CH6 Static Magnetic Field

- Electric force $\vec{F} = q\vec{E}(N)$
- Magnetic force $\vec{F}_m = q\vec{u} \times \vec{B}(N)$
- Elecmagnetic fore $\vec{F} = q(\vec{E} + \vec{u} \times \vec{B})$

(N) ~ Lorentz's force equation

Electromagnetic

Free space

- Static Electric Field

$$\vec{\nabla}\cdot\vec{D}=\rho$$

$$\vec{\nabla}\times\vec{E}=0$$

- Static Magnetic Field

$$\vec{\nabla}\cdot\vec{B}=0$$

$$\vec{\nabla}\times\vec{B}=\mu_o\vec{J}$$

$\vec{\nabla}\cdot\vec{J}=0$ Steady current

$\mu_o=4\pi\times10^{-7}\,(Henry/m)$

Permeability of free space

Electromagnetic

$$\vec{\nabla}\cdot\vec{B}=0 \quad\Rightarrow\quad \oint_s \vec{B}\cdot d\vec{s}=0$$

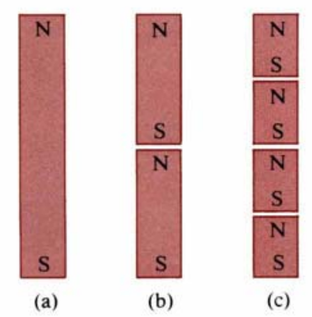


FIGURE 6–1
Successive division of a bar magnet.

- No magnetic flow sources
- Magnetic flux lines always close
- Low of conservation of magnetic flux

- Each magnets has a north pole south
- Magnetic poles cannot be isolated

$$\vec{\nabla}\times\vec{B}=\mu_o\vec{J}\Rightarrow\int_s(\vec{\nabla}\times\vec{B})\cdot d\vec{s}=\mu_o\int_s\vec{J}\cdot d\vec{s}$$

$$\oint_c\vec{B}\cdot d\vec{\ell}=\mu_o I$$ Ampere's circuital law

summary

$$\vec{\nabla}\cdot\vec{B}=0 \qquad \oint_s\vec{B}\cdot d\vec{s}=0$$

$$\vec{\nabla}\times\vec{B}=\mu_o\vec{J} \qquad \oint_c\vec{B}\cdot d\vec{\ell}=\mu_o I$$

EX 6-1

- Inside conductor

$$\vec{B}_1=\hat{a}_\phi B_{\phi1} \qquad d\vec{\ell}=\hat{a}_\phi r_1 d\phi$$

$$\oint_{c1}\vec{B}_1 d\vec{\ell}=\int_0^{2\pi}B_{\phi1}r_1 d\phi=2\pi r_1 B_{\phi1}$$

$$I_1=\frac{I}{\pi b^2}\pi r_1^2=\left(\frac{r_1}{b}\right)^2 I$$

$$\vec{B}_1=\hat{a}_\phi B_{\phi1}=\hat{a}_\phi\frac{\mu_o r_1 I}{2\pi b^2} \quad r_1\le b$$

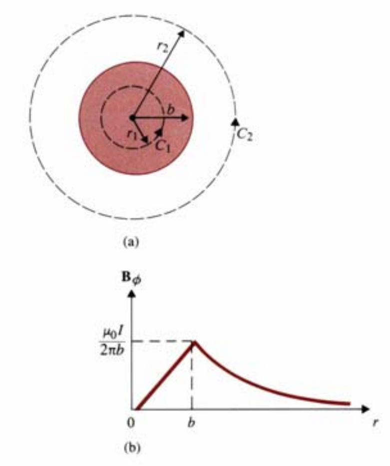
- Outside conductor

$$\vec{B}_2=\hat{a}_\phi B_{\phi2} \qquad d\vec{\ell}=\hat{a}_\phi r_2 d\phi$$

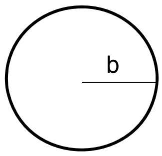
$$\oint_{c2}\vec{B}_2 d\vec{\ell}=2\pi r_2 B_{\phi2}$$

C_2 outside conductor encloses I

$$\vec{B}_2=\hat{a}_\phi B_{\phi2}=\hat{a}_\phi\frac{\mu_o I}{2\pi r_2} \quad r_2\ge b$$



If



空心圓柱(柱座標)

$$\vec{J}_s = \hat{a}_z J_s (A/m)$$

$$I = 2\pi b J_s$$

$$B = \begin{cases} 0 & r < b \\ \hat{a}_\phi \dfrac{\mu_o b}{r} J_s & r > b \end{cases}$$

EX 6-2 (Toroidal Coil)

A circular contour C with radius r
(b - a) < r < (b + a)

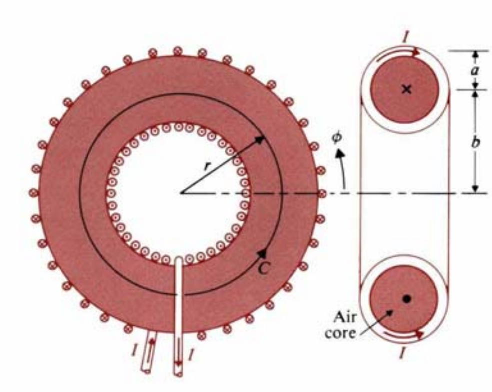
$$\oint \vec{B} \cdot d\vec{\ell} = 2\pi r B_\phi = \mu_o NI$$

$$(1)\vec{B} = \hat{a}_\phi B_\phi = \hat{a}_\phi \frac{\mu_o NI}{2\pi r}$$

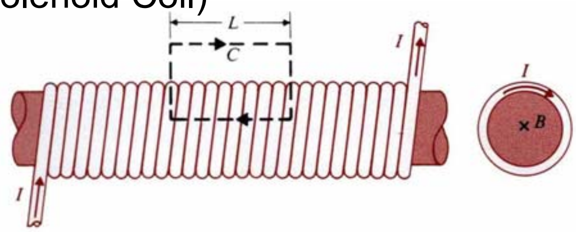
(b - a) < r < (b + a)

$$(2)\vec{B} = 0 \qquad r < (b-a)$$
$$\& \, r > (b+a)$$

(No source)



EX 6-3 (Solenoid Coil)



(a) Direct application of Ampere Law

$$BL = \mu_o nLI$$

$$B = \mu_o nI$$

(b) Special case of torid Ex 6-2, $b \to \infty$

$$B = \mu_o \left(\frac{N}{2\pi b} \right) I$$

$$B = \mu_o nI$$

6-3 Vector Magnetic Potential

$\boxed{\vec{\nabla} \cdot \vec{B} = 0} \Rightarrow \vec{B} = \vec{\nabla} \times \vec{A},$ $\quad \vec{A}$: magnetic potential [Vector]

c.f $\boxed{\vec{\nabla} \times \vec{E} = 0} \Rightarrow \vec{E} = -\vec{\nabla}\phi,$ $\quad \phi$: electric potential [Scalar]

$$\vec{\nabla} \times \vec{B} = \mu_o \vec{J}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A} = \mu_o \vec{J}$$

取 $\boxed{\vec{\nabla} \cdot \vec{A} = 0}$ $\Rightarrow$ $\boxed{\vec{\nabla}^2 \vec{A} = -\mu_o \vec{J}}$

Coulomb gauge

Vector Poisson's equation

In Cartesian coordinates,

Vector

$$\begin{cases} \vec{\nabla}^2 A_x = -\mu_o J_x \\ \vec{\nabla}^2 A_y = -\mu_o J_y \\ \vec{\nabla}^2 A_z = -\mu_o J_z \end{cases} \Rightarrow A_x = \frac{\mu_o}{4\pi}\int_{u'} \frac{J_x}{r} du' \Rightarrow \boxed{\vec{A} = \frac{\mu_o}{4\pi}\int_{u'} \frac{\vec{J}}{r} du' (Wb/m)}$$

$$c.f. \quad \vec{\nabla}^2 \phi = -\frac{\rho}{\varepsilon_o} \Rightarrow \phi = \frac{\mu_o}{4\pi}\int_{u'} \frac{\rho}{r} du'$$

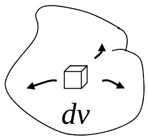
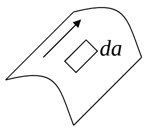
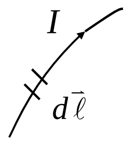
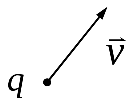
Magnetic Flux Φ through a given area S which is bounded by contour C

$$\Phi = \int_s \vec{B} \cdot d\vec{s} \qquad (Web)$$

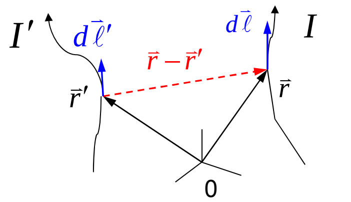
$$\Phi = \int_s (\vec{\nabla} \times \vec{A}) \cdot d\vec{s} = \oint_c \vec{A} \cdot d\vec{\ell} \qquad (Web)$$

