hich implies c 4 m.

c f W 0 then 16 4c hich implies c 4 m.

75. **THINK** o er must be supplied in order to lift the elevator ith load up ard at a constant speed.

EXPRESS or the elevator load system to move up ard at a constant speed ero acceleration the applied force F must e actly balance the gravitational force on the system i.e. $F = F_g = \ m_{\text{elev}} + m_{\text{load}} \ g$. he po er re uired can then be calculated using . 17 48 $P = Fv$.

ANALYZE ith $m_{\text{elev}} = 4500\,\text{kg}$ $m_{\text{load}} = 1800\,\text{kg}$ and $v = 3.80\,\text{m s}$ e find the po er to be

$$P = Fv = \ m_{\text{elev}} + m_{\text{load}} \ gv = \ 4500\,\text{kg} + 1800\,\text{kg} \ \ 9.8\,\text{m s}^2 \ \ 3.80\,\text{m s} \ = 235\,\text{k} \quad .$$

LEARN he po er re uired is proportional to the speed at hich the system moves the greater the speed the greater the po er that must be supplied.

76. a he component of the force of gravity e erted on the ice block of mass m along the incline is mg sin θ here $\theta = \sin^{-1}(0.91/1.5)$ gives the angle of inclination for the inclined plane. ince the ice block slides do n ith uniform velocity the orker must e ert a force $\vec{F}$ "uphill" with a magnitude equal to mg sin θ. onse uently

$$F = mg \sin\theta = \ 45\ \text{kg} \ \ 9.8\ \text{m s}^2 \ \left(\frac{0.91\,\text{m}}{1.5\,\text{m}}\right) = 2.7 \times 10^2 \quad .$$

(b) Since the "downhill" displacement is opposite to $\vec{F}$ the ork done by the orker is

$$W_1 = -\left(2.7 \times 10^2 \quad \right) \ 1.5\ \text{m} \ = -4.0 \times 10^2 \ .$$

c ince the displacement has a vertically do n ard component of magnitude 0.91 m in the same direction as the force of gravity e find the ork done by gravity to be

$$W_2 = \ 45\ \text{kg} \ \left(9.8\ \text{m} \ \ \text{s}^2\right) \ 0.91\ \text{m} \ = 4.0 \times 10^2 \ .$$

d ince $\vec{F}_N$ is perpendicular to the direction of motion of the block and cos 90° = 0 ork done by the normal force is $W_3 = 0$ by . 7 7.

e he resultant force $\vec{F}_{\text{net}}$ is ero since there is no acceleration. hus its ork is ero as can be checked by adding the above results $W_1 + W_2 + W_3 = 0$.

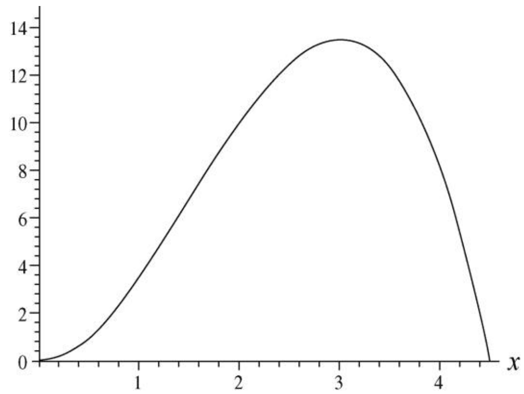
77. a o estimate the area under the curve bet een x = 1 m and x = 3 m hich should yield the value for the work done), one can try "counting squares" (or half s uares or thirds of s uares bet een the curve and the a is. stimates bet een 5 and 8 are typical for this crude procedure.

b uation 7 32 gives

$$\int_1^3 \frac{a}{x^2}\, dx = \frac{a}{3} - \frac{a}{1} = 6$$

here $a = -9$ m^2 is given in the problem statement.

78. a sing . 7 32 the ork becomes $W = \frac{9}{2}x^2 - x^3$ units understood . he plot is sho n belo



b e see from the graph that its peak value occurs at x = 3.00 m. his can be verified by taking the derivative of W and setting e ual to ero or simply by noting that this is here the force vanishes.

c he ma imum value is $W = \frac{9}{2}\ 3.00^{\ 2} - \ 3.00^{\ 3} = 13.50$.

d e see from the graph or from our analytic e pression that W = 0 at x = 4.50 m.

e he case is at rest hen $v = 0$. ince $W = \Delta K = mv^2\ \ 2$ the condition implies $W = 0$. his happens at x = 4.50 m.

79. **THINK** bo sliding in the x direction is slo ed do n by a steady ind in the $-x$ direction. he problem involves graphical analysis.

EXPRESS ig. 7 51 represents $x\ t$ the position of the lunch bo as a function of time. t is convenient to fit the curve to a concave do n ard parabola

$$x\ t\ = \frac{1}{10} t\ 10 - t\ = t - \frac{1}{10}t^2.$$

y taking one and t o derivatives e find the velocity and acceleration to be

$$v\ t\ = \frac{dx}{dt} = 1 - \frac{t}{5} \quad \text{in m s} \qquad a = \frac{d^2x}{dt^2} = -\frac{1}{5} = -0.2 \quad \text{in m s}^2 .$$

he e uations imply that the initial speed of the bo is $v_i = v\ 0\ = 1.0\ \text{m s}$ and the constant force by the ind is

$$F = ma = \ 2.0\ \text{kg}\ \ -0.2\ \text{m s}^2\ = -0.40\ \ .$$

he corresponding ork is given by units understood

$$W\ t\ = F \cdot x\ t\ = -0.04t\ 10 - t\ .$$

he initial kinetic energy of the lunch bo is

$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2}\ 2.0\ \text{kg}\ \ 1.0\ \text{m s}\ ^2 = 1.0\ \ .$$

ith $\Delta K = K_f - K_i = W$ the kinetic energy at a later time is given by in units

$$K\ t\ = K_i + W = 1 - 0.04t\ 10 - t$$

ANALYZE a hen t = 1.0 s the above e pression gives

$$K\ 1\text{s}\ = 1 - 0.04\ 1\ \ 10 - 1\ = 1 - 0.36 = 0.64 \approx 0.6$$

here the second significant figure is not to be taken too seriously.

b t t = 5.0 s the above method gives $K\ 5.0\ \text{s}\ = 1 - 0.04\ 5\ \ 10 - 5\ = 1 - 1 = 0.$

c he ork done by the force from the ind from t = 1.0 s to t = 5.0 s is

$$W = K\ 5.0\ - K\ 1.0\ \text{s}\ = 0 - 0.6 \approx -0.6\ \ .$$

LEARN he result in c can also be obtained by evaluating $W\ t\ = -0.04t\ 10 - t$ directly at t = 5.0 s and t = 1.0 s and subtracting

$$W\ 5\ - W\ 1\ = -0.04\ 5\ \ 10 - 5\ - \left[-0.04\ 1\ \ 10 - 1\ \right] = -1 -\ -0.36\ = -0.64 \approx -0.6\ \ .$$

ote that at t = 5.0 s K = 0 the bo comes to a stop and then reverses its direction subse uently ith x decreasing .

80. he problem indicates that units are understood so the result of . 7 23 is in oules. one numerically using features available on many modern calculators the result is roughly 0.47 . or the interested student it might be orth hile to uote the "exact" answer (in terms of the "error function"):

$$\int_{.15}^{1.2} e^{2x^2}\, dx = \quad \sqrt{2\pi}\ \ \text{erf}\ 6\sqrt{2}\ \ 5\ \ -\text{erf}\ 3\sqrt{2}\ \ 20\quad .$$

81. a he ork done by the spring force is $W_s = \frac{1}{2}k\ \ x_i^2 - x_f^2$. y energy conservation hen the block is at $x_f = 0$ the energy stored in the spring is completed converted to the kinetic energy of the block $W_s = K = \frac{1}{2}mv^2$. olving for v e obtain

$$\frac{1}{2}k\ \ x_i^2 - x_f^2\ \ = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{500\quad \text{m}}{4.00\ \text{kg}}}\left(0.300\ \text{m}\right) = 3.35\ \text{m s}.$$

b he ork done by the spring is

$$W_s = \frac{1}{2}kx_i^2 = \frac{1}{2}\ \ 500\quad \text{m}\ \ 0.300\ \text{m}\ ^2 = 22.5\quad .$$

c he instantaneous po er due to the spring can be ritten as

$$P = Fv = \ \ kx\ \ \sqrt{\frac{k}{m}\left(x_i^2 - x^2\right)}$$

t the release point x_i the po er is ero.

d imilarly at $x = 0$ e also have $P = 0$.

e he position here the po er is ma imum can be found by differentiating P ith respect to x setting $dP\ dx = 0$

$$\frac{dP}{dx} = \frac{k^2\ \ x_i^2 - 2x^2}{\sqrt{\dfrac{k\ \ x_i^2 - x^2}{m}}} = 0$$

hich gives $x = \dfrac{x_i}{\sqrt{2}} = \dfrac{0.300\ \text{m}}{\sqrt{2}} = 0.212\ \text{m}$.

82. (a) Applying Newton's second law to the x directed uphill and y normal to the inclined plane a es e obtain

$$F - mg\sin\theta = ma$$
$$F_N - mg\cos\theta = 0.$$

he second e uation allo s us to solve for the angle the inclined plane makes ith the hori ontal

$$\theta = \cos^{-1}\left(\frac{F_N}{mg}\right) = \cos^{-1}\left(\frac{13.41}{4.00\ \text{kg}\ \ 9.8\ \text{m s}^2}\right) = 70.0°$$

rom the e uation for the a is e find the acceleration of the block to be

$$a = \frac{F}{m} - g\sin\theta = \frac{50}{4.00\text{ kg}} - \ 9.8\text{ m s}^2 \ \sin 70.0° = 3.29\text{ m s}^2$$

sing the kinematic e uation $v^2 = v_0^2 + 2ad$ the speed of the block hen d = 3.00 m is

$$v = \sqrt{2ad} = \sqrt{2\ 3.29\text{ m s}^2\ \ 3.00\text{ m}} = 4.44\text{ m s}$$

83. a he ork done by the spring force ith spring constant $k = 18$ cm $= 1800$ m is

$$W_s = \frac{1}{2}k\ x_i^2 - x_f^2\ = -\frac{1}{2}kx_f^2 = -\frac{1}{2}\ 1800\ \ \text{m}\ \ 7.60\times10^{-3}\text{m}\ ^2 = -5.20\times10^{-2}$$

b or $x'_f = 2x_f$ the ork done by the spring force is $W'_s = -\frac{1}{2}kx_f'^2 = -\frac{1}{2}k\ 2x_f\ ^2$ so the additional ork done is

$$\Delta W = W'_s - W_s = -\frac{1}{2}k\ 2x_f\ ^2 - \left(-\frac{1}{2}kx_f^2\right) = -\frac{3}{2}kx_f^2 = 3W_s = 3\ -5.20\times10^{-2}\ \ = -0.156$$

84. a he displacement of the ob ect is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \ -4.10\hat{\text{i}} + 3.30\hat{\ } + 5.40\ \hat{\text{k}}\ -\ 2.70\hat{\text{i}} - 2.90\hat{\ } + 5.50\ \hat{\text{k}}\ = \ -6.80\hat{\text{i}} + 6.20\hat{\ } - 0.10\ \hat{\text{k}}$$

he ork done by $\vec{F} = \ 2.00\hat{\text{i}} + 9.00\hat{\ } + 5.30\ \hat{\text{k}}$ is in units

$$W = \vec{F}\cdot\Delta\vec{r} = \ 2.00\hat{\text{i}} + 9.00\hat{\ } + 5.30\ \hat{\text{k}}\ \cdot\ -6.80\hat{\text{i}} + 6.20\hat{\ } - 0.10\ \hat{\text{k}}\ = 41.7$$

b he average po er due to the force during the time interval is

$$P = \frac{W}{\Delta t} = \frac{41.7}{2.10\text{ s}} = 19.8$$

c he magnitudes of the position vectors are in units

$$r_1 = \ \vec{r}_1\ = \sqrt{\ 2.70\ ^2 + \ -2.90\ ^2 + \ 5.50\ ^2} = 6.78$$
$$r_2 = \ \vec{r}_2\ = \sqrt{\ -4.10\ ^2 + \ 3.30\ ^2 + \ 5.40\ ^2} = 7.54$$

and their dot product is

$$\vec{r}_1 \cdot \vec{r}_2 = \ 2.70\hat{\text{i}} - 2.90\hat{\ } + 5.50\ \hat{\text{k}}\ \cdot\ -4.10\hat{\text{i}} + 3.30\hat{\ } + 5.40\ \hat{\text{k}}$$
$$= \ 2.70\ \ -4.10\ +\ -2.90\ \ 3.30\ +\ 5.50\ \ 5.40\ = 9.06$$

sing $\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos\theta$ the angle bet een $\vec{r}_1$ and $\vec{r}_2$ is

$$\theta = \cos^{-1}\left(\frac{\vec{r}_1 \cdot \vec{r}_2}{r_1 r_2}\right) = \cos^{-1}\left(\frac{9.06}{6.78\ \ 7.54}\right) = 79.8^\circ$$

85. he ork done by the force is in units

$$W = \vec{F} \cdot \vec{d} = \ -5.00\hat{\text{i}} + 5.00\hat{\ } + 4.00\ \hat{\text{k}}\ \cdot\ 2.00\hat{\text{i}} + 2.00\hat{\ } + 7.00\ \hat{\text{k}}\ = 28$$

y energy conservation $W = \Delta K = \frac{1}{2} m\ v_f^2 - v_i^2$. hus the final speed of the particle is

$$v_f = \sqrt{v_i^2 + \frac{2W}{m}} = \sqrt{\ 4.00\text{ m s}\ ^2 + \frac{2\ \ 28}{2.00\text{ kg}}} = 6.63\text{ m s}\ .$$

Chapter

1. **THINK** compressed spring stores potential energy. his e ercise e plores the relationship bet een the energy stored and the spring constant.

EXPRESS he potential energy stored by the spring is given by $U = kx^2$ 2 here k is the spring constant and x is the displacement of the end of the spring from its position hen the spring is in e uilibrium. hus the spring constant is $k = 2U$ x^2.

ANALYZE ubstituting $U = 25$ and $x = 7.5$ m $= 0.075$ cm into the above e pression e find the spring constant to be

$$k = \frac{2U}{x^2} = \frac{2\ 25}{0.075\ \text{m}\ ^2} = 8.9 \times 10^3 \quad \text{m.}$$

LEARN he spring constant k has units m. he uantity provides a measure of stiffness of the spring for a given x the greater the value of k the greater the potential energy U.

2. e use . 7 12 for W_g and . 8 9 for U.

a he displacement bet een the initial point and A is hori ontal so ϕ = 90.0 and $W_g = 0$ since cos 90.0 = 0 .

b he displacement bet een the initial point and B has a vertical component of h 2 do n ard same direction as $\vec{F}_g$ so e obtain

$$W_g = \vec{F}_g \cdot \vec{d} = \frac{1}{2}mgh = \frac{1}{2}\ 825\ \text{kg}\ \ 9.80\ \text{m s}^2\ \ 42.0\ \text{m}\ = 1.70 \times 10^5 \quad .$$

c he displacement bet een the initial point and C has a vertical component of h do n ard same direction as $\vec{F}_g$ so e obtain

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = \ 825\ \text{kg}\ \ 9.80\ \text{m s}^2\ \ 42.0\ \text{m}\ = 3.40 \times 10^5 \quad .$$

d ith the reference position at C e obtain

$$U_B = \frac{1}{2}mgh = \frac{1}{2}\ 825\ \text{kg}\ \ 9.80\ \text{m s}^2\ \ 42.0\ \text{m}\ = 1.70 \times 10^5 \quad .$$

e imilarly e find

$$U_A = mgh = \; 825 \text{ kg} \;\; 9.80 \text{ m s}^2 \;\; 42.0 \text{ m} \; = 3.40 \times 10^5 \quad .$$

f ll the ans ers are proportional to the mass of the ob ect. f the mass is doubled all ans ers are doubled.

3. a oting that the vertical displacement is 10.0 m – 1.50 m = 8.50 m do n ard same direction as $\vec{F}_g$. 7 12 yields

$$W_g = mgd\cos\phi = \; 2.00 \text{ kg} \;\; 9.80 \text{ m s}^2 \;\; 8.50 \text{ m} \; \cos 0° = 167 \quad .$$

b ne approach hich is fairly trivial is to use . 8 1 but e feel it is instructive to instead calculate this as ΔU here U = mgy ith up ard understood to be the y direction . he result is

$$\Delta U = mg \;\; y_f - y_i \;\; = \; 2.00 \text{ kg} \;\; 9.80 \text{ m s}^2 \;\; 1.50 \text{ m} - 10.0 \text{ m} \; = -167 \quad .$$

c n part b e used the fact that $U_i = mgy_i$ =196 .

d n part b e also used the fact $U_f = mgy_f$ = 29 .

e he computation of W_g does not use the ne information that U = 100 at the ground so e again obtain W_g = 167 .

f s a result of . 8 1 e must again find $\Delta U = -W_g$ = –167 .

g ith this ne information that U_0 = 100 here y = 0 e have

$$U_i = mgy_i \quad U_0 = 296 \quad .$$

h ith this ne information that U_0 = 100 here y = 0 e have

$$U_f = mgy_f \quad U_0 = 129 \quad .$$

e can check part f by subtracting the ne U_i from this result.

4. a he only force that does ork on the ball is the force of gravity the force of the rod is perpendicular to the path of the ball and so does no ork. n going from its initial position to the lo est point on its path the ball moves vertically through a distance e ual to the length L of the rod so the ork done by the force of gravity is

$$W = mgL = \; 0.341 \text{ kg} \;\; 9.80 \text{ m s}^2 \;\; 0.452 \text{ m} \; = 1.51 \quad .$$

b n going from its initial position to the highest point on its path the ball moves vertically through a distance e ual to L but this time the displacement is up ard opposite the direction of the force of gravity. he ork done by the force of gravity is

$$W = -mgL = - \ 0.341 \text{ kg} \ \ 9.80 \text{ m s}^2 \ \ 0.452 \text{ m} \ = -1.51 \ .$$

c he final position of the ball is at the same height as its initial position. he displacement is hori ontal perpendicular to the force of gravity. he force of gravity does no ork during this displacement.

d he force of gravity is conservative. he change in the gravitational potential energy of the ball arth system is the negative of the ork done by gravity

$$\Delta U = -mgL = - \ 0.341 \text{ kg} \ \ 9.80 \text{ m s}^2 \ \ 0.452 \text{ m} \ = -1.51$$

as the ball goes to the lo est point.

e ontinuing this line of reasoning e find

$$\Delta U = +mgL = \ 0.341 \text{ kg} \ \ 9.80 \text{ m s}^2 \ \ 0.452 \text{ m} \ = 1.51$$

as it goes to the highest point.

f ontinuing this line of reasoning e have $\Delta U = 0$ as it goes to the point at the same height.

g he change in the gravitational potential energy depends only on the initial and final positions of the ball not on its speed any here. he change in the potential energy is the *same* since the initial and final positions are the same.

5. **THINK** s the ice flake slides do n the frictionless bo l its potential energy changes due to the ork done by the gravitational force.

EXPRESS he force of gravity is constant so the ork it does is given by $W = \vec{F} \cdot \vec{d}$ here $\vec{F}$ is the force and $\vec{d}$ is the displacement. he force is vertically do n ard and has magnitude mg here m is the mass of the flake so this reduces to $W = mgh$ here h is the height from hich the flake falls. he force of gravity is conservative so the change in gravitational potential energy of the flake arth system is the negative of the ork done $\Delta U = -W$.

ANALYZE a he ice flake falls a distance $h = r = 22.0 \text{ cm} = 0.22 \text{ m}$. herefore the ork done by gravity is

$$W = mgr = \ 2.00 \times 10^{-3} \text{ kg} \ \ 9.8 \ \text{m/s}^2 \ \ 22.0 \times 10^{-2} \text{ m} \ = 4.31 \times 10^{-3} \ .$$

b he change in gravitational potential energy is $\Delta U = -W = -4.31 \times 10^{-3}$.

c he potential energy hen the flake is at the top is greater than hen it is at the bottom by ΔU. f $U = 0$ at the bottom then $U =$ 4.31×10^{-3} at the top.

d f $U = 0$ at the top then $U = -4.31 \times 10^{-3}$ at the bottom.

e ll the ans ers are proportional to the mass of the flake. f the mass is doubled all ans ers are doubled.

LEARN hile the potential energy depends on the reference point location here $U = 0$ the change in potential energy ΔU does not. n both c and d e find $\Delta U = -4.31\times10^{-3}$.

6. e use . 7 12 for W_g and . 8 9 for U.

a he displacement bet een the initial point and Q has a vertical component of $h - R$ do n ard same direction as $\vec{F}_g$ so ith $h = 5R$ e obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 4mgR = 4\ 3.20\times10^{-2}\ \text{kg}\ \ 9.80\ \text{m s}^2\ \ 0.12\ \text{m}\ = 0.15\ \ .$$

b he displacement bet een the initial point and the top of the loop has a vertical component of $h - 2R$ do n ard same direction as $\vec{F}_g$ so ith $h = 5R$ e obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 3mgR = 3\ 3.20\times10^{-2}\ \text{kg}\ \ 9.80\ \text{m s}^2\ \ 0.12\ \text{m}\ = 0.11\ \ .$$

c ith $y = h = 5R$ at P e find

$$U = 5mgR = 5\ 3.20\times10^{-2}\ \text{kg}\ \ 9.80\ \text{m s}^2\ \ 0.12\ \text{m}\ = 0.19\ \ .$$

d ith $y = R$ at Q e have

$$U = mgR = \ 3.20\times10^{-2}\ \text{kg}\ \ 9.80\ \text{m s}^2\ \ 0.12\ \text{m}\ = 0.038\ \ .$$

e ith $y = 2R$ at the top of the loop e find

$$U = 2mgR = 2\ 3.20\times10^{-2}\ \text{kg}\ \ 9.80\ \text{m s}^2\ \ 0.12\ \text{m}\ = 0.075\ \ .$$

f he ne information $v_i \neq 0$ is not involved in any of the preceding computations the above results are unchanged.

