CH6 Static Magnetic Field

- Electric force $\vec{F} = q\vec{E}(N)$
- Magnetic force $\vec{F}_m = q\vec{u} \times \vec{B}(N)$
- Elecmagnetic fore $\vec{F} = q(\vec{E} + \vec{u} \times \vec{B})$

(N) ~ Lorentz's force equation

Free space

- Static Electric Field

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = 0$$

- Static Magnetic Field

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_o \vec{J}$$

$\vec{\nabla} \cdot \vec{J} = 0$ Steady current

$\mu_o = 4\pi \times 10^{-7} (Henry/m)$

Permeability of free space

Electromagnetic

$$\vec{\nabla}\cdot\vec{B}=0 \quad\Rightarrow\quad \oint_s \vec{B}\cdot d\vec{s}=0$$

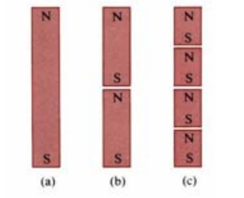


FIGURE 6–1
Successive division of a bar magnet.

- No magnetic flow sources
- Magnetic flux lines always close
- Low of conservation of magnetic flux

- Each magnets has a north pole south
- Magnetic poles cannot be isolated

Electromagnetic

$$\vec{\nabla}\times\vec{B}=\mu_o\vec{J} \Rightarrow \int_s(\vec{\nabla}\times\vec{B})\cdot d\vec{s}=\mu_o\int_s\vec{J}\cdot d\vec{s}$$

$$\oint_c \vec{B}\cdot d\vec{\ell}=\mu_o I$$ Ampere's circuital law

summary

$$\vec{\nabla}\cdot\vec{B}=0 \qquad \oint_s \vec{B}\cdot d\vec{s}=0$$

$$\vec{\nabla}\times\vec{B}=\mu_o\vec{J} \qquad \oint_c \vec{B}\cdot d\vec{\ell}=\mu_o I$$

EX 6-1

Electromagnetic

- Inside conductor

$$\vec{B}_1=\hat{a}_\phi B_{\phi1} \qquad d\vec{\ell}=\hat{a}_\phi r_1 d\phi$$

$$\oint_{c1}\vec{B}_1 d\vec{\ell}=\int_0^{2\pi}B_{\phi1}r_1 d\phi=2\pi r_1 B_{\phi1}$$

$$I_1=\frac{I}{\pi b^2}\pi r_1^2=\left(\frac{r_1}{b}\right)^2 I$$

$$\vec{B}_1=\hat{a}_\phi B_{\phi1}=\hat{a}_\phi\frac{\mu_o r_1 I}{2\pi b^2} \quad r_1\le b$$

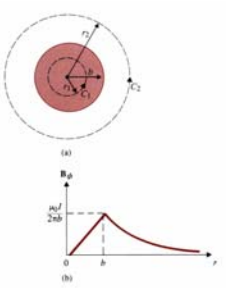
- Outside conductor

$$\vec{B}_2=\hat{a}_\phi B_{\phi2} \qquad d\vec{\ell}=\hat{a}_\phi r_2 d\phi$$

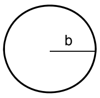
$$\oint_{c2}\vec{B}_2 d\vec{\ell}=2\pi r_2 B_{\phi2}$$

C_2 outside conductor encloses I

$$\vec{B}_2=\hat{a}_\phi B_{\phi2}=\hat{a}_\phi\frac{\mu_o I}{2\pi r_2} \quad r_2\ge b$$



If



空心圓柱(柱座標)

$\vec{J}_s = \hat{a}_z J_s (A/m)$

$I = 2\pi b J_s$

$$B = \begin{cases} 0 & r < b \\ \hat{a}_\phi \dfrac{\mu_o b}{r} J_s & r > b \end{cases}$$

EX 6-2 (Toroidal Coil)

A circular contour C with radius r

(b - a) < r < (b + a)

$\oint \vec{B} \cdot d\vec{\ell} = 2\pi r B_\phi = \mu_o NI$

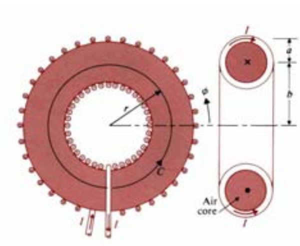
$(1)\vec{B} = \hat{a}_\phi B_\phi = \hat{a}_\phi \dfrac{\mu_o NI}{2\pi r}$

(b - a) < r < (b + a)

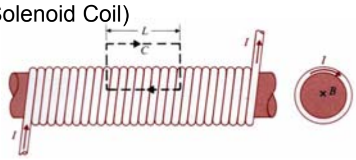
$(2)\vec{B} = 0 \quad r < (b-a)$

$\& \, r > (b+a)$

(No source)



EX 6-3 (Solenoid Coil)



(a) Direct application of Ampere Law

$BL = \mu_o nLI$

$B = \mu_o nI$

(b) Special case of torid Ex 6-2, $b \to \infty$

$B = \mu_o \left(\dfrac{N}{2\pi b} \right) I$

$B = \mu_o nI$

6-3 Vector Magnetic Potential

$\boxed{\nabla \cdot \vec{B} = 0} \Rightarrow \vec{B} = \nabla \times \vec{A},$ $\quad \vec{A}$: magnetic potential [Vector]

c.f $\boxed{\nabla \times \vec{E} = 0} \Rightarrow \vec{E} = -\nabla \phi,$ $\quad \phi$: electric potential [Scalar]

$$\nabla \times \vec{B} = \mu_o \vec{J}$$

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_o \vec{J}$$

取 $\boxed{\nabla \cdot \vec{A} = 0} \Rightarrow \boxed{\nabla^2 \vec{A} = -\mu_o \vec{J}}$

Coulomb gauge

Vector Poisson's equation

In Cartesian coordinates,

$$\begin{cases} \nabla^2 A_x = -\mu_o J_x \\ \nabla^2 A_y = -\mu_o J_y \\ \nabla^2 A_z = -\mu_o J_z \end{cases} \Rightarrow A_x = \frac{\mu_o}{4\pi} \int_{u'} \frac{J_x}{r} du' \Rightarrow \boxed{\vec{A} = \frac{\mu_o}{4\pi} \int_{u'} \frac{\vec{J}}{r} du' \left(Wb/m \right)}$$

Vector

$$c.f. \quad \nabla^2 \phi = -\frac{\rho}{\varepsilon_o} \Rightarrow \phi = \frac{\mu_o}{4\pi} \int_{u'} \frac{\rho}{r} du'$$

Magnetic Flux Φ through a given area S which is bounded by contour C

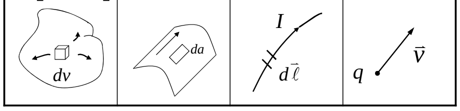
$$\Phi = \int_s \vec{B} \cdot d\vec{s} \qquad (Web)$$

$$\Phi = \int_s (\nabla \times \vec{A}) \cdot d\vec{s} = \oint_c \vec{A} \cdot d\vec{\ell} \qquad (Web)$$

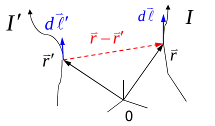
6-4 Biot-Savart Law and applications

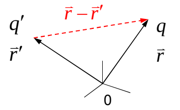
Magnetic: Vector source

	3-dim	2-dim	1-dim	0-dim
Current distribution	Volume current density	Surface current density	current	
	$\vec j\left[\frac{coul}{\sec\cdot m^2}\right]$	$\vec j_s\left[\frac{coul}{\sec\cdot m}\right]$	$I\left[\frac{coul}{\sec}\right]$	$q[coul]$
Current element	$\vec j dv\left[\frac{coul}{\sec}\cdot m\right]$	$\vec j_s da\left[\frac{coul}{\sec}\cdot m\right]$	$Id\vec\ell\left[\frac{coul}{\sec}\cdot m\right]$	$q\vec v\left[\frac{coul}{\sec}\cdot m\right]$



Biot-Savart Law：[Valid in steady current]





$d\vec F = \frac{\mu_o}{4\pi} I d\vec\ell \times \left[I' d\vec\ell' \times \frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3}\right]$ c.f. $\vec F_q = \frac{1}{4\pi\varepsilon_o} qq' \frac{\vec r - \vec r'}{|\vec r - \vec r'|^3}$

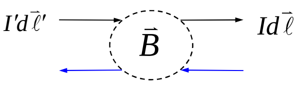
因次分析 $[\mu_o]\cdot v^2 = \frac{1}{[\varepsilon_0]}$

Biot-Sarvart Law：

$$d\vec B = \frac{\mu_o}{4\pi} I' d\vec\ell' \times \frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3}$$

$$d\vec F = I d\vec\ell \times d\vec B$$

Action at a distance：$\vec B$ field



$\oint_s \vec{B}\cdot d\vec{a} = 0$ ← Gauss thm. → $\vec{\nabla}_{\vec{r}}\cdot\vec{B}(\vec{r}) = 0$

特殊式

$\vec{B} = \vec{\nabla}_{\vec{r}}\times\vec{A}(\vec{r})$

$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi}\int_{a.s.}\frac{\vec{j}(\vec{r}')}{|\vec{r}-\vec{r}'|}dv'$

Biot-Sarvart Law

$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\int_{a.s.}\vec{j}(\vec{r}')\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dv'$

$\vec{\nabla}^2_{\vec{r}}\vec{A}(\vec{r}) = -\mu_o\vec{j}$

Poission Eqe.