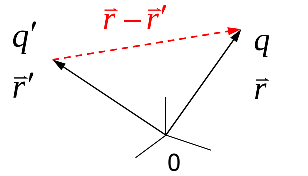
6-4 Biot-Savart Law and applications

Magnetic: Vector source

	3-dim	2-dim	1-dim	0-dim
Current distribution	Volume current density	Surface current density	current	
	$\vec{j}\left[\frac{coul}{\sec\cdot m^2}\right]$	$\vec{j}_s\left[\frac{coul}{\sec\cdot m}\right]$	$I\left[\frac{coul}{\sec}\right]$	$q[coul]$
Current element	$\vec{j}dv\left[\frac{coul}{\sec}\cdot m\right]$	$\vec{j}_s da\left[\frac{coul}{\sec}\cdot m\right]$	$Id\vec{\ell}\left[\frac{coul}{\sec}\cdot m\right]$	$q\vec{v}\left[\frac{coul}{\sec}\cdot m\right]$
				

Biot-Savart Law：[Valid in steady current]





$$d\vec{F} = \frac{\mu_o}{4\pi} Id\vec{\ell}\times\left[I'd\vec{\ell}'\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right] \quad \text{c.f.} \quad \vec{F}_q = \frac{1}{4\pi\varepsilon_o} qq'\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

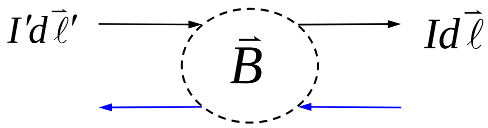
因次分析 $[\mu_o]\cdot v^2 = \frac{1}{[\varepsilon_0]}$

Biot-Sarvart Law：

$$d\vec{F} = Id\vec{\ell} \times d\vec{B}$$

$$d\vec{B} = \frac{\mu_o}{4\pi} I' d\vec{\ell}' \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

Action at a distance：$\vec{B}$ field



$$\oint_s \vec{B} \cdot d\vec{a} = 0$$

Gauss thm.

$$\vec{\nabla}_{\vec{r}} \cdot \vec{B}(\vec{r}) = 0$$

特殊式

$$\vec{B} = \vec{\nabla}_{\vec{r}} \times \vec{A}(\vec{r})$$

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{a.s.} \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} dv'$$

Biot-Sarvart Law

$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{a.s.} \vec{j}(\vec{r}') \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dv'$$

$$\vec{\nabla}_{\vec{r}}^2 \vec{A}(\vec{r}) = -\mu_o \vec{j}$$

Poission Eqe.

$$\oint_p \vec{B} \cdot d\vec{\ell} = \mu_o \int_s \vec{j} \cdot d\vec{a}$$

Stoke Thm.

$$\vec{\nabla}_{\vec{r}} \times \vec{B}(\vec{r}) = \mu_o \vec{j}(\vec{r})$$

Ampere's Law

Electromagnetic

$$\vec{B}(\vec{r})=\frac{\mu_o}{4\pi}\int_{a.s.}\vec{j}(\vec{r}\,')\times\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}dv'$$

$$(1)\nabla_{\vec{r}}\cdot\vec{B}(\vec{r})=0$$

PF: $$\nabla_{\vec{r}}\cdot\vec{B}=\frac{\mu_o}{4\pi}\nabla_{\vec{r}}\cdot\left[\int_{a.s.}\vec{j}(\vec{r}\,')\times\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}dv'\right]$$

$$\nabla\cdot(\vec{A}\times\vec{B})=(\nabla\times\vec{A})\cdot\vec{B}-(\nabla\times\vec{B})\cdot\vec{A}$$

$$=\frac{\mu_o}{4\pi}\int_{a.s.}\nabla_{\vec{r}}\cdot\left[\vec{j}(\vec{r}\,')\times\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\right]dv'$$

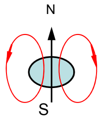
$$=\frac{\mu_o}{4\pi}\int_{a.s.}\left\{\underbrace{\left[\nabla_{\vec{r}}\times\vec{j}(\vec{r}\,')\right]}_{=0}\cdot\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}-\underbrace{\left[\nabla_{\vec{r}}\times\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\right]}_{=0}\cdot\vec{j}(\vec{r}\,')\right\}dv'$$

$$=0$$

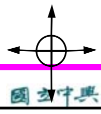
$$\nabla\times\left(\frac{\vec{e}_r}{r^2}\right)=0$$

C.F

$\nabla\cdot\vec{B}=0$ 封閉磁迴路



$\nabla\cdot\vec{E}=\dfrac{\rho}{\varepsilon_o}$ 孤立電單極



Electromagnetic

From $\nabla_{\vec{r}}\cdot\vec{B}(\vec{r})=0$

$$(2)\vec{B}=\nabla_{\vec{r}}\times\vec{A}(\vec{r})\qquad \vec{A}=\frac{\mu_o}{4\pi}\int_{a.s.}\frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}dv'$$

$$\vec{B}=\frac{\mu_o}{4\pi}\int_{a.s.}\vec{j}(\vec{r}\,')\times\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}dv'$$

$$\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}=-\nabla_{\vec{r}}\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right)$$

$$=\frac{\mu_o}{4\pi}\int_{a.s.}\left\{\nabla_{\vec{r}}\times\left(\frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\right)-\frac{1}{|\vec{r}-\vec{r}\,'|}\underbrace{\left(\nabla_{\vec{r}}\times\vec{j}(\vec{r}\,')\right)}_{0}\right\}dv'$$

$$\nabla\times(f\vec{A})=(\nabla f)\times\vec{A}+f(\nabla\times\vec{A})$$

$$=\nabla_{\vec{r}}\times\left\{\frac{\mu_o}{4\pi}\int_{a.s.}\left(\frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\right)dv'\right\}$$

$$=\nabla_{\vec{r}}\times\vec{A}(\vec{r})$$

其中 $$\vec{A}(\vec{r})=\frac{\mu_o}{4\pi}\int_{a.s.}\left(\frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\right)dv'$$

Electromagnetic

From $\vec{\nabla}_{\vec{r}} \times \vec{A}(\vec{r}) = \vec{B}(\vec{r}) \Rightarrow \begin{cases} \vec{\nabla}^2 \vec{A}(\vec{r}) = -\mu_o \vec{j}(\vec{r}) \\ \vec{\nabla} \times \vec{B}(\vec{r}) = \mu_o \vec{j}(\vec{r}) \\ \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - (\vec{\nabla} \cdot \vec{\nabla})\vec{A}(\vec{r}) \end{cases}$

Steady current $\vec{\nabla} \cdot \vec{j}(\vec{r}) = 0$

Static magnetic field

$c.f.\quad \vec{\nabla}^2 \phi = -\frac{\rho}{\varepsilon_o}, \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_o}$

$$pf: \ \vec{\nabla}_{\vec{r}} \times [\vec{\nabla}_{\vec{r}} \times \vec{A}(\vec{r})] = \vec{\nabla}_{\vec{r}}[\vec{\nabla}_{\vec{r}} \cdot \vec{A}(\vec{r})] - (\vec{\nabla}_{\vec{r}} \cdot \vec{\nabla}_{\vec{r}})\vec{A}(\vec{r})$$

其中： $\vec{\nabla}_{\vec{r}} \cdot \vec{A}(\vec{r}) = \vec{\nabla}_{\vec{r}} \cdot \left\{ \frac{\mu_o}{4\pi} \int_{a.s.} \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} dv' \right\} = \frac{\mu_o}{4\pi} \int_{a.s.} \vec{\nabla}_{\vec{r}} \cdot \left[\frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} \right] dv'$

$$= \frac{\mu_o}{4\pi} \int_{a.s.} \left\{ \vec{j}(\vec{r}') \cdot \vec{\nabla}_{\vec{r}} \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) + \frac{1}{|\vec{r} - \vec{r}'|} [\vec{\nabla}_{\vec{r}} \cdot \vec{j}(\vec{r}')] \right\} dv'$$

Steady state $\vec{\nabla} \cdot \vec{j} = 0$

$$= \frac{\mu_o}{4\pi} \oint_{s \to \infty} \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} \cdot d\vec{a}' = 0$$

$\because \lim_{r \to \infty} \frac{1}{|\vec{r} - \vec{r}'|} = 0$

$\vec{\nabla} \cdot \vec{A} = 0$ Coulomb Gauge

Electromagnetic

$$\vec{\nabla}_{\vec{r}} \cdot \left[\frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} \right] = \vec{\nabla}_{\vec{r}} \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) \cdot \vec{j}(\vec{r}') + \frac{1}{|\vec{r} - \vec{r}'|} [\vec{\nabla}_{\vec{r}} \cdot \vec{j}(\vec{r}')]$$

$$\vec{\nabla}_{\vec{r}} \times \vec{B}(\vec{r}) = -(\vec{\nabla}_{\vec{r}} \cdot \vec{\nabla}_{\vec{r}})\vec{A}(\vec{r}) = -\vec{\nabla}^2_{\vec{r}} \vec{A}(\vec{r})$$

其中： $\vec{\nabla}^2_{\vec{r}} \vec{A}(\vec{r}) = \vec{\nabla}^2_{\vec{r}} \left\{ \frac{\mu_o}{4\pi} \int_{a.s.} \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} dv' \right\} = \frac{\mu_o}{4\pi} \int_{a.s.} \vec{\nabla}^2_{\vec{r}} \left(\frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} \right) dv'$

$$= -\mu_o \vec{j}(\vec{r})$$

$= \vec{j}(\vec{r}') \vec{\nabla}^2_{\vec{r}} \left(\frac{1}{|\vec{r} - \vec{r}'|} \right)$