7. he main challenge for students in this type of problem seems to be orking out the trigonometry in order to obtain the height of the ball relative to the lo point of the

s ing $h = L - L\cos\theta$ for angle θ measured from vertical as sho n in ig. 8 34 . nce this relation hich e ill not derive here since e have found this to be most easily illustrated at the blackboard is established then the principal results of this problem follo from . 7 12 for W_g and . 8 9 for U .

a he vertical component of the displacement vector is do n ard ith magnitude h so e obtain

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = mgL\ 1 - \cos\theta$$
$$= 5.00\text{ kg}\ \ 9.80\text{ m s}^2\ \ 2.00\text{ m}\ \ 1 - \cos 30°\ = 13.1\ .$$

b rom . 8 1 e have $\Delta U = -W_g = -mgL\ 1 - \cos\theta\ = -13.1$.

c ith $y = h$. 8 9 yields $U = mgL\ 1 - \cos\theta\ = 13.1$.

d s the angle increases e intuitively see that the height h increases and less obviously from the mathematics e see that cos θ decreases so that 1 – cos θ increases so the ans ers to parts a and c increase and the absolute value of the ans er to part b also increases.

8. a he force of gravity is constant so the ork it does is given by $W = \vec{F}\cdot\vec{d}$ here $\vec{F}$ is the force and $\vec{d}$ is the displacement. he force is vertically do n ard and has magnitude mg here m is the mass of the sno ball. he e pression for the ork reduces to $W = mgh$ here h is the height through hich the sno ball drops. hus

$$W = mgh = 1.50\text{ kg}\ \ 9.80\text{ m s}^2\ \ 12.5\text{ m}\ = 184\ .$$

b he force of gravity is conservative so the change in the potential energy of the sno ball arth system is the negative of the ork it does $\Delta U = -W = -184$.

c he potential energy hen it reaches the ground is less than the potential energy hen it is fired by ΔU so $U = -184$ hen the sno ball hits the ground.

9. e use . 8 17 representing the conservation of mechanical energy hich neglects friction and other dissipative effects .

a n roblem 9 2 e found $U_A = mgh$ ith the reference position at C . eferring again to ig. 8 29 e see that this is the same as U_0 hich implies that $K_A = K_0$ and thus that

$$v_A = v_0 = 17.0\text{ m s}.$$

b n the solution to roblem 9 2 e also found $U_B = mgh/2$. n this case e have

$$K_0 + U_0 = K_B + U_B$$

$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_B^2 + mg\left(\frac{h}{2}\right)$$

hich leads to

$$v_B = \sqrt{v_0^2 + gh} = \sqrt{17.0\ \text{m s}\ ^2 + \ 9.80\ \text{m s}^2\ \ 42.0\ \text{m}} = 26.5\ \text{m s.}$$

c imilarly $v_C = \sqrt{v_0^2 + 2gh} = \sqrt{17.0\ \text{m s}\ ^2 + 2\ 9.80\ \text{m s}^2\ \ 42.0\ \text{m}} = 33.4\ \text{m s.}$

(d) To find the "final" height, we set $K_f = 0$. n this case e have

$$K_0 + U_0 = K_f + U_f$$

$$\frac{1}{2}mv_0^2 + mgh = 0 + mgh_f$$

hich yields $h_f = h + \frac{v_0^2}{2g} = 42.0\ \text{m} + \frac{17.0\ \text{m s}\ ^2}{2\ 9.80\ \text{m s}^2} = 56.7\ \text{m.}$

e t is evident that the above results do not depend on mass. hus a different mass for the coaster must lead to the same results.

10. e use . 8 17 representing the conservation of mechanical energy hich neglects friction and other dissipative effects .

a n the solution to roblem 9 3 to hich this problem refers e found $U_i = mgy_i$ = 196 and $U_f = mgy_f$ = 29.0 assuming the reference position is at the ground . ince $K_i = 0$ in this case e have

$$0 + 196 \quad = K_f + 29.0$$

hich gives $K_f = 167$ and thus leads to $v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2\ 167}{2.00\ \text{kg}}} = 12.9\ \text{m s.}$

b f e proceed algebraically through the calculation in part a e find $K_f = -\Delta U = mgh$ here $h = y_i - y_f$ and is positive valued. hus

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gh}$$

as e might also have derived from the e uations of able 2 1 particularly . 2 16 . he fact that the ans er is independent of mass means that the ans er to part b is identical to that of part a that is $v = 12.9\ \text{m s}$.

c f $K_i \neq 0$ then e find K_f = mgh K_i here K_i is necessarily positive valued . his represents a larger value for K_f than in the previous parts and thus leads to a larger value for v.

11. **THINK** s the ice flake slides do n the frictionless bo l its potential energy decreases discussed in roblem 8 5 . y conservation of mechanical energy its kinetic energy must increase.

EXPRESS f K_i is the kinetic energy of the flake at the edge of the bo l K_f is its kinetic energy at the bottom U_i is the gravitational potential energy of the flake arth system ith the flake at the top and U_f is the gravitational potential energy ith it at the bottom then

$$K_f + U_f = K_i + U_i .$$

aking the potential energy to be ero at the bottom of the bo l then the potential energy at the top is U_i = mgr here r = 0.220 m is the radius of the bo l and m is the mass of the flake. K_i = 0 since the flake starts from rest. ince the problem asks for the speed at the bottom e rite $K_f = mv^2$ 2.

ANALYZE a nergy conservation leads to

$$K_f + U_f = K_i + U_i \quad \Rightarrow \quad \frac{1}{2}mv^2 + 0 = 0 + mgr .$$

he speed is $v = \sqrt{2gr} = 2.08$ m s .

b ince the e pression for speed is $v = \sqrt{2gr}$ hich does not contain the mass of the flake the speed ould be the same 2.08 m s regardless of the mass of the flake.

c he final kinetic energy is given by $K_f = K_i + U_i - U_f$. f K_i is greater than before then K_f ill also be greater. his means the final speed of the flake is greater.

LEARN he mechanical energy conservation principle can also be e pressed as $\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$ hich implies $\Delta K = -\Delta U$ i.e. the increase in kinetic energy is e ual to the negative of the change in potential energy.

12. e use . 8 18 representing the conservation of mechanical energy. e choose the reference position for computing U to be at the ground belo the cliff it is also regarded as the “final” position in our calculations.

a sing . 8 9 the initial potential energy is given by U_i = mgh here h = 12.5 m and $m = 1.50$ kg . hus e have

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv^2 + 0$$

hich leads to the speed of the sno ball at the instant before striking the ground

$$v = \sqrt{\frac{2}{m}\left(\frac{1}{2}mv_i^2 + mgh\right)} = \sqrt{v_i^2 + 2gh}$$

here v_i = 14.0 m s is the magnitude of its initial velocity not ust one component of it . hus e find v = 21.0 m s.

b s noted above v_i is the magnitude of its initial velocity and not ust one component of it therefore there is no dependence on launch angle. he ans er is again 21.0 m s.

c t is evident that the result for v in part a does not depend on mass. hus changing the mass of the sno ball does not change the result for v.

13. **THINK** s the marble moves vertically up ard its gravitational potential energy increases. his energy comes from the release of elastic potential energy stored in the spring.

EXPRESS e take the reference point for gravitational potential energy to be at the position of the marble hen the spring is compressed. he gravitational potential energy hen the marble is at the top of its motion is $U_g = mgh$. n the other had the energy stored in the spring is $U_s = kx^2$ 2. pplying mechanical energy conservation principle allo s us to solve the problem.

ANALYZE a he height of the highest point is h = 20 m. ith initial gravitational potential energy set to ero e find

$$\Delta U_g = mgh = 5.0\times10^{-3}\text{ kg} \;\; 9.8\text{ m s}^2 \;\; 20\text{ m} = 0.98 \;\; .$$

b ince the kinetic energy is ero at the release point and at the highest point then conservation of mechanical energy implies ΔU_g ΔU_s = 0 here ΔU_s is the change in the spring s elastic potential energy. herefore $\Delta U_s = -\Delta U_g = -0.98$.

c e take the spring potential energy to be ero hen the spring is rela ed. hen our result in the previous part implies that its initial potential energy is U_s = 0.98 . his must be $\frac{1}{2}kx^2$ here k is the spring constant and x is the initial compression. onse uently

$$k = \frac{2U_s}{x^2} = \frac{0.98}{0.080\text{ m}^{\;\;2}} = 3.1\times10^2 \quad \text{m} = 3.1 \quad \text{cm.}$$

LEARN n general the marble has both kinetic and potential energies

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + mgy$$

t the ma imum height $y_{ma} = h$ $v = 0$ and $mgh = kx^2$ 2 or $h = \frac{kx^2}{2mg}$.

14. e use . 8 18 representing the conservation of mechanical energy hich neglects friction and other dissipative effects .

a he change in potential energy is $\Delta U = mgL$ as it goes to the highest point. hus e have

$$\Delta K + \Delta U = 0$$
$$K_{top} - K_0 + mgL = 0$$

hich upon re uiring $K_{top} = 0$ gives $K_0 = mgL$ and thus leads to

$$v_0 = \sqrt{\frac{2K_0}{m}} = \sqrt{2gL} = \sqrt{2\ 9.80\ \text{m s}^2\ \ 0.452\ \text{m}} = 2.98\ \text{m s}.$$

b e also found in roblem 9 4 that the potential energy change is $\Delta U = -mgL$ in going from the initial point to the lo est point the bottom . hus

$$\Delta K + \Delta U = 0$$
$$K_{bottom} - K_0 - mgL = 0$$

hich ith $K_0 = mgL$ leads to $K_{bottom} = 2mgL$. herefore

$$v_{bottom} = \sqrt{\frac{2K_{bottom}}{m}} = \sqrt{4gL} = \sqrt{4\ 9.80\ \text{m s}^2\ \ 0.452\ \text{m}} = 4.21\ \text{m s}.$$

c ince there is no change in height going from initial point to the rightmost point then $\Delta U = 0$ hich implies $\Delta K = 0$. onse uently the speed is the same as hat it as initially

$$v_{right} = v_0 = 2.98\ \text{m s}.$$

d t is evident from the above manipulations that the results do not depend on mass. hus a different mass for the ball must lead to the same results.

15. **THINK** he truck ith failed brakes is moving up an escape ramp. n order for it to come to a complete stop all of its kinetic energy must be converted into gravitational potential energy.

EXPRESS e ignore any ork done by friction. n units the initial speed of the truck ust before entering the escape ramp is v_i = 130 1000 3600 = 36.1 m s. hen the truck comes to a stop its kinetic and potential energies are K_f = 0 and U_f = mgh. e apply mechanical energy conservation to solve the problem.

ANALYZE a nergy conservation implies $K_f + U_f = K_i + U_i$. ith U_i = 0 and $K_i = \frac{1}{2} m v_i^2$ e obtain

$$\frac{1}{2} m v_i^2 + 0 = 0 + mgh \quad \Rightarrow \quad h = \frac{v_i^2}{2g} = \frac{36.1\text{ m s}\ ^2}{2\ 9.8\text{ m s}^2} = 66.5\text{ m.}$$

f L is the minimum length of the ramp then $L\sin\theta = h$ or L sin 15 = 66.5 m so that $L =$ 66.5 m sin15° = 257 m. hat is the ramp must be about 2.6×10^2 m long if friction is negligible.

b he minimum length is $L = \dfrac{h}{\sin\theta} = \dfrac{v_i^2}{2g\sin\theta}$ hich does not depend on the mass of the truck. hus the ans er remains the same if the mass is reduced.

c f the speed is decreased then h and L both decrease note that h is proportional to the s uare of the speed and that L is proportional to h .

LEARN he greater the speed of the truck the longer the ramp re uired. his length can be shortened considerably if the friction bet een the tires and the ramp surface is factored in.

16. e place the reference position for evaluating gravitational potential energy at the rela ed position of the spring. e use x for the spring s compression measured positively do n ard so x 0 means it is compressed .

a ith x = 0.190 m . 7 26 gives

$$W_s = -\frac{1}{2} kx^2 = -7.22 \quad \approx -7.2$$

for the ork done by the spring force. sing e ton s third la e see that the ork done on the spring is 7.2 .

b s noted above W_s = –7.2 .

c nergy conservation leads to

$$K_i + U_i = K_f + U_f \quad \Rightarrow \quad 0 + mgh_0 = \frac{1}{2} kx^2 - mgx$$

hich ith m = 0.70 kg yields h_0 = 0.86 m.

d ith a ne value for the height $h_0' = 2h_0 = 1.72$ m e solve for a ne value of x using the uadratic formula taking its positive root so that x 0 .

$$mgh_0' = -mgx + \frac{1}{2}kx^2 \;\Rightarrow\; x = \frac{mg + \sqrt{(mg)^2 + 2mgkh_0'}}{k}$$

hich yields x = 0.26 m.

17. a t Q the block hich is in circular motion at that point e periences a centripetal acceleration v^2 R left ard. e find v^2 from energy conservation

$$K_P + U_P = K_Q + U_Q$$
$$0 + mgh = \frac{1}{2}mv^2 + mgR$$

sing the fact that h = 5R e find mv^2 = 8mgR. hus the hori ontal component of the net force on the block at Q is

$$F = mv^2\ R = 8mg = 8\ 0.032 \text{ kg}\ \ 9.8 \text{ m s}^2 = 2.5\ \ .$$

he direction is to the left in the same direction as $\vec{a}$.

b he do n ard component of the net force on the block at Q is the do n ard force of gravity

$$F = mg = 0.032 \text{ kg}\ \ 9.8 \text{ m s}^2 = 0.31\ \ .$$

c o barely make the top of the loop the centripetal force there must e ual the force of gravity

$$\frac{mv_t^2}{R} = mg \;\Rightarrow\; mv_t^2 = mgR\,.$$

his re uires a different value of h than hat as used above.

$$K_P + U_P = K_t + U_t$$
$$0 + mgh = \frac{1}{2}mv_t^2 + mgh_t$$
$$mgh = \frac{1}{2}\ mgR\ + mg\ 2R$$

onse uently h = 2.5R = 2.5 0.12 m = 0.30 m.

d he normal force F_N for speeds v_t greater than $\sqrt{gR}$ hich are the only possibilities for non ero F_N — see the solution in the previous part obeys

$$F_N = \frac{mv_t^2}{R} - mg$$

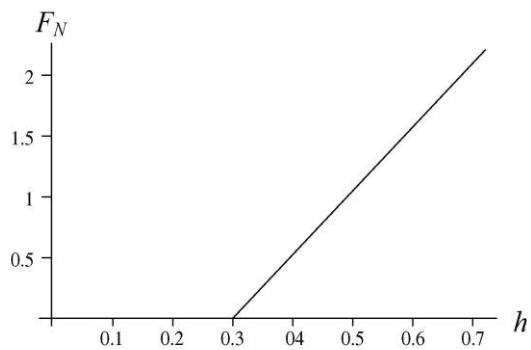
from e ton s second la . ince v_t^2 is related to h by energy conservation

$$K_P + U_P = K_t + U_t \Rightarrow gh = \frac{1}{2}v_t^2 + 2gR$$

then the normal force as a function for h so long as $h \geq 2.5R$ — see the solution in the previous part becomes

$$F_N = \frac{2mgh}{R} - 5mg\ .$$

hus the graph for $h \geq 2.5R = 0.30$ m consists of a straight line of positive slope $2mg\ R$ hich can be set to some convenient values for graphing purposes . ote that for $h \leq 2.5R$ the normal force is ero.



18. e use . 8 18 representing the conservation of mechanical energy. he reference position for computing U is the lo est point of the s ing it is also regarded as the “final” position in our calculations.

a he potential energy is $U = mgL\ 1 - \cos\ \theta$ at the position sho n in ig. 8 34 hich e consider to be the initial position . hus e have

$$K_i + U_i = K_f + U_f$$

$$0 + mgL\ 1 - \cos\theta\ = \frac{1}{2}mv^2 + 0$$

hich leads to

$$v = \sqrt{\frac{2mgL\ 1 - \cos\theta}{m}} = \sqrt{2gL\ 1 - \cos\theta}\ .$$

lugging in L = 2.00 m and θ = 30.0 e find v = 2.29 m s.

b t is evident that the result for v does not depend on mass. hus a different mass for the ball must not change the result.

19. e convert to units and choose up ard as the y direction. lso the rela ed position of the top end of the spring is the origin so the initial compression of the spring defining an e uilibrium situation bet een the spring force and the force of gravity is y_0 = –0.100 m and the additional compression brings it to the position y_1 = –0.400 m.

a hen the stone is in the e uilibrium $a = 0$ position e ton s second la becomes

$$\vec{F}_{\text{net}} = ma$$
$$F_{\text{spring}} - mg = 0$$
$$-k \quad -0.100 \quad - \quad 8.00 \quad 9.8 \quad = 0$$

here ooke s la . 7 21 has been used. his leads to a spring constant e ual to $k = 784$ m .

b ith the additional compression and release the acceleration is no longer ero and the stone ill start moving up ard turning some of its elastic potential energy stored in the spring into kinetic energy. he amount of elastic potential energy at the moment of release is using . 8 11

$$U = \frac{1}{2}ky_1^2 = \frac{1}{2} \; 784 \quad \text{m} \quad -0.400 \;^2 = 62.7 \quad .$$

c ts ma imum height y_2 is beyond the point that the stone separates from the spring entering free fall motion . s usual it is characteri ed by having momentarily ero speed. f e choose the y_1 position as the reference position in computing the gravitational potential energy then

$$K_1 + U_1 = K_2 + U_2$$
$$0 + \frac{1}{2}ky_1^2 = 0 + mgh$$

here h = $y_2 - y_1$ is the height above the release point. hus mgh the gravitational potential energy is seen to be e ual to the previous ans er 62.7 and e proceed ith the solution in the ne t part.

d e find $h = ky_1^2/2mg = 0.800$ m or 80.0 cm.

20. a e take the reference point for gravitational energy to be at the lo est point of the s ing. et θ be the angle measured from vertical. hen the height y of the pendulum "bob" (the object at the end of the pendulum, which in this problem is the stone) is given by $L \;\; 1 - \cos\theta \;\; = y$. ence the gravitational potential energy is

$$mg\,y = mgL\ 1 - \cos\theta\ .$$

hen $\theta = 0$ the string at its lo est point e are told that its speed is 8.0 m s its kinetic energy there is therefore 64 using . 7 1 . t $\theta = 60$ its mechanical energy is

$$E_{\text{mech}} = \frac{1}{2}\ mv^2 \quad mgL\ 1 - \cos\theta\ .$$

nergy conservation since there is no friction re uires that this be e ual to 64 . olving for the speed e find $v = 5.0$ m s.

b e no set the above e pression again e ual to 64 ith θ being the unkno n but ith ero speed hich gives the condition for the maximum point, or "turning point" that it reaches . his leads to $\theta_{\text{ma}} = 79°$.

c s observed in our solution to part a the total mechanical energy is 64 .

21. e use . 8 18 representing the conservation of mechanical energy hich neglects friction and other dissipative effects . he reference position for computing U and height h) is the lowest point of the swing; it is also regarded as the "final" position in our calculations.

a areful e amination of the figure leads to the trigonometric relation $h = L - L\cos\theta$ hen the angle is measured from vertical as sho n. hus the gravitational potential energy is $U = mgL\ 1 - \cos\theta_0$ at the position sho n in ig. 8 34 the initial position . hus e have

$$K_0 + U_0 = K_f + U_f$$

$$\frac{1}{2}mv_0^2 + mgL\left(1 - \cos\theta_0\right) = \frac{1}{2}mv^2 + 0$$

hich leads to

$$v = \sqrt{\frac{2}{m}\left[\frac{1}{2}mv_0^2 + mgL\ 1 - \cos\theta_0\ \right]} = \sqrt{v_0^2 + 2gL\ 1 - \cos\theta_0}$$

$$= \sqrt{\ 8.00\text{ m s}\ ^2 + 2\ 9.80\text{ m s}^2\ \ 1.25\text{ m}\ \ 1 - \cos 40°\ } = 8.35\text{ m s}.$$

b e look for the initial speed re uired to barely reach the hori ontal position — described by $v_h = 0$ and $\theta = 90$ or $\theta = -90$ if one prefers but since cos $-\phi$ = cos ϕ the sign of the angle is not a concern .

$$K_0 + U_0 = K_h + U_h$$

$$\frac{1}{2}mv_0^2 + mgL\left(1 - \cos\theta_0\right) = 0 + mgL$$

hich yields

$$v_0 = \sqrt{2gL\cos\theta_0} = \sqrt{2\ 9.80\text{ m s}^2\ \ 1.25\text{ m}\ \cos 40°} = 4.33\text{ m s}.$$

c or the cord to remain straight then the centripetal force at the top must be at least e ual to gravitational force

$$\frac{mv_t^2}{r} = mg \Rightarrow mv_t^2 = mgL$$

here e recogni e that $r = L$. e plug this into the e pression for the kinetic energy at the top here $\theta = 180$.

$$K_0 + U_0 = K_t + U_t$$

$$\frac{1}{2}mv_0^2 + mgL\,(1 - \cos\theta_0) = \frac{1}{2}mv_t^2 + mg(1 - \cos 180°)$$

$$\frac{1}{2}mv_0^2 + mgL\,(1 - \cos\theta_0) = \frac{1}{2}\ mgL\ + mg\ 2L$$

hich leads to

$$v_0 = \sqrt{gL\ 3 + 2\cos\theta_0} = \sqrt{\ 9.80 \text{ m s}^2\ \ 1.25 \text{ m}\ \ 3 + 2\cos 40°} = 7.45 \text{ m s.}$$

d he more initial potential energy there is the less initial kinetic energy there needs to be in order to reach the positions described in parts b and c . ncreasing θ_0 amounts to increasing U_0 so e see that a greater value of θ_0 leads to smaller results for v_0 in parts b and c .

22. rom hapter 4 e kno the height h of the skier s ump can be found from $v_y^2 = 0 = v_{0y}^2 - 2gh$ here $v_{0y} = v_0 \sin 28°$ is the upward component of the skier's “launch velocity.” To find v_0 e use energy conservation.

a he skier starts at rest $y = 20$ m above the point of “launch” so energy conservation leads to

$$mgy = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gy} = 20 \text{ m/s}$$

hich becomes the initial speed v_0 for the launch. ence the above e uation relating h to v_0 yields

$$h = \frac{(v_0 \sin 28°)^2}{2g} = 4.4 \text{ m} .$$

b e see that all reference to mass cancels from the above computations so a ne value for the mass ill yield the same result as before.