$\oint_p \vec{B}\cdot d\vec{\ell} = \mu_o\int_s \vec{j}\cdot d\vec{a}$ ← Stoke Thm. ← $\vec{\nabla}_{\vec{r}}\times\vec{B}(\vec{r}) = \mu_o\vec{j}(\vec{r})$

Ampere's Law

$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\int_{a.s.}\vec{j}(\vec{r}')\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dv'$$

(1) $\vec{\nabla}_{\vec{r}}\cdot\vec{B}(\vec{r}) = 0$

PF: $\vec{\nabla}_{\vec{r}}\cdot\vec{B} = \frac{\mu_o}{4\pi}\vec{\nabla}_{\vec{r}}\cdot\left[\int_{a.s.}\vec{j}(\vec{r}')\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dv'\right]$

$\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = (\vec{\nabla}\times\vec{A})\cdot\vec{B} - (\vec{\nabla}\times\vec{B})\cdot\vec{A}$

$$= \frac{\mu_o}{4\pi}\int_{a.s.}\vec{\nabla}_{\vec{r}}\cdot\left[\vec{j}(\vec{r}')\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right]dv'$$

$$= \frac{\mu_o}{4\pi}\int_{a.s.}\left\{\underbrace{\left[\vec{\nabla}_{\vec{r}}\times\vec{j}(\vec{r}')\right]}_{=0}\cdot\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} - \underbrace{\left[\vec{\nabla}_{\vec{r}}\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right]}_{=0}\cdot\vec{j}(\vec{r}')\right\}dv'$$

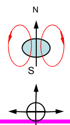
$= 0$

$\vec{\nabla}\times(\frac{\vec{e}_r}{r^2}) = 0$

C.F

$\vec{\nabla}\cdot\vec{B} = 0$ 封閉磁迴路

$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_o}$ 孤立電單極



From $\vec{\nabla}_{\vec{r}}\cdot\vec{B}(\vec{r}) = 0$

(2) $\vec{B} = \vec{\nabla}_{\vec{r}}\times\vec{A}(\vec{r})$ $\quad \vec{A} = \frac{\mu_o}{4\pi}\int_{a.s.}\frac{\vec{j}(\vec{r}')}{|\vec{r}-\vec{r}'|}dv'$

$$\vec{B} = \frac{\mu_o}{4\pi}\int_{a.s.}\vec{j}(\vec{r}')\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dv' \quad \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = -\vec{\nabla}_{\vec{r}}\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$$

$$= \frac{\mu_o}{4\pi}\int_{a.s.}\left\{\vec{\nabla}_{\vec{r}}\times\left(\frac{\vec{j}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right) - \frac{1}{|\vec{r}-\vec{r}'|}\left(\vec{\nabla}_{\vec{r}}\times\vec{j}(\vec{r}')\right)\right\}dv' \quad (\vec{\nabla}_{\vec{r}}\times\vec{j}(\vec{r}') = 0)$$

$$= \vec{\nabla}_{\vec{r}}\times\left\{\frac{\mu_o}{4\pi}\int_{a.s.}\left(\frac{\vec{j}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right)dv'\right\}$$

$\vec{\nabla}\times(f\vec{A}) = (\vec{\nabla}f)\times\vec{A} + f(\vec{\nabla}\times\vec{A})$

$= \vec{\nabla}_{\vec{r}}\times\vec{A}(\vec{r})$

其中 $\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi}\int_{a.s.}\left(\frac{\vec{j}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right)dv'$

Electromagnetic

From $\vec{\nabla}_{\vec{r}} \times \vec{A}(\vec{r}) = \vec{B}(\vec{r}) \Rightarrow \begin{cases} \vec{\nabla}^2 \vec{A}(\vec{r}) = -\mu_o \vec{j}(\vec{r}) \\ \vec{\nabla} \times \vec{B}(\vec{r}) = \mu_o \vec{j}(\vec{r}) \\ \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - (\vec{\nabla} \cdot \vec{\nabla})\vec{A}(\vec{r}) \end{cases}$

Steady current $\vec{\nabla} \cdot \vec{j}(\vec{r}) = 0$

Static magnetic field

c.f. $\vec{\nabla}^2 \phi = -\frac{\rho}{\varepsilon_o}$, $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_o}$

$$pf: \ \vec{\nabla}_{\vec{r}} \times [\vec{\nabla}_{\vec{r}} \times \vec{A}(\vec{r})] = \vec{\nabla}_{\vec{r}} [\vec{\nabla}_{\vec{r}} \cdot \vec{A}(\vec{r})] - (\vec{\nabla}_{\vec{r}} \cdot \vec{\nabla}_{\vec{r}})\vec{A}(\vec{r})$$

其中：$\vec{\nabla}_{\vec{r}} \cdot \vec{A}(\vec{r}) = \vec{\nabla}_{\vec{r}} \cdot \left\{ \frac{\mu_o}{4\pi} \int_{a.s.} \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} dv' \right\} = \frac{\mu_o}{4\pi} \int_{a.s.} \vec{\nabla}_{\vec{r}} \cdot \left[\frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} \right] dv'$

$$= \frac{\mu_o}{4\pi} \int_{a.s.} \left\{ \vec{j}(\vec{r}') \cdot \vec{\nabla}_{\vec{r}} \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) + \frac{1}{|\vec{r} - \vec{r}'|} \left[\vec{\nabla}_{\vec{r}} \cdot \vec{j}(\vec{r}') \right] \right\} dv'$$

Steady state $\vec{\nabla} \cdot \vec{j} = 0$

$$= \frac{\mu_o}{4\pi} \oint_{s \to \infty} \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} \cdot d\vec{a}' = 0$$

$$\because \lim_{r \to \infty} \frac{1}{|\vec{r} - \vec{r}'|} = 0$$

$\vec{\nabla} \cdot \vec{A} = 0$ Coulomb Gauge

Electromagnetic

$$\vec{\nabla}_{\vec{r}} \cdot \left[\frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} \right] = \vec{\nabla}_{\vec{r}} \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) \cdot \vec{j}(\vec{r}') + \frac{1}{|\vec{r} - \vec{r}'|} \left[\vec{\nabla}_{\vec{r}} \cdot \vec{j}(\vec{r}') \right]$$

$$\vec{\nabla}_{\vec{r}} \times \vec{B}(\vec{r}) = -(\vec{\nabla}_{\vec{r}} \cdot \vec{\nabla}_{\vec{r}})\vec{A}(\vec{r}) = -\vec{\nabla}_{\vec{r}}^2 \vec{A}(\vec{r})$$

其中：

$$\vec{\nabla}_{\vec{r}}^2 \vec{A}(\vec{r}) = \vec{\nabla}_{\vec{r}}^2 \left\{ \frac{\mu_o}{4\pi} \int_{a.s.} \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} dv' \right\} = \frac{\mu_o}{4\pi} \int_{a.s.} \vec{\nabla}_{\vec{r}}^2 \left(\frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} \right) dv'$$

$$= -\mu_o \vec{j}(\vec{r})$$

$$= \vec{j}(\vec{r}')\vec{\nabla}_{\vec{r}}^2 \left(\frac{1}{|\vec{r} - \vec{r}'|} \right)$$

$$= \vec{\nabla}^2 \left(\frac{1}{r} \right) = -4\pi\delta^3(\vec{r})$$

$$\vec{\nabla}^2 \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) = -4\pi\delta^3(\vec{r} - \vec{r}')$$

$\vec{\nabla}_{\vec{r}} \times \vec{B}(\vec{r}) = \mu_o \vec{j}(\vec{r})$: Ampere' Law

$\vec{\nabla}_{\vec{r}}^2 \vec{A}(\vec{r}) = -\mu_o \vec{j}(\vec{r})$: Poission Equ.

Electromagnetic

$$\oint_s \vec{B}(\vec{r}) \cdot d\vec{a} = 0$$

$$pf: \vec{\nabla} \cdot \vec{B}(\vec{r}) = 0$$

Gauss Thm. $\int_v \vec{\nabla} \cdot \vec{B}(\vec{r}) dv = 0$

$$\oint_s \vec{B}(\vec{r}) \cdot d\vec{a} = 0$$

$$\oint_C \vec{B}(\vec{r}) \cdot d\vec{\ell} = \mu_o \int_s \vec{j}(\vec{r}) \cdot d\vec{a}$$

$$pf: \vec{\nabla} \cdot \vec{B}(\vec{r}) = \mu_o \vec{j}(\vec{r})$$

$$\int_s [\vec{\nabla} \cdot \vec{B}(\vec{r})] \cdot d\vec{a} = \mu_o \int \vec{j}(\vec{r}) \cdot d\vec{a}$$

$$\oint_C \vec{B}(\vec{r}) \cdot d\vec{\ell} = \mu_o \int \vec{j}(\vec{r}) \cdot d\vec{a}$$

Example 6-4

(a) $\overline{A}=\hat{a}_z \frac{\mu_o I}{4\pi}\int_{-L}^{L}\frac{dz'}{\sqrt{z'^2+\rho^2}}$