$= \vec{\nabla}^2 \left(\frac{1}{r} \right) = -4\pi\delta^3(\vec{r})$

$\vec{\nabla}^2 \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) = -4\pi\delta^3(\vec{r} - \vec{r}')$

$\vec{\nabla}_{\vec{r}} \times \vec{B}(\vec{r}) = \mu_o \vec{j}(\vec{r})$: Ampere' Law

$\vec{\nabla}^2_{\vec{r}} \vec{A}(\vec{r}) = -\mu_o \vec{j}(\vec{r})$: Poission Equ.

$$\oint_s \vec{B}(\vec{r})\cdot d\vec{a}=0$$

$$pf:\vec{\nabla}\cdot\vec{B}(\vec{r})=0$$

Gauss Thm. $$\int_v \vec{\nabla}\cdot\vec{B}(\vec{r})dv=0$$

$$\oint_s \vec{B}(\vec{r})\cdot d\vec{a}=0$$

$$\oint_C \vec{B}(\vec{r})\cdot d\vec{\ell}=\mu_o\int_s \vec{j}(\vec{r})\cdot d\vec{a}$$

$$pf:\vec{\nabla}\cdot\vec{B}(\vec{r})=\mu_o\vec{j}(\vec{r})$$

$$\int_s[\vec{\nabla}\cdot\vec{B}(\vec{r})]\cdot d\vec{a}=\mu_o\int\vec{j}(\vec{r})\cdot d\vec{a}$$

$$\oint_C \vec{B}(\vec{r})\cdot d\vec{\ell}=\mu_o\int\vec{j}(\vec{r})\cdot d\vec{a}$$

Example 6-4

(a) $$\vec{A}=\hat{a}_z\frac{\mu_o I}{4\pi}\int_{-L}^{L}\frac{dz'}{\sqrt{z'^2+\rho^2}}$$

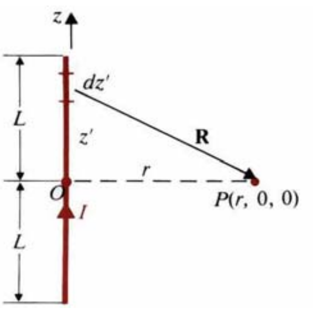
$$=\hat{a}_z\frac{\mu_o I}{4\pi}\left[\ln(z'+\sqrt{z'^2+\rho^2})\right]_{-L}^{L}$$

$$=\hat{a}_z\frac{\mu_o I}{4\pi}\ln\frac{\sqrt{L^2+\rho^2}+L}{\sqrt{L^2+\rho^2}-L}$$

$$\vec{B}=\vec{\nabla}\times\vec{A}=\vec{\nabla}\times(\hat{a}_z A_z)$$

$$=\hat{a}_\rho\frac{1}{\rho}\frac{\partial A_z}{\partial\phi}-\hat{a}_\phi\frac{\partial A_z}{\partial\rho}$$

$$cylindrical\quad sym.\frac{\partial A_z}{\partial\phi}=0$$



$$d\vec{\ell}'=\hat{a}_z dz'$$

$$R=\sqrt{z'^2+\rho^2}$$

$$\vec{B} = -\hat{a}_\phi \frac{\partial}{\partial \rho}\left[\frac{\mu_o I}{4\pi}\ln\frac{\sqrt{L^2+\rho^2}+L}{\sqrt{L^2+\rho^2}-L}\right]$$

$$= \hat{a}_\phi \frac{\mu_o IL}{2\pi\rho\sqrt{L^2+\rho^2}}$$

$if\ \rho \ll L;$

$$\vec{B} = \hat{a}_\phi \frac{\mu_o I}{2\pi\rho}$$

$$\vec{r}-\vec{r}' = \hat{a}_\rho \rho - \hat{a}_z z'$$

$$d\vec{\ell}\times(\vec{r}-\vec{r}') = \hat{a}_z z' \times (\hat{a}_\rho \rho - \hat{a}_z z')$$

$$= \hat{a}_\phi \rho dz'$$

$$\vec{B} = \int d\vec{B} = \hat{a}_\phi \frac{\mu_o I}{4\pi}\int_{-L}^{L}\frac{\rho dz'}{(z'^2+\rho^2)^{3/2}}$$

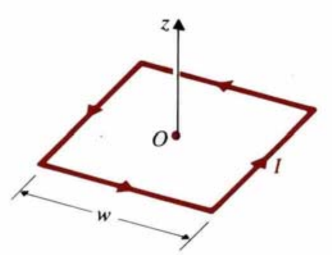
$$= \hat{a}_\phi \frac{\mu_o IL}{2\pi\rho\sqrt{L^2+\rho^2}}$$

Example 6-5

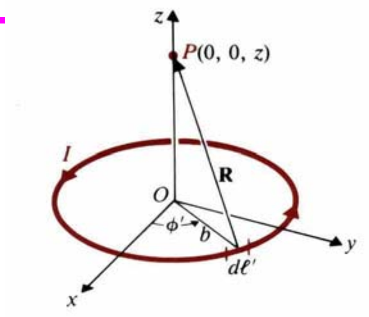
From 6-4

$$L = \rho = w/2$$

$$\vec{B} = \hat{a}_z \frac{\mu_o I}{\sqrt{2}\pi w}\times 4 = \hat{a}_z \frac{2\sqrt{2}\mu_o I}{\pi w}$$



Example 6-6



$$d\vec{\ell}' = \hat{a}_\phi b d\phi'$$

$$\vec{R} = \vec{r} - \vec{r}' = z\hat{a}_z - b\hat{a}_\rho$$

$$|\vec{r} - \vec{r}'| = (z^2 + b^2)^{1/2}$$

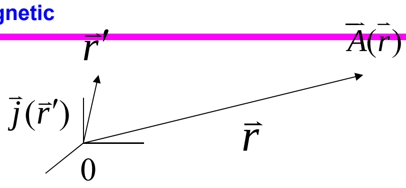
$$d\vec{\ell}' \times |\vec{r} - \vec{r}'| = \hat{a}_\phi b d\phi' \times (z\hat{a}_z - b\hat{a}_\rho)$$

$$= \hat{a}_\rho bz d\phi' + \hat{a}_z b^2 d\phi'$$

$\hat{a}_\rho$ is canceled due to cylindrical sym.

$$\vec{B} = \frac{\mu_o I}{4\pi}\int_0^{2\pi} \hat{a}_z \frac{b^2 d\phi'}{(z^2+b^2)^{3/2}} = \hat{a}_z \frac{\mu_o I b^2}{2(z^2+b^2)^{3/2}} (T)$$

6-5 Magnetic Dipole



$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi}\int_v \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} dv'$$

$$= \underbrace{\frac{\mu_o}{4\pi}\frac{\int_v \vec{j}(\vec{r}')dv'}{r}}_{2^0 \text{ pole} = 0} + \underbrace{\frac{\mu_o}{4\pi}\frac{\int_v \vec{j}(\vec{r}')(\vec{r}'\cdot\vec{a}_r')dv'}{r^2}}_{2' \text{ pole} = 0} + \dots$$

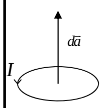
$$\nabla \cdot \vec{j}(\vec{r}) = 0$$

$$\int_v \vec{j}(\vec{r}')(\vec{r}'\cdot\vec{a}_r')dv' \equiv \vec{m} \times \vec{a}_r$$

$$\vec{m} = \frac{1}{2}\int_v \vec{r}' \times \vec{j}(\vec{r}')dv' = \text{電流} \cdot \text{面積}$$

$$\vec{m} = I d\vec{a}$$

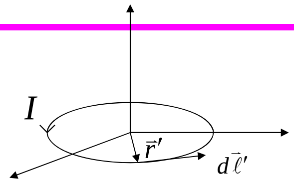
Electromagnetic



$$\vec{m} = \frac{1}{2}\int_{dp} \vec{r}\,' \times \vec{j}(\vec{r}\,')dv'$$

$$= \frac{1}{2}\int_{dp} \vec{r}\,' \times (Id\vec{\ell}') = I\frac{1}{2}\int_{dp} \vec{r}\,' \times d\vec{\ell}'$$

$$= Id\vec{a}$$

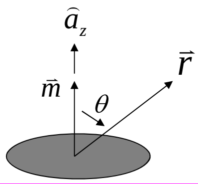


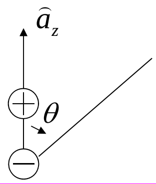
$$d\vec{a} = \frac{1}{2}\vec{r}' \times d\vec{\ell}'$$

$$\vec{A} = \frac{\mu_o}{4\pi}\frac{\vec{m}\times\hat{a}_r}{r^2} = \frac{\mu_o}{4\pi}\frac{m\sin\theta}{r^2}\hat{a}_\varphi$$

c.f.

$$\phi = \frac{1}{4\pi\varepsilon_o}\frac{\vec{P}\cdot\vec{a}_r}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{P\cos\theta}{r^2}$$





Electromagnetic

$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{\mu_o}{4\pi}m\frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\hat{a}_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ 0 & 0 & r\sin\theta\frac{\sin\theta}{r^2} \end{vmatrix}$$

$$= \frac{\mu_o}{4\pi}m\frac{1}{r^2\sin\theta}\left[\hat{a}_r\frac{\partial}{\partial\theta}(\frac{\sin^2\theta}{r}) - r\hat{a}_\theta\frac{\partial}{\partial r}(\frac{\sin^2\theta}{r})\right]$$

$$= \frac{\mu_o}{4\pi}m\frac{\hat{a}_r 2\cos\theta + \hat{a}_\theta\sin\theta}{r^3}$$

$$\frac{\partial}{\partial\theta}(\frac{\sin^2\theta}{r}) = \frac{2\sin\theta\cos\theta}{r}$$