23. a s the string reaches its lo est point its original potential energy $U = mgL$ measured relative to the lo est point is converted into kinetic energy. hus

$$mgL = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gL} \ .$$

ith L = 1.20 m e obtain $v = \sqrt{2gL} = \sqrt{2\ 9.80\ \text{m s}^2\ \ 1.20\ \text{m}} = 4.85\ \text{m s}$.

b n this case the total mechanical energy is shared bet een kinetic $\frac{1}{2}mv_b^2$ and potential mgy_b. e note that y_b = 2r here r = L – d = 0.450 m. nergy conservation leads to

$$mgL = \frac{1}{2}mv_b^2 + mgy_b$$

hich yields $v_b = \sqrt{2gL - 2g(2r)} = 2.42\ \text{m/s}$.

24. e denote m as the mass of the block h = 0.40 m as the height from hich it dropped measured from the rela ed position of the spring and x as the compression of the spring measured do n ard so that it yields a positive value . ur reference point for the gravitational potential energy is the initial position of the block. he block drops a total distance h x and the final gravitational potential energy is $-mg\ h\ \ x$. he spring potential energy is $\frac{1}{2}kx^2$ in the final situation and the kinetic energy is ero both at the beginning and end. ince energy is conserved

$$K_i + U_i = K_f + U_f$$

$$0 = -mg\ \ h + x\ \ + \frac{1}{2}kx^2$$

hich is a second degree e uation in x. sing the uadratic formula its solution is

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k} \ .$$

o mg = 19.6 h = 0.40 m and $k = 1960$ /m and e choose the positive root so that x 0.

$$x = \frac{19.6 + \sqrt{19.6^2 + 2(19.6)(0.40)(1960)}}{1960} = 0.10\ \text{m} \ .$$

25. ince time does not directly enter into the energy formulations e return to hapter 4 or able 2 1 in hapter 2 to find the change of height during this t = 6.0 s flight.

$$\Delta y = v_{0_y}t - \frac{1}{2}gt^2$$

his leads to $\Delta y = -32\ \text{m}$. herefore $\Delta U = mg\Delta y = -318\ \ \approx -3.2\times10^{-2}$.

26. a ith energy in oules and length in meters e have

$$\Delta U = U(x) - U(0) = -\int_0^x (6x' - 12)dx' \ .$$

herefore ith U 0 = 27 e obtain U x ritten simply as U by integrating and rearranging

$$U = 27 + 12x - 3x^2 \ .$$

b e can ma imi e the above function by orking through the $dU \quad dx = 0$ condition or e can treat this as a force e uilibrium situation — hich is the approach e sho .

$$F = 0 \Rightarrow 6x_{eq} - 12 = 0$$

hus x_{eq} = 2.0 m and the above e pression for the potential energy becomes U = 39 .

c sing the uadratic formula or using the polynomial solver on an appropriate calculator e find the negative value of x for hich U = 0 to be x = –1.6 m.

d imilarly e find the positive value of x for hich U = 0 to be x = 5.6 m.

27. a o find out hether or not the vine breaks it is sufficient to e amine it at the moment ar an s ings through the lo est point hich is hen the vine — if it didn t break — ould have the greatest tension. hoosing up ard positive e ton's second la leads to

$$T - mg = m\frac{v^2}{r}$$

here r = 18.0 m and $m = W/g = 688/9.8 = 70.2$ kg . e find the v^2 from energy conservation here the reference position for the potential energy is at the lo est point .

$$mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v^2 = 2gh$$

here h = 3.20 m. ombining these results e have

$$T = mg + m\frac{2gh}{r} = mg\left(1 + \frac{2h}{r}\right)$$

hich yields 933 . hus the vine does not break.

b ounding to an appropriate number of significant figures e see the ma imum tension is roughly 9.3×10^2 .

28. rom the slope of the graph e find the spring constant

$$k = \frac{\Delta F}{\Delta x} = 0.10 \quad /\text{cm} = 10 \quad /\text{m}.$$

a uating the potential energy of the compressed spring to the kinetic energy of the cork at the moment of release e have

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow v = x\sqrt{\frac{k}{m}}$$

hich yields v = 2.8 m s for m = 0.0038 kg and x = 0.055 m.

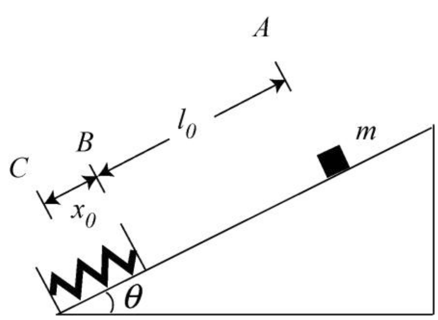
b he ne scenario involves some potential energy at the moment of release. ith d = 0.015 m energy conservation becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}kd^2 \Rightarrow v = \sqrt{\frac{k}{m}\left(x^2 - d^2\right)}$$

hich yields v = 2.7 m s.

29. **THINK** s the block slides do n the inclined plane it compresses the spring then stops momentarily before sliding back up again.

EXPRESS e refer to its starting point as A the point here it first comes into contact ith the spring as B and the point here the spring is compressed by $x_0 = 0.055$ m as C see the figure belo . oint C is our reference point for computing gravitational potential energy. lastic potential energy of the spring is ero hen the spring is rela ed.



nformation given in the second sentence allo s us to compute the spring constant. rom ooke s la e find

$$k = \frac{F}{x} = \frac{270}{0.02 \text{ m}} = 1.35 \times 10^4 \quad /\text{m}.$$

he distance bet een points A and B is l_0 and e note that the total sliding distance $l_0 + x_0$ is related to the initial height h_A of the block measured relative to C by $\sin\theta = \dfrac{h_A}{l_0 + x_0}$ here the incline angle θ is 30 .

ANALYZE a echanical energy conservation leads to

$$K_A + U_A = K_C + U_C \quad \Rightarrow \quad 0 + mgh_A = \frac{1}{2}kx_0^2$$

hich yields

$$h_A = \frac{kx_0^2}{2mg} = \frac{1.35\times10^4 \quad m \quad 0.055\ \text{m}\ ^2}{2\ \ 12\ \text{kg}\ \ 9.8\ \text{m}\ \text{s}^2} = 0.174\ \text{m}.$$

herefore the total distance traveled by the block before coming to a stop is

$$l_0 + x_0 = \frac{h_A}{\sin 30^\circ} = \frac{0.174\ \text{m}}{\sin 30^\circ} = 0.347\ \text{m} \approx 0.35\ \text{m}.$$

b rom this result e find $l_0 = x_0 = 0.347\ \text{m} - 0.055\ \text{m} = 0.292\ \text{m}$ hich means that the block has descended a vertical distance

$$\Delta y \ = h_A - h_B = l_0 \sin\theta = \ 0.292\ \text{m}\ \sin 30^\circ = 0.146\ \text{m}$$

in sliding from point A to point B. hus using . 8 18 e have

$$0 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B \quad \Rightarrow \quad \frac{1}{2}mv_B^2 = mg \ \ \Delta y$$

hich yields $v_B = \sqrt{2g \ \ \Delta y} = \sqrt{2\ 9.8\ \text{m}\ \text{s}^2 \ \ 0.146\ \text{m}} = 1.69\ \text{m}\ \text{s} \approx 1.7\ \text{m}\ \text{s}$.

LEARN nergy is conserved in the process. he total energy of the block at position B is

$$E_B = \frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}\ 12\ \text{kg}\ \ 1.69\ \text{m}\ \text{s}\ ^2 + \ 12\ \text{kg}\ \ 9.8\ \text{m}\ \text{s}^2\ \ 0.028\ \text{m}\ = 20.4$$

hich is e ual to the elastic potential energy in the spring

$$\frac{1}{2}kx_0^2 = \frac{1}{2}\ 1.35\times10^4 \quad \text{m}\ \ 0.055\ \text{m}\ ^2 = 20.4 \quad .$$

30. e take the original height of the bo to be the y = 0 reference level and observe that in general the height of the bo hen the bo has moved a distance d do nhill is $y = -d\sin 40°$.

a sing the conservation of energy e have

$$K_i + U_i = K + U \Rightarrow 0 + 0 = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kd^2.$$

herefore ith d = 0.10 m e obtain v = 0.81 m s.

b e look for a value of $d \neq 0$ such that K = 0.

$$K_i + U_i = K + U \Rightarrow 0 + 0 = 0 + mgy + \frac{1}{2}kd^2.$$

hus e obtain $mgd\sin 40° = \frac{1}{2}kd^2$ and find d = 0.21 m.

c he uphill force is caused by the spring ooke s la and has magnitude kd = 25.2 . he do nhill force is the component of gravity $mg\sin 40°$ = 12.6 . hus the net force on the bo is 25.2 – 12.6 = 12.6 uphill ith

$$a = F\ m = 12.6 \quad 2.0\text{ kg} = 6.3\text{ m s}^2.$$

d he acceleration is up the incline.

31. he reference point for the gravitational potential energy U_g and height h is at the block hen the spring is ma imally compressed. hen the block is moving to its highest point it is first accelerated by the spring later it separates from the spring and finally reaches a point here its speed v_f is momentarily ero. he x a is is along the incline pointing uphill so x_0 for the initial compression is negative valued its origin is at the rela ed position of the spring. e use units so k = 1960 m and x_0 = –0.200 m.

a he elastic potential energy is $\frac{1}{2}kx_0^2 = 39.2$.

b ince initially U_g = 0 the change in U_g is the same as its final value mgh here m = 2.00 kg. hat this must e ual the result in part a is made clear in the steps sho n in the ne t part. hus $\Delta U_g = U_g$ = 39.2 .

c he principle of mechanical energy conservation leads to

$$K_0 + U_0 = K_f + U_f$$

$$0 + \frac{1}{2}kx_0^2 = 0 + mgh$$

hich yields h = 2.00 m. he problem asks for the distance *along the incline* so e have $d = h$ sin 30 = 4.00 m.

32. he ork re uired is the change in the gravitational potential energy as a result of the chain being pulled onto the table. ividing the hanging chain into a large number of infinitesimal segments each of length dy e note that the mass of a segment is $m L \; dy$ and the change in potential energy of a segment hen it is a distance y belo the table top is

$$dU = \; m\,L \; g\,y \; dy = -\; m\,L \; gy\,dy$$

since y is negative valued e have y up ard and the origin is at the tabletop . he total potential energy change is

$$\Delta U = -\frac{mg}{L}\int_{-L\;4}^{0} y\,dy = \frac{1}{2}\frac{mg}{L}\;\; L/4\;^{2} = mgL/32.$$

he ork re uired to pull the chain onto the table is therefore

$$W = \Delta U = mgL\;32 = \; 0.012 \text{ kg} \;\; 9.8 \text{ m s}^2 \;\; 0.28 \text{ m} \;\; 32 = 0.0010 \; .$$

33. ll heights h are measured from the lo er end of the incline hich is our reference position for computing gravitational potential energy mgh . ur x a is is along the incline ith x being uphill so spring compression corresponds to x 0 and its origin being at the rela ed end of the spring. he height that corresponds to the canister s initial position ith spring compressed amount x = 0.200 m is given by $h_1 = \; D + x \; \sin\theta$ here $\theta = 37°$.

a nergy conservation leads to

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad 0 + mg\;\; D + x\;\; \sin\theta + \frac{1}{2}kx^2 = \frac{1}{2}mv_2^2 + mgD\sin\theta$$

hich yields using the data m = 2.00 kg and k = 170 m

$$v_2 = \sqrt{2gx\sin\theta + kx^2/m} = 2.40 \text{ m/s}.$$

b n this case energy conservation leads to

$$K_1 + U_1 = K_3 + U_3$$
$$0 + mg\;\; D + x\;\; \sin\theta + \frac{1}{2}kx^2 = \frac{1}{2}mv_3^2 + 0$$

hich yields $v_3 = \sqrt{2g\;\; D + x\;\; \sin\theta + kx^2 \;\; m} = 4.19 \text{ m s}.$

34. et $\vec{F}_N$ be the normal force of the ice on him and m is his mass. he net in ard force is mg cos $\theta - F_N$ and according to e ton s second la this must be e ual to mv^2 R here v is the speed of the boy. t the point here the boy leaves the ice $F_N = 0$ so g cos $\theta = v^2$ R. e ish to find his speed. f the gravitational potential energy is taken to be ero hen he is at the top of the ice mound then his potential energy at the time sho n is

$$U = -mgR\ 1 - \cos\ \theta\ .$$

e starts from rest and his kinetic energy at the time sho n is $\frac{1}{2}mv^2$. hus conservation of energy gives

$$0 = \tfrac{1}{2}mv^2 - mgR\ 1 - \cos\theta$$

or $v^2 = 2gR\ 1 - \cos\ \theta$. e substitute this e pression into the e uation developed from the second la to obtain g cos $\theta = 2g\ 1 - \cos\ \theta$. his gives cos $\theta = 2\ 3$. he height of the boy above the bottom of the mound is

$$h = R\cos\theta = \frac{2}{3}R = \frac{2}{3}\ 13.8\text{ m} = 9.20\text{ m}.$$

35. a he final elastic potential energy is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}\ 431\quad\text{m}\ \ 0.210\text{ m}^{\ 2} = 9.50\ \ .$$

ltimately this must come from the original gravitational energy in the system mgy (where we are measuring y from the lowest "elevation" reached by the block, so

$$y = \ d\quad x\ \sin 30\ \ .$$

hus

$$mg\ d\quad x\ \sin 30\ = 9.50\qquad\Rightarrow\qquad d = 0.396\text{ m}.$$

b he block is still accelerating due to the component of gravity along the incline mgsin 30 for a fe moments after coming into contact ith the spring hich e erts the Hooke's law force kx), until the Hooke's law force is strong enough to cause the block to begin decelerating. his point is reached hen

$$kx = mg\sin 30$$

hich leads to $x = 0.0364$ m = 3.64 cm this is long before the block finally stops 36.0 cm before it stops .

36. he distance the marble travels is determined by its initial speed and the methods of hapter 4 and the initial speed is determined using energy conservation by the original compression of the spring. e denote h as the height of the table and x as the hori ontal

distance to the point here the marble lands. hen $x = v_0\ t$ and $h = \frac{1}{2}gt^2$ since the vertical component of the marble's "launch velocity" is zero). From these we find $x = v_0\sqrt{2\,h/g}$. e note from this that the distance to the landing point is directly proportional to the initial speed. e denote v_{01} be the initial speed of the first shot and D_1 = 2.20 – 0.27 m = 1.93 m be the hori ontal distance to its landing point similarly v_{02} is the initial speed of the second shot and D = 2.20 m is the hori ontal distance to its landing spot. hen

$$\frac{v_{02}}{v_{01}} = \frac{D}{D_1} \quad \Rightarrow \quad v_{02} = \frac{D}{D_1}v_{01}$$

hen the spring is compressed an amount ℓ the elastic potential energy is $\frac{1}{2}k\ell^2$. hen the marble leaves the spring its kinetic energy is $\frac{1}{2}mv_0^2$. echanical energy is conserved $\frac{1}{2}mv_0^2 = \frac{1}{2}k\ell^2$ and e see that the initial speed of the marble is directly proportional to the original compression of the spring. f ℓ_1 is the compression for the first shot and ℓ_2 is the compression for the second then $v_{02} = (\ell_2/\ell_1)v_{01}$. elating this to the previous result e obtain

$$\ell_2 = \frac{D}{D_1}\ell_1 = \left(\frac{2.20\text{ m}}{1.93\text{ m}}\right)1.10\text{ cm} = 1.25\text{ cm}.$$

37. onsider a differential element of length dx at a distance x from one end the end that remains stuck of the cord. s the cord turns vertical its change in potential energy is given by

$$dU = -\ \lambda dx\ gx$$

here $\lambda = m\ \ h$ is the mass unit length and the negative sign indicates that the potential energy decreases. ntegrating over the entire length e obtain the total change in the potential energy

$$\Delta U = \int dU = -\int_0^h \lambda gx dx = -\frac{1}{2}\lambda gh^2 = -\frac{1}{2}mgh.$$

ith m = 15 g and h = 25 cm e have $\Delta U = -0.018$.

38. n this problem the mechanical energy the sum of K and U remains constant as the particle moves.

a ince mechanical energy is conserved $U_B + K_B = U_A + K_A$ the kinetic energy of the particle in region A $3.00\text{ m} \le x \le 4.00\text{ m}$ is

$$K_A = U_B - U_A + K_B = 12.0\quad - 9.00\quad + 4.00\quad = 7.00\quad .$$

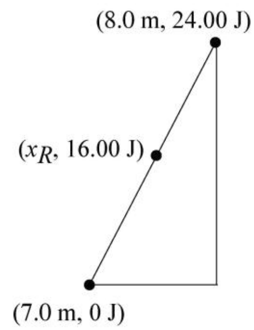
ith $K_A = mv_A^2\ \ 2$ the speed of the particle at $x = 3.5$ m ithin region A is

$$v_A = \sqrt{\frac{2K_A}{m}} = \sqrt{\frac{2\ \ 7.00}{0.200\text{ kg}}} = 8.37\text{ m s.}$$

b t $x = 6.5$ m $U = 0$ and $K = U_B + K_B = 12.0\ \ + 4.00\ \ = 16.0$ by mechanical energy conservation. herefore the speed at this point is

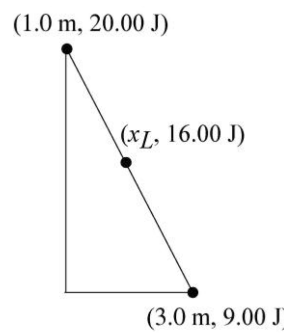
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\ 16.0}{0.200\text{ kg}}} = 12.6\text{ m s.}$$

c t the turning point the speed of the particle is ero. et the position of the right turning point be x_R. rom the figure sho n on the right e find x_R to be



$$\frac{16.00\ \ -0}{x_R - 7.00\text{ m}} = \frac{24.00\ \ -16.00}{8.00\text{ m} - x_R} \quad\Rightarrow\quad x_R = 7.67\text{ m.}$$

d et the position of the left turning point be x_L. rom the figure sho n e find x_L to be



$$\frac{16.00\ \ -20.00}{x_L - 1.00\text{ m}} = \frac{9.00\ \ -16.00}{3.00\text{ m} - x_L} \quad\Rightarrow\quad x_L = 1.73\text{ m.}$$

39. rom the figure e see that at x = 4.5 m the potential energy is U_1 = 15 . f the speed is v = 7.0 m s then the kinetic energy is

$$K_1 = mv^2\ 2 = \ 0.90\text{ kg}\ \ 7.0\text{ m s}\ ^2\ 2 = 22\ \ .$$

 he total energy is $E_1 = U_1\ \ K_1$ = 15 22 = 37 .

a t x = 1.0 m the potential energy is U_2 = 35 . y energy conservation e have K_2 = 2.0   0. his means that the particle can reach there ith a corresponding speed

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2\ \ 2.0}{0.90\text{ kg}}} = 2.1\text{ m s.}$$

b he force acting on the particle is related to the potential energy by the negative of the slope

$$F_x = -\frac{\Delta U}{\Delta x}$$

rom the figure e have $F_x = -\frac{35 \quad -15}{2\text{ m} - 4\text{ m}} = +10$.

c ince the magnitude $F_x > 0$ the force points in the x direction.

d t x = 7.0 m the potential energy is U_3 = 45 hich e ceeds the initial total energy E_1. hus the particle can never reach there. t the turning point the kinetic energy is ero. et een x = 5 and 6 m the potential energy is given by

$$U \; x \; = 15 + 30 \; x - 5 \qquad 5 \le x \le 6.$$

hus the turning point is found by solving $37 = 15 + 30 \; x - 5$ hich yields x = 5.7 m.

e t x = 5.0 m the force acting on the particle is

$$F_x = -\frac{\Delta U}{\Delta x} = -\frac{45 - 15}{6 - 5 \;\; \text{m}} = -30 \quad .$$

he magnitude is $F_x = 30$.

f he fact that $F_x < 0$ indicated that the force points in the $-x$ direction.

40. a he force at the e uilibrium position $r = r_e$ is

$$F = -\left.\frac{dU}{dr}\right|_{r = r_e} = 0 \quad \Rightarrow \quad -\frac{12A}{r_e^{13}} + \frac{6B}{r_e^{7}} = 0$$

hich leads to the result

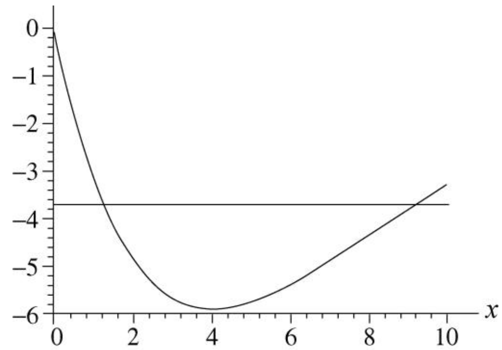
$$r_e = \left(\frac{2A}{B}\right)^{\frac{1}{6}} = 1.12\left(\frac{A}{B}\right)^{\frac{1}{6}}.$$

b his defines a minimum in the potential energy curve as can be verified either by a graph or by taking another derivative and verifying that it is concave up ard at this point hich means that for values of r slightly smaller than r_e the slope of the curve is negative so the force is positive repulsive .

c nd for values of r slightly larger than r_e the slope of the curve must be positive so the force is negative attractive .

41. a he energy at x = 5.0 m is $E = K \quad U$ = 2.0 – 5.7 = –3.7 .

b plot of the potential energy curve units understood and the energy E the hori ontal line is sho n for $0 \le x \le 10$ m.



c he problem asks for a graphical determination of the turning points hich are the points on the curve corresponding to the total energy computed in part a . he result for the smallest turning point determined to be honest by more careful means is x = 1.3 m.

d nd the result for the largest turning point is x = 9.1 m.

e ince $K = E - U$ then ma imi ing K involves finding the minimum of U. graphical determination suggests that this occurs at x = 4.0 m hich plugs into the e pression $E - U = -3.7 - \ \ -4xe^{-x\ 4}$ to give $K = 2.16 \quad \approx 2.2$. lternatively one can measure from the graph from the minimum of the U curve up to the level representing the total energy E and thereby obtain an estimate of K at that point.

f s mentioned in the previous part the minimum of the U curve occurs at x = 4.0 m.

g he force understood to be in ne tons follo s from the potential energy using . 8 20 and ppendi if students are unfamiliar ith such derivatives .

$$F = \frac{dU}{dx} = (4 - x)e^{-x\ 4}$$

h his revisits the considerations of parts d and e since e are returning to the minimum of $U\ x$ — but no ith the advantage of having the analytic result of part g . e see that the location that produces F = 0 is e actly x = 4.0 m.