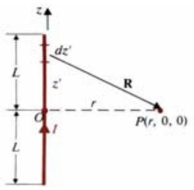
$$=\hat{a}_z \frac{\mu_o I}{4\pi}\left[\ln(z'+\sqrt{z'^2+\rho^2})\right]_{-L}^{L}$$

$$=\hat{a}_z \frac{\mu_o I}{4\pi}\ln\frac{\sqrt{L^2+\rho^2}+L}{\sqrt{L^2+\rho^2}-L}$$

$$\overline{B}=\overline{\nabla}\times\overline{A}=\overline{\nabla}\times(\hat{a}_z A_z)$$

$$=\hat{a}_\rho \frac{1}{\rho}\frac{\partial A_z}{\partial\phi}-\hat{a}_\phi\frac{\partial A_z}{\partial\rho}$$

$$cylindrical \quad sym. \frac{\partial A_z}{\partial\phi}=0$$



$$d\overline{\ell}'=\hat{a}_z dz'$$

$$R=\sqrt{z'^2+\rho^2}$$

$$\overline{B}=-\hat{a}_\phi \frac{\partial}{\partial\rho}\left[\frac{\mu_o I}{4\pi}\ln\frac{\sqrt{L^2+\rho^2}+L}{\sqrt{L^2+\rho^2}-L}\right]$$

$$=\hat{a}_\phi \frac{\mu_o IL}{2\pi\rho\sqrt{L^2+\rho^2}}$$

$if\ \rho \ll L;$

$$\overline{B}=\hat{a}_\phi\frac{\mu_o I}{2\pi\rho}$$

$$\vec{r}-\vec{r}'=\hat{a}_\rho\rho-\hat{a}_z z'$$

$$d\vec{\ell}\times(\vec{r}-\vec{r}')=\hat{a}_z z'\times(\hat{a}_\rho\rho-\hat{a}_z z')$$

$$=\hat{a}_\phi \rho dz'$$

$$\overline{B}=\int d\overline{B}=\hat{a}_\phi\frac{\mu_o I}{4\pi}\int_{-L}^{L}\frac{\rho dz'}{(z'^2+\rho^2)^{3/2}}$$

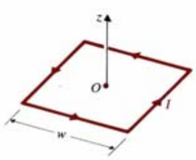
$$=\hat{a}_\phi\frac{\mu_o IL}{2\pi\rho\sqrt{L^2+\rho^2}}$$

Example 6-5

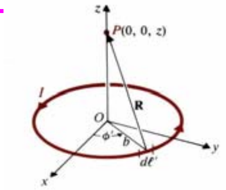
From 6-4

$$L=\rho=w/2$$

$$\overline{B}=\hat{a}_z\frac{\mu_o I}{\sqrt{2}\pi w}\times 4=\hat{a}_z\frac{2\sqrt{2}\mu_o I}{\pi w}$$



Example 6-6



$$d\vec{\ell}' = \hat{a}_\phi b d\phi'$$

$$\vec{R} = \vec{r} - \vec{r}' = z\hat{a}_z - b\hat{a}_\rho$$

$$|\vec{r} - \vec{r}'| = (z^2 + b^2)^{1/2}$$

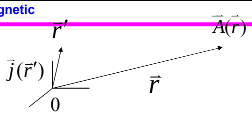
$$d\vec{\ell}' \times |\vec{r} - \vec{r}'| = \hat{a}_\phi b d\phi' \times (z\hat{a}_z - b\hat{a}_\rho)$$

$$= \hat{a}_\rho bz d\phi' + \hat{a}_z b^2 d\phi'$$

$\hat{a}_\rho$ is canceled due to cylindrical sym.

$$\vec{B} = \frac{\mu_o I}{4\pi} \int_0^{2\pi} \hat{a}_z \frac{b^2 d\phi'}{(z^2 + b^2)^{3/2}} = \hat{a}_z \frac{\mu_o I b^2}{2(z^2 + b^2)^{3/2}} (T)$$

6-5 Magnetic Dipole



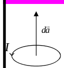
$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int_v \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} dv'$$

$$= \underbrace{\frac{\mu_o}{4\pi} \frac{\int_v \vec{j}(\vec{r}') dv'}{r}}_{2^0 \text{ pole} = 0} + \underbrace{\frac{\mu_o}{4\pi} \frac{\int_v \vec{j}(\vec{r}')(\vec{r}' \cdot \vec{a}_r') dv'}{r^2}}_{2' \text{ pole} = 0} + \ldots$$

$$\nabla \cdot \vec{j}(\vec{r}) = 0 \qquad \int_v \vec{j}(\vec{r}')(\vec{r}' \cdot \vec{a}_r') dv' \equiv \vec{m} \times \hat{a}_r$$

$$\vec{m} = \frac{1}{2} \int_v \vec{r}' \times \vec{j}(\vec{r}') dv' = \text{電流} \cdot \text{面積}$$

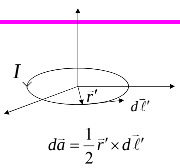
$$\vec{m} = I d\vec{a}$$



$$\vec{m} = \frac{1}{2} \int_{dp} \vec{r}' \times \vec{j}(\vec{r}') dv'$$

$$= \frac{1}{2} \int_{dp} \vec{r}' \times (I d\vec{\ell}') = I \frac{1}{2} \int_{dp} \vec{r}' \times d\vec{\ell}'$$

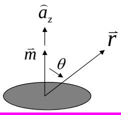
$$= I d\vec{a}$$

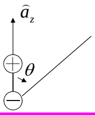


$$d\vec{a} = \frac{1}{2} \vec{r}' \times d\vec{\ell}'$$

$$\vec{A} = \frac{\mu_o}{4\pi} \frac{\vec{m} \times \hat{a}_r}{r^2} = \frac{\mu_o}{4\pi} \frac{m \sin\theta}{r^2} \hat{a}_\varphi$$

c.f. $\phi = \frac{1}{4\pi\varepsilon_o} \frac{\vec{P} \cdot \vec{a}_r}{r^2} = \frac{1}{4\pi\varepsilon_o} \frac{P\cos\theta}{r^2}$





Electromagnetic

$$\vec{B} = \vec{\nabla} \times \vec{A} = \frac{\mu_o}{4\pi} m \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\hat{a}_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ 0 & 0 & r\sin\theta\frac{\sin\theta}{r^2} \end{vmatrix}$$

$$= \frac{\mu_o}{4\pi} m \frac{1}{r^2 \sin\theta} \left[\hat{a}_r \frac{\partial}{\partial\theta}(\frac{\sin^2\theta}{r}) - r\hat{a}_\theta \frac{\partial}{\partial r}(\frac{\sin^2\theta}{r}) \right]$$

$$= \frac{\mu_o}{4\pi} m \frac{\hat{a}_r 2\cos\theta + \hat{a}_\theta \sin\theta}{r^3}$$

$$\frac{\partial}{\partial\theta}(\frac{\sin^2\theta}{r}) = \frac{2\sin\theta\cos\theta}{r}$$

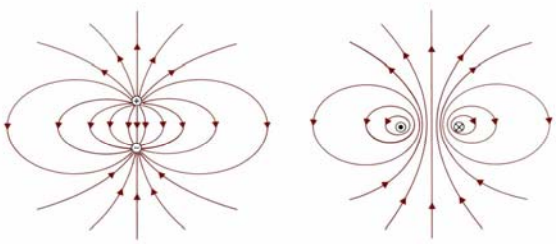
$$\frac{\partial}{\partial r}(\frac{\sin^2\theta}{r}) = -\frac{\sin^2\theta}{r^2}$$

c.f.

$$\vec{E} = -\vec{\nabla}\phi = \frac{1}{4\pi\varepsilon_o}(-1)P\left[\hat{a}_r \frac{\partial}{\partial r}(\frac{\cos\theta}{r^2}) + \hat{a}_\theta \frac{1}{r}\frac{\partial}{\partial\theta}(\frac{\cos\theta}{r^2}) \right]$$

$$= \frac{P}{4\pi\varepsilon_o}\left[\hat{a}_r \frac{2\cos\theta}{r^3} + \hat{a}_\theta \frac{\sin\theta}{r^3} \right]$$

Electromagnetic



(a) Electric dipole. (b) Magnetic dipole.

FIGURE 6–9
Electric field lines of an electric dipole and magnetic flux lines of a magnetic dipole.

Electromagnetic

Scalar Magnetic Potential

$$\vec{\nabla} \times \vec{B} = \mu_o \vec{J} \quad if \quad \vec{J} = 0 \quad \vec{\nabla} \times \vec{B} = 0$$

$\vec{B} = -\mu_o \vec{\nabla}\phi_m$, ϕ_m : Scalar Magnetic Potential

$$\phi_{m2} - \phi_{m1} = -\int_{P_1}^{P_2} \frac{1}{\mu_o} \vec{B} \cdot d\vec{\ell} \Rightarrow \phi_m = \frac{1}{4\pi}\int_{v'} \frac{\rho_m}{r} dv'$$

(not physical)

$$\vec{m} = q_m \vec{d} = \hat{a}_n IS$$

$$\phi_m = \frac{\vec{m} \cdot \hat{a}_r}{4\pi r^2}$$

$if \quad \vec{J} \neq 0,$ $\vec{B}$: Non conservative (path dependent)

6-6 Magnetization and Equivalent Current Density

磁性物質	Components	source
Conductor	free electron	$\vec{j}_f = \sigma \vec{E}$
Non-conductor	polarized ion	$\vec{j}_m = \vec{\nabla} \times \vec{M}$

定義

$$\overline{M} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{n\Delta v} m_k}{\Delta v} \left(A/m \right) = \frac{mag.\ dipole\ monent}{體積}$$

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int_v \overline{M}(\vec{r}') \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dv'$$

$$\frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} = \vec{\nabla}_r \left(\frac{-1}{|\vec{r} - \vec{r}'|} \right) = \vec{\nabla}_{r'} \left(\frac{1}{|\vec{r} - \vec{r}'|} \right)$$

$$\overline{M}(\vec{r}') \times \vec{\nabla}_{r'} \left(\frac{1}{|\vec{r} - \vec{r}'|} \right)$$

$$= \vec{\nabla}_{r'} \times \left[\left(\frac{1}{|\vec{r} - \vec{r}'|} \right) (-\overline{M}(\vec{r}')) \right] + \frac{1}{|\vec{r} - \vec{r}'|} \left[\vec{\nabla}_{r'} \times \overline{M}(\vec{r}') \right]$$

$$\vec{\nabla} \times (f \vec{A}) = \vec{\nabla} f \times \vec{A} + f (\vec{\nabla} \times \vec{A})$$