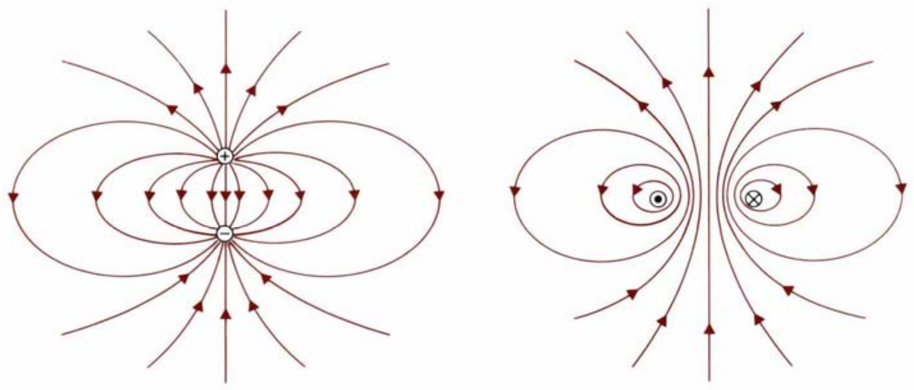
$$\frac{\partial}{\partial r}(\frac{\sin^2\theta}{r}) = -\frac{\sin^2\theta}{r^2}$$

c.f.

$$\vec{E} = -\vec{\nabla}\phi = \frac{1}{4\pi\varepsilon_o}(-1)P\left[\hat{a}_r\frac{\partial}{\partial r}(\frac{\cos\theta}{r^2}) + \hat{a}_\theta\frac{1}{r}\frac{\partial}{\partial\theta}(\frac{\cos\theta}{r^2})\right]$$

$$= \frac{P}{4\pi\varepsilon_o}\left[\hat{a}_r\frac{2\cos\theta}{r^3} + \hat{a}_\theta\frac{\sin\theta}{r^3}\right]$$

Electromagnetic



(a) Electric dipole. (b) Magnetic dipole.

FIGURE 6–9
Electric field lines of an electric dipole and magnetic flux lines of a magnetic dipole.

Electromagnetic

Scalar Magnetic Potential

$$\vec{\nabla}\times\vec{B}=\mu_o\vec{J} \qquad if \quad \vec{J}=0 \quad \vec{\nabla}\times\vec{B}=0$$

$$\vec{B}=-\mu_o\vec{\nabla}\phi_m,$$ ϕ_m : Scalar Magnetic Potential

$$\phi_{m2}-\phi_{m1}=-\int_{p_1}^{p_2}\frac{1}{\mu_o}\vec{B}\cdot d\vec{\ell}\Rightarrow\phi_m=\frac{1}{4\pi}\int_{v'}\frac{\rho_m}{r}dv'$$

(not physical)

$$\vec{m}=q_m\vec{d}=\hat{a}_n IS$$

$$\phi_m=\frac{\vec{m}\cdot\hat{a}_r}{4\pi r^2}$$

$if \quad \vec{J}\neq 0,$ $\vec{B}$: Non conservative (path dependent)

6-6 Magnetization and Equivalent Current Density

磁性物質	Components	source
Conductor	free electron	$\vec{j}_f = \sigma \vec{E}$
Non-conductor	polarized ion	$\vec{j}_m = \vec{\nabla} \times \vec{M}$

定義 $\overline{M} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{n\Delta v} m_k}{\Delta v} (A/m) = \frac{mag.\ dipole\ moment}{\text{體積}}$

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int_v \overline{M}(\vec{r}') \times \underbrace{\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}_{} dv'$$

$$\rightarrow = \vec{\nabla}_r \left(\frac{-1}{|\vec{r}-\vec{r}'|}\right) = \vec{\nabla}_{r'} \left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$$

$$\overline{M}(\vec{r}') \times \vec{\nabla}_{r'} \left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$$

$$= \vec{\nabla}_{r'} \times \left[\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)(-\overline{M}(\vec{r}')) \right] + \frac{1}{|\vec{r}-\vec{r}'|} \left[\vec{\nabla}_{r'} \times \overline{M}(\vec{r}') \right]$$

$$\vec{\nabla} \times (f\vec{A}) = \vec{\nabla} f \times \vec{A} + f(\vec{\nabla} \times \vec{A})$$

$$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{v'} \vec{\nabla}_{\vec{r}'} \times \left[\frac{-\overline{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} \right] dv' + \frac{\mu_o}{4\pi} \int_{v'} \frac{\vec{\nabla}_{\vec{r}'} \times \overline{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} dv'$$

$$= \frac{\mu_o}{4\pi} \oint_{S'} da' \times \left[\frac{-\overline{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} \right] + \frac{\mu_o}{4\pi} \int_{v'} \frac{\vec{\nabla}_{\vec{r}'} \times \overline{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} dv'$$

$$= \frac{\mu_o}{4\pi} \oint_{S'} da' \frac{\overline{M}(\vec{r}') \times \vec{n}'}{|\vec{r}-\vec{r}'|} + \frac{\mu_o}{4\pi} \int_{v'} \frac{\vec{\nabla}_{\vec{r}'} \times \overline{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} dv'$$

$$= \frac{\mu_o}{4\pi} \oint_{S'} da' \frac{\vec{j}_{ms}}{|\vec{r}-\vec{r}'|} + \frac{\mu_o}{4\pi} \int_{v'} \frac{\vec{j}_m}{|\vec{r}-\vec{r}'|} dv'$$

$$\left[\begin{array}{l} \vec{j}_m = \vec{\nabla} \times \overline{M} (A/m^2) \\ \vec{j}_{ms} = \overline{M} \times \hat{a}_n (A/m) \end{array} \right]$$

c.f. $\rho_p = -\vec{\nabla} \cdot \vec{P}; \quad \rho_{sp} = \hat{a}_n \cdot \vec{P}$

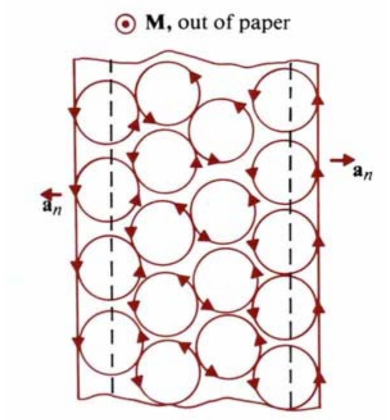


FIGURE 6–10
A cross section of a magnetized material.

EX : 6-8

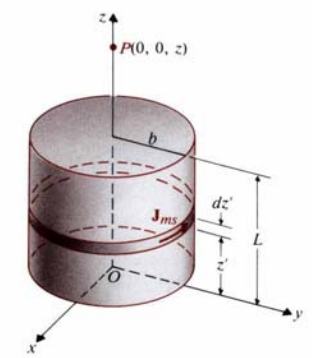
$$\vec{j}_{ms} = \overrightarrow{M} \times \hat{a}_n = (M_o \hat{a}_z) \times \hat{a}_r = M_o \hat{a}_\varphi$$

$$\vec{B} = \hat{a}_z \frac{\mu_o I b^2}{2(z^2 + b^2)^{3/2}}$$

$$d\vec{B} = \hat{a}_z \frac{\mu_o M_o b^2 dz'}{2\left[(z - z')^2 + b^2\right]^{3/2}}$$

$$\vec{B} = \int_0^L d\vec{B}$$

$$= \hat{a}_z \frac{\mu_o M_o}{2} \left[\frac{z}{\sqrt{z^2 + b^2}} - \frac{z - L}{\sqrt{(z - L)^2 + b^2}} \right]$$



Electromagnetic

靜磁學（包含導體與磁性材料）

$\vec{\nabla}\cdot\vec{B}=0$

$\vec{\nabla}\times\vec{B}=\mu_o(\vec{j}_f+\vec{j}_m)$

$=\mu_o(\vec{j}_f+\vec{\nabla}\times\vec{M})$

$\vec{\nabla}\times\left[\frac{1}{\mu_o}\vec{B}-\vec{M}\right]=\vec{j}_f$

$\vec{\nabla}\times\vec{H}=\vec{j}_f$

$\therefore\vec{H}=\frac{1}{\mu_o}\vec{B}-\vec{M}=\frac{1}{\mu_o\mu_r}\vec{B}$

C.F. $\vec{\nabla}\times\vec{E}=0$

$\vec{\nabla}\cdot\vec{E}=\frac{1}{\varepsilon_o}(\rho_f+\rho_p)$

$=\frac{1}{\varepsilon_o}(\rho_f-\vec{\nabla}\cdot\vec{P})$

$\vec{\nabla}\cdot\left[\varepsilon_o\vec{E}+\vec{P}\right]=\rho_f$

$\vec{\nabla}\cdot\vec{D}=\rho_f$

$\vec{D}=\varepsilon_o\vec{E}+\vec{P}$

Electromagnetic

Static Magnetic

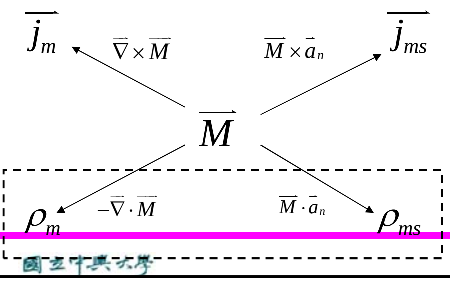
Source : $\vec{j}_f$, μ (permeativity)