42. ince the velocity is constant $\vec{a} = 0$ and the hori ontal component of the orker s push F cos θ here θ = 32 must e ual the friction force magnitude $f_k = \mu_k F_N$. lso the vertical forces must cancel implying

$$W_{\text{applied}} = \ 8.0 \quad 0.70\text{m} \ = 5.6$$

hich is solved to find F = 71 .

a he ork done on the block by the orker is using . 7 7

$$W = Fd\cos\theta = (71\quad)(9.2\text{ m})\cos 32° = 5.6\times 10^2\ .$$

b ince $f_k = \mu_k\ \ mg\quad F\sin\theta$ e find $\Delta E_{\text{th}} = f_k d =\ 60\quad 9.2\text{ m} = 5.6\times 10^2$.

43. a sing . 7 8 e have $W_{\text{applied}} =\ 8.0\quad 0.70\text{ m} = 5.6$.

b sing . 8 31 the thermal energy generated is $\Delta E_{\text{th}} = f_k d =\ 5.0\quad 0.70\text{ m} = 3.5$.

44. a he ork is $W = Fd =\ 35.0\quad 3.00\text{ m} = 105$.

b he total amount of energy that has gone to thermal forms is see . 8 31 and . 6 2

$$\Delta E_{\text{th}} = \mu_k mgd =\ 0.600\quad 4.00\text{ kg}\quad 9.80\text{ m s}^2\quad 3.00\text{ m} = 70.6\ .$$

f 40.0 has gone to the block then 70.6 – 40.0 = 30.6 has gone to the floor.

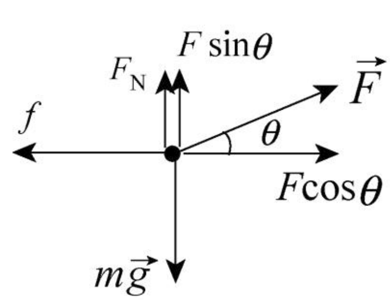
c Much of the work (105 J) has been "wasted" due to the 70.6 J of thermal energy generated but there still remains 105 – 70.6 = 34.4 that has gone into increasing the kinetic energy of the block. t has not gone into increasing the potential energy of the block because the floor is presumed to be hori ontal.

45. **THINK** ork is done against friction hile pulling a block along the floor at a constant speed.

EXPRESS lace the x a is along the path of the block and the y a is normal to the floor. he free body diagram is sho n belo . he x and the y component of e ton s second la are

$$x\qquad F\cos\theta - f\quad = 0$$
$$y\quad F_N\quad F\sin\theta - mg = 0$$

here m is the mass of the block F is the force e erted by the rope f is the magnitude of the kinetic friction force and θ is the angle bet een that force and the hori ontal.



he ork done on the block by the force in the rope is $W = Fd\cos\theta$. imilarly the increase in thermal energy of the block floor system due to the frictional force is given by . 8 29 $\Delta E_{\text{th}} = fd$.

ANALYZE a ubstituting the values given e find the ork done on the block by the rope's force to be

$$W = Fd\cos\theta = \ 7.68 \quad 4.06\,\text{m} \ \cos 15.0^\circ = 30.1 \ .$$

b he increase in thermal energy is $\Delta E_{\text{th}} = fd = \ 7.42 \quad 4.06\,\text{m} \ = 30.1$.

c e can use e ton s second la of motion to obtain the frictional and normal forces then use $\mu_k = f\,F_N$ to obtain the coefficient of friction. he x component of Newton's law gives

$$f = F\cos\theta = \ 7.68 \quad \cos 15.0^\circ = 7.42 \ .$$

imilarly the y component yields

$$F_N = mg - F\sin\theta = \ 3.57\ \text{kg} \ \ 9.8\ \text{m s}^2 \ - \ 7.68 \quad \sin 15.0^\circ = 33.0 \ .$$

hus the coefficient of kinetic friction is

$$\mu_k = \frac{f}{F_N} = \frac{7.42}{33.0} = 0.225.$$

LEARN n this problem the block moves at a constant speed so that $\Delta K = 0$ i.e. no change in kinetic energy. he ork done by the e ternal force is converted into thermal energy of the system $W = \Delta E_{\text{th}}$.

46. e ork this using nglish units ith g = 32 ft s but for consistency e convert the eight to pounds

$$mg = \ 9.0\ \text{o} \ \left(\frac{1\ \text{1b}}{16\,\text{o}}\right) = 0.56\,\text{lb}$$

hich implies $m = 0.018\ \text{lb}\cdot\text{s}^2\ \text{ft}$ hich can be phrased as 0.018 slug as e plained in ppendi . nd e convert the initial speed to feet per second

$$v_i = \ 81.8\,\text{mi/h} \ \left(\frac{5280\ \text{ft/mi}}{3600\ \text{s/h}}\right) = 120\ \text{ft/s}$$

or a more "direct" conversion from Appendix D can be used. Equation 8 30 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. hus

$$\Delta E_{\text{th}} = \frac{1}{2} m \; v_i^2 - v_f^2 \; + mg \; y_i - y_f \; = \frac{1}{2} \; 0.018 \; 120^2 - 110^2 \; + 0 = 20 \text{ ft} \cdot \text{lb}.$$

47. e use units so m = 0.075 kg. uation 8 33 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. Thus,

$$\begin{aligned}\Delta E_{\text{th}} &= \frac{1}{2} m \; v_i^2 - v_f^2 \; + mg \; y_i - y_f \\ &= \frac{1}{2} \; 0.075 \text{ kg} \quad 12 \text{ m s} \;^2 - \; 10.5 \text{ m s} \;^2 \; + \; 0.075 \text{ kg} \quad 9.8 \text{ m s}^2 \quad 1.1 \text{ m} - 2.1 \text{ m} \\ &= 0.53 \quad .\end{aligned}$$

48. e use . 8 31 to obtain $\Delta E_{\text{th}} = f_k d = \; 10 \quad 5.0 \text{ m} \; = 50$ and . 7 8 to get

$$W = Fd = \; 2.0 \quad 5.0 \text{ m} \; = 10 \quad .$$

imilarly . 8 31 gives

$$\begin{aligned} W &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ 10 &= 35 + \Delta U + 50 \end{aligned}$$

hich yields $\Delta U = -75$. y . 8 1 then the ork done by gravity is $W = -\Delta U = 75$.

49. **THINK** s the bear slides do n the tree its gravitational potential energy is converted into both kinetic energy and thermal energy.

EXPRESS e take the initial gravitational potential energy to be $U_i = mgL$ here L is the length of the tree and final gravitational potential energy at the bottom to be $U_f = 0$. o solve this problem e note that the changes in the mechanical and thermal energies must sum to ero.

ANALYZE a ubstituting the values given the change in gravitational potential energy is

$$\Delta U = U_f - U_i = -mgL = - \; 25 \text{ kg} \quad 9.8 \text{ m s}^2 \quad 12 \text{ m} \; = -2.9 \times 10^3 \quad .$$

b he final speed is $v_f = 5.6 \text{ m s}$. herefore the kinetic energy is

$$K_f = \frac{1}{2} m v_f^2 = \frac{1}{2} \; 25 \text{ kg} \quad 5.6 \text{ m s} \;^2 = 3.9 \times 10^2 \quad .$$

c he change in thermal energy is $\Delta E_{\text{th}} = fL$ here f is the magnitude of the average frictional force therefore from $\Delta E_{\text{th}} + \Delta K + \Delta U = 0$ e find f to be

$$f = -\frac{\Delta K + \Delta U}{L} = -\frac{3.9 \times 10^2 \quad -2.9 \times 10^3}{12 \text{ m}} = 2.1 \times 10^2 \quad .$$

LEARN n this problem no e ternal ork is done to the bear. herefore

$$W=\Delta E_{\text{th}}+\Delta E_{\text{mech}}=\Delta E_{\text{th}}+\Delta K+\Delta U=0$$

hich implies $\Delta K=-\Delta U-\Delta E_{\text{th}}=-\Delta U-fL$. hus $\Delta E_{\text{th}}=fL$ can be interpreted as the additional change decrease in kinetic energy due to frictional force.

50. uation 8 33 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. hus

$$\begin{aligned}\Delta E_{\text{th}} &= \frac{1}{2}m\ v_i^2-v_f^2\ +mg\ y_i-y_f\\ &=\frac{1}{2}\ 60\text{ kg}\quad 24\text{ m s}^{\ 2}-\ 22\text{ m s}^{\ 2}\ +\ 60\text{ kg}\ \ 9.8\text{ m s}^2\ \ 14\text{ m}\\ &=1.1\times10^4\quad .\end{aligned}$$

hat the angle of 25 is no here used in this calculation is indicative of the fact that energy is a scalar uantity.

51. a he initial potential energy is

$$U_i=mgy_i=\ 520\text{ kg}\ \left(9.8\,\text{m/s}^2\right)\ 300\text{ m}\ =1.53\times10^6$$

here y is up ard and y = 0 at the bottom so that $U_f = 0$.

b ince $f_k = \mu_k F_N = \mu_k mg\cos\theta$ e have $\Delta E_{\text{th}}=f_kd=\mu_k mgd\cos\theta$ from . 8 31. o the hillside surface of length d = 500 m is treated as an hypotenuse of a 3 4 5 triangle so cos $\theta = x\ d$ here x = 400 m. herefore

$$\Delta E_{\text{th}}=\mu_k mgd\frac{x}{d}=\mu_k mgx=\ 0.25\ \ 520\ \ 9.8\ \ 400\ =5.1\times10^5\quad .$$

c sing . 8 31 ith W = 0 e find

$$K_f=K_i+U_i-U_f-\Delta E_{\text{th}}=0+\ 1.53\times10^6\quad -0-\ 5.1\times10^6\quad =1.02\times10^6\quad .$$

d rom $K_f=mv^2\ \ 2$ e obtain v = 63 m s.

52. a n appropriate picture once friction is included for this problem is igure 8 3 in the te tbook. e apply . 8 31 $\Delta E_{\text{th}} = f_k d$ and relate initial kinetic energy K_i to the resting potential energy U_r

$$K_i + U_i\ \ = f_kd + K_r + U_r\ \ \Rightarrow\ \ 20.0\quad\ 0 = f_kd\ \ \ 0\ \ \ \frac{1}{2}kd^2$$

here f_k = 10.0 and k = 400 m. e solve the e uation for d using the uadratic formula or by using the polynomial solver on an appropriate calculator ith d = 0.292 m being the only positive root.

b e apply . 8 31 again and relate U_r to the second kinetic energy K_s it has at the unstretched position.

$$K_r + U_r = f_k d + K_s + U_s \quad \Rightarrow \quad \frac{1}{2}kd^2 = f_k d \quad K_s \quad 0$$

sing the result from part a this yields K_s = 14.2 .

53. a he vertical forces acting on the block are the normal force up ard and the force of gravity do n ard. ince the vertical component of the block s acceleration is ero e ton s second la re uires $F_N = mg$ here m is the mass of the block. hus $f = \mu_k F_N = \mu_k mg$. he increase in thermal energy is given by $\Delta E_{\text{th}} = fd = \mu_k mgD$ here D is the distance the block moves before coming to rest. sing . 8 29 e have

$$\Delta E_{\text{th}} = (0.25)(3.5\,\text{kg})\left(9.8\,\text{m/s}^2\right)(7.8\,\text{m}) = 67 \;\; .$$

b he block has its ma imum kinetic energy K_{ma} ust as it leaves the spring and enters the region here friction acts. herefore the ma imum kinetic energy e uals the thermal energy generated in bringing the block back to rest 67 .

c he energy that appears as kinetic energy is originally in the form of potential energy in the compressed spring. hus $K_{\text{ma}} = U_i = \frac{1}{2}kx^2$ here k is the spring constant and x is the compression. hus

$$x = \sqrt{\frac{2K_{\text{ma}}}{k}} = \sqrt{\frac{2(67 \;\;)}{640 \;\; /\text{m}}} = 0.46\,\text{m}.$$

54. a sing the force analysis sho n in hapter 6 e find the normal force $F_N = mg\cos\theta$ here mg = 267 hich means

$$f_k = \mu_k F_N = \mu_k mg \cos\,\theta.$$

hus . 8 31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta = (0.10)(267)(6.1)\cos 20° = 1.5\times 10^2 \;\; .$$

b he potential energy change is

$$\Delta U = mg \; -d \sin\,\theta \; = \; 267 \quad -6.1\text{ m} \; \sin 20 \; = -5.6\times 10^2 \;\; .$$

he initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{267}{9.8\,\text{m s}^2}\right) 0.457\,\text{m s}^2 = 2.8 .$$

herefore using . 8 33 ith $W = 0$ the final kinetic energy is

$$K_f = K_i - \Delta U - \Delta E_{\text{th}} = 2.8 - \left(-5.6\times10^2\right) - 1.5\times10^2 = 4.1\times10^2 .$$

onse uently the final speed is $v_f = \sqrt{2K_f/m} = 5.5$ m/s.

55. a ith x = 0.075 m and $k = 320$ /m . 7 26 yields $W_s = -\frac{1}{2}kx^2 = -0.90$. or later reference this is e ual to the negative of ΔU.

b naly ing forces e find $F_N = mg$ hich means $f_k = \mu_k F_N = \mu_k mg$. ith $d = x$. 8 31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgx = 0.25 \quad 2.5 \quad 9.8 \quad 0.075 = 0.46 .$$

c uation 8 33 ith $W = 0$ indicates that the initial kinetic energy is

$$K_i = \Delta U + \Delta E_{\text{th}} = 0.90 + 0.46 = 1.36$$

hich leads to $v_i = \sqrt{2K_i/m} = 1.0$ m/s.

56. nergy conservation as e pressed by . 8 33 ith $W = 0$ leads to

$$\Delta E_{\text{th}} = K_i - K_f + U_i - U_f \;\Rightarrow\; f_k d = 0 - 0 + \frac{1}{2}kx^2 - 0$$

$$\Rightarrow\; \mu_k mgd = \frac{1}{2}\;200 \quad \text{m} \;\; 0.15\,\text{m}^{\;2} \;\Rightarrow\; \mu_k \;\; 2.0\,\text{kg} \;\; 9.8\,\text{m s}^2 \;\; 0.75\,\text{m} = 2.25$$

hich yields μ_k = 0.15 as the coefficient of kinetic friction.

57. ince the valley is frictionless the only reason for the speed being less hen it reaches the higher level is the gain in potential energy $\Delta U = mgh$ here h = 1.1 m. liding along the rough surface of the higher level the block finally stops since its remaining kinetic energy has turned to thermal energy $\Delta E_{\text{th}} = f_k d = \mu mgd$ here $\mu = 0.60$. hus . 8 33 ith W = 0 provides us ith an e uation to solve for the distance d

$$K_i = \Delta U + \Delta E_{\text{th}} = mg\left(h + \mu d\right)$$

here $K_i = mv_i^2$ 2 and v_i = 6.0 m s. ividing by mass and rearranging e obtain

$$d = \frac{v_i^2}{2\mu g} - \frac{h}{\mu} = 1.2\,\text{m}.$$

58. his can be orked entirely by the methods of hapters 2–6 but e ill use energy methods in as many steps as possible.

a y a force analysis of the style done in hapter 6 e find the normal force has magnitude $F_N = mg \cos\theta$ here $\theta = 40$ hich means $f_k = \mu_k F_N = \mu_k mg \cos\theta$ here $\mu_k = 0.15$. hus . 8 31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd \cos\theta.$$

lso elementary trigonometry leads us to conclude that $\Delta U = mgd \sin\theta$. . 8 33 ith $W = 0$ and $K_f = 0$ provides an e uation for determining d

$$K_i = \Delta U + \Delta E_{\text{th}}$$
$$\frac{1}{2} m v_i^2 = mgd\left(\sin\theta + \mu_k \cos\theta\right)$$

here $v_i = 1.4\,\text{m/s}$. ividing by mass and rearranging e obtain

$$d = \frac{v_i^2}{2g\ \sin\theta + \mu_k \cos\theta} = 0.13\,\text{m}.$$

b o that e kno here on the incline it stops $d' = 0.13$ 0.55 = 0.68 m from the bottom e can use . 8 33 again ith $W = 0$ and no ith $K_i = 0$ to describe the final kinetic energy at the bottom

$$K_f = -\Delta U - \Delta E_{\text{th}}$$
$$\frac{1}{2} m v^2 = mgd'\left(\sin\theta - \mu_k \cos\theta\right)$$

hich — after dividing by the mass and rearranging — yields

$$v = \sqrt{2gd'\left(\sin\theta - \mu_k \cos\theta\right)} = 2.7\,\text{m/s}.$$

c n part a it is clear that d increases if μ_k decreases — both mathematically since it is a positive term in the denominator and intuitively less friction — less energy "lost"). In part b there are t o terms in the e pression for v that imply that it should increase if μ_k ere smaller the increased value of $d' = d_0$ d and that last factor $\sin\theta - \mu_k \cos\theta$, hich indicates that less is being subtracted from $\sin\theta$ hen μ_k is less so the factor itself increases in value .

59. a he ma imum height reached is h. he thermal energy generated by air resistance as the stone rises to this height is $\Delta E_{\text{th}} = fh$ by . 8 31. e use energy conservation in the form of . 8 33 ith $W = 0$

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i$$

and e take the potential energy to be ero at the thro ing point ground level . he initial kinetic energy is $K_i = \frac{1}{2} m v_0^2$ the initial potential energy is $U_i = 0$ the final kinetic energy is $K_f = 0$ and the final potential energy is $U_f = wh$ here $w = mg$ is the eight of the stone. hus $wh + fh = \frac{1}{2} m v_0^2$ and e solve for the height

$$h = \frac{m v_0^2}{2\ w + f} = \frac{v_0^2}{2g\ 1 + f \quad w} .$$

umerically e have ith $m =$ 5.29 9.80 m s^2 = 0.54 kg

$$h = \frac{20.0 \text{ m s }^2}{2\ 9.80 \text{ m s}^2\ \ 1 + 0.265\ 5.29} = 19.4 \text{ m} .$$

b e notice that the force of the air is do n ard on the trip up and up ard on the trip do n since it is opposite to the direction of motion. ver the entire trip the increase in thermal energy is $\Delta E_{\text{th}} = 2fh$. he final kinetic energy is $K_f = \frac{1}{2} m v^2$ here v is the speed of the stone ust before it hits the ground. he final potential energy is $U_f = 0$. hus using . 8 31 ith $W = 0$ e find

$$\frac{1}{2} m v^2 + 2 fh = \frac{1}{2} m v_0^2 .$$

e substitute the e pression found for h to obtain

$$\frac{2 f v_0^2}{2g\ 1 + f \quad w} = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$$

hich leads to

$$v^2 = v_0^2 - \frac{2 f v_0^2}{mg\ 1 + f \quad w} = v_0^2 - \frac{2 f v_0^2}{w\ 1 + f \quad w} = v_0^2 \left(1 - \frac{2f}{w + f} \right) = v_0^2\ \frac{w - f}{w + f}$$

here w as substituted for mg and some algebraic manipulations ere carried out. herefore

$$v = v_0\sqrt{\frac{w-f}{w+f}} = \;20.0\text{ m s}\;\sqrt{\frac{5.29\quad -0.265}{5.29\quad +0.265}} = 19.0\text{ m s}\,.$$

60. e look for the distance along the incline d hich is related to the height ascended by $\Delta h = d \sin\theta$. y a force analysis of the style done in hapter 6 e find the normal force has magnitude $F_N = mg\cos\theta$, hich means $f_k = \mu_k mg\cos\theta$. hus . 8 33 ith W = 0 leads to

$$\begin{aligned} 0 &= K_f - K_i + \Delta U + \Delta E_{\text{th}} \\ &= 0 - K_i + mgd\sin\theta + \mu_k mgd\cos\theta \end{aligned}$$

hich leads to

$$d = \frac{K_i}{mg(\sin\theta + \mu_k\cos\theta)} = \frac{128}{(4.0)(9.8)(\sin 30° + 0.30\cos 30°)} = 4.3\text{m}.$$

61. efore the launch the mechanical energy is $\Delta E_{\text{mech }0} = 0$. t the ma imum height h here the speed of the beetle vanishes the mechanical energy is $\Delta E_{\text{mech }1} = mgh$. he change of the mechanical energy is related to the e ternal force by

$$\Delta E_{\text{mech}} = \Delta E_{\text{mech }1} - \Delta E_{\text{mech }0} = mgh = F_{avg}d\cos\phi$$

here F_{avg} is the average magnitude of the e ternal force on the beetle.

a rom the above e uation e have

$$F_{avg} = \frac{mgh}{d\cos\phi} = \frac{4.0\times10^{-6}\text{ kg}\;\;9.80\text{ m s}^2\;\;0.30\text{ m}}{7.7\times10^{-4}\text{ m}\;\;\cos 0°} = 1.5\times10^{-2}\quad.$$

b ividing the above result by the mass of the beetle e obtain

$$a = \frac{F_{avg}}{m} = \frac{h}{d\cos\phi}\;g = \frac{0.30\text{ m}}{7.7\times10^{-4}\text{ m}\;\;\cos 0°}\;g = 3.8\times10^{2}g.$$