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{v'} \vec{\nabla}_{\vec{r}'} \times \left[\frac{-\overline{M}(\vec{r}')}{|\vec{r} - \vec{r}'|} \right] dv' + \frac{\mu_o}{4\pi} \int_{v'} \frac{\vec{\nabla}_{\vec{r}'} \times \overline{M}(\vec{r}')}{|\vec{r} - \vec{r}'|} dv'$$

$$= \frac{\mu_o}{4\pi} \oint_{S'} da' \times \left[\frac{-\overline{M}(\vec{r}')}{|\vec{r} - \vec{r}'|} \right] + \frac{\mu_o}{4\pi} \int_{v'} \frac{\vec{\nabla}_{\vec{r}'} \times \overline{M}(\vec{r}')}{|\vec{r} - \vec{r}'|} dv'$$

$$= \frac{\mu_o}{4\pi} \oint_{S'} da' \frac{\overline{M}(\vec{r}') \times \vec{n}'}{|\vec{r} - \vec{r}'|} + \frac{\mu_o}{4\pi} \int_{v'} \frac{\vec{\nabla}_{\vec{r}'} \times \overline{M}(\vec{r}')}{|\vec{r} - \vec{r}'|} dv'$$

$$= \frac{\mu_o}{4\pi} \oint_{S'} da' \frac{\vec{j}_{ms}}{|\vec{r} - \vec{r}'|} + \frac{\mu_o}{4\pi} \int_{v'} \frac{\vec{j}_m}{|\vec{r} - \vec{r}'|} dv'$$

$$\begin{bmatrix} \vec{j}_m = \vec{\nabla} \times \overline{M} \left(A/m^2 \right) \\ \vec{j}_{ms} = \overline{M} \times \hat{a}_n \left(A/m \right) \end{bmatrix}$$

c.f. $\rho_p = -\vec{\nabla} \cdot \vec{P}; \quad \rho_{sp} = \hat{a}_n \cdot \vec{P}$

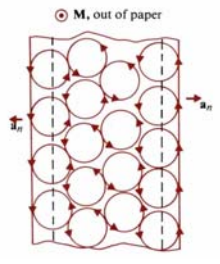


FIGURE 6–10
A cross section of a magnetized material.

EX : 6-8

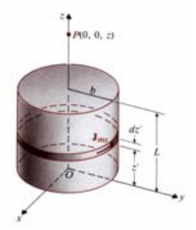
$$\vec{j}_{ms} = \overline{M} \times \hat{a}_n = (M_o \hat{a}_z) \times \hat{a}_r = M_o \hat{a}_\varphi$$

$$\vec{B} = \hat{a}_z \frac{\mu_o I b^2}{2(z^2 + b^2)^{3/2}}$$

$$d\vec{B} = \hat{a}_z \frac{\mu_o M_o b^2 dz'}{2\left[(z - z')^2 + b^2\right]^{3/2}}$$

$$\vec{B} = \int_0^L d\vec{B}$$

$$= \hat{a}_z \frac{\mu_o M_o}{2} \left[\frac{z}{\sqrt{z^2 + b^2}} - \frac{z - L}{\sqrt{(z - L)^2 + b^2}} \right]$$



靜磁學（包含導體與磁性材料）

$$\overline{\nabla} \cdot \overline{B} = 0$$

$$\overline{\nabla} \times \overline{B} = \mu_o (\vec{j}_f + \vec{j}_m)$$

$$= \mu_o (\vec{j}_f + \overline{\nabla} \times \overline{M})$$

$$\overline{\nabla} \times \left[\frac{1}{\mu_o} \overline{B} - \overline{M} \right] = \vec{j}_f$$

$$\overline{\nabla} \times \overline{H} = \vec{j}_f$$

$$\therefore \overline{H} = \frac{1}{\mu_o} \overline{B} - \overline{M} = \frac{1}{\mu_o \mu_r} \overline{B}$$

C.F.

$$\overline{\nabla} \times \overline{E} = 0$$

$$\overline{\nabla} \cdot \overline{E} = \frac{1}{\varepsilon_o} (\rho_f + \rho_p)$$

$$= \frac{1}{\varepsilon_o} (\rho_f - \overline{\nabla} \cdot \overline{P})$$

$$\overline{\nabla} \cdot \left[\varepsilon_o \overline{E} + \overline{P} \right] = \rho_f$$

$$\overline{\nabla} \cdot \overline{D} = \rho_f$$

$$\overline{D} = \varepsilon_o \overline{E} + \overline{P}$$

Static Magnetic

Source : $\vec{j}_f$, μ (permeativity)

$\overline{\nabla} \times \overline{H} = \vec{j}_f$

Conductor : $\overline{H} \rightarrow \overline{B} \rightarrow \Phi_m \rightarrow L$

$\overline{B} = \mu \overline{H}$ $\Phi_m = \int \overline{B} \cdot d\vec{s}$ $\frac{1}{L} = \frac{I_f}{\Phi_m}$

Magnetic material :

$\overline{M} \rightarrow \overline{j_m}$: $\overline{\nabla} \times \overline{M}$

$\overline{M} \rightarrow \overline{j_{ms}}$: $\overline{M} \times \vec{a}_n$

$\overline{M} \rightarrow \rho_m$: $-\overline{\nabla} \cdot \overline{M}$

$\overline{M} \rightarrow \rho_{ms}$: $\overline{M} \cdot \vec{a}_n$

Copy 電學

數學上等效，無物理 → (ρ_m, ρ_{ms})

$$d\phi_m = \frac{\vec{M}\cdot\hat{a}_r}{4\pi r^2}$$

$$\phi_m = \frac{1}{4\pi}\int_{v'}\frac{\vec{M}\cdot\hat{a}_r}{r^2}dv'$$

$$= \frac{1}{4\pi}\oint_{s'}\frac{\vec{M}\cdot\hat{a}_n}{r}ds' + \frac{1}{4\pi}\int_{v'}\frac{-(\vec{\nabla}\times\vec{M})}{r}dv'$$

$$\rho_{ms} = \vec{M}\cdot\hat{a}_n \; ; \; \rho_m = -\vec{\nabla}\cdot\vec{M}$$

◎ $\vec{j}_{ms}$, $\vec{j}_m$, $\vec{A} = \frac{\mu_0}{4\pi}\left[\oint_{s'}\frac{\vec{j}_{ms}}{|\vec{r}-\vec{r}'|}da' + \int_{v'}\frac{\vec{j}_m}{|\vec{r}-\vec{r}'|}dv'\right]$, $\vec{B} = \vec{\nabla}\times\vec{A}$

◎ ρ_{ms} , ρ_m , $\phi_m = \frac{1}{4\pi}\left[\oint_{s'}\frac{\rho_{ms}}{|\vec{r}-\vec{r}'|}da' + \int_{v'}\frac{\rho_m}{|\vec{r}-\vec{r}'|}dv'\right]$, $\vec{H} = -\vec{\nabla}\phi_m \Rightarrow \vec{B} = \frac{1}{\mu}\vec{H}$

Ex 6-9

$$\rho_{ms} = \begin{cases} M_o & top\ face \\ -M_o & bottom\ face \\ 0 & side\ wall \end{cases}$$

$$\rho_m = 0 \qquad inside$$

$$q_m = \pi b^2\rho_{ms} = \pi b^2 M_0$$

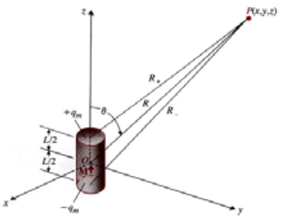
$$\phi_m = \frac{q_m}{4\pi}\left(\frac{1}{R_+} - \frac{1}{R_-}\right)$$

$$R >> b \; [Dipole]$$

$$\phi_m = \frac{q_m L\cos\theta}{4\pi R^2} = \frac{(\pi b^2 M_0)L\cos\theta}{4\pi R^2} = \frac{M_T\cos\theta}{4\pi R^2} \; ; \; M_T = \pi b^2 L M_0$$

$$\vec{B} = -\mu_0\nabla\phi_m = \frac{\mu_0 M_T}{4\pi R^3}\left[\hat{a}_r 2\cos\theta + \hat{a}_\varphi\sin\theta\right]$$

與（Dipole 類似）電場



6-7

$$\vec{\nabla}\times\vec{H} = \vec{J}_f$$

$$\left(\vec{H} = \frac{B}{\mu_0} - \vec{M}\right)$$

$$\int_s(\vec{\nabla}\times\vec{H})\cdot d\vec{S} = \int_s\vec{J}\cdot d\vec{S}$$

$$\oint\vec{H}\cdot d\ell = I$$

Ampere's circuital law

$$\vec{B} = \mu_0(1+\chi_m)\vec{H} = \mu_0\mu_r\vec{H}$$

$$\boxed{\vec{H} = \frac{1}{\mu}\vec{B}} \; ; \; \boxed{u_r = 1+\chi_m = \frac{\mu}{\mu_0}}$$

relative permeability

Electrostatics	Magnetostatics
E	**B**
D	**H**
ϵ	$\frac{1}{\mu}$
P	−**M**
ρ	**J**
V	**A**
·	×
×	·

6-8 Magnetic Circuits

Electric circuit : Voltage / Current source ; V, I, …

Magnetic circuit : Transformer / Generator / Motor …

$\vec{\nabla}\cdot\vec{B}=0$

$\vec{\nabla}\times\vec{H}=\vec{J}$; closed path c to enclose N turns of I

$\oint_c \vec{H}\cdot d\vec{l}=NI=V_m$ (m.m.f) magnetomotive force [Amp]

Ex 6-10

Sol :

$\oint_c \vec{H}\cdot d\vec{l}=NI_o$

$\vec{B}$ is identical in different material ; $\boxed{\vec{\nabla}\cdot\vec{B}=0}$