↓ $\vec{\nabla}\times\vec{H}=\vec{j}_f$

Conductor : $\vec{H}\rightarrow\vec{B}\rightarrow\Phi_m\rightarrow L$

$\vec{B}=\mu\vec{H}$ $\Phi_m=\int\vec{B}\cdot d\vec{s}$ $\frac{1}{L}=\frac{I_f}{\Phi_m}$

Magnetic material :



Copy 電學
數學上等效，無物理

$$d\phi_m = \frac{\vec{M}\cdot\hat{a}_r}{4\pi r^2}$$

$$\phi_m = \frac{1}{4\pi}\int_{v'}\frac{\vec{M}\cdot\hat{a}_r}{r^2}dv'$$

$$= \frac{1}{4\pi}\oint_{s'}\frac{\vec{M}\cdot\hat{a}_n}{r}ds' + \frac{1}{4\pi}\int_{v'}\frac{-\left(\vec{\nabla}\times\vec{M}\right)}{r}dv'$$

$$\rho_{ms} = \vec{M}\cdot\hat{a}_n\ ;\ \rho_m = -\vec{\nabla}\cdot\vec{M}$$

◎ $\vec{j}_{ms}$, $\vec{j}_m$, $\vec{A} = \frac{\mu_0}{4\pi}\left[\oint_{s'}\frac{\vec{j}_{ms}}{\left|\vec{r}-\vec{r}'\right|}da' + \int_{v'}\frac{\vec{j}_m}{\left|\vec{r}-\vec{r}'\right|}dv'\right]$, $\vec{B} = \vec{\nabla}\times\vec{A}$

◎ ρ_{ms} , ρ_m , $\phi_m = \frac{1}{4\pi}\left[\oint_{s'}\frac{\rho_{ms}}{\left|\vec{r}-\vec{r}'\right|}da' + \int_{v'}\frac{\rho_m}{\left|\vec{r}-\vec{r}'\right|}dv'\right]$, $\vec{H} = -\vec{\nabla}\phi_m \Rightarrow \vec{B} = \frac{1}{\mu}\vec{H}$

Ex 6-9

$$\rho_{ms} = \begin{cases} M_o & top\ \ face \\ -M_o & bottom\ \ face \\ 0 & side\ wall \end{cases}$$

$\rho_m = 0$ inside

$$q_m = \pi b^2 \rho_{ms} = \pi b^2 M_0$$

$$\phi_m = \frac{q_m}{4\pi}\left(\frac{1}{R_+} - \frac{1}{R_-}\right)$$

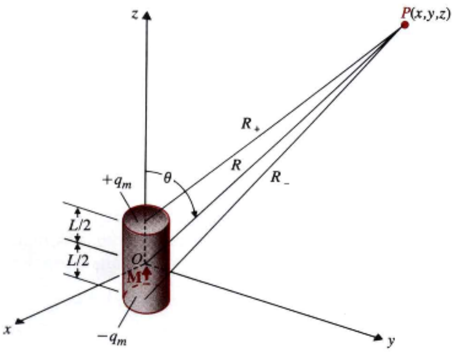
$R >> b$ [Dipole]

$$\phi_m = \frac{q_m L\cos\theta}{4\pi R^2} = \frac{\left(\pi b^2 M_0\right)L\cos\theta}{4\pi R^2} = \frac{M_T\cos\theta}{4\pi R^2}\ ;\ M_T = \pi b^2 L M_0$$

$$\vec{B} = -\mu_0\nabla\phi_m = \frac{\mu_0 M_T}{4\pi R^3}\left[\hat{a}_r 2\cos\theta + \hat{a}_\varphi\sin\theta\right]$$

電場

與（**Dipole** 類似）



6-7

$$\vec{\nabla}\times\vec{H}=\vec{J}_f$$

$$\left(\vec{H}=\frac{\vec{B}}{\mu_0}-\vec{M}\right)$$

$$\int_s\left(\vec{\nabla}\times\vec{H}\right)\cdot d\vec{S}=\int_s\vec{J}\cdot d\vec{S}$$

$$\oint\vec{H}\cdot d\ell=I$$

Ampere's circuital law

$$\vec{B}=\mu_0(1+\chi_m)\vec{H}$$
$$=\mu_0\mu_r\vec{H}$$

$$\boxed{\vec{H}=\frac{1}{\mu}\vec{B}}\ ;\ \boxed{u_r=1+\chi_m=\frac{\mu}{\mu_0}}$$

relative permeability

Electrostatics	Magnetostatics
E	**B**
D	**H**
ϵ	$\frac{1}{\mu}$
P	$-$**M**
ρ	**J**
V	**A**
$\cdot$	$\times$
$\times$	$\cdot$

6-8 Magnetic Circuits

Electric circuit : Voltage / Current source ; V, I, …

Magnetic circuit : Transformer / Generator / Motor …

$\vec{\nabla}\cdot\vec{B}=0$

$\vec{\nabla}\times\vec{H}=\vec{J}$; closed path c to enclose N turns of I

$\oint_c\vec{H}\cdot d\vec{l}=NI=V_m$ (m.m.f) magnetomotive force [Amp]

Electromagnetic

Ex 6-10

Sol :

$\oint_c \vec{H}\cdot d\vec{l} = NI_o$

$\vec{B}$ is identical in different material ; $\boxed{\vec{\nabla}\cdot\vec{B}=0}$

$\vec{B}_f = \vec{B}_g = \hat{a}_\varphi B_f$ f : ferromagnetic; g : gap

$\vec{H}_f = \hat{a}_\varphi \frac{B_f}{\mu}; \vec{H}_g = \hat{a}_\varphi \frac{B_f}{\mu_o}$

Ampere law

$$\frac{B_f}{\mu}(2\pi r_o - l_g) + \frac{B_f}{\mu_o} l_g = NI_o$$

$$\vec{B}_f = \hat{a}_\varphi \frac{\mu_0 \mu N I_o}{\mu_0(2\pi r_o - l_g)+\mu l_g}$$

$$\vec{H}_f = \hat{a}_\varphi \frac{\mu_0 N I_o}{\mu_0(2\pi r_o - l_g)+\mu l_g}; \vec{H}_g = \hat{a}_\varphi \frac{\mu N I_o}{\mu_0(2\pi r_o - l_g)+\mu l_g}; \frac{H_g}{H_f} = \frac{\mu}{\mu_0}$$

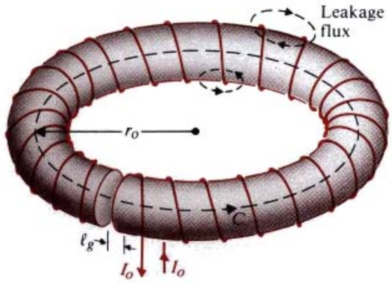


FIGURE 6–13
Coil on ferromagnetic toroid with air gap

Electromagnetic

Magnetic Flux $\Phi \approx B_f S$; S : cross-section

$$B_f = \frac{\mu_0 \mu N I_o}{\mu_0(2\pi r_o - l_g)+\mu l_g} = \frac{NI_o}{\left(\frac{2\pi r_o - l_g}{\mu}\right)+\frac{l_g}{\mu_o}}$$

$$\Phi = B_f \cdot S = \frac{NI_o}{\left(\frac{2\pi r_o - l_g}{\mu S}\right)+\frac{l_g}{\mu_o S}} = \frac{V_m}{R_f + R_g}$$

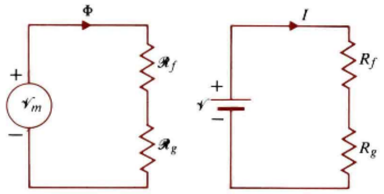
$R_f = \frac{2\pi r_o - l_g}{\mu S} = \frac{l_f}{\mu S}; l_f = 2\pi r_o - l_g$: length of ferromagnetic core.

$R_g = \frac{l_g}{\mu_o S}$: Reluctance $\begin{cases} R_f & \text{:ferromagnetic core} \\ R_g & \text{:air gap} \end{cases}$

Analog to : [Electric circuit]

$$I = \frac{v}{R_f + R_g}$$

Electromagnetic



(a) Magnetic circuit. (b) Electric circuit.

FIGURE 6–14 Equivalent magnetic circuit and analogous electric circuit for toroidal coil with air gap in Fig. 6–13.