62. e ill refer to the point here it first encounters the “rough region” as point C this is the point at a height h above the reference level . rom . 8 17 e find the speed it has at point C to be

$$v_C = \sqrt{v_A^2 - 2gh} = \sqrt{\;8.0\;^2 - 2\;\;9.8\;\;2.0\;} = 4.980 \approx 5.0\text{ m s}.$$

hus we see that its kinetic energy right at the beginning of its “rough slide” (heading uphill to ards B is

$$K_C = \frac{1}{2}\;m\;\;4.980\text{ m s}\;^2 = 12.4m$$

(with SI units understood). Note that we "carry along" the mass (as if it were a known uantity as e ill see it ill cancel out shortly. sing . 8 37 and . 6 2 ith F_N = $mg\cos\theta$ and $y = d\sin\theta$ e note that if d L the block does not reach point B this kinetic energy ill turn entirely into thermal and potential energy

$$K \;= mgy \quad f_k d \;\Rightarrow\; 12.4m = mgd\sin\theta \quad \mu_k mgd\cos\theta.$$

ith μ_k = 0.40 and θ = 30 e find d = 1.49 m hich is greater than L given in the problem as 0.75 m so our assumption that d L is incorrect. hat is its kinetic energy as it reaches point B he calculation is similar to the above but ith d replaced by L and the final v^2 term being the unkno n instead of assumed ero

$$\frac{1}{2} m v^2 = K_C - \; mgL\sin\theta \quad \mu_k mgL\cos\theta \;.$$

his determines the speed ith hich it arrives at point B

$$\begin{aligned} v_B &= \sqrt{v_C^2 - 2gL \;\sin\theta + \mu_k \cos\theta} \\ &= \sqrt{\;4.98\text{ m s}\;^2 - 2\;9.80\text{ m s}^2\;\;0.75\text{ m}\;\;\sin 30° + 0.4\cos 30°\;} = 3.5\text{ m s}. \end{aligned}$$

63. e observe that the last line of the problem indicates that static friction is not to be considered a factor in this problem. he friction force of magnitude f = 4400 mentioned in the problem is kinetic friction and as mentioned is constant and directed up ard and the thermal energy change associated ith it is $\Delta E_{\text{th}} = fd$. 8 31 here d = 3.7 m in part a but ill be replaced by x the spring compression in part b .

a ith W = 0 and the reference level for computing U = mgy set at the top of the rela ed spring . 8 33 leads to

$$U_i = K + \Delta E_{\text{th}} \Rightarrow v = \sqrt{2d\left(g - \frac{f}{m}\right)}$$

hich yields $v = 7.4\text{ m/s}$ for m = 1800 kg.

b e again utili e . 8 33 ith W = 0 no relating its kinetic energy at the moment it makes contact ith the spring to the system energy at the bottom most point. sing the same reference level for computing U = mgy as e did in part a e end up ith gravitational potential energy e ual to $mg\,-x$ at that bottom most point here the spring ith spring constant $k = 1.5 \times 10^5$ /m is fully compressed.

$$K = mg(-x) + \frac{1}{2}kx^2 + fx$$

here $K=\frac{1}{2}mv^2=4.9\times10^4$ using the speed found in part a . sing the abbreviation ξ = $mg-f$ = 1.3 × 10^4 the uadratic formula yields

$$x=\frac{\xi\pm\sqrt{\xi^2+2kK}}{k}=0.90\,\text{m}$$

here e have taken the positive root.

c e relate the energy at the bottom most point to that of the highest point of rebound a distance d' above the rela ed position of the spring . e assume d' x. e no use the bottom most point as the reference level for computing gravitational potential energy.

$$\frac{1}{2}kx^2=mgd'+fd'\Rightarrow d'=\frac{kx^2}{2(mg+d)}=2.8\,\text{m}.$$

d he non conservative force 8 1 is friction and the energy term associated ith it is the one that keeps track of the total distance traveled hereas the potential energy terms coming as they do from conservative forces depend on positions — but not on the paths that led to them . e assume the elevator comes to final rest at the e uilibrium position of the spring ith the spring compressed an amount d_e given by

$$mg=kd_e\ \Rightarrow d_e\ =\frac{mg}{k}=0.12\,\text{m}.$$

n this part e use that final rest point as the reference level for computing gravitational potential energy so the original U = mgy becomes mg d_e d . n that final position then the gravitational energy is ero and the spring energy is kd_e^2 2 . hus . 8 33 becomes

$$mg\left(d_e\ +d\right)=\frac{1}{2}kd_e^2\ +fd_{\text{total}}$$

$$(1800)(9.8)(0.12+3.7)=\frac{1}{2}\left(1.5\times10^5\right)(0.12)^2+(4400)d_{\text{total}}$$

hich yields d_{total} = 15 m.

64. n the absence of friction e have a simple conversion as it moves along the inclined ramps of energy bet een the kinetic form . 7 1 and the potential form . 8 9 . long the hori ontal plateaus ho ever there is friction that causes some of the kinetic energy to dissipate in accordance ith . 8 31 along ith . 6 2 here μ_k = 0.50 and $F_N = mg$ in this situation . hus after it slides do n a vertical distance d it has gained $K=\frac{1}{2}mv^2=mgd$ some of hich ΔE_{th} = $\mu_k\,mgd$ is dissipated so that the value of kinetic energy at the end of the first plateau ust before it starts descending to ards the lo est plateau is

$$K = mgd - \mu_k mgd = \frac{1}{2} mgd \, .$$

n its descent to the lo est plateau it gains $mgd/2$ more kinetic energy but as it slides across it "loses" $\mu_k mgd/2$ of it. herefore as it starts its climb up the right ramp it has kinetic energy e ual to

$$K = \frac{1}{2} mgd + \frac{1}{2} mgd - \frac{1}{2} \mu_k mgd = \frac{3}{4} mgd \, .$$

etting this e ual to . 8 9 to find the height to hich it climbs e get $H =$ d. hus the block momentarily stops on the inclined ramp at the right at a height of

$$H = 0.75d = 0.75 \quad 40 \text{ cm} \quad = 30 \text{ cm}$$

measured from the lo est plateau.

65. he initial and final kinetic energies are ero and e set up energy conservation in the form of . 8 33 ith $W = 0$ according to our assumptions. ertainly it can only come to a permanent stop some here in the flat part but the uestion is hether this occurs during its first pass through going right ard or its second pass through going left ard or its third pass through going right ard again and so on. f it occurs during its first pass through then the thermal energy generated is $\Delta E_{\text{th}} = f_k d$ here $d \le L$ and $f_k = \mu_k mg$. f it occurs during its second pass through then the total thermal energy is $\Delta E_{\text{th}} = \mu_k mg \; L \quad d$ here e again use the symbol d for ho far through the level area it goes during that last pass so $0 \le d \le L$. enerali ing to the n^{th} pass through e see that

$$\Delta E_{\text{th}} = \mu_k mg \quad n - 1 \; L \quad d \, .$$

n this ay e have

$$mgh = \mu_k mg\big((n-1)L + d\big)$$

hich simplifies hen $h = L$ 2 is inserted to

$$\frac{d}{L} = 1 + \frac{1}{2\mu_k} - n.$$

he first t o terms give $1 + 1/2\mu_k = 3.5$ so that the re uirement $0 \le d/L \le 1$ demands that $n = 3$. e arrive at the conclusion that $d/L = \frac{1}{2}$ or

$$d = \frac{1}{2} L = \frac{1}{2} \; 40 \text{ cm} \; = 20 \text{ cm}$$

and that this occurs on its third pass through the flat region.

66. a uation 8 9 gives $U = mgh =$ 3.2 kg 9.8 m s^2 3.0 m = 94 .

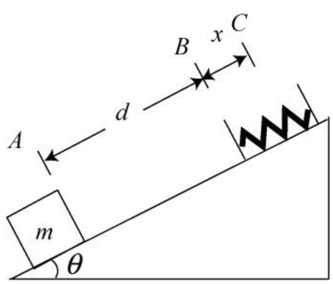
b he mechanical energy is conserved so K = 94 .

c he speed from solving . 7 1 is

$$v = \sqrt{2K\ \ m} = \sqrt{2\ 94\qquad 32\ \text{kg}} = 7.7\ \text{m s}.$$

67. **THINK** s the block is pro ected up the inclined plane its kinetic energy is converted into gravitational potential energy and elastic potential energy of the spring. he block compresses the spring stopping momentarily before sliding back do n again.

EXPRESS et A be the starting point and the reference point for computing gravitational potential energy $U_A = 0$. he block first comes into contact ith the spring at B. he spring is compressed by an additional amount x at C as sho n in the figure belo .



y energy conservation $K_A + U_A = K_B + U_B = K_C + U_C$. ote that

$$U = U_g + U_s = mgy + \frac{1}{2}kx^2$$

i.e. the total potential energy is the sum of gravitational potential energy and elastic potential energy of the spring.

ANALYZE a t the instant hen $x_C = 0.20$ m the vertical height is

$$y_C = \ d + x_C\ \sin\theta = \ 0.60\ \text{m} + 0.20\ \text{m}\ \sin 40° = 0.514\ \text{m}.$$

pplying energy conservation principle gives

$$K_A + U_A = K_C + U_C \Rightarrow \ 16\quad + 0 = K_C + mgy_C + \frac{1}{2}kx_C^2$$

from hich e obtain

$$K_C = K_A - mgy_C - \frac{1}{2}kx_C^2$$
$$= 16 \quad - 1.0\,\text{kg} \quad 9.8\,\text{m s}^2 \quad 0.514\ \text{m} \; - \frac{1}{2}\ 200 \qquad \text{m} \quad 0.20\ \text{m}\ ^2 = 6.96 \qquad \approx 7.0 \quad .$$

b t the instant hen $x'_C = 0.40\ \text{m}$ the vertical height is

$$y'_C = \ d + x'_C\ \sin\theta = \ 0.60\ \text{m} + 0.40\ \text{m}\ \sin 40° = 0.64\ \text{m}.$$

pplying energy conservation principle e have $K'_A + U'_A = K'_C + U'_C$. ince $U'_A = 0$ the initial kinetic energy that gives $K'_C = 0$ is

$$K'_A = U'_C = mgy'_C + \frac{1}{2}kx'^2_C$$
$$= \ 1.0\,\text{kg} \quad 9.8\,\text{m s}^2 \quad 0.64\ \text{m} \ + \frac{1}{2}\ 200 \qquad \text{m} \quad 0.40\ \text{m}\ ^2$$
$$= 22 \quad .$$

LEARN omparing the results found in a and b e see that more kinetic energy is re uired to move the block higher in the inclined plane to achieve a greater spring compression.

68. a t the point of ma imum height here y = 140 m the vertical component of velocity vanishes but the hori ontal component remains hat it as hen it as launched if e neglect air friction . ts kinetic energy at that moment is

$$K = \frac{1}{2}(0.55\,\text{kg})v_x^2 .$$

lso its potential energy ith the reference level chosen at the level of the cliff edge at that moment is $U = mgy$ = 755 . hus by mechanical energy conservation

$$K = K_i - U = 1550 - 755 \Rightarrow v_x = \sqrt{\frac{2(1550 - 755)}{0.55}} = 54\ \text{m s}.$$

b s mentioned $v_x = v_{ix}$ so that the initial kinetic energy

$$K_i = \frac{1}{2}m\left(v_{ix}^2 + v_{iy}^2\right)$$

can be used to find v_{iy}. e obtain $v_{iy} = 52\ \text{m/s}$.

c pplying . 2 16 to the vertical direction ith y up ard e have

$$v_y^2 = v_{iy}^2 - 2g\Delta y \quad \Rightarrow \quad 65\,\text{m s}^{\;2} = 52\,\text{m s}^{\;2} - 2\;9.8\,\text{m s}^2\;\Delta y$$

hich yields $\Delta y = -76$ m. he minus sign tells us it is belo its launch point.

69. **THINK** he t o blocks are connected by a cord. s block B falls block A moves up the incline.

EXPRESS f the larger mass block B m_B = 2.0 kg falls a vertical distance $d = 0.25$ m then the smaller mass blocks A m_A = 1.0 kg must increase its height by $h = d\sin 30°$. he change in gravitational potential energy is

$$\Delta U = -m_B g d + m_A g h .$$

y mechanical energy conservation $\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$ the change in kinetic energy of the system is $\Delta K = -\Delta U$.

ANALYZE ince the initial kinetic energy is ero the final kinetic energy is

$$\begin{aligned} K_f &= \Delta K = m_B g d - m_A g h = m_B g d - m_A g d \sin\theta \\ &= \;m_B - m_A \sin\theta\;\, g d = \;2.0\,\text{kg} - 1.0\,\text{kg}\;\sin 30°\;\;9.8\,\text{m s}^2\;\;0.25\text{ m} \\ &= 3.7\;\;. \end{aligned}$$

LEARN rom the above e pression e see that in the special case here $m_B = m_A \sin\theta$ the t o block system ould remain stationary. n the other hand if $m_A \sin\theta > m_B$ block A ill slide do n the incline ith block B moving vertically up ard.

70. e use conservation of mechanical energy the mechanical energy must be the same at the top of the s ing as it is initially. e ton s second la is used to find the speed and hence the kinetic energy at the top. here the tension force T of the string and the force of gravity are both do n ard to ard the center of the circle. e notice that the radius of the circle is $r = L - d$ so the la can be ritten

$$T + mg = mv^2/(L-d)$$

here v is the speed and m is the mass of the ball. hen the ball passes the highest point ith the least possible speed the tension is ero. hen

$$mg = m\frac{v^2}{L-d} \;\Rightarrow\; v = \sqrt{g(L-d)}\ .$$

e take the gravitational potential energy of the ball arth system to be ero hen the ball is at the bottom of its s ing. hen the initial potential energy is mgL. he initial kinetic energy is ero since the ball starts from rest. he final potential energy at the top of the s ing is $2mg\ L-d$ and the final kinetic energy is $\frac{1}{2}mv^2=\frac{1}{2}mg(L-d)$ using the above result for v. onservation of energy yields

$$mgL=2mg(L-d)+\frac{1}{2}mg(L-d)\ \Rightarrow\ d=3L/5\ .$$

ith L = 1.20 m e have d = 0.60 1.20 m = 0.72 m.

otice that if d is greater than this value so the highest point is lo er then the speed of the ball is greater as it reaches that point and the ball passes the point. f d is less the ball cannot go around. hus the value e found for d is a lo er limit.

71. **THINK** s the block slides do n the frictionless incline its gravitational potential energy is converted to kinetic energy so the speed of the block increases.

EXPRESS y energy conservation $K_A+U_A=K_B+U_B$. hus the change in kinetic energy as the block moves from points A to B is

$$\Delta K=K_B-K_A=-\Delta U=-\ U_B-U_A\ \ .$$

n both circumstances e have the same potential energy change. hus $\Delta K_1=\Delta K_2$.

ANALYZE ith $\Delta K_1=\Delta K_2$ the speed of the block at the second time is given by

$$\frac{1}{2}mv_{B\,1}^2-\frac{1}{2}mv_{A\,1}^2=\frac{1}{2}mv_{B\,2}^2\ \frac{1}{2}mv_{A\,2}^2$$

or

$$v_{B\,2}=\sqrt{v_{B\,1}^2-v_{A\,1}^2+v_{A\,2}^2}=\sqrt{\ 2.60\,\text{m s}\ ^2-\ 2.00\,\text{m s}\ ^2+\ 4.00\,\text{m s}\ ^2}=4.33\,\text{m s}\ .$$

LEARN he speed of the block at A is greater the second time $v_{A\,2}>v_{A\,1}$. his can happen if the block slides do n from a higher position ith greater initial gravitational potential energy.

72. a e take the gravitational potential energy of the skier arth system to be ero hen the skier is at the bottom of the peaks. he initial potential energy is U_i = mgH here m is the mass of the skier and H is the height of the higher peak. he final potential energy is U_f = mgh here h is the height of the lo er peak. he skier initially has a kinetic energy of K_i = 0 and the final kinetic energy is $K_f=\frac{1}{2}mv^2$ here v is the speed of the skier at the top of the lo er peak. he normal force of the slope on the skier does no ork and friction is negligible so mechanical energy is conserved

$$U_i + K_i = U_f + K_f \;\Rightarrow\; mgH = mgh + \frac{1}{2}mv^2.$$

hus

$$v = \sqrt{2g\ H-h} = \sqrt{2\ 9.8\,\text{m s}^2\ \ 850\ \text{m} - 750\ \text{m}} = 44\ \text{m s}.$$

b e recall from analy ing ob ects sliding do n inclined planes that the normal force of the slope on the skier is given by $F_N = mg\cos\theta$ here θ is the angle of the slope from the hori ontal 30 for each of the slopes sho n. he magnitude of the force of friction is given by $f = \mu_k F_N = \mu_k mg\cos\theta$. he thermal energy generated by the force of friction is $fd = \mu_k mgd\cos\theta$ here d is the total distance along the path. ince the skier gets to the top of the lo er peak ith no kinetic energy the increase in thermal energy is e ual to the decrease in potential energy. hat is $\mu_k mgd\cos\theta = mg\ H-h$. onse uently

$$\mu_k = \frac{H-h}{d\cos\theta} = \frac{850\ \text{m} - 750\ \text{m}}{3.2\times10^3\ \text{m}\ \cos 30°} = 0.036.$$

73. **THINK** s the cube is pushed across the floor both the thermal energies of floor and the cube increase because of friction.

EXPRESS y la of conservation of energy e have $W = \Delta E_{\text{mech}} + \Delta E_{\text{th}}$ for the floor cube system. ince the speed is constant $\Delta K = 0$. 8 33 an application of the energy conservation concept implies

$$W = \Delta E_{\text{mech}} + \Delta E_{\text{th}} = \Delta E_{\text{th}} = \Delta E_{\text{th cube}} + \Delta E_{\text{th floor}}.$$

ANALYZE ith $W =$ 15 3.0 m = 45 and e are told that $\Delta E_{\text{th cube}}$ = 20 then e conclude that $\Delta E_{\text{th floor}}$ = 25 .

LEARN he applied ork here has all been converted into thermal energies of the floor and the cube. he amount of thermal energy transferred to a material depends on its thermal properties as e shall discuss in hapter 18.

74. e take her original elevation to be the $y = 0$ reference level and observe that the top of the hill must conse uently have $y_A = R\ 1 - \cos 20$ = 1.2 m here R is the radius of the hill. he mass of the skier is $m = 600\quad 9.8\ \text{m s}^2 = 61\ \text{kg}$.

a pplying energy conservation . 8 17 e have

$$K_B + U_B = K_A + U_A \Rightarrow K_B + 0 = K_A + mgy_A.$$

sing $K_B = \frac{1}{2}(61\,\text{kg})(8.0\,\text{m/s})^2$ e obtain $K_A = 1.2\times10^3$. hus e find the speed at the hilltop is

$$v_A = \sqrt{\frac{2K_A}{m}} = \sqrt{\frac{2\ 1.2\times 10^3}{61\,\mathrm{kg}}} = 6.4\ \mathrm{m\ \ s}.$$

ote ne might ish to check that the skier stays in contact ith the hill — hich is indeed the case here. or instance at A e find $v^2\ r \approx 2$ m s^2 hich is considerably less than g.

b ith K_A = 0 e have

$$K_B + U_B = K_A + U_A \Rightarrow K_B + 0 = 0 + mgy_A$$

hich yields K_B = 724 and the corresponding speed is

$$v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{2\ 724}{61\,\mathrm{kg}}} = 4.9\ \mathrm{m\ \ s}.$$

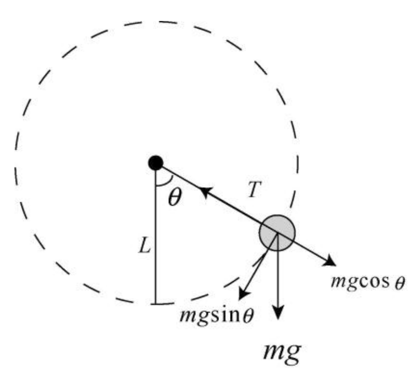
c pressed in terms of mass e have

$$K_B + U_B = K_A + U_A \Rightarrow$$
$$\frac{1}{2}mv_B^2 + mgy_B = \frac{1}{2}mv_A^2 + mgy_A\ .$$

hus the mass m cancels and e observe that solving for speed does not depend on the value of mass or eight .

75. **THINK** his problem deals ith pendulum motion. he kinetic and potential energies of the ball attached to the rod change ith position but the mechanical energy remains conserved throughout the process.

EXPRESS et L be the length of the pendulum. he connection bet een angle θ measured from vertical and height h measured from the lo est point hich is our choice of reference position in computing the gravitational potential energy mgh is given by $h = L\ 1 - \cos\ \theta$.



he free body diagram is sho n above. he initial height is at $h_1 = 2L$ and at the lo est point e have $h_2 = 0$. he total mechanical energy is conserved throughout.

ANALYZE a nitially the ball is at $h_1 = 2L$ ith $K_1 = 0$ and $U_1 = mgh_1 = mg\ 2L$. t the lo est point $h_2 = 0$ e have $K_2 = \frac{1}{2}mv_2^2$ and $U_2 = 0$. sing energy conservation in the form of . 8 17 leads to

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad 0 + 2mgL = \frac{1}{2}mv_2^2 + 0$$

his leads to $v_2 = 2\sqrt{gL}$. ith $L = 0.62$ m e have

$$v_2 = 2\sqrt{\ 9.8\ \text{m s}^2\ \ 0.62\ \text{m}\ } = 4.9\ \text{m s}.$$

b t the lo est point the ball is in circular motion ith the center of the circle above it so $\vec{a} = v^2\ \ r$ up ard here $r = L$. e ton s second la leads to

$$T - mg = m\frac{v^2}{r} \Rightarrow T = m\left(g + \frac{4gL}{L}\right) = 5\ mg.$$

ith $m = 0.092$ kg the tension is $T = 4.5$.

c he pendulum is no started ith ero speed at $\theta_i = 90°$ that is $h_i = L$ and e look for an angle θ such that $T = mg$. hen the ball is moving through a point at angle θ as can be seen from the free body diagram sho n above e ton s second la applied to the a is along the rod yields

$$\frac{mv^2}{r} = T - mg\cos\theta = mg\ 1 - \cos\theta$$

hich since $r = L$ implies $v^2 = gL\ 1 - \cos\ \theta$ at the position e are looking for. nergy conservation leads to

$$K_i + U_i = K + U$$
$$0 + mgL = \frac{1}{2}mv^2 + mgL\ \ 1 - \cos\theta$$
$$gL = \frac{1}{2}\ gL\ 1 - \cos\theta\ \ + gL\ \ 1 - \cos\theta$$

here e have divided by mass in the last step. implifying e obtain

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 71°.$$

d ince the angle found in c is independent of the mass the result remains the same if the mass of the ball is changed.