$\vec{B}_f=\vec{B}_g=\hat{a}_\varphi \vec{B}_f$ f : ferromagnetic, g : gap

$\vec{H}_f=\hat{a}_\varphi\frac{B_f}{\mu};\vec{H}_g=\hat{a}_\varphi\frac{B_f}{\mu_o}$

Ampere law

$\frac{B_f}{\mu}\left(2\pi r_o-l_g\right)+\frac{B_f}{\mu_o}l_g=NI_o$

$\vec{B}_f=\hat{a}_\varphi\frac{\mu_0\mu NI_o}{\mu_0\left(2\pi r_o-l_g\right)+\mu l_g}$

$\vec{H}_f=\hat{a}_\varphi\frac{\mu_0 NI_o}{\mu_0\left(2\pi r_o-l_g\right)+\mu l_g};\vec{H}_g=\hat{a}_\varphi\frac{\mu NI_o}{\mu_0\left(2\pi r_o-l_g\right)+\mu l_g};\frac{H_g}{H_f}=\frac{\mu}{\mu_0}$

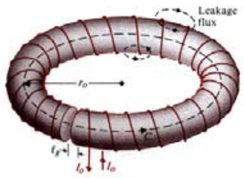


FIGURE 6-13
Coil on ferromagnetic toroid with air gap

Magnetic Flux $\Phi\approx B_f S$; S : cross-section

$B_f=\frac{\mu_0\mu NI_o}{\mu_0\left(2\pi r_o-l_g\right)+\mu l_g}=\frac{NI_o}{\left(\frac{2\pi r_o-l_g}{\mu}\right)+\frac{l_g}{\mu_o}}$

$\Phi=B_f\cdot S=\frac{NI_o}{\left(\frac{2\pi r_o-l_g}{\mu S}\right)+\frac{l_g}{\mu_o S}}=\frac{V_m}{R_f+R_g}$

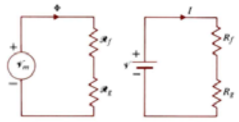
$R_f=\frac{2\pi r_o-l_g}{\mu S}=\frac{l_f}{\mu S};l_f=2\pi r_o-l_g$: length of ferromagnetic core.

$R_g=\frac{l_g}{\mu_o S}$: Reluctance $\begin{cases}R_f & \text{:ferromagnetic core}\\ R_g & \text{:air gap}\end{cases}$

Analog to : [Electric circuit]

$I=\frac{v}{R_f+R_g}$

Electromagnetic



(a) Magnetic circuit. (b) Electric circuit.

FIGURE 6-14
Equivalent magnetic circuit and analogous electric circuit for toroidal coil with air gap in Fig. 6-13.

Magnetic circuit	Electric circuit
$\Phi = \frac{V_m}{R_f + R_g}; R = \frac{l}{\mu S}$	$I = \frac{v}{R_f + R_g}; R = \frac{l}{\sigma S}$
mmf $V_m (= NI)$	**emf** V
mag. flux Φ	**electric current ,** I
reluctance R	**resistance ,** R
Permeability μ	**conductivity ,** σ

Electromagnetic

An exact analysis of magnetic circuits is difficult

◎Leakage Fluxes ◎Fringing effect ◎ $\vec{B} = \mu(\vec{B}, \vec{H})\vec{H}$

2 conditions must be satisfied

$$\left.\begin{aligned} H_g l_g + H_f l_f &= NI_o \\ B_f = B_g &= \mu_o H_g \end{aligned}\right\} \Rightarrow B_f + \mu_o \frac{l_f}{l_g} H_f = \frac{\mu_o}{l_g} NI_o$$

Similar to

Kirchhoff's voltage Law

$$\sum_j N_j I_j = \sum_k R_k \Phi_k$$

Kirchhoff's current Law

$$\sum_j \Phi_j = 0 \qquad \vec{\nabla} \bullet \vec{B} = 0$$

Electromagnetic

EX.6-11

$R_1 = \frac{l_1}{\mu S_c}$

$R_2 = \frac{l_2}{\mu S_c}$

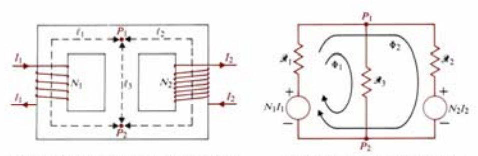
$R_3 = \frac{l_3}{\mu S_c}$

K.V.L. (Time Independent)

Loop1: $N_1 I_1 = (R_1 + R_3)\Phi_1 + R_1 \Phi_2$

Loop2: $N_1 I_1 - N_2 I_2 = R_1 \Phi_1 + (R_1 + R_2)\Phi_2$

$$\Rightarrow \Phi_1 = \frac{R_2 N_1 I_1 + R_1 N_2 I_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$



(a) Magnetic core with current-carrying windings. (b) Magnetic circuit for loop analysis.

FIGURE 6-15
A magnetic circuit (Example 6-11).

6-9 Behavior of Magnetic Materials

$\vec{M} = \chi_m \vec{H}$, χ_m : magnetic susceptibility

$\vec{H} = \frac{1}{\mu}\vec{B}$, $\mu_r = 1 + \chi_m = \frac{\mu}{\mu_o}$

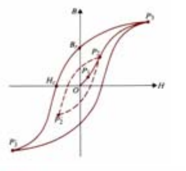


FIGURE 6–17
Hysteresis loops in the B–H plane for ferromagnetic material.

◎Diamagetic: $\mu_r \le 1$ (χ_m : small negative number)

◎Paramagnetic: $\mu_r \ge 1$ (χ_m : small positive number)

◎Ferromagnetic: $\mu_r >> 1$ (χ_m : large positive number)

6-10 Boundary Conditions for Magnetostatic Field

$\vec{\nabla} \bullet \vec{B} = 0$

$B_{1n} = B_{2n}$

$\mu_1 H_{1n} = \mu_2 H_{2n}$

$\vec{\nabla} \times \vec{H} = \vec{J}$

$\oint_C \vec{H} \cdot d\vec{l} = I \quad (bc = da = \Delta h \to 0)$

$\oint_{abcda} \vec{H} \cdot d\vec{l} = \vec{H}_1 \cdot \Delta\vec{W} + \vec{H}_2 \cdot (-\Delta\vec{W}) = J_{sn}\Delta W$

$\Rightarrow H_{1t} - H_{2t} = J_{sn}$

$\hat{a}_{n2} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_S$

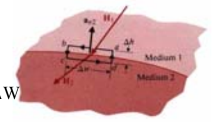


FIGURE 6–19
Closed path about the interface of two media for determining the boundary condition of H_t.

Ex 6-12

B_n component

$\mu_2 H_2 \cos\alpha_2 = \mu_1 H_1 \cos\alpha_1$

H_t component

$H_2 \sin\alpha_2 = H_1 \sin\alpha_1$

$\frac{\tan\alpha_2}{\tan\alpha_1} = \frac{\mu_2}{\mu_1}$

or $\alpha_2 = \tan^{-1}(\frac{\mu_2}{\mu_1}\tan\alpha_1)$

Magnitude of $\vec{H}_2$

$H_2 = \sqrt{H^2_{2t} + H^2_{2n}} = \sqrt{(H_2 \sin\alpha_2)^2 + (H_2 \cos\alpha_2)^2}$

$= H_1\left[\sin^2\alpha_1 + (\frac{\mu_1}{\mu_2}\cos\alpha_1)^2\right]^{1/2}$

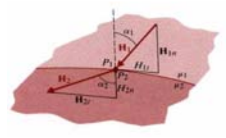


FIGURE 6–20
Boundary conditions for magnetostatic field at an interface (Example 6–12).

Similar to E-field

$\mu_2 \gg \mu_1$, $\alpha_2 = 90^o$

$\mu_1 \gg \mu_2$, $\alpha_2 = 0^o$

$\vec{H}$ In ferromagnetic parallel interface

$\vec{H}$ Originates in a ferromangeitc ,

Flux perpendicular to interface

Ex 6-13

Surface current $\vec{J}_{ms} = M_0 \hat{a}_\varphi$

Example 6-8 [p246]

$$\vec{B}_{po} = \hat{a}_z \frac{\mu_0 M_0}{2}\left[\frac{L}{\sqrt{(L/2)^2 + b^2}}\right]$$

$$\vec{B}_{p1} = \hat{a}_z \frac{\mu_0 M_0}{2}\left[\frac{L}{\sqrt{(L)^2 + b^2}}\right] = \vec{B}_{p'1}$$

$$\underbrace{\vec{B}_{p1} = \vec{B}_{p'1}}_{\text{End}} < \underbrace{\vec{B}_{po}}_{\text{Center}}$$

at interface quantity

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

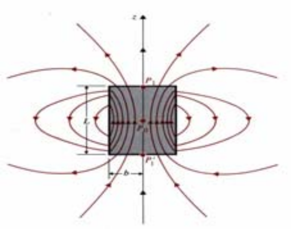


FIGURE 6–21
Magnetic flux lines around a cylindrical bar magnet (Example 6–13).

6-11 Inductances & Inductors

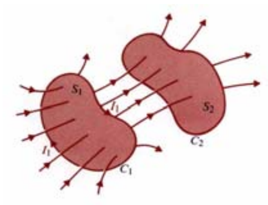


FIGURE 6–22
Two magnetically coupled loops.