Magnetic circuit	Electric circuit
$\Phi = \dfrac{V_m}{R_f + R_g} ; R = \dfrac{l}{\mu S}$	$I = \dfrac{v}{R_f + R_g} ; R = \dfrac{l}{\sigma S}$
mmf $V_m (= NI)$	**emf** V
mag. flux Φ	**electric current ,** I
reluctance R	**resistance ,** R
Permeability μ	**conductivity ,** σ

Electromagnetic

An exact analysis of magnetic circuits is difficult

◎Leakage Fluxes ◎Fringing effect ◎ $\bar{B} = \mu(\vec{B}, \vec{H})\vec{H}$

2 conditions must be satisfied

$$\left.\begin{aligned} H_g l_g + H_f l_f &= NI_o \\ B_f = B_g &= \mu_o H_g \end{aligned}\right\} \Rightarrow B_f + \mu_o \frac{l_f}{l_g} H_f = \frac{\mu_o}{l_g} NI_o$$

Similar to

Kirchhoff's voltage Law

$$\sum_j N_j I_j = \sum_k R_k \Phi_k$$

Kirchhoff's current Law

$$\sum_j \Phi_j = 0 \qquad \vec{\nabla} \bullet \vec{B} = 0$$

EX.6-11

$$R_1 = \frac{l_1}{\mu S_c}$$

$$R_2 = \frac{l_2}{\mu S_c}$$

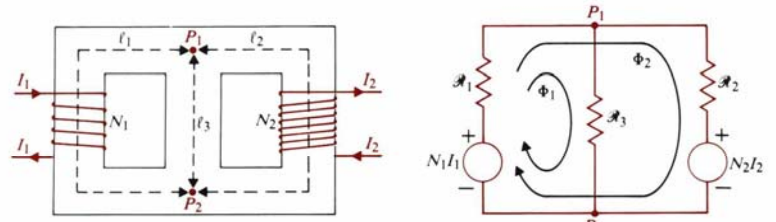
$$R_3 = \frac{l_3}{\mu S_c}$$

K.V.L. (Time Independent)

Loop1: $N_1 I_1 = (R_1 + R_3)\Phi_1 + R_1 \Phi_2$

Loop2: $N_1 I_1 - N_2 I_2 = R_1 \Phi_1 + (R_1 + R_2)\Phi_2$

$$\Rightarrow \quad \Phi_1 = \frac{R_2 N_1 I_1 + R_1 N_2 I_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$



(a) Magnetic core with current-carrying windings. (b) Magnetic circuit for loop analysis.

FIGURE 6–15
A magnetic circuit (Example 6–11).

6-9 Behavior of Magnetic Materials

$\vec{M} = \chi_m \vec{H}$, χ_m : magnetic susceptibility

$\vec{H} = \frac{1}{\mu}\vec{B}$, $\mu_r = 1 + \chi_m = \frac{\mu}{\mu_o}$

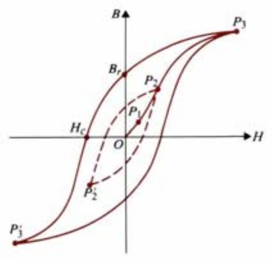


FIGURE 6–17
Hysteresis loops in the B–H plane for ferromagnetic material.

◎Diamagetic: $\mu_r \le 1$ (χ_m : small negative number)

◎Paramagnetic: $\mu_r \ge 1$ (χ_m : small positive number)

◎Ferromagnetic: $\mu_r >> 1$ (χ_m : large positive number)

6-10 Boundary Conditions for Magnetostatic Field

$\vec{\nabla} \bullet \vec{B} = 0$

$B_{1n} = B_{2n}$

$\mu_1 H_{1n} = \mu_2 H_{2n}$

$\vec{\nabla} \times \vec{H} = \vec{J}$

$\oint_C \vec{H} \cdot d\vec{l} = I \quad (bc = da = \Delta h \to 0)$

$\oint_{abcda} \vec{H} \cdot d\vec{l} = \vec{H}_1 \cdot \Delta\vec{W} + \vec{H}_2 \cdot (-\Delta\vec{W}) = J_{sn} \Delta W$

$\Rightarrow H_{1t} - H_{2t} = J_{sn}$

$\hat{a}_{n2} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_S$

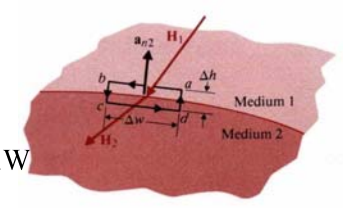


FIGURE 6-19
Closed path about the interface of two media for determining the boundary condition of H_t.

Ex 6-12

B_n component

$\mu_2 H_2 \cos\alpha_2 = \mu_1 H_1 \cos\alpha_1$

H_t component

$H_2 \sin\alpha_2 = H_1 \sin\alpha_1$

$\frac{\tan\alpha_2}{\tan\alpha_1} = \frac{\mu_2}{\mu_1}$

or $\alpha_2 = \tan^{-1}(\frac{\mu_2}{\mu_1} \tan\alpha_1)$

Magnitude of $\vec{H}_2$

$H_2 = \sqrt{H^2_{2t} + H^2_{2n}} = \sqrt{(H_2 \sin\alpha_2)^2 + (H_2 \cos\alpha_2)^2}$

$= H_1 \left[\sin^2\alpha_1 + (\frac{\mu_1}{\mu_2} \cos\alpha_1)^2 \right]^{1/2}$

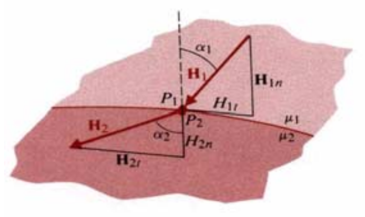


FIGURE 6-20
Boundary conditions for magnetostatic field at an interface (Example 6-12).

Similar to E-field

$\mu_2 \gg \mu_1 \ , \ \alpha_2 = 90^o$

$\mu_1 \gg \mu_2 \ , \ \alpha_2 = 0^o$

$\vec{H}$ In ferromagnetic parallel interface

$\vec{H}$ Originates in a ferromangeitc ,

Flux perpendicular to interface

Ex 6-13

Surface current $\vec{J}_{ms} = M_0 \hat{a}_\varphi$

Example 6-8 [p246]

$$\vec{B}_{po} = \hat{a}_z \frac{\mu_0 M_0}{2}\left[\frac{L}{\sqrt{(L/2)^2 + b^2}}\right]$$

$$\vec{B}_{p1} = \hat{a}_z \frac{\mu_0 M_0}{2}\left[\frac{L}{\sqrt{(L)^2 + b^2}}\right] = \vec{B}_{p'1}$$

$$\underbrace{\vec{B}_{p1} = \vec{B}_{p'1}}_{\text{End}} < \underbrace{\vec{B}_{po}}_{\text{Center}}$$

at interface quantity

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

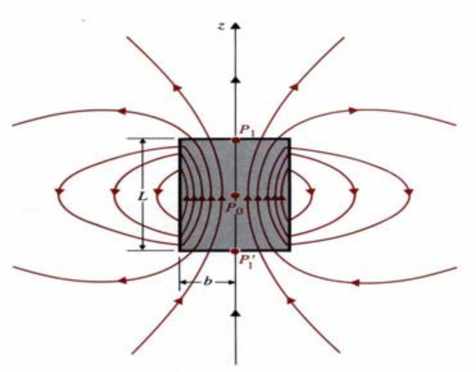


FIGURE 6–21
Magnetic flux lines around a cylindrical bar magnet (Example 6–13).

6-11 Inductances & Inductors

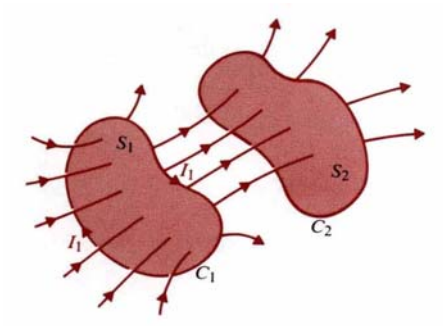


FIGURE 6–22
Two magnetically coupled loops.

Mutual flux $\Phi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{S}_2$

$$\Phi_{12} = L_{12} I_1$$

L_{12} ： mutual inductance

between loops C_1 and C_2

If loop C_2 has N_2 turns ,

$$\Lambda_{12} = N_2 \Phi_{12}$$

Generalizes to

$$\Lambda_{12} = L_{12} I_1$$

$$\boxed{L_{12} = \frac{\Lambda_{12}}{I_1}} \Longrightarrow \boxed{L_{12} = \frac{d\Lambda_{12}}{dI_1}\ (H)}$$

Electromagnetic

Some of $\vec{B}$ produced by I_1 links only with C_1 loop itself, not with C_2

$$\Lambda_{11}=N_1\Phi_{11}>N_1\Phi_{12}$$

Self inductance of C_1 loop

$$\boxed{L_{11}=\frac{\Lambda_{11}}{I_1}} \Longrightarrow \boxed{L_{11}=\frac{d\Lambda_{11}}{dI_1}}$$

Procedure for Finding Inductance

1. Appropriate coordinate system
2. Find
$$\vec{B}=\frac{\mu_0}{4\pi}\int_{v'}\vec{J}(\vec{r}')\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dv'$$
3. $$\Phi=\int_S\vec{B}\cdot d\vec{S}$$
4. $$\Lambda=N\Phi$$
5. $$L=\frac{\Lambda}{I}$$

Electromagnetic

EX 6-14

$$\vec{B}=B_\varphi\hat{a}_\varphi$$

$$d\vec{l}=rd\varphi\,\hat{a}_\varphi$$

$$\oint_C\vec{B}\cdot d\vec{l}=\int_0^{2\pi}B_\varphi rd\varphi$$

$$=2\pi rB_\varphi$$

total current NI

$$2\pi rB_\varphi=\mu_0NI$$

$$B_\varphi=\frac{\mu_0NI}{2\pi r}$$

$$\Phi=\int_S\vec{B}\cdot d\vec{s}$$

$$=\int_S\left(\hat{a}_\varphi\frac{\mu_0NI}{2\pi r}\right)\cdot\left(\hat{a}_\varphi hdr\right)$$

$$=\frac{\mu_0NIh}{2\pi}\ln\left(\frac{b}{a}\right)$$

flux linkage

$$\wedge=N\Phi$$

$$=\frac{\mu_0N^2Ih\cdot\ln\left(\frac{b}{a}\right)}{2\pi}$$

$$L=\frac{\wedge}{I}=\frac{\pi_0N^2h\cdot\ln\left(\frac{b}{a}\right)}{2\pi}$$

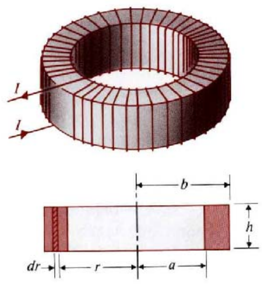


FIGURE 6–23
A closely wound toroidal coil (Example 6–14).