LEARN t a given angle θ ith respect to the vertical the tension in the rod is

$$T = m\left(\frac{v^2}{r} + g\cos\theta\right)$$

he tangential acceleration $a_t = g\sin\theta$ is hat causes the speed and therefore the kinetic energy to change ith time. onetheless mechanical energy is conserved.

76. a he table sho s that the force is 3.0 i hile the displacement is in the x direction $\vec{d}$ = 3.0 m i and it is – 3.0 i hile the displacement is in the $-x$ direction. sing . 7 8 for each part of the trip and adding the results e find the ork done is 18 . his is not a conservative force field if it had been then the net ork done ould have been ero since it returned to here it started .

b his ho ever is a conservative force field as can be easily verified by calculating that the net ork done here is ero.

c he t o integrations that need to be performed are each of the form $\int 2x\,dx$ so that e are adding t o e uivalent terms here each e uals x^2 evaluated at x = 4 minus its value at x = 1 . hus the ork done is 2 $4^2 - 1^2$ = 30 .

d his is another conservative force field as can be easily verified by calculating that the net ork done here is ero.

e he forces in b and d are conservative.

77. **THINK** his problem involves graphical analyses. rom the graph of potential energy as a function of position the conservative force can de deduced.

EXPRESS he connection bet een the potential energy function $U\ x$ and the conservative force $F\ x$ is given by . 8 22 $F\ x = -dU\ dx$. positive slope of $U\ x$ at a point means that $F\ x$ is negative and vice versa.

ANALYZE a he force at x = 2.0 m is

$$F = -\frac{dU}{dx} \approx -\frac{\Delta U}{\Delta x} = -\frac{U\ x = 4\text{ m}\ - U\ x = 1\text{ m}}{4.0\text{ m} - 1.0\text{ m}} = -\frac{-17.5\ \ -\ -2.8}{4.0\text{ m} - 1.0\text{ m}} = 4.9\ \ .$$

b ince the slope of $U\ x$ at x = 2.0 m is negative the force points in the x direction but there is some uncertainty in reading the graph hich makes the last digit not very significant .

c t x = 2.0 m e estimate the potential energy to be

$$U\ x=2.0\ \text{m}\ \approx U\ x=1.0\ \text{m}\ +\ -4.9\quad \text{m}\ \ 1.0\ \text{m}\ =-7.7$$

hus the total mechanical energy is

$$E=K+U=\frac{1}{2}mv^2+U=\frac{1}{2}\ 2.0\,\text{kg}\ \ -1.5\,\text{m s}\ ^2+\ -7.7\quad =-5.5\ \ .$$

gain there is some uncertainty in reading the graph hich makes the last digit not very significant. t that level –5.5 on the graph e find t o points here the potential energy curve has that value — at x ≈ 1.5 m and x ≈ 13.5 m. herefore the particle remains in the region 1.5 x 13.5 m. he left boundary is at x = 1.5 m.

d rom the above results the right boundary is at x = 13.5 m.

e t x = 7.0 m e read $U \approx -17.5$. hus if its total energy calculated in the previous part is $E \approx -5.5$ then e find

$$\frac{1}{2}mv^2=E-U\approx 12\quad \Rightarrow v=\sqrt{\frac{2}{m}\ \ E-U\ }\approx 3.5\ \text{m/s}$$

here there is certainly room for disagreement on that last digit for the reasons cited above.

LEARN ince the total mechanical energy is negative the particle is bounded by the potential ith its motion confined to the region 1.5 m x 13.5 m. t the turning points 1.5 m and 13.5 m kinetic energy is ero and the particle is momentarily at rest.

78. a ince the speed of the crate of mass m increases from 0 to 1.20 m s relative to the factory ground the kinetic energy supplied to it is

$$K=\frac{1}{2}mv^2=\frac{1}{2}\ 300\,\text{kg}\ \ 120\,\text{m s}\ ^2=216\ \ .$$

b he magnitude of the kinetic frictional force is

$$f=\mu F_N=\mu mg=\ 0.400\ \ 300\,\text{kg}\ \ 9.8\,\text{m s}^2\ \ =1.18\times 10^3\ \ .$$

c et the distance the crate moved relative to the conveyor belt before it stops slipping be d. hen from . 2 16 $v^2 = 2ad = 2\,f\ \ m\ d$ e find

$$\Delta E_{\text{th}}=fd=\frac{1}{2}mv^2=K.$$

hus the total energy that must be supplied by the motor is

$$W = K + \Delta E_{\text{th}} = 2K = 2 \quad 216 \quad = 432 \quad .$$

d he energy supplied by the motor is the ork W it does on the system and must be greater than the kinetic energy gained by the crate computed in part b . his is due to the fact that part of the energy supplied by the motor is being used to compensate for the energy dissipated ΔE_{th} hile it as slipping.

79. **THINK** s the car slides do n the incline due to the presence of frictional force some of its mechanical energy is converted into thermal energy.

EXPRESS he incline angle is $\theta = 5.0°$. hus the change in height bet een the car s highest and lo est points is Δy = – 50 m sin θ = – 4.4 m. e take the lo est point the car s final reported location to correspond to the y = 0 reference level. he change in potential energy is given by $\Delta U = mg\Delta y$.

s for the kinetic energy e first convert the speeds to units $v_0 = 8.3$ m/s and $v = 11.1$ m/s. he change in kinetic energy is $\Delta K = \frac{1}{2} m \; v_f^2 - v_i^2$. he total change in mechanical energy is $\Delta E_{\text{mech}} = \Delta K + \Delta U$.

ANALYZE a ubstituting the values given e find ΔE_{mech} to be

$$\begin{aligned}\Delta E_{\text{mech}} &= \Delta K + \Delta U = \frac{1}{2} m \; v_f^2 - v_i^2 \; + mg\Delta y \\ &= \frac{1}{2} \; 1500\,\text{kg} \left[\; 11.1\,\text{m} \;\; \text{s} \; ^2 - \; 8.3\,\text{m} \;\; \text{s} \; ^2 \right] + \; 1500\,\text{kg} \;\; 9.8\,\text{m} \;\; \text{s}^2 \;\; -4.4\,\text{m} \\ &= -23940 \quad \approx -2.4 \times 10^4\end{aligned}$$

hat is the mechanical energy decreases due to friction by 2.4×10^4 .

b sing . 8 31 and . 8 33 e find $\Delta E_{\text{th}} = f_k d = -\Delta E_{\text{mech}}$. ith d = 50 m e solve for f_k and obtain

$$f_k = \frac{-\Delta E_{\text{mech}}}{d} = \frac{- \; -2.4 \times 10^4}{50\ \text{m}} = 4.8 \times 10^2 \quad .$$

LEARN he amount of mechanical energy lost is proportional to the frictional force in the absence of friction mechanical energy ould have been conserved.

80. e note that in one second the block slides d = 1.34 m up the incline hich means its height increase is $h = d \sin \theta$ here

$$\theta = \tan^{-1}\left(\frac{30}{40}\right) = 37°.$$

e also note that the force of kinetic friction in this inclined plane problem is $f_k = \mu_k mg\cos\theta$ here μ_k = 0.40 and m = 1400 kg. hus using . 8 31 and . 8 33 e find

$$W = mgh + f_k d = mgd\left(\sin\theta + \mu_k \cos\theta\right)$$

or $W = 1.69 \times 10^4$ for this one second interval. hus the po er associated ith this is

$$P = \frac{1.69\times10^4}{1\text{ s}} = 1.69\times10^4 \quad \approx 1.7\times10^4 \quad .$$

81. a he remark in the problem statement that the forces can be associated ith potential energies is illustrated as follo s the ork from x = 3.00 m to x = 2.00 m is

$$W = F_2\,\Delta x = 5.00 \quad -1.00\text{ m} = -5.00$$

so the potential energy at x = 2.00 m is U_2 = 5.00 .

b o it is evident from the problem statement that E_{ma} = 14.0 so the kinetic energy at x = 2.00 m is

$$K_2 = E_{ma} - U_2 = 14.0 - 5.00 = 9.00 \quad .$$

c he ork from x = 2.00 m to x = 0 is $W = F_1\,\Delta x$ = 3.00 –2.00 m = – 6.00 so the potential energy at x = 0 is

$$U_0 = 6.00 \quad U_2 = 6.00 \quad 5.00 \quad = 11.0 \quad .$$

d imilar reasoning to that presented in part a then gives

$$K_0 = E_{ma} - U_0 = 14.0 - 11.0 \quad = 3.00 \quad .$$

e he ork from x = 8.00 m to x = 11.0 m is $W = F_3\,\Delta x$ = – 4.00 3.00 m = –12.0 so the potential energy at x = 11.0 m is U_{11} = 12.0 .

f he kinetic energy at x = 11.0 m is therefore

$$K_{11} = E_{ma} - U_{11} = 14.0 - 12.0 \quad = 2.00 \quad .$$

g o e have $W = F_4\,\Delta x$ = –1.00 1.00 m = –1.00 so the potential energy at x = 12.0 m is

$$U_{12} = 1.00 \quad U_{11} = 1.00 \quad 12.0 \quad = 13.0 \quad .$$

h hus the kinetic energy at x = 12.0 m is

$$K_{12} = E_{ma} - U_{12} = 14.0 - 13.0 = 1.00 .$$

i here is no ork done in this interval from x = 12.0 m to x = 13.0 m so the ans ers are the same as in part g U_{12} = 13.0 .

here is no ork done in this interval from x = 12.0 m to x = 13.0 m so the ans ers are the same as in part h K_{12} = 1.00 .

k Although the plot is not shown here, it would look like a "potential well" with piece ise sloping sides from x = 0 to x = 2 units understood the graph of U is a decreasing line segment from 11 to 5 and from x = 2 to x = 3 it then heads do n to ero here it stays until x = 8 here it starts increasing to a value of 12 at x = 11 and then in another positive slope line segment it increases to a value of 13 at x = 12 . or $x > 12$ its value does not change (this is the "top of the well").

l The particle can be thought of as "falling" down the $0 < x$ 3 slopes of the ell gaining kinetic energy as it does so and certainly is able to reach x = 5. ince U = 0 at x = 5 then its initial potential energy 11 has completely converted to kinetic no $K = 11.0$.

m his is not sufficient to climb up and out of the ell on the large x side x 8 but does allow it to reach a "height" of 11 at x = 10.8 m. s discussed in section 8 5 this is a "turning point" of the motion.

n Next it "falls" back down and rises back up the small x slope until it comes back to its original position. tating this more carefully hen it is momentarily stopped at x = 10.8 m it is accelerated to the left by the force $\vec{F}_3$ it gains enough speed as a result that it eventually is able to return to x = 0 here it stops again.

82. a t x = 5.00 m the potential energy is ero and the kinetic energy is

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \ 2.00 \text{ kg} \ \ 3.45 \text{ m s}^{\ 2} = 11.9 .$$

he total energy therefore is great enough to reach the point x = 0 here U = 11.0 with a little "left over" (11.9 J – 11.0 = 0.9025 . his is the kinetic energy at x = 0 hich means the speed there is

$$v = \sqrt{2 \ 0.9025 \qquad 2 \text{ kg}} = 0.950 \text{ m s}.$$

t has no come to a stop therefore so it has not encountered a turning point.

b he total energy 11.9 is e ual to the potential energy in the scenario here it is initially moving right ard at x = 10.9756 ≈ 11.0 m. his point may be found by interpolation or simply by using the ork kinetic energy theorem

$$K_f = K_i + W = 0 \quad \Rightarrow \quad 11.9025 \quad -4\ d = 0 \quad \Rightarrow \quad d = 2.9756 \approx 2.98$$

hich hen added to x = 8.00 the point here F_3 begins to act gives the correct result . This provides a turning point for the particle's motion.

83. **THINK** nergy is transferred from an e ternal agent to the block so that its speed continues to increase.

EXPRESS ccording to . 8 25 the ork done by the e ternal force is $W = \Delta E_{\text{mech}} = \Delta K + \Delta U$. hen there is no change in potential energy $\Delta U = 0$ the e pression simplifies to

$$W = \Delta E_{\text{mech}} = \Delta K = \frac{1}{2} m\ v_f^2 - v_i^2 \ .$$

he average po er or average rate of ork done is given by $P_{\text{avg}} = W \quad \Delta t$.

ANALYZE a ubstituting the values given the change in mechanical energy is

$$\Delta E_{\text{mech}} = \Delta K = \frac{1}{2} m\ v_f^2 - v_i^2 \ = \frac{1}{2}\ 15\,\text{kg} \quad 30\,\text{m s}^{\ 2} - 10\,\text{m s}^{\ 2} \ = 6000 \quad = 6.0\times 10^3$$

b rom the above e have W = 6.0 × 10^3 . lso from hapter 2 e kno that $\Delta t = \Delta v / a = 10$ s. hus using . 7 42 the average rate at hich energy is transferred to the block is

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{6.0\times 10^3}{10.0\,\text{s}} = 600 \quad .$$

c and d he constant applied force is F = ma = 30 and clearly in the direction of motion so . 7 48 provides the results for instantaneous po er

$$P = \vec{F}\cdot\vec{v} = \begin{cases} 300 & \text{for } v = 10\ \text{m/s} \\ 900 & \text{for } v = 30\ \text{m/s} \end{cases}$$

LEARN he average of these t o values found in c and d agrees ith the result in part b . ote that the e pression for the instantaneous rate used above can be derived from

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2} m v^2\right) = m\vec{v}\cdot\frac{d\vec{v}}{dt} = m\vec{v}\cdot\vec{a} = \vec{F}\cdot\vec{v}$$

84. a o stretch the spring an e ternal force e ual in magnitude to the force of the spring but opposite to its direction is applied. ince a spring stretched in the positive x direction e erts a force in the negative x direction the applied force must be $F = 52.8x + 38.4x^2$ in the x direction. he ork it does is

$$W = \int_{0.50}^{1.00} 52.8x + 38.4x^2 \; dx = \left(\frac{52.8}{2}x^2 + \frac{38.4}{3}x^3 \right)\Bigg|_{0.50}^{1.00} = 31.0 \quad .$$

b he spring does 31.0 of ork and this must be the increase in the kinetic energy of the particle. ts speed is then

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(31.0 \;)}{2.17\,\text{kg}}} = 5.35\,\text{m/s}.$$

c he force is conservative since the ork it does as the particle goes from any point x_1 to any other point x_2 depends only on x_1 and x_2 not on details of the motion bet een x_1 and x_2.

85. **THINK** his problem deals ith the concept of hydroelectric generator – kinetic energy of ater can be converted into electrical energy.

EXPRESS y energy conservation the change in kinetic energy of ater in one second is

$$\Delta K = -\Delta U = mgh = \rho Vgh = \; 10^3\,\text{kg} \;\; \text{m}^3 \;\; 1200\,\text{m}^3 \;\; 9.8\,\text{m} \;\; \text{s}^2 \;\; 100\,\text{m} \; = 1.176 \times 10^9$$

 nly 3 4 of this amount is transferred to electrical energy.

ANALYZE he po er generation assumed constant so average po er is the same as instantaneous po er is

$$P_{\text{avg}} = \frac{3 \;\; 4 \;\; \Delta K}{t} = \frac{3 \;\; 4 \;\; 1.176 \times 10^9}{1.0\,\text{s}} = 8.82 \times 10^8 \quad .$$

LEARN ydroelectricity is the most idely used rene able energy it accounts for almost 20% of the world's electricity supply.

86. a t B the speed is from . 8 17

$$v = \sqrt{v_0^2 + 2gh_1} = \sqrt{\;7.0\,\text{m s}\;^2 + 2\;\;9.8\,\text{m s}^2\;\;6.0\,\text{m}\;} = 13\,\text{m s}.$$

a ere hat matters is the difference in heights bet een A and C

$$v = \sqrt{v_0^2 + 2g\;\;h_1 - h_2\;} = \sqrt{\;7.0\,\text{m s}\;^2 + 2\;\;9.8\,\text{m s}^2\;\;4.0\,\text{m}\;} = 11.29\,\text{m s} \approx 11\,\text{m s}.$$

c sing the result from part b e see that its kinetic energy right at the beginning of its "rough slide" (heading horizontally toward D is $\frac{1}{2}\,m\;11.29\;\text{m s}\;^2 = 63.7m$ ith units understood). Note that we "carry along" the mass (as if it were a known quantity);

as e ill see it ill cancel out shortly. sing . 8 31 and . 6 2 ith $F_N = mg$ e note that this kinetic energy ill turn entirely into thermal energy

$$63.7m = \mu_k mgd$$

if d L. ith $\mu_k = 0.70$ e find $d = 9.3$ m hich is indeed less than L given in the problem as 12 m . e conclude that the block stops before passing out of the "rough" region and thus does not arrive at point D .

87. **THINK** e have a ball attached to a rod that moves in a vertical circle. he total mechanical energy of the system is conserved.

EXPRESS et position A be the reference point for potential energy $U_A = 0$. he total mechanical energies at A B and C are

$$E_A = \frac{1}{2}mv_A^2 + U_A = \frac{1}{2}mv_0^2$$
$$E_B = \frac{1}{2}mv_B^2 + U_B = \frac{1}{2}mv_B^2 - mgL$$
$$E_D = \frac{1}{2}mv_D^2 + U_D = mgL$$

here $v_D = 0$. he problem can be analy ed by applying energy conservation $E_A = E_B = E_D$.

ANALYZE a he condition $E_A = E_D$ gives

$$\frac{1}{2}mv_0^2 = mgL \quad \Rightarrow \quad v_0 = \sqrt{2gL}$$

b o find the tension in the rod hen the ball passes through B e first calculate the speed at B. sing $E_B = E_D$ e find

$$\frac{1}{2}mv_B^2 - mgL = mgL$$

or $v_B = \sqrt{4gL}$. he direction of the centripetal acceleration is up ard at that moment as is the tension force. Thus, Newton's second law gives

$$T - mg = \frac{mv_B^2}{r} = \frac{m\ 4gL}{L} = 4mg$$

or $T = 5mg$.

c he difference in height bet een C and D is L, so the "loss" of mechanical energy hich goes into thermal energy is $-mgL$.

d he difference in height bet een B and D is $2L$, so the total "loss" of mechanical energy hich all goes into thermal energy is $-2mgL$.

LEARN n alternative ay to calculate the energy loss in d is to note that

$$E'_B = \frac{1}{2}mv_B'^2 + U_B = 0 - mgL = -mgL$$

hich gives

$$\Delta E = E'_B - E_A = -mgL - mgL = -2mgL.$$

88. a he initial kinetic energy is $K_i = \frac{1}{2}(1.5)(3)^2 = 6.75$.

b he ork of gravity is the negative of its change in potential energy. t the highest point all of K_i has converted into U if e neglect air friction so e conclude the ork of gravity is -6.75 .

c nd e conclude that $\Delta U = 6.75$.

d he potential energy there is $U_f = U_i + \Delta U = 6.75$.

e f $U_f = 0$ then $U_i = U_f - \Delta U = -6.75$.

f ince $mg\Delta y = \Delta U$ e obtain $\Delta y = 0.459$ m.

89. a y mechanical energy conversation the kinetic energy as it reaches the floor hich e choose to be the $U = 0$ level is the sum of the initial kinetic and potential energies

$$K = K_i + U_i = \frac{1}{2}\ 2.50\text{ kg}\ \ 3.00\text{ m s}\ ^2\ \ 2.50\text{ kg}\ \ 9.80\text{ m s}^2\ \ 4.00\text{ m}\ = 109\ .$$

or later use e note that the speed ith hich it reaches the ground is

$$v = \sqrt{2K/m}\ \ = 9.35\text{ m s}.$$

b hen the drop in height is 2.00 m instead of 4.00 m the kinetic energy is

$$K = \frac{1}{2}\ 2.50\text{ kg}\ \ 3.00\text{ m s}\ ^2\ \ 2.50\text{ kg}\ \ 9.80\text{ m s}^2\ \ 2.00\text{ m}\ = 60.3\ .$$

c simple ay to approach this is to imagine the can being *launched* from the ground at $t = 0$ ith a speed 9.35 m s see above and calculate the height and speed at t = 0.200 s using . 2 15 and . 2 11

$$y = \ 9.35 \text{ m s} \ \ 0.200 \text{ s} \ - \frac{1}{2} \ \ 9.80 \text{ m s}^2 \ \ 0.200 \text{ s} \ ^2 = 1.67 \text{ m}$$

$$v = 9.35 \text{ m s} - \ 9.80 \text{ m s}^2 \ \ 0.200 \text{ s} \ = 7.39 \text{ m s.}$$

he kinetic energy is $K = \frac{1}{2} \ \ 2.50 \text{ kg} \ \ \ 7.39 \text{ m s} \ ^2 = 68.2$.

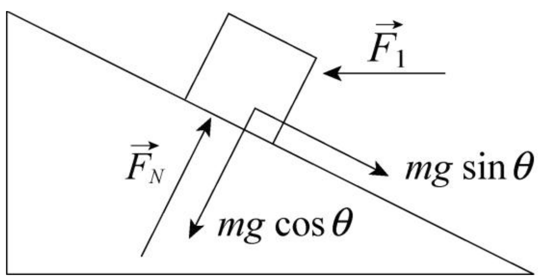
d he gravitational potential energy is

$$U = mgy = \ 2.5 \text{ kg} \ \ 9.8 \text{ m s}^2 \ \ 1.67 \text{ m} \ = 41.0 \ \ .$$

90. he free body diagram for the trunk is sho n belo . he x and y applications of e ton s second la provide t o e uations

$$F_1 \cos \theta - f_k - mg \sin \theta \ = ma$$

$$F_N - F_1 \sin \theta - mg \cos \theta \ = 0.$$



a he trunk is moving up the incline at constant velocity so $a = 0$. sing $f_k = \mu_k F_N$ e solve for the push force F_1 and obtain

$$F_1 = \frac{mg(\sin\theta + \mu_k \cos\theta)}{\cos\theta - \mu_k \sin\theta}.$$

he ork done by the push force $\vec{F}_1$ as the trunk is pushed through a distance ℓ up the inclined plane is therefore

$$W_1 = F_1 \ell \cos\theta = \frac{(mg\ell \cos\theta)(\sin\theta + \mu_k \cos\theta)}{\cos\theta - \mu_k \sin\theta}$$

$$= \frac{(50\text{ kg})(9.8\text{ m/s}^2)(6.0\text{ m})(\cos 30^\circ)(\sin 30^\circ + (0.20)\cos 30^\circ)}{\cos 30^\circ - (0.20)\sin 30^\circ}$$

$$= 2.2 \times 10^3 \ \ .$$

b he increase in the gravitational potential energy of the trunk is

$$\Delta U = mg\ell \sin\theta = 50\,\text{kg}\ \ 9.8\,\text{m s}^2\ \ 6.0\,\text{m}\ \sin 30° = 1.5\times 10^3\ .$$

ince the speed and therefore the kinetic energy of the trunk is unchanged . 8 33 leads to

$$W_1 = \Delta U + \Delta E_{\text{th}}.$$

hus using more precise numbers than are sho n above the increase in thermal energy generated by the kinetic friction is $2.24 \times 10^3 - 1.47 \times 10^3 = 7.7 \times 10^2$. n alternate ay to this result is to use $\Delta E_{\text{th}} = f_k \ell$. 8 31 .