Mutual flux $\Phi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{S}_2$

$\Phi_{12} = L_{12} I_1$

L_{12} : mutual inductance between loops C_1 and C_2

If loop C_2 has N_2 turns ,

$\Lambda_{12} = N_2 \Phi_{12}$

Generalizes to

$\Lambda_{12} = L_{12} I_1$

$$\boxed{L_{12} = \frac{\Lambda_{12}}{I_1}} \Longrightarrow \boxed{L_{12} = \frac{d\Lambda_{12}}{dI_1}\ (H)}$$

Some of $\vec{B}$ produced by I_1 links only with C_1 loop itself, not with C_2

$\Lambda_{11} = N_1 \Phi_{11} > N_1 \Phi_{12}$

Self inductance of C_1 loop

$$\boxed{L_{11} = \frac{\Lambda_{11}}{I_1}} \Longrightarrow \boxed{L_{11} = \frac{d\Lambda_{11}}{dI_1}}$$

Procedure for Finding Inductance

1. Appropriate coordinate system
2. Find $\vec{B} = \frac{\mu_0}{4\pi}\int_{V'} \vec{J}(\vec{r}') \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dv'$
3. $\Phi = \int_S \vec{B} \cdot d\vec{S}$
4. $\Lambda = N\Phi$
5. $L = \frac{\Lambda}{I}$

Electromagnetic

EX 6-14

$$\vec{B} = B_\varphi \hat{a}_\varphi$$

$$d\vec{l} = r d\varphi \hat{a}_\varphi$$

$$\oint_C \vec{B}\cdot d\vec{l} = \int_0^{2\pi} B_\varphi r d\varphi = 2\pi r B_\varphi$$

total current NI

$$2\pi r B_\varphi = \mu_0 NI$$

$$B_\varphi = \frac{\mu_0 NI}{2\pi r}$$

$$\Phi = \int_S \vec{B}\cdot d\vec{s} = \int_S \left(\hat{a}_\varphi \frac{\mu_0 NI}{2\pi r}\right)\cdot(\hat{a}_\varphi h dr) = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$$

flux linkage

$$\wedge = N\Phi = \frac{\mu_0 N^2 Ih\cdot\ln\left(\frac{b}{a}\right)}{2\pi}$$

$$L = \frac{\wedge}{I} = \frac{\pi_0 N^2 h\cdot\ln\left(\frac{b}{a}\right)}{2\pi}$$

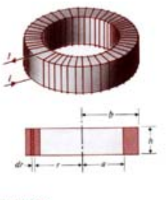


FIGURE 6-23
A closely wound toroidal coil (Example 6-14).

Electromagnetic

EX 6-15 Long solenoid

From(Ex6-3) p231

$$B = \mu_0 nI$$

$$\Phi = BS = \mu_0 nIS$$

$$\wedge' = n\Phi = \mu_0 n^2 SI$$

Inductance per unit length

$$L' = \mu_0 n^2 s$$

$$l >> s$$

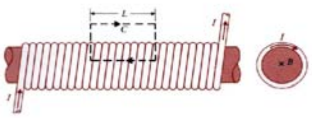


FIGURE 6-4
A current-carrying long solenoid (Example 6-3).

$$L \;\alpha\; N^2$$

$in\ Ex6-14$

$Ex6-15$

Electromagnetic

EX 6-16

a) Inside inner conductor.

$$0 \le r \le a$$

$$\vec{B}_1 = \hat{a}_\varphi B_{\varphi 1} = \hat{a}_\varphi \frac{\mu_0 rI}{2\pi a^2}$$

b) Between inner & outer conductors

$$a \le r \le b$$

$$\vec{B}_2 = \hat{a}_{\varphi 2} B_{\varphi 2} = \hat{a}_\varphi \frac{\mu_0 I}{2\pi r}$$

$$d\Phi' = \int_r^a B_{\varphi 1} dr + \int_a^b B_{\varphi 2} dr = \frac{\mu_0 I}{2\pi a^2}\int_r^a r dr + \frac{\mu_0 I}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I}{4\pi a^2}(a^2 - r^2) + \frac{\mu_0 I}{2\pi}\ln(\frac{b}{a})$$

Current in annular ring

$$\left(\frac{2\pi r dr}{\pi a^2}\right) \Rightarrow \frac{2rdr}{a^2}$$

$$d\wedge' = \frac{2rdr}{a^2} d\Phi'$$

$$\wedge' = \int_{r=0}^{r=a} d\wedge' = \frac{\mu_0 I}{\pi a^2}\left[\frac{1}{2a^2}\int_0^a (a^2 - r^2) r dr + (\ln\frac{b}{a})\int_0^a r dr\right] = \frac{\mu_0 I}{2\pi}\left(\frac{1}{4} + \ln\frac{b}{a}\right)$$

$$L' = \frac{\wedge'}{I} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln(\frac{b}{a}) \ [H/m]$$

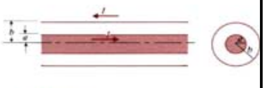


FIGURE 6-24
Two views of a coaxial transmission line (Example 6-16).

Electromagnetic

EX 6-17

Internal $L'_{self} = \frac{\mu_0}{8\pi}$

2 wires : $L'_i = 2\cdot\frac{\mu_0}{8\pi} = \frac{\mu_0}{4\pi}$

external : $xz-plane,\ only\ y-comp.$

$$B_{y1} = \frac{\mu_0 I}{2\pi x}$$

$$B_{y2} = \frac{\mu_0 I}{2\pi(d-x)}$$

$$\Phi' = \int_a^{d-a} (B_{y1}+B_{y2})dx$$

$$= \int_a^b \frac{\mu_0 I}{2\pi}\left[\frac{1}{x}+\frac{1}{d-x}\right]dx$$

$$= \frac{\mu_0 I}{\pi}\ln(\frac{d-a}{a}) \cong \frac{\mu_0 I}{\pi}\ln(\frac{d}{a})$$

$$L'_e = \frac{\Phi'}{I} = \frac{\mu_0}{\pi}\ln(\frac{d}{a})$$

total

$$L' = L'_i + L'_e = \frac{\mu_0}{\pi}\left[\frac{1}{4}+\ln\left(\frac{d}{a}\right)\right]$$

FIGURE 6-25
A two-wire transmission line (Example 6-17).

Electromagnetic

$L_{12} = L_{21}$?

$$L_{12} = \frac{N_2}{I_1}\int_{S_2}\overline{B}_1 \bullet d\overline{S}_2$$

$$(\overline{B}_1 = \overline{\nabla}\times\overline{A}_1)$$

$$L_{12} = \frac{N_2}{I_1}\int_{S_2}(\overline{\nabla}\times\overline{A}_1)\bullet d\overline{S}_2$$

$$= \frac{N_2}{I_1}\oint_{C_1}\overline{A}_1 \bullet d\overline{\ell}_2$$

$$(\overline{A}_1 = \frac{\mu_o}{4\pi}N_1 I_1 \oint_{C_1}\frac{d\overline{\ell}_1}{R})$$

$$\Rightarrow L_{12} = \frac{\mu_o N_1 N_2}{4\pi}\oint_{C_1}\oint_{C_2}\frac{d\overline{\ell}_1\cdot d\overline{\ell}_2}{R}$$

Neumann Formula

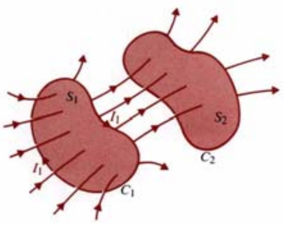


FIGURE 6-22
Two magnetically coupled loops.

Electromagnetic

EX : 6-18

$$\Phi_{12} = \mu(\frac{N_1}{\ell_1})(\pi a^2)I_1$$

Outer coil has N_2 turns,

$$\wedge_{12} = N_2\Phi_{12} = \frac{\mu}{\ell_1}N_1N_2\pi a^2 I_1$$

$$L_{12} = \frac{\wedge_{12}}{I_1} = \frac{\mu}{\ell_1}N_1N_2\pi a^2$$

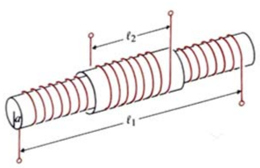


FIGURE 6-26
A solenoid with two windings (Example 6-18).

EX : 6-19

Find B_2 is caused by long wire I_2.

$$\vec{B}_2 = \hat{a}_\varphi \frac{\mu_o I_2}{2\pi r}$$

$$\wedge_{21} = \Phi_{21},$$

$$\wedge_{21} = \int_{S_1} \vec{B}_2 \bullet d\vec{s}_1$$

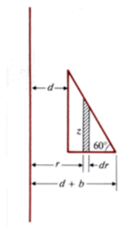
$$d\vec{s}_1 = \hat{a}_\varphi z dr$$

$$*z = \left[(d+b) - r\right]$$

$$\wedge_{21} = \frac{\sqrt{3}\mu_o I_2}{2\pi} \int_d^{d+b} \frac{1}{r}\left[(d+b) - r\right] dr$$

$$= \frac{\sqrt{3}\mu_o I_2}{2\pi}\left[(d+b)\ln\left(1+\frac{b}{d}\right) - b\right]$$

$$L_{21} = \frac{\wedge_{21}}{I_2} = \frac{\sqrt{3}\mu_o}{2\pi}\left[(d+b)\ln\left(1+\frac{b}{d}\right) - b\right]$$