EX 6-15 Long solenoid

From(Ex6-3) p231

$$B = \mu_0 nI$$

$$\Phi = BS = \mu_0 nIS$$

$$\wedge' = n\Phi = \mu_0 n^2 SI$$

Inductance per unit length

$$L' = \mu_0 n^2 s$$

$$l >> s$$

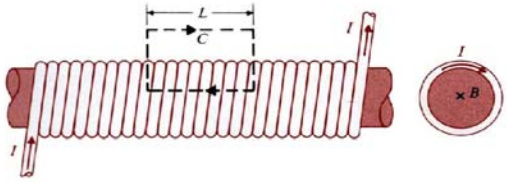


FIGURE 6–4
A current-carrying long solenoid (Example 6–3).

$$L \; \alpha \; N^2$$

$$in \; Ex6-14$$

$$Ex6-15$$

EX 6-16

a) Inside inner conductor.

$$0 \le r \le a$$

$$\vec{B}_1 = \hat{a}_\varphi B_{\varphi 1} = \hat{a}_\varphi \frac{\mu_0 rI}{2\pi a^2}$$

b) Between inner & outer conductors

$$a \le r \le b$$

$$\vec{B}_2 = \hat{a}_{\varphi 2} B_{\varphi 2} = \hat{a}_\varphi \frac{\mu_0 I}{2\pi r}$$

$$d\Phi' = \int_r^a B_{\varphi 1} dr + \int_a^b B_{\varphi 2} dr$$

$$= \frac{\mu_0 I}{2\pi a^2} \int_r^a rdr + \frac{\mu_0 I}{2\pi} \int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 I}{4\pi a^2}(a^2 - r^2) + \frac{\mu_0 I}{2\pi} \ln(\frac{b}{a})$$

Current in annular ring

$$\left(\frac{2\pi rdr}{\pi a^2}\right) \Rightarrow \frac{2rdr}{a^2}$$

$$d\wedge' = \frac{2rdr}{a^2} d\Phi'$$

$$\wedge' = \int_{r=0}^{r=a} d\wedge'$$

$$= \frac{\mu_0 I}{\pi a^2}\left[\frac{1}{2a^2}\int_0^a (a^2 - r^2) rdr + (\ln\frac{b}{a})\int_0^a rdr\right]$$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{1}{4} + \ln\frac{b}{a}\right)$$

$$L' = \frac{\wedge'}{I} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln(\frac{b}{a}) \; [H/m]$$

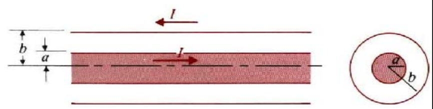


FIGURE 6–24
Two views of a coaxial transmission line (Example 6–16).

Electromagnetic

EX 6-17

Internal $L'self = \frac{\mu_0}{8\pi}$

2 wires : $L_i' = 2 \cdot \frac{\mu_0}{8\pi} = \frac{\mu_0}{4\pi}$

external : $xz - plane,\ only\ y - comp.$

$$B_{y1} = \frac{\mu_0 I}{2\pi x}$$

$$B_{y2} = \frac{\mu_0 I}{2\pi(d-x)}$$

$$\Phi' = \int_a^{d-a} (B_{y1} + B_{y2})dx$$

$$= \int_a^b \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} + \frac{1}{d-x}\right]dx$$

$$= \frac{\mu_0 I}{\pi}\ln(\frac{d-a}{a}) \cong \frac{\mu_0 I}{\pi}\ln(\frac{d}{a})$$

$$L_e' = \frac{\Phi'}{I} = \frac{\mu_0}{\pi}\ln(\frac{d}{a})$$

total

$$L' = L_i' + L_e' = \frac{\mu_0}{\pi}\left[\frac{1}{4} + \ln\left(\frac{d}{a}\right)\right]$$

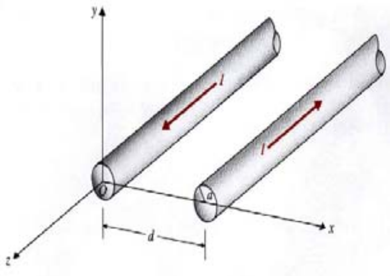


FIGURE 6-25
A two-wire transmission line (Example 6-17).

Electromagnetic

$L_{12} = L_{21}$?

$$L_{12} = \frac{N_2}{I_1}\int_{S_2} \overline{B_1} \bullet d\overline{S_2}$$

$$(\overline{B_1} = \overline{\nabla} \times \overline{A_1})$$

$$L_{12} = \frac{N_2}{I_1}\int_{S_2} (\overline{\nabla} \times \overline{A_1}) \bullet d\overline{S_2}$$

$$= \frac{N_2}{I_1}\oint_{C_1} \overline{A_1} \bullet d\overline{\ell_2}$$

$$(\overline{A_1} = \frac{\mu_o}{4\pi} N_1 I_1 \oint_{C_1} \frac{d\overline{\ell_1}}{R})$$

$$\Rightarrow L_{12} = \frac{\mu_o N_1 N_2}{4\pi}\oint_{C_1}\oint_{C_2} \frac{d\overline{\ell_1} \cdot d\overline{\ell_2}}{R}$$

Neumann Formula

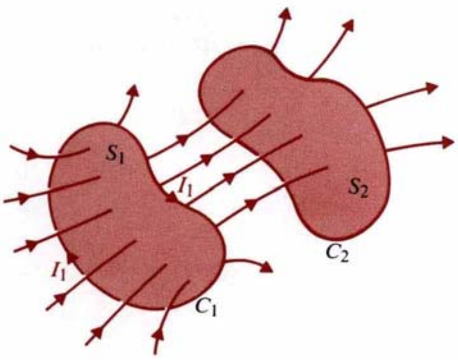


FIGURE 6–22
Two magnetically coupled loops.

EX：6-18

$$\Phi_{12} = \mu(\frac{N_1}{\ell_1})(\pi a^2)I_1$$

Outer coil has N_2 turns,

$$\wedge_{12} = N_2\Phi_{12} = \frac{\mu}{\ell_1}N_1N_2\pi a^2 I_1$$

$$L_{12} = \frac{\wedge_{12}}{I_1} = \frac{\mu}{\ell_1}N_1N_2\pi a^2$$

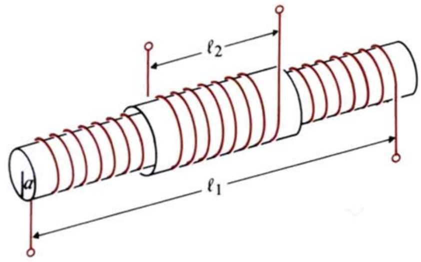


FIGURE 6–26
A solenoid with two windings (Example 6–18).

EX：6-19

Find B_2 is caused by long wire I_2.

$$\overline{B}_2 = \hat{a}_\varphi \frac{\mu_o I_2}{2\pi r}$$

$$\wedge_{21} = \Phi_{21},$$

$$\wedge_{21} = \int_{S_1} \overline{B}_2 \bullet d\vec{s}_1$$

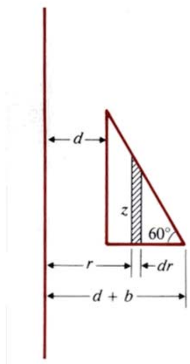
$$d\vec{s}_1 = \hat{a}_\varphi z dr$$

$$*z = \left[(d+b) - r\right]$$

$$\wedge_{21} = \frac{\sqrt{3}\mu_o I_2}{2\pi}\int_d^{d+b}\frac{1}{r}\left[(d+b)-r\right]dr$$

$$= \frac{\sqrt{3}\mu_o I_2}{2\pi}\left[(d+b)\ln\left(1+\frac{b}{d}\right)-b\right]$$

$$L_{21} = \frac{\wedge_{21}}{I_2} = \frac{\sqrt{3}\mu_o}{2\pi}\left[(d+b)\ln\left(1+\frac{b}{d}\right)-b\right]$$