91. he initial height of the $2M$ block sho n in ig. 8 69 is the $y = 0$ level in our computations of its value of U_g. s that block drops the spring stretches accordingly. lso the kinetic energy K_{sys} is evaluated for the *system* that is for a total moving mass of $3M$.

a he conservation of energy . 8 17 leads to

$$K_i + U_i = K_{sys} + U_{sys} \Rightarrow 0 \quad 0 = K_{sys} + 2M\,g\,{-0.090} \quad \frac{1}{2}\,k\ 0.090^{\ 2}.$$

hus ith $M = 2.0$ kg e obtain $K_{sys} = 2.7$.

b he kinetic energy of the $2M$ block represents a fraction of the total kinetic energy

$$\frac{K_{2M}}{K_{sys}} = \frac{2M\ v^2\ \ 2}{3M\ v^2\ \ 2} = \frac{2}{3}.$$

herefore $K_{2M} = \frac{2}{3}\ 2.7 \quad = 1.8$.

c ere e let $y = -d$ and solve for d.

$$K_i + U_i = K_{sys} + U_{sys} \Rightarrow 0 \quad 0 = 0 + 2M\,g\,{-d} \quad \frac{1}{2}\,kd^2.$$

hus ith $M = 2.0$ kg e obtain $d = 0.39$ m.

92. y energy conservation $mgh = mv^2\ \ 2$ the speed of the volcanic ash is given by $v = \sqrt{2gh}$. n our present problem the height is related to the distance on the $\theta = 10$ slope $d = 920$ m by the trigonometric relation $h = d\sin\theta$. hus

$$v = \sqrt{2\ 9.8\,\text{m s}^2\ \ 920\,\text{m}\ \ \sin 10°} = 56\,\text{m s}.$$

93. a he assumption is that the slope of the bottom of the slide is hori ontal like the ground. useful analogy is that of the pendulum of length $R = 12$ m that is pulled

left ard to an angle θ corresponding to being at the top of the slide at height h = 4.0 m and released so that the pendulum s ings to the lo est point ero height gaining speed $v = 6.2\,\text{m/s}$. actly as e ould analy e the trigonometric relations in the pendulum problem e find

$$h = R(1-\cos\theta) \Rightarrow \theta = \cos^{-1}\left(1-\frac{h}{R}\right) = 48°$$

or 0.84 radians. he slide representing a circular arc of length $s = R\theta$ is therefore 12 m 0.84 = 10 m long.

b o find the magnitude f of the frictional force e use . 8 31 ith W = 0

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + fs \end{aligned}$$

so that ith m = 25 kg e obtain f = 49 .

c he assumption is no longer that the slope of the bottom of the slide is hori ontal but rather that the slope of the top of the slide is vertical and 12 m to the left of the center of curvature . eturning to the pendulum analogy this corresponds to releasing the pendulum from hori ontal at θ_1 = 90 measured from vertical and taking a snapshot of its motion a fe moments later hen it is at angle θ_2 ith speed v = 6.2 m s. he difference in height bet een these t o positions is ust as e ould figure for the pendulum of length R

$$\Delta h = R(1-\cos\theta_2) - R(1-\cos\theta_1) = -R\cos\theta_2$$

here e have used the fact that cos θ_1 = 0. hus ith Δh = –4.0 m e obtain θ_2 = 70.5 hich means the arc subtends an angle of $\Delta\theta$ = 19.5 or 0.34 radians. ultiplying this by the radius gives a slide length of s' = 4.1 m.

d e again find the magnitude f' of the frictional force by using . 8 31 ith W = 0

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + f's' \end{aligned}$$

so that e obtain $f' = 1.2 \times 10^2$.

94. e use $P = Fv$ to compute the force

$$F = \frac{P}{v} = \frac{92\times10^6}{(32.5\,\text{knot})\left(1.852\,\frac{\text{km/h}}{\text{knot}}\right)\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)} = 5.5\times10^6 \quad .$$

95. his can be orked entirely by the methods of hapters 2–6 but e ill use energy methods in as many steps as possible.

a y a force analysis in the style of hapter 6 e find the normal force has magnitude $F_N = mg\cos\theta$ here $\theta = 39$ hich means $f_k = \mu_k mg\cos\theta$ here $\mu_k = 0.28$. hus . 8 31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta.$$

lso elementary trigonometry leads us to conclude that $\Delta U = -mgd\sin\theta$ here $d = 3.7$ m. ince $K_i = 0$. 8 33 ith $W = 0$ indicates that the final kinetic energy is

$$K_f = -\Delta U - \Delta E_{\text{th}} = mgd\ \sin\theta - \mu_k\cos\theta$$

hich leads to the speed at the bottom of the ramp

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gd\,(\sin\theta - \mu_k\cos\theta)} = 5.5\ \text{m/s}.$$

b his speed begins its hori ontal motion here $f_k = \mu_k mg$ and $\Delta U = 0$. t slides a distance d' before it stops. ccording to . 8 31 ith $W = 0$

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= 0 - \frac{1}{2}mv^2 + 0 + \mu_k mgd' \\ &= -\frac{1}{2}\left(2gd\,(\sin\theta - \mu_k\cos\theta)\right) + \mu_k gd' \end{aligned}$$

here e have divided by mass and substituted from part a in the last step. herefore

$$d' = \frac{d(\sin\theta - \mu_k\cos\theta)}{\mu_k} = 5.4\ \text{m}.$$

c e see from the algebraic form of the results above that the ans ers do not depend on mass. 90 kg crate should have the same speed at the bottom and sliding distance across the floor to the e tent that the friction relations in hapter 6 are accurate. nterestingly since g does not appear in the relation for d' the sliding distance ould seem to be the same if the e periment ere performed on ars

96. a he loss of the initial $K = \frac{1}{2}mv^2 = \frac{1}{2}$ 70 kg 10 m s ^{2} is 3500 or 3.5 k .

b his is dissipated as thermal energy $\Delta E_{\text{th}} = 3500$ = 3.5 k .

97. . 8 33 gives $mgy_f = K_i + mgy_i - \Delta E_{\text{th}}$ or

$$0.50\text{ kg}\ \ 9.8\text{ m s}^2\ \ 0.80\text{ m}\ = \frac{1}{2}\ \ 0.50\text{ kg}\ \ 4.00\ \ \text{s}^{\ 2}\quad 0.50\text{ kg}\ \ 9.8\text{ m s}^2\ \ 0\ - \Delta E_{\text{th}}$$

hich yields ΔE_{th} = 4.00 – 3.92 = 0.080 .

98. ince the period T is 2.5 rev s $^{-1}$ = 0.40 s then . 4 33 leads to v = 3.14 m s. he frictional force has magnitude using . 6 2

$$f = \mu_k F_N = \ 0.320\ \ 180\ \ = 57.6\ .$$

he po er dissipated by the friction must e ual that supplied by the motor so . 7 48 gives P = 57.6 3.14 m s = 181 .

99. o s im at constant velocity the s immer must push back against the ater ith a force of 110 . elative to him the ater is going at 0.22 m s to ard his rear in the same direction as his force. sing . 7 48 his po er output is obtained

$$P = \vec{F}\cdot\vec{v} = Fv = (110\quad)(0.22\text{ m/s}) = 24\quad .$$

100. he initial kinetic energy of the automobile of mass m moving at speed v_i is $K_i = \frac{1}{2}mv_i^2$ here m = 16400 9.8 = 1673 kg. sing . 8 31 and . 8 33 this relates to the effect of friction force f in stopping the auto over a distance d by $K_i = fd$ here the road is assumed level so ΔU = 0 . ith

$$v_i = (113\ \text{km h}) = (113\ \text{km h})\ 1000\ \text{m km}\ \ 1\text{ h }3600\text{ s}\ = 31.4\text{ m s}$$

e obtain

$$d = \frac{K_i}{f} = \frac{mv_i^2}{2f} = \frac{(1673\text{kg})\ \ 31.4\text{ m s}\ ^2}{2(8230\quad)} = 100\text{ m}.$$

101. ith the potential energy reference level set at the point of thro ing e have ith units understood

$$\Delta E = mgh - \frac{1}{2}mv_0^2 = m\left((9.8)(8.1) - \frac{1}{2}(14)^2\right)$$

hich yields ΔE = –12 for m = 0.63 kg. This “loss” of mechanical energy is presumably due to air friction.

102. a he internal energy the climber must convert to gravitational potential energy is

$$\Delta U = mgh = (90\text{ kg})\left(9.80\text{ m s}^2\right)(8850\text{ m}) = 7.8\times10^6 \quad .$$

b he number of candy bars this corresponds to is

$$N = \frac{7.8\times10^6}{1.25\times10^6 \quad /\text{bar}} \approx 6.2\,\text{bars}.$$

103. a he acceleration of the sprinter is using . 2 15

$$a = \frac{2\Delta x}{t^2} = \frac{(2)(7.0\,\text{m})}{(1.6\,\text{s})^2} = 5.47\,\text{m/s}^2\,.$$

onse uently the speed at t = 1.6s is $v = at = \left(5.47\,\text{m/s}^2\right)(1.6\,\text{s}) = 8.8\,\text{m/s}.$ lternatively . 2 17 could be used.

b he kinetic energy of the sprinter of eight w and mass m = w g is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 = \frac{1}{2}\left(670 \quad 9.8\text{ m s}^2\right)(8.8\text{ m s})^2 = 2.6\times10^3 \quad .$$

c he average po er is

$$P_{\text{avg}} = \frac{\Delta K}{\Delta t} = \frac{2.6\times10^3}{1.6\text{ s}} = 1.6\times10^3 \quad .$$

104. rom . 8 6 e find ith units understood

$$U(\xi) = -\int_0^{\xi}\left(-3x - 5x^2\right)dx = \frac{3}{2}\xi^2 + \frac{5}{3}\xi^3.$$

a sing the above formula e obtain U 2 $\approx$ 19 .

b hen its speed is v = 4 m s its mechanical energy is $\frac{1}{2}mv^2 + U(5)$. his must e ual the energy at the origin

$$\frac{1}{2}mv^2 + U(5) = \frac{1}{2}mv_o^2 + U(0)$$

so that the speed at the origin is

$$v_o = \sqrt{v^2 + \frac{2}{m}\left(U(5) - U(0)\right)}.$$

hus ith U 5 = 246 U 0 = 0 and m = 20 kg e obtain v_o = 6.4 m s.

c ur original formula for U is changed to

$$U \;x\; = -8 + \frac{3}{2}x^2 + \frac{5}{3}x^3$$

in this case. herefore U 2 = 11 . ut e still have v_0 = 6.4 m s since that calculation only depended on the difference of potential energy values specifically U 5 – U 0 .

105. a Resolving the gravitational force into components and applying Newton's second la as ell as . 6 2 e find

$$F_{\text{machine}} - mg\sin\theta - \mu_k mg\cos\theta = ma.$$

n the situation described in the problem e have $a = 0$ so

$$F_{\text{machine}} = mg\sin\theta \quad \mu_k mg\cos\theta = 372 \quad .$$

hus the ork done by the machine is $F_{\text{machine}}d = 744 \quad = 7.4 \times 10^2$.

b he thermal energy generated is $\mu_k mg\cos\theta)\, d = 240 \quad = 2.4 \times 10^2$.

106. a t the highest point the velocity $v = v_x$ is purely hori ontal and is e ual to the hori ontal component of the launch velocity see section 4 6 $v_{0x} = v_0 \cos\theta$ here $\theta = 30°$ in this problem. uation 8 17 relates the kinetic energy at the highest point to the launch kinetic energy

$$K_0 \quad = mgy \quad \frac{1}{2}\, mv^2 = \frac{1}{2}\, mv_{0x}{}^2 \quad \frac{1}{2}\, mv_{0y}{}^2$$

ith $y = 1.83$ m. ince the $mv_{0x}{}^2$ 2 term on the left hand side cancels the mv^2 2 term on the right hand side this yields $v_{0y} = \sqrt{2gy} \approx 6$ m s. ith $v_{0y} = v_0 \sin\theta$ e obtain

$$v_0 = 11.98 \text{ m s} \approx 12 \text{ m s}.$$

b nergy conservation including no the energy stored elastically in the spring . 8 11 also applies to the motion along the mu le through a distance d that corresponds to a vertical height increase of $d\sin\theta$

$$\frac{1}{2}\, kd^2 = K_0 \quad mg\,d\sin\theta \quad \Rightarrow \quad d = 0.11 \text{ m}.$$

107. he ork done by $\vec{F}$ is the negative of its potential energy change see . 8 6 so $U_B = U_A - 25 = 15$.

108. a e assume his mass is bet een m_1 = 50 kg and m_2 = 70 kg corresponding to a eight bet een 110 lb and 154 lb . is increase in gravitational potential energy is therefore in the range

$$m_1 gh \le \Delta U \le m_2 gh \quad \Rightarrow \quad 2\times10^5 \le \Delta U \le 3\times10^5$$

in units here h = 443 m.

b he problem only asks for the amount of internal energy that converts into gravitational potential energy so this result is the same as in part a . ut if e ere to consider his *total* internal energy "output" much of hich converts to heat e can e pect that e ternal climb is uite different from taking the stairs.

109. a e implement . 8 37 as

$$K_f \quad = K_i \quad mgy_i - f_k d = 0 \quad 60 \text{ kg} \quad 9.8 \text{ m s}^2 \quad 4.0 \text{ m} \quad - 0 = 2.35 \times 10^3 \quad .$$

b o it applies ith a non ero thermal term

$$K_f = K_i \quad mgy_i - f_k d = 0 \quad 60 \text{ kg} \quad 9.8 \text{ m s}^2 \quad 4.0 \text{ m} \quad - \quad 500 \quad 4.0 \text{ m} \quad = 352 \quad .$$

110. e take the bottom of the incline to be the y = 0 reference level. he incline angle is $\theta = 30°$. he distance along the incline d measured from the bottom is related to height y by the relation $y = d \sin \theta$.

a sing the conservation of energy e have

$$K_0 + U_0 = K_{\text{top}} + U_{\text{top}} \Rightarrow \frac{1}{2} m v_0^2 + 0 = 0 + mgy$$

ith $v_0 = 5.0 \text{ m/s}$. his yields y = 1.3 m from hich e obtain d = 2.6 m.

b n analysis of forces in the manner of hapter 6 reveals that the magnitude of the friction force is $f_k = \mu_k mg \cos \theta$. o e rite . 8 33 as

$$K_0 + U_0 = K_{\text{top}} + U_{\text{top}} + f_k d$$
$$\frac{1}{2} m v_0^2 + 0 = 0 + mgy + f_k d$$
$$\frac{1}{2} m v_0^2 = mgd \sin\theta + \mu_k mgd \cos\theta$$

hich — upon canceling the mass and rearranging — provides the result for d

$$d = \frac{v_0^2}{2g(\mu_k \cos\theta + \sin\theta)} = 1.5 \text{ m} .$$

c he thermal energy generated by friction is $f_k d = \mu_k mgd \cos\theta = 26$.

d he slide back do n from the height y = 1.5 sin 30 is also described by . 8 33. ith ΔE_{th} again e ual to 26 e have

$$K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}} + f_k d \Rightarrow 0 + mgy = \frac{1}{2} mv_{\text{bot}}^2 + 0 + 26$$

from hich e find $v_{\text{bot}} = 2.1 \text{ m/s}$.

111. uation 8 8 leads directly to $\Delta y = \dfrac{68000}{9.4 \text{ kg} \;\; 9.8 \text{ m s}^2}$ = 738 m.

112. e assume his initial kinetic energy hen he umps is negligible. hen his initial gravitational potential energy measured relative to here he momentarily stops is hat becomes the elastic potential energy of the stretched net neglecting air friction . hus

$$U_{\text{net}} = U_{\text{grav}} = mgh$$

here h = 11.0 m 1.5 m = 12.5 m. ith m = 70 kg e obtain U_{net} = 8580 .

113. e use units so m = 0.030 kg and d = 0.12 m.

a ince there is no change in height and e assume no changes in elastic potential energy then ΔU = 0 and e have

$$\Delta E_{\text{mech}} = \Delta K = -\frac{1}{2} mv_0^2 = -3.8 \times 10^3$$

here v_0 = 500 m s and the final speed is ero.

b y . 8 33 ith W = 0 e have $\Delta E_{\text{th}} = 3.8 \times 10^3$ hich implies

$$f = \frac{\Delta E_{\text{th}}}{d} = 3.1 \times 10^4$$

using . 8 31 ith f_k replaced by f effectively generali ing that e uation to include a greater variety of dissipative forces than ust those obeying . 6 2 .

114. a he kinetic energy K of the automobile of mass m at t = 30 s is

$$K=\frac{1}{2}mv^2=\frac{1}{2}(1500\,\text{kg})\left((72\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)\right)^2=3.0\times10^5 \ .$$

b he average po er re uired is

$$P_{\text{avg}}=\frac{\Delta K}{\Delta t}=\frac{3.0\times10^5}{30\,\text{s}}=1.0\times10^4 \ .$$

c ince the acceleration a is constant the po er is $P = Fv = mav = ma\ at = ma^2t$ using . 2 11. y contrast from part b the average po er is $P_{\text{avg}}=\frac{mv^2}{2t}$ hich becomes $\frac{1}{2}ma^2t$ hen $v = at$ is again utili ed. hus the instantaneous po er at the end of the interval is t ice the average po er during it

$$P=2P_{\text{avg}}=(2)(1.0\times10^4 \ )=2.0\times10^4 \ .$$

115. a he initial kinetic energy is $K_i=\ 1.5\,\text{kg}\ \ 20\,\text{m s}\ ^2\ \ 2=300$.

b t the point of ma imum height the vertical component of velocity vanishes but the hori ontal component remains what it was when it was “shot” (if we neglect air friction). ts kinetic energy at that moment is

$$K=\frac{1}{2}\ 1.5\,\text{kg}\ [\ 20\,\text{m s}\ \cos 34°]^2=206 \ .$$

hus $\Delta U = K_i - K = 300 \ - 206 \ = 93.8$.

c ince $\Delta U = mg\Delta y$ e obtain $\Delta y=\frac{94}{1.5\,\text{kg}\ \ 9.8\,\text{m s}^2}=6.38\,\text{m}$.

116. a he rate of change of the gravitational potential energy is

$$\frac{dU}{dt}=mg\frac{dy}{dt}=-mg|v|=-(68)(9.8)(59)=-3.9\times10^4 \ /\text{s}.$$

hus the gravitational energy is being reduced at the rate of 3.9×10^4 .

b ince the velocity is constant the rate of change of the kinetic energy is ero. hus the rate at hich the mechanical energy is being dissipated is the same as that of the gravitational potential energy 3.9×10^4 .

117. a he effect of sliding friction is described in terms of energy dissipated as sho n in . 8 31. e have

$$\Delta E = K + \frac{1}{2}k(0.08)^2 - \frac{1}{2}k(0.10)^2 = -f_k\,(0.02)$$

here distances are in meters and energies are in oules. ith k = 4000 m and $f_k = 80$ e obtain K = 5.6 .

b n this case e have d = 0.10 m. hus

$$\Delta E = K + 0 - \frac{1}{2}k(0.10)^2 = -f_k(0.10)$$

hich leads to K = 12 .

c e can approach this t o ays. ne ay is to e amine the dependence of energy on the variable d

$$\Delta E = K + \frac{1}{2}k(d_0 - d)^2 - \frac{1}{2}kd_0^2 = -f_k d$$

here d_0 = 0.10 m and solving for K as a function of d

$$K = -\frac{1}{2}kd^2 + (kd_0)d - f_k d\,.$$

n this first approach e could ork through the $dK \;\; d\,d = 0$ condition or ith the special capabilities of a graphing calculator to obtain the ans er $K_{ma} = \frac{1}{2k}(kd_0 - f_k)^2$.

n the second and perhaps easier approach e note that K is ma imum here v is ma imum — hich is here $a = 0 \Rightarrow$ e uilibrium of forces. hus the second approach simply solves for the e uilibrium position

$$\left|F_{\text{spring}}\right| = f_k \Rightarrow kx = 80.$$

hus ith k = 4000 m e obtain x = 0.02 m. ut $x = d_0 - d$ so this corresponds to d = 0.08 m. hen the methods of part a lead to the ans er K_{ma} = 12.8 ≈ 13 .

118. e ork this in units and convert to horsepo er in the last step. hus

$$v = (80\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right) = 22.2\,\text{m/s}.$$

he force F needed to propel the car of eight w and mass $m = w\;g$ is found from Newton's second law:

$$F_{\text{net}} = F_P - F = ma = \frac{wa}{g}$$

here F = 300 1.8v^2 in units. herefore the po er re uired is

$$P = \vec{F}_P \cdot \vec{v} = \left(F + \frac{wa}{g}\right)v = \left(300 + 1.8(22.2)^2 + \frac{(12000)(0.92)}{9.8}\right)(22.2) = 5.14\times10^4$$
$$= \left(5.14\times10^4 \quad\right)\left(\frac{1\text{ hp}}{746}\right) = 69\text{ hp.}$$

119. **THINK** e apply energy method to analy e the pro ectile motion of a ball.