6-12 Magnetic Energy

Loop 1 $V_1 = L_1 \dfrac{di_1}{dt}$

$$W_1 = \int V_1 \, i_1 dt$$

$$= L_1 \int_0^{I_1} i_1 di_1$$

$$= \frac{1}{2} L_1 I_1^2 = \frac{1}{2}\Phi_1 L_1$$

Loop 2 : C_1 & C_2

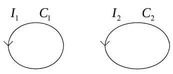
$$W_{21} = \int V_{21} I_1 dt$$

$$= L_{21} I_1 \int_0^{I_2} di_2$$

$$= L_{21} I_1 I_2$$

Similary

$$W_{22} = \frac{1}{2} L_2 I_2^2$$



Total work at C_2

$$W_2 = W_1 + W_{12} + W_{22}$$

$$= \frac{1}{2} L_1 I_1^2 + L_1 I_1 I_2 + \frac{1}{2} L_2 I_2^2$$

$$= \frac{1}{2}\sum_{j=1}^{2}\sum_{k=1}^{2} L_{jk} I_j I_k$$

$$W_m = \frac{1}{2} L I^2$$

Generalizing $I_1, I_2, I_3, \ldots I_N$,

$$W_m = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} L_{jk} I_j I_k$$

Consider K^{th} loop of N coupled loops

$$dW_k = V_k i_k dt$$

$$= i_k d\varphi_k$$

$$V_k = \frac{d\varphi_k}{dt}$$

Magnetic energy

$$dW_m = \sum_{k=1}^{N} dW_k = \sum_{k=1}^{N} i_k d\varphi_k$$

Total magnetic energy $i_k = \alpha I_k$ $\quad \phi_k = \alpha \Phi_k$

$$W_m = \int dW_m = \sum_{k=1}^{N} I_k \Phi_k \int_0^1 \alpha d\alpha$$

$$= \frac{1}{2}\sum_{k=1}^{N} I_k \Phi_k$$

$$W_m = \frac{1}{2}\sum_{k=1}^{N} I_k \Phi_k$$

$$\Phi_k = \sum_{j=1}^{N} L_{jk} I_j$$

6-12.1 Wm in terms of Field Quantities

$$\Phi_k = \int_{S_k} \overline{B}\cdot\hat{a}_n dS'_k = \oint_{C_k} \overline{A}\cdot d\ell_k'$$

$$W_m = \frac{1}{2}\sum_{k=1}^{N} \Delta I_k \oint_{C_k} \overline{A}\cdot d\ell_k'$$

$$\Delta I_k d\ell_k' = J(\Delta a_k')d\ell_k' = \overline{J}\Delta v_k'$$

$$N \to \infty,\ \Delta v_k' \to dv'$$

$$W_m = \frac{1}{2}\int_{v'} \overline{A}\cdot\overline{J} dv'$$

Vector identity

$$\overline{\nabla}\cdot(\overline{A}\times\overline{B}) = \overline{B}\cdot(\overline{\nabla}\times\overline{A}) - \overline{A}\cdot(\overline{\nabla}\times\overline{B})$$

取 $\overline{A}=\overline{A}$; $\overline{B}=\overline{H}$

$$\Rightarrow \overline{A}\cdot(\overline{\nabla}\times\overline{B}) = \overline{H}\cdot(\overline{\nabla}\times\overline{A}) - \overline{\nabla}\cdot(\overline{A}\times\overline{H})$$

其中 $\overline{\nabla}\times\overline{H} = \overline{J}$ $\quad \overline{\nabla}\times\overline{A} = \overline{B}$

$$\Rightarrow \overline{A}\cdot\overline{J} = \overline{H}\cdot\overline{B} - \overline{\nabla}\cdot(\overline{A}\times\overline{H})$$

$$W_m = \frac{1}{2}\int_{v'}(\overline{H}\cdot\overline{B})dv'$$

$$-\frac{1}{2}\oint_{S'}(\overline{A}\times\overline{H})\cdot\hat{a}_n ds'$$

All space

$$\lim_{s'\to\infty}\left(\frac{1}{r}\frac{1}{r^2}\right)r^2 \to 0$$

$$W_m = \frac{1}{2}\int_{v'}(\overline{H}\cdot\overline{B})dv'$$

$$W_m = \frac{1}{2}\int_{v'}(\overline{H}\cdot\overline{B})dv'$$

$$\overline{H} = \frac{\overline{B}}{\mu}$$

$$\boxed{W_m = \frac{1}{2}\int_{v'}\frac{B^2}{\mu}dv'}$$

or

$$\boxed{W_m = \frac{1}{2}\int_{v'}\mu H^2 dv'}$$

c.f.

$$W_e = \frac{1}{2}\int_{v'}(\overline{E}\cdot\overline{D})dv'$$

$$W_e = \frac{1}{2}\int_{v'}\varepsilon E^2 dv' = \frac{1}{2}\int_{v'}\frac{D^2}{\varepsilon}dv'$$

Magnetic energy density Wm

$$W_m = \int_{v'} w_m dv'$$

$$w_m = \frac{1}{2}\overline{H}\cdot\overline{B} = \frac{B^2}{2\mu} = \frac{1}{2}\mu H^2$$

$$\boxed{L = \frac{2W_m}{I^2}}$$

Ex 6-20 (Ref. Ex 6-16)

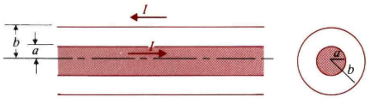
Wm in inner conductor

$$W'_{m1} = \frac{1}{2\mu_0}\int_0^a B_{\phi_1}^2 2\pi r dr = \frac{\mu_0 I^2}{4\pi a^4}\int_0^a r^3 dr = \frac{\mu_0 I^2}{16\pi}$$

Wm between inner & outer

$$W'_{m2} = \frac{1}{2\mu_0}\int_a^b B_{\phi_2}^2 2\pi r dr = \frac{\mu_0 I^2}{4\pi}\int_a^b \frac{1}{r}dr = \frac{\mu_0 I^2}{4\pi}\ln\left(\frac{b}{a}\right)$$

Hence, $L' = \frac{2}{I^2}(W'_{m1} + W'_{m2}) = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$



6-13 Magnetic forces & Torques

$\vec{F}_m = q\vec{u}\times\vec{B}$

$\vec{B} = B_0\hat{a}_z$; $\vec{J} = J_0\hat{a}_y = Nq\vec{u}$

$V_h = -\int_0^d E_h dx = u_0 B_0 d$

electron move toward to x-dir.

Creating a transverse $\vec{E}$-field. $\vec{E}_h$

Steady state, net force is Zero.

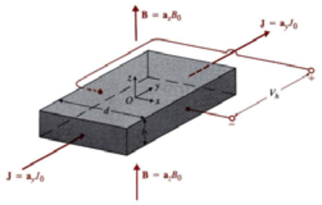
$\vec{E}_h + \vec{u}\times\vec{B} = 0$

$\vec{E}_h = -\vec{u}\times\vec{B}$; *Hall effect.*

$\vec{E}_h$: *Hall feild.*

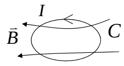
$N\text{-}type: \vec{u} = -u_0\hat{a}_y$

$\vec{E}_h = -(-u_0\hat{a}_y)\times B_0\hat{a}_z = u_0 B_0\hat{a}_x$

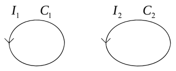


6-13.2 Force & Torques

$d\overline{F_m} = Id\vec{l}\times\overline{B}$



$\overline{F_m} = I\oint_c d\vec{l}\times\overline{B}$



$\overline{B_{21}} : I_2$ *source*

$\overline{F_{21}} : I_1$ *field*

$\overline{F_{21}} = I_1\oint_{c1} d\vec{l_1}\times\overline{B_{21}}$

$$\overline{B_{21}} = \frac{\mu_0 I_2}{4\pi}\oint_{c2}\frac{d\hat{l_2}\times\widehat{a_{\Re 21}}}{\Re_{21}^{\ 2}}$$

$$\overline{F_{21}} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_{c1}\oint_{c2}\frac{d\hat{l_1}(d\hat{l_2}\bullet\widehat{a_{\Re 21}})}{\Re_{21}^{\ 2}}$$

$$\overline{F_{21}} = \frac{\mu_0}{4\pi} I_2 I_1 \oint_{c1}\oint_{c2}\frac{d\hat{l_2}(d\hat{l_1}\bullet\widehat{a_{\Re 21}})}{\Re_{21}^{\ 2}}$$

$$-[d\hat{l_1}\times(d\hat{l_2}\times\widehat{a_{\Re 21}})] \stackrel{?}{=} d\hat{l_2}\times(d\hat{l_1}\times\widehat{a_{\Re 21}})$$

$-\overline{F_{21}} \stackrel{?}{=} \overline{F_{12}}$: Newton 3rd Law

Vector triple product

$$\frac{d\hat{l_1}\times(d\hat{l_2}\times\widehat{a_{\Re 21}})}{\Re_{21}^{\ 2}} = \frac{d\hat{l_2}(d\hat{l_1}\bullet\widehat{a_{\Re 21}})}{\Re_{21}^{\ 2}} - \frac{\widehat{a_{\Re 21}}(d\hat{l_1}\bullet d\hat{l_2})}{\Re_{21}^{\ 2}}$$

1st term

$$\oint_{c1}\oint_{c2}\frac{d\hat{l_2}(d\hat{l_1}\bullet\widehat{a_{\Re 21}})}{\Re_{21}^{\ 2}} = \oint_{c2} d\overline{l_2}\oint_{c1}\frac{d\hat{l_1}\bullet\widehat{a_{\Re 21}}}{\Re_{21}^{\ 2}} = \oint_{c2} d\overline{l_2}\oint_{c1} d\hat{l_1}(-\overline{\nabla_1}\frac{1}{\Re_{21}})$$

$$= -\oint_{c2} d\overline{l_2}\oint_{c1} d(\frac{1}{\Re_{21}}) = 0$$

代回 $\overline{F_{21}} = -\frac{\mu_0}{4\pi} I_1 I_2 \oint_{c1}\oint_{c2}\frac{\widehat{a_{\Re 21}}(d\hat{l_1}\bullet d\hat{l_2})}{\Re_{21}^{\ 2}} = -\overline{F_{12}}$

$\widehat{a_{\Re 21}} = -\widehat{a_{\Re 12}}$, Newton 3rd Law Hold

$Ex6-21$

$\overline{F_{12}}'$ force on wire 2

$\overline{F_{12}}' = I_2(\widehat{a_z} \times \overline{B_{12}})$

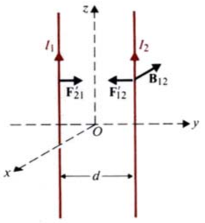
$\overline{B_{12}}$ source at wire1(I_1)

$\overline{B_{12}} = -\widehat{a_x}\dfrac{\mu_0 I_1}{2\pi d}$

$\overline{F_{12}}' = -\widehat{a_y}\dfrac{\mu_0 I_1 I_2}{2\pi d}$

Attraction

[Same polarity of current I_1 & I_2]



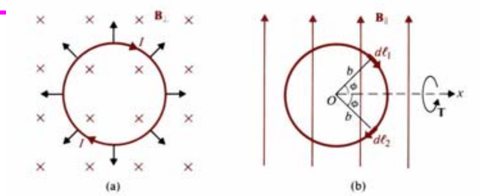


FIGURE 6-30
A circular loop in a uniform magnetic field $\mathbf{B} = \mathbf{B}_\perp + \mathbf{B}_{\|}$.