EXPRESS e choose the initial position at the indo to be our reference point for calculating the potential energy. he initial energy of the ball is $E_0 = \frac{1}{2}mv_0^2$. t the top of its flight the vertical component of the velocity is ero and the hori ontal component neglecting air friction is the same as it as hen it as thro n $v_x = v_0 \cos\theta$. t a position h belo the indo the energy of the ball is

$$E = K + U = \frac{1}{2}mv^2 - mgh$$

here v is the speed of the ball.

ANALYZE a he kinetic energy of the ball at the top of the flight is

$$K_{\text{top}} = \frac{1}{2}mv_x^2 = \frac{1}{2}m\ v_0\cos\theta\ ^2 = \frac{1}{2}\ 0.050\,\text{kg}\quad 8.0\,\text{m s}\ \cos30^\circ\ ^2 = 1.2\quad .$$

b hen the ball is h = 3.0 m belo the indo by energy conservation e have

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 - mgh$$

or

$$v = \sqrt{v_0^2 + 2gh} = \sqrt{\ 8.0\,\text{m s}\ ^2 + 2\ 9.8\,\text{m s}^2\quad 3.0\,\text{m}\ } = 11.1\,\text{m s}.$$

c s can be seen from our e pression above $v = \sqrt{v_0^2 + 2gh}$ hich is independent of the mass m.

d imilarly the speed v is independent of the initial angle θ.

LEARN ur results demonstrate that the uantity v in the kinetic energy formula is the magnitude of the velocity vector it does not depend on direction. n addition mass cancels out in the energy conservation e uation so that v is independent of m.

120. a n the initial situation the elongation as using . 8 11

$$x_i = \sqrt{2\ 1.44\ \ 3200} = 0.030 \text{ m} \text{ or } 3.0 \text{ cm} .$$

n the ne t situation the elongation is only 2.0 cm or 0.020 m so e no have less stored energy relative to hat e had initially . pecifically

$$\Delta U = \frac{1}{2}\ 3200\ \ \text{m}\ 0.020 \text{ m}^2 - 1.44\ = -0.80\ .$$

b he elastic stored energy for $x|$ = 0.020 m does not depend on hether this represents a stretch or a compression. he ans er is the same as in part a ΔU = –0.80 .

c o e have $x|$ = 0.040 m hich is greater than x_i so this represents an increase in the potential energy relative to hat e had initially . pecifically

$$\Delta U = \frac{1}{2}\ 3200\ \ \text{m}\ 0.040 \text{ m}^2 - 1.44\ = \ 1.12\ \ \approx 1.1\ .$$

121. a ith P = 1.5 = 1.5 × 10^6 assumed constant and t = 6.0 min = 360 s the ork kinetic energy theorem becomes

$$W = Pt = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right).$$

he mass of the locomotive is then

$$m = \frac{2Pt}{v_f^2 - v_i^2} = \frac{(2)\left(1.5\times 10^6\ \ \right)(360\,\text{s})}{(25\,\text{m/s})^2 - (10\,\text{m/s})^2} = 2.1\times 10^6 \text{ kg}.$$

b ith t arbitrary e use $Pt = \frac{1}{2}m\left(v^2 - v_i^2\right)$ to solve for the speed v = $v\ t$ as a function of time and obtain

$$v(t) = \sqrt{v_i^2 + \frac{2Pt}{m}} = \sqrt{(10)^2 + \frac{(2)\left(1.5\times 10^6\right)t}{2.1\times 10^6}} = \sqrt{100 + 1.5t}$$

in units v in m s and t in s .

c he force $F\ t$ as a function of time is

$$F(t) = \frac{P}{v(t)} = \frac{1.5\times10^6}{\sqrt{100+1.5t}}$$

in units F in and t in s .

d he distance d the train moved is given by

$$d = \int_0^t v\ t'\ dt' = \int_0^{360}\left(100+\frac{3}{2}t\right)^{1\ 2} dt = \frac{4}{9}\left(100+\frac{3}{2}t\right)^{3\ 2}\Bigg|_0^{360} = 6.7\times10^3\ \text{m.}$$

122. **THINK** shuffleboard disk is accelerated over some distance by an e ternal force but it eventually comes to rest due to the frictional force.

EXPRESS n the presence of frictional force the ork done on a system is $W = \Delta E_{\text{mech}} + \Delta E_{\text{th}}$ here $\Delta E_{\text{mech}} = \Delta K + \Delta U$ and $\Delta E_{\text{th}} = f_k d$. n our situation ork has been done by the cue only to the first 2.0 m and not to the subse uent 12 m of distance traveled.

ANALYZE a uring the final d = 12 m of motion $W = 0$ and e use

$$K_1 + U_1 = K_2 + U_2 + f_k d$$
$$\frac{1}{2}mv^2 + 0 = 0 + 0 + f_k d$$

here $m = 0.42$ kg and v = 4.2 m s. his gives f_k = 0.31 . herefore the thermal energy change is $\Delta E_{\text{th}} = f_k d = 3.7$.

b sing f_k = 0.31 for the entire distance d_{total} = 14 m e obtain

$$\Delta E_{\text{th total}} = f_k d_{\text{total}} = \ 0.31\quad 14\ \text{m}\ = 4.3$$

for the thermal energy generated by friction.

c uring the initial d' = 2 m of motion e have

$$W = \Delta E_{\text{mech}} + \Delta E'_{\text{th}} = \Delta K + \Delta U + f_k d' = \frac{1}{2}mv^2 + 0 + f_k d'$$

hich essentially combines . 8 31 and . 8 33. hus the ork done on the disk by the cue is

$$W = \frac{1}{2}mv^2 + f_k d' = \frac{1}{2}\ 0.42\ \text{kg}\ \ 4.2\ \text{m s}\ ^2 + \ 0.31\quad 2.0\ \text{m}\ = 4.3\ .$$

LEARN ur ans er in c is the same as that in b . his is e pected because all the ork done becomes thermal energy at the end.

123. he ater has gained

$$\Delta K = \frac{1}{2} \;\; 10\text{ kg} \;\; 13\text{ m s}^{\;2} - \frac{1}{2} \;\; 10\text{ kg} \;\; 3.2\text{ m s}^{\;2} = 794$$

of kinetic energy and it has lost $\Delta U =$ 10 kg 9.8 m s^2 15 m = 1470 .

of potential energy the lack of agreement bet een these t o values is presumably due to transfer of energy into thermal forms . he ratio of these values is 0.54 = 54 . he mass of the ater cancels hen e take the ratio so that the assumption stated at the end of the problem m = 10 kg is not needed for the final result.

124. a he integral see . 8 6 here the value of U at $x = \infty$ is re uired to vanish is straightfor ard. he result is $U\ x\ = -Gm_1m_2\ x$.

b ne approach is to use . 8 5 hich means that e are effectively doing the integral of part a all over again. nother approach is to use our result from part a and thus use . 8 1 . ither ay e arrive at

$$W = \frac{G\,m_1\,m_2}{x_1} - \frac{G\,m_1\,m_2}{x_1 + d} = \frac{G\,m_1\,m_2\,d}{x_1\ x_1 \quad d}\ .$$

125. a uring one second the decrease in potential energy is

$$-\Delta U = mg\ -\Delta y\ = \ 5.5\times 10^6\text{ kg}\ \left(9.8\,\text{m/s}^2\right)\ 50\text{ m}\ = 2.7\times 10^9$$

here y is up ard and $\Delta y = y_f - y_i$.

b he information relating mass to volume is not needed in the computation. y . 8 40 and the relation = s the result follo s

$$P = \ 2.7\times 10^9 \quad\ 1\text{ s}\ = 2.7\times 10^9 \quad .$$

c ne year is e uivalent to 24 × 365.25 = 8766 h hich e rite as 8.77 kh. hus the energy supply rate multiplied by the cost and by the time is

$$2.7\times 10^9 \quad\ 8.77\text{ kh}\ \left(\frac{1\text{ cent}}{1\text{ k}\quad\text{h}}\right) = 2.4\times 10^{10}\text{cents} \ = \ 2.4\times 10^8.$$

126. he connection bet een angle θ measured from vertical and height h measured from the lo est point hich is our choice of reference position in computing the

gravitational potential energy is given by $h = L\ 1 - \cos\ \theta$ here L is the length of the pendulum.

a e use energy conservation in the form of . 8 17.

$$K_1 + U_1 = K_2 + U_2$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_2^2 + mgL(1 - \cos\theta_2)$$

ith L = 1.4 m θ_1 = 30 and θ_2 = 20 e have

$$v_2 = \sqrt{2gL(\cos\theta_2 - \cos\theta_1)} = 1.4 \text{ m/s}.$$

b he ma imum speed v_3 is at the lo est point. ur formula for h gives h_3 = 0 hen θ_3 = 0 as e pected. rom

$$K_1 + U_1 = K_3 + U_3$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_3^2 + 0$$

e obtain $v_3 = 1.9 \text{ m/s}$.

c e look for an angle θ_4 such that the speed there is $v_4 = v_3/3$. o be as accurate as possible e proceed algebraically substituting $v_3^2 = 2gL(1 - \cos\theta_1)$ at the appropriate place and plug numbers in at the end. nergy conservation leads to

$$K_1 + U_1 = K_4 + U_4$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_4^2 + mgL(1 - \cos\theta_4)$$
$$mgL(1 - \cos\theta_1) = \frac{1}{2}m\frac{v_3^2}{9} + mgL(1 - \cos\theta_4)$$
$$-gL\cos\theta_1 = \frac{1}{2}\frac{2gL(1 - \cos\theta_1)}{9} - gL\cos\theta_4$$

here in the last step e have subtracted out mgL and then divided by m. hus e obtain

$$\theta_4 = \cos^{-1}\left(\frac{1}{9} + \frac{8}{9}\cos\theta_1\right) = 28.2° \approx 28°.$$

127. uating the mechanical energy at his initial position as he emerges from the canon here e set the reference level for computing potential energy to his energy as he lands e obtain

$$K_i = K_f + U_f$$

$$\frac{1}{2}(60\,\text{kg})(16\,\text{m/s})^2 = K_f + (60\,\text{kg})\left(9.8\,\text{m/s}^2\right)(3.9\,\text{m})$$

hich leads to $K_f = 5.4 \times 10^3$.

128. a his part is essentially a free fall problem hich can be easily done ith hapter 2 methods. nstead choosing energy methods e take $y = 0$ to be the ground level.

$$K_i + U_i = K + U \Rightarrow 0 + mgy_i = \frac{1}{2}mv^2 + 0$$

herefore $v = \sqrt{2gy_i} = 9.2\,\text{m/s}$ here $y_i = 4.3$ m.

b . 8 29 provides $\Delta E_{\text{th}} = f_k d$ for thermal energy generated by the kinetic friction force. e apply . 8 31

$$K_i + U_i = K + U \Rightarrow 0 + mgy_i = \frac{1}{2}mv^2 + 0 + f_k d\,.$$

ith $d = y_i$ $m = 70$ kg and $f_k = 500$ this yields $v = 4.8$ m s.

129. e ant to convert at least in theory the ater that falls through $h = 500$ m into electrical energy. he problem indicates that in one year a volume of ater e ual to $A\Delta z$ lands in the form of rain on the country here $A = 8 \times 10^{12}$ m^2 and $\Delta z = 0.75$ m. ultiplying this volume by the density $\rho = 1000$ kg m^3 leads to

$$m_{\text{total}} = \rho A \Delta z = (1000)\left(8 \times 10^{12}\right)(0.75) = 6 \times 10^{15}\,\text{kg}$$

for the mass of rain ater. ne third of this "falls" to the ocean, so it is $m = 2 \times 10^{15}$ kg that e ant to use in computing the gravitational potential energy mgh hich ill turn into electrical energy during the year . ince a year is e uivalent to 3.2×10^7 s e obtain

$$P_{\text{avg}} = \frac{\left(2 \times 10^{15}\right)(9.8)(500)}{3.2 \times 10^7} = 3.1 \times 10^{11} \quad .$$

130. he spring is rela ed at $y = 0$ so the elastic potential energy . 8 11 is $U_{\text{el}} = \frac{1}{2}ky^2$. he total energy is conserved and is ero determined by evaluating it at its initial position . e note that U is the same as ΔU in these manipulations. hus e have

$$0 = K + U_g + U_e \quad \Rightarrow \quad K = -U_g - U_e$$

here U_g = mgy = 20 y ith y in meters so that the energies are in oules . e arrange the results in a table

position y	–0.05	–0.10	–0.15	–0.20
K	a 0.75	d 1.0	g 0.75	0
U_g	b –1.0	e –2.0	h –3.0	k –4.0
U_e	c 0.25	f 1.0	i 2.25	l 4.0

131. et the amount of stretch of the spring be x. or the ob ect to be in e uilibrium

$$kx - mg = 0 \Rightarrow x = mg/k.$$

hus the gain in elastic potential energy for the spring is

$$\Delta U_e = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{m^2g^2}{2k}$$

hile the loss in the gravitational potential energy of the system is

$$-\Delta U_g = mgx = mg\left(\frac{mg}{k}\right) = \frac{m^2g^2}{k}$$

hich e see by comparing ith the previous e pression is e ual to 2ΔU_e. he reason hy $\left|\Delta U_g\right| \neq \Delta U_e$ is that since the ob ect is slo ly lo ered an up ard e ternal force e.g. due to the hand must have been e erted on the ob ect during the lo ering process preventing it from accelerating do n ard. his force does *negative* ork on the ob ect reducing the total mechanical energy of the system.

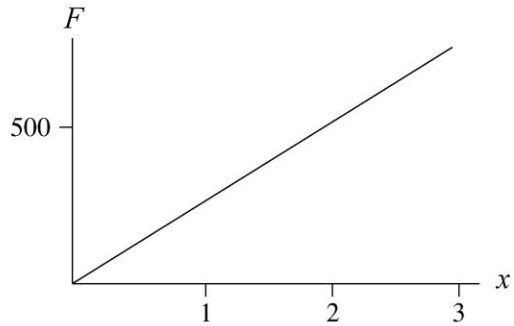
132. (a) The compression is "spring like" so the maximum force relates to the distance x by ooke s la

$$F = kx \Rightarrow x = \frac{750}{2.5\times10^5} = 0.0030\,\text{m}.$$

(b) The work is what produces the "spring like" potential energy associated with the compression. hus using . 8 11

$$W = \frac{1}{2}kx^2 = \frac{1}{2}\;2.5\times10^5\;\;0.0030\;^2 = 1.1\;.$$

c y e ton s third la the force F e erted by the tooth is e ual and opposite to the "spring like" force exerted by the licorice, so the graph of F is a straight line of slope k. e plot F in ne tons versus x in millimeters both are taken as positive.

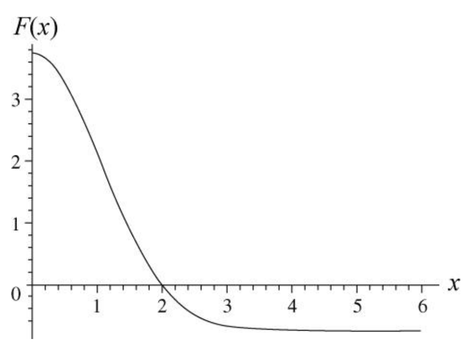


d s mentioned in part b the spring potential energy e pression is relevant. o hether or not e can ignore dissipative processes is a deeper uestion. n other ords it seems unlikely that — if the tooth at any moment ere to reverse its motion — that the licorice could "spring back" to its original shape. Still, to the extent that $U = \frac{1}{2}kx^2$ applies the graph is a parabola not sho n here hich has its verte at the origin and is either concave up ard or concave do n ard depending on ho one ishes to define the sign of F the connection being $F = -dU\ dx$.

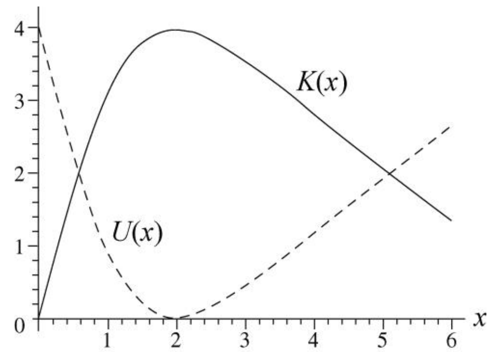
e s a crude estimate the area under the curve is roughly half the area of the entire plotting area 8000 by 12 mm . his leads to an appro imate ork of $\frac{1}{2}$ 8000 0.012 m $\approx$ 50 . stimates in the range $40 \le W \le 50$ are acceptable.

f ertainly dissipative effects dominate this process and e cannot assign it a meaningful potential energy.

133. a he force units understood from . 8 20 is plotted in the graph belo .



b he potential energy U and the kinetic energy $K\ x$ are sho n in the ne t. he potential energy curve begins at 4 and drops until about $x = 2$ the kinetic energy curve is the one that starts at ero and rises until about $x = 2$.



134. he style of reasoning used here is presented in ection 8 5.

a he hori ontal line representing E_1 intersects the potential energy curve at a value of $r \approx 0.07$ nm and seems not to intersect the curve at larger r though this is some hat unclear since U r is graphed only up to $r = 0.4$ nm . hus if m ere propelled to ards M from large r ith energy E_1 it would "turn around" at 0.07 nm and head back in the direction from hich it came.

b he line representing E_2 has t o intersection points $r_1 \approx 0.16$ nm and $r_2 \approx 0.28$ nm ith the U r plot. hus if m starts in the region r_1 r r_2 ith energy E_2 it ill bounce back and forth bet een these t o points presumably forever.

c t $r = 0.3$ nm the potential energy is roughly $U = -1.1 \times 10^{-19}$.

d ith M m the kinetic energy is essentially ust that of m. ince $E = 1 \times 10^{-19}$ its kinetic energy is $K = E - U \approx 2.1 \times 10^{-19}$.

e ince force is related to the slope of the curve e must crudely estimate $|F| \approx 1 \times 10^{-9}$ at this point. he sign of the slope is positive so by . 8 20 the force is negative valued. his is interpreted to mean that the atoms are attracted to each other.

f ecalling our remarks in the previous part e see that the sign of F is positive meaning it s repulsive for r 0.2 nm.

g nd the sign of F is negative attractive for r 0.2 nm.

h t $r = 0.2$ nm the slope hence F vanishes.

135. he distance traveled up the incline can be calculated using the kinematic e uations discussed in hapter 2

$$v^2 = v_0^2 + 2a\Delta x \rightarrow \Delta x = 200 \text{ m}.$$

his corresponds to an increase in height e ual to $y =$ 200 m $\sin\theta = 17$ m here $\theta = 5.0°$. e take its initial height to be $y = 0$.

a . 8 24 leads to

$$W_{\text{app}} = \Delta E = \frac{1}{2}m\left(v^2 - v_0^2\right) + mgy \ .$$

herefore $\Delta E = 8.6 \times 10^3$.

b rom the above manipulation e see W_{app} = 8.6 × 10^3 . lso from hapter 2 e kno that $\Delta t = \Delta v/a = 10$ s. hus using . 7 42

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{8.6 \times 10^3}{10} = 860$$

here the ans er has been rounded off from the 856 value that is provided by the calculator .

c and d aking into account the component of gravity along the incline surface the applied force is ma mg sin θ = 43 and clearly in the direction of motion so . 7 48 provides the results for instantaneous po er

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 430 & \text{for } v = 10 \text{ m s} \\ 1300 & \text{for } v = 30 \text{ m s} \end{cases}$$

here these ans ers have been rounded off from 428 and 1284 respectively . e note that the average of these t o values agrees ith the result in part b .

136. a onservation of mechanical energy leads to

$$K_i + U_i = K_f + U_f \quad \Rightarrow \quad 0 + \frac{1}{2}ky_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}k \ \ y_f - y_i \ \ ^2 + mgy_f$$

here $y_i = 0.25$ m is the initial depression of the spring and $y_f - y_i$ is the displacement of the spring from its e uilibrium position hen the block is at y_f. hus the kinetic energy of the block can be ritten as

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}k\left[y_i^2 - \ \ y_f - y_i \ \ ^2\right] - mgy_f \ .$$

or $y_f = 0$ the kinetic energy is $K_f = 0$ as e pected since this corresponds to the initial release point.

b t $y_f = 0.050$ m e have

$$K_f = \frac{1}{2}k\left[y_i^2 - \ y_f - y_i\ ^2\right] - mgy_f$$
$$= \frac{1}{2}\ 620 \quad \text{m}\left[\ 0.250\ \text{m}\ ^2 - \ 0.050\ \text{m} - 0.250\ \text{m}\ ^2\right] - \ 50 \quad 0.050\ \text{m}\ = 4.48$$

c t $y_f = 0.100$ m e have

$$K_f = \frac{1}{2}k\left[y_i^2 - \ y_f - y_i\ ^2\right] - mgy_f$$
$$= \frac{1}{2}\ 620 \quad \text{m}\left[\ 0.250\ \text{m}\ ^2 - \ 0.100\ \text{m} - 0.250\ \text{m}\ ^2\right] - \ 50 \quad 0.100\ \text{m}\ = 7.40$$

d imilarly the kinetic energy at $y_f = 0.150$ m is

$$K_f = \frac{1}{2}k\left[y_i^2 - \ y_f - y_i\ ^2\right] - mgy_f$$
$$= \frac{1}{2}\ 620 \quad \text{m}\left[\ 0.250\ \text{m}\ ^2 - \ 0.150\ \text{m} - 0.250\ \text{m}\ ^2\right] - \ 50 \quad 0.150\ \text{m}\ = 8.78$$

e t $y_f = 0.200$ m the kinetic energy of the block is

$$K_f = \frac{1}{2}k\left[y_i^2 - \ y_f - y_i\ ^2\right] - mgy_f$$
$$= \frac{1}{2}\ 620 \quad \text{m}\left[\ 0.250\ \text{m}\ ^2 - \ 0.200\ \text{m} - 0.250\ \text{m}\ ^2\right] - \ 50 \quad 0.200\ \text{m}\ = 8.60$$

f he spring returns to its uncompressed state once $y_f \geq y_i$. ince the block becomes detached from the spring beyond that point at its ma imum height $K = 0$ and e have

$$\frac{1}{2}ky_i^2 = mgy_{\text{ma}} \quad \Rightarrow \quad y_{\text{ma}} = \frac{ky_i^2}{2mg} = \frac{620 \quad \text{m}\ \ 0.250\ \text{m}\ ^2}{2\ 50} = 0.388\ \text{m}.$$