$\vec{B} = \overline{B_\perp} + \overline{B_{//}}$

$\overline{B_\perp}$: expand loop 6-30(a)
no net force to more loop

$\overline{B_{//}}$: produce on upward force $d\overline{F_1}$ on $d\vec{l_1}$
downward force $d\overline{F_2}$ on $d\vec{l_2}$
$d\overline{F_1} = -d\overline{F_2}$

$$d\overline{T} = \widehat{a_x}(dF)2b\sin\phi$$
$$= \widehat{a_I}(IdlB_{//}\sin\phi)2b\sin\phi$$
$$= \widehat{a_I}\,2Ib^2B_{//}\sin^2\phi\,d\phi$$
$$dF = |dF_1| = |dF_2| \quad ; \quad dl = |dl_1| = |dl_2| = bd\phi$$
$$\overline{T} = \int d\overline{T} = \widehat{a_x}2Ib^2B_{//}\int_0^\pi \sin^2\phi\,d\phi$$
$$= \widehat{a_x}I(\pi b^2)B_{//}$$

Magnetic dipole moment

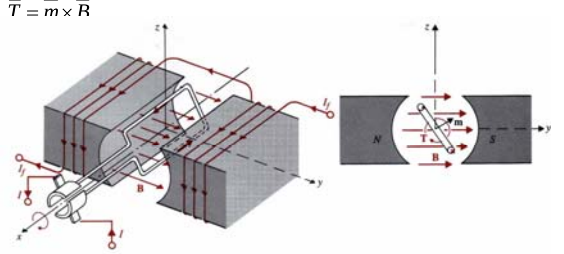
$\overline{m} = \widehat{a_I} I(\pi b^2) = \widehat{a_N} IS$

Hence,

$\overline{T} = \overline{m} \times \overline{B}$

∘DC-motor

∘Torque rotates at clockwise + X-dir



(a) Perspective view. (b) Schematic view from +x direction.

$Ex6-22$

$\overline{B_\perp} = \widehat{a_Z} B_Z$; $\overline{B_{//}} = \widehat{a_X} B_x + \widehat{a_y} B_y$

$\overline{B_{//}}$ produes the following forces

$\overline{F_1} = Ib_1 \widehat{a_X} \times (\widehat{a_x} B_x + \widehat{a_y} B_y) = \widehat{a_Z} Ib_1 B_y = -\overline{F_3}$

$\overline{F_2} = Ib_2 (-\widehat{a_y}) \times (\widehat{a_X} B_x + \widehat{a_y} B_y) = \widehat{a_Z} Ib_2 B_x = -\overline{F_4}$

$\overline{F_{NET}} = \sum_{i=1}^{4} \overline{F_i} = 0$

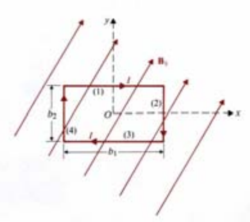
$\overline{T_{13}} = \widehat{a_x} Ib_1 b_2 B_y$;$[\overline{F_1} \,\&\, \overline{F_3}]$

$\overline{T_{24}} = -\widehat{a_y} Ib_1 b_2 B_x$;$[\overline{F_2} \,\&\, \overline{F_4}]$

$\overline{T} = \overline{T_{13}} + \overline{T_{24}} = Ib_1 b_2 (\widehat{a_x} B_y - \widehat{a_y} B_x)$

$\overline{m} = -\widehat{a_z} Ib_1 b_2$

$\overline{T} = \overline{m} \times \overline{B} = \overline{m} \times (\widehat{a_x} B_y - \widehat{a_y} B_x)$



6−13.3 Forces and Torques in terms of Wm

∘ Constant Flux Linkages

[Source Spply No energy]

$\overline{F_\Phi} \bullet d\vec{l} = -dW_m = -(\vec{\nabla} W_m) \bullet d\vec{l}$

$\overline{F_\Phi} = -\vec{\nabla} W_m$

rotate about z-axis

$(T_\Phi)_Z = -\dfrac{\partial W_m}{\partial \phi}$

∘ Constant Currents

[*Current* sorce → increase Wm]

$dW_s = dW + dW_m$

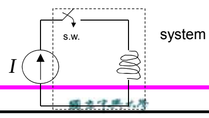
$dW_m = \dfrac{1}{2} dW_s$

$dW = \overline{F_I} \bullet d\vec{l} = dW_m = (\vec{\nabla} W_m) \bullet d\vec{l}$

$\overline{F_I} = \vec{\nabla} W_m$

$(T_I)_Z = \dfrac{\partial W_m}{\partial \phi}$

S.W.(OFF) S.W.(ON)



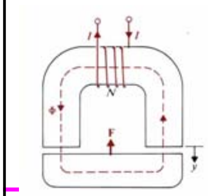
$Ex6-23$

• Contant Flux

$$dW_m = d(W_m)_{air} = 2(\frac{B^2}{2\mu_0}Sdy)$$

$$= \frac{\Phi^2}{\mu_0 S}dy$$

$$\overline{F_\Phi} = \hat{a}_y(-\frac{dW_m}{dy}) = -\hat{a}_y\frac{\Phi^2}{\mu_0 S}$$



• Constant Current

$$W_m = \frac{1}{2}LI^2 \quad \text{Core} : \Re c$$

$$\Phi = \frac{NI}{\Re c + 2(\frac{y}{\mu_0 S})} \quad \text{2Gap}:2\frac{y}{\mu_0 S}$$

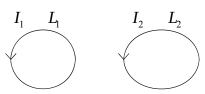
$$L = \frac{N\Phi}{I} = \frac{N^2}{\Re c + 2(\frac{y}{\mu_0 S})}$$

$$\overline{F_I} = \hat{a}_y\frac{I^2}{2}\frac{dL}{dy} = -\hat{a}_y\frac{1}{\mu_0 S}[\frac{N^2}{\Re c + 2(\frac{y}{\mu_0 S})}]^2$$

$$= -\hat{a}_y\frac{\Phi^2}{\mu_0 S}$$

$(6-13.4)$force and toraues in terms of mutual inductance

Two coils



$$W_m = \frac{1}{2}L_1 I_1^2 + L_{12}I_1 I_2 + \frac{1}{2}L_2 I_2^2$$

Conotant currents

$$\overline{F_1} = I_1 I_2(\overline{\nabla} L_{12})$$

$$(T_I)_Z = I_1 I_2\frac{\partial L_{12}}{\partial \phi}$$

$Ex6-24$ $\quad I_1$: *source* [Ex6-7,p239]

$$\overline{A_{12}} = \hat{a}_\varphi\frac{\mu_0 N_1 I_1 b_1^2}{4R^2}\sin\theta = \hat{a}_\varphi\frac{\mu_0 N_1 I_1 b_1^2 b_2}{4[Z^2+b_2^2]^{3/2}}$$

$$\Phi_{12} = \oint_{c2}\overline{A_{12}}\bullet d\overline{l_2} = \int_0^{2\pi}A_{12}b_2 d\phi = \frac{\mu_0 N_1 I_1 b_1^2 b_2^2 \pi}{2[Z^2+b_2^2]^{3/2}}$$

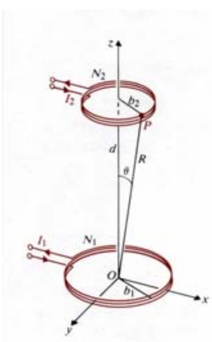
$$A_{12} = \frac{N_2\Phi_{12}}{I_1} = \frac{\mu_0 N_1 N_2 \pi b_1^2 b_2^2}{2[Z^2+b_2^2]^{3/2}}$$

$$\overline{F_{12}} = \hat{a}_Z I_1 I_2\frac{dL_{12}}{dZ}\Big|_{Z=d}$$

$$\overline{F_{12}} = -\hat{a}_Z I_1 I_2\frac{3\mu_0 N_1 N_2 \pi b_1^2 b_2^2 d}{2(d^2+b_2^2)^{5/2}}$$

$$d \gg b_2 \; ; \; m_1 = N_1 I_1 \pi b_1^2 \; ; \; m_2 = N_2 I_2 \pi b_2^2$$

$$\overline{F_{12}} \simeq -\hat{a}_Z\frac{3\mu_0 m_1 m_2}{2\pi d^4} \quad \textit{attraction}$$



Home Work #6

David Cheng: Chapter6

P6-2, P6-4, P6-5, P6-6, P6-10,P6-11,P6-12
P6-13,P6-15,P6-18,P6-19,P6-22,P6-26,

P6-27,P6-29,P6-32,P6-37,P6-38,P6-39,
P6-40,P6-41,P6-42,P6-43,P6-44,P6-46,
P6-50,P6-53
Due: 2 weeks