6-12 Magnetic Energy

Loop 1 $V_1 = L_1 \dfrac{di_1}{dt}$

$$W_1 = \int V_1 \, i_1 dt$$

$$= L_1 \int_0^{I_1} i_1 di_1$$

$$= \frac{1}{2} L_1 I_1^2 = \frac{1}{2} \Phi_1 L_1$$

Loop 2 : C_1 & C_2

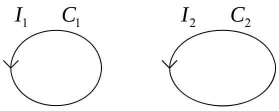
$$W_{21} = \int V_{21} I_1 dt$$

$$= L_{21} I_1 \int_0^{I_2} di_2$$

$$= L_{21} I_1 I_2$$

Similary

$$W_{22} = \frac{1}{2} L_2 I_2^2$$



Total work at C_2

$$W_2 = W_1 + W_{12} + W_{22}$$

$$= \frac{1}{2} L_1 I_1^2 + L_1 I_1 I_2 + \frac{1}{2} L_2 I_2^2$$

$$= \frac{1}{2} \sum_{j=1}^{2} \sum_{k=1}^{2} L_{jk} I_j I_k$$

$$W_m = \frac{1}{2} L I^2$$

Generalizing I_1, I_2, I_3, … I_N,

$$W_m = \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} L_{jk} I_j I_k$$

Consider K[th] loop of N coupled loops

$$dW_k = V_k i_k dt$$

$$= i_k d\varphi_k$$

$$V_k = \frac{d\varphi_k}{dt}$$

Magnetic energy

$$dW_m = \sum_{k=1}^{N} dW_k = \sum_{k=1}^{N} i_k d\varphi_k$$

Total magnetic energy $i_k = \alpha I_k$ $\phi_k = \alpha \Phi_k$

$$W_m = \int dW_m = \sum_{k=1}^{N} I_k \Phi_k \int_0^1 \alpha d\alpha$$

$$= \frac{1}{2} \sum_{k=1}^{N} I_k \Phi_k$$

$$W_m = \frac{1}{2} \sum_{k=1}^{N} I_k \Phi_k$$

$$\Phi_k = \sum_{j=1}^{N} L_{jk} I_j$$

6-12.1 Wm in terms of Field Quantities

$$\Phi_k = \int_{S_k} \overline{B}\cdot \hat{a}_n dS_k' = \oint_{C_k} \overline{A}\cdot d\ell_k'$$

$$W_m = \frac{1}{2}\sum_{k=1}^{N} \Delta I_k \oint_{C_k} \overline{A}\cdot d\ell_k'$$

$$\Delta I_k d\ell_k' = J(\Delta a_k')d\ell_k' = \overline{J}\Delta v_k'$$

$$N \to \infty,\ \Delta v_k' \to dv'$$

$$W_m = \frac{1}{2}\int_{V'} \overline{A}\cdot\overline{J} dv'$$

Vector identity

$$\overline{\nabla}\cdot(\overline{A}\times\overline{B}) = \overline{B}\cdot(\overline{\nabla}\times\overline{A}) - \overline{A}\cdot(\overline{\nabla}\times\overline{B})$$

取 $\overline{A}=\overline{A}$; $\overline{B}=\overline{H}$

$$\Rightarrow \overline{A}\cdot(\overline{\nabla}\times\overline{B}) = \overline{H}\cdot(\overline{\nabla}\times\overline{A}) - \overline{\nabla}\cdot(\overline{A}\times\overline{H})$$

其中 $\overline{\nabla}\times\overline{H}=\overline{J}$ $\overline{\nabla}\times\overline{A}=\overline{B}$

$$\Rightarrow \overline{A}\cdot\overline{J} = \overline{H}\cdot\overline{B} - \overline{\nabla}\cdot(\overline{A}\times\overline{H})$$

$$W_m = \frac{1}{2}\int_{V'} (\overline{H}\cdot\overline{B})dv' - \frac{1}{2}\oint_{S'} (\overline{A}\times\overline{H})\cdot\hat{a}_n ds'$$

All space

$$\lim_{s'\to\infty}\left(\frac{1}{r}\frac{1}{r^2}\right)r^2 \to 0$$

$$W_m = \frac{1}{2}\int_{V'} (\overline{H}\cdot\overline{B})dv'$$

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$$W_m = \frac{1}{2}\int_{V'} (\overline{H}\cdot\overline{B})dv'$$

$$\overline{H} = \frac{\overline{B}}{\mu}$$

$$\boxed{W_m = \frac{1}{2}\int_{V'} \frac{B^2}{\mu} dv'}$$

or

$$\boxed{W_m = \frac{1}{2}\int_{V'} \mu H^2 dv'}$$

c.f.

$$W_e = \frac{1}{2}\int_{V'} (\overline{E}\cdot\overline{D})dv'$$

$$W_e = \frac{1}{2}\int_{V'} \varepsilon E^2 dv' = \frac{1}{2}\int_{V'} \frac{D^2}{\varepsilon} dv'$$

Magnetic energy density Wm

$$W_m = \int_{V'} W_m dv'$$

$$W_m = \frac{1}{2}\overline{H}\cdot\overline{B} = \frac{B^2}{2\mu} = \frac{1}{2}\mu H^2$$

$$\boxed{L = \frac{2W_m}{I^2}}$$

Ex 6-20 (Ref. Ex 6-16)

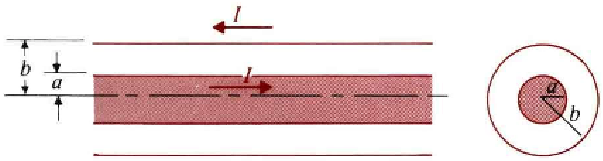
W_m in inner conductor

$$W'_{m1}=\frac{1}{2\mu_0}\int_0^a B_{\phi_1}^2\,2\pi r dr=\frac{\mu_0 I^2}{4\pi a^4}\int_0^a r^3 dr=\frac{\mu_0 I^2}{16\pi}$$

W_m between inner & outer

$$W'_{m2}=\frac{1}{2\mu_0}\int_a^b B_{\phi_2}^2\,2\pi r dr=\frac{\mu_0 I^2}{4\pi}\int_a^b \frac{1}{r}dr=\frac{\mu_0 I^2}{4\pi}\ln\left(\frac{b}{a}\right)$$

Hence, $L'=\frac{2}{I^2}\left(W'_{m1}+W'_{m2}\right)=\frac{\mu_0}{8\pi}+\frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$



6-13 Magnetic forces & Torques

$$\vec{F}_m=q\vec{u}\times\vec{B}$$

$$\vec{B}=B_0\hat{a}_z\ ;\ \vec{J}=J_0\hat{a}_y=Nq\vec{u}$$

$$V_h=-\int_0^d E_h dx=u_0 B_0 d$$

electron move toward to x-dir.

Creating a transverse $\vec{E}$-field. $\vec{E}_h$

Steady state, net force is Zero.

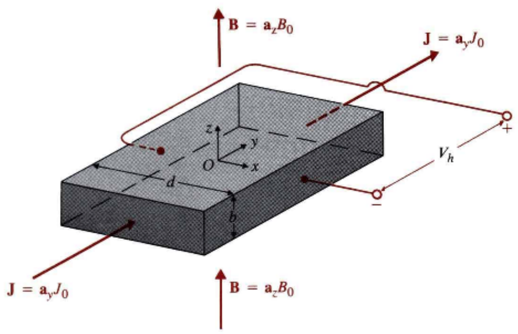
$$\vec{E}_h+\vec{u}\times\vec{B}=0$$

$$\vec{E}_h=-\vec{u}\times\vec{B}\ ;\ Hall\ effect.$$

$$\vec{E}_h : Hall\ \ feild.$$

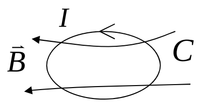
$$N\text{-}type : \vec{u}=-u_0\hat{a}_y$$

$$\vec{E}_h=-\left(-u_0\hat{a}_y\right)\times B_0\hat{a}_z=u_0B_0\hat{a}_x$$

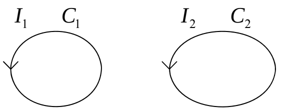


6-13.2 Force & Torques

$d\overrightarrow{F_m} = Id\vec{l}\times\vec{B}$



$\overrightarrow{F_m} = I\oint_c d\vec{l}\times\vec{B}$



$\overrightarrow{B_{21}} : I_2$ *source*

$\overrightarrow{F_{21}} : I_1$ *field*

$\overrightarrow{F_{21}} = I_1\oint_{c1} d\vec{l}_1\times\overrightarrow{B_{21}}$

$$\overrightarrow{B_{21}} = \frac{\mu_0 I_2}{4\pi}\oint_{c2}\frac{d\hat{l}_2\times\widehat{a_{\Re 21}}}{\Re_{21}{}^2}$$

$$\overrightarrow{F_{21}} = \frac{\mu_0}{4\pi}I_1 I_2\oint_{c1}\oint_{c2}\frac{d\hat{l}_1(d\hat{l}_2\bullet\widehat{a_{\Re 21}})}{\Re_{21}{}^2}$$

$$\overrightarrow{F_{21}} = \frac{\mu_0}{4\pi}I_2 I_1\oint_{c1}\oint_{c2}\frac{d\hat{l}_2(d\hat{l}_1\bullet\widehat{a_{\Re 21}})}{\Re_{21}{}^2}$$

$$-[d\hat{l}_1\times(d\hat{l}_2\times\widehat{a_{\Re 21}})] \overset{?}{=} d\hat{l}_2\times(d\hat{l}_1\times\widehat{a_{\Re 21}})$$

$-\overrightarrow{F_{21}} \overset{?}{=} \overrightarrow{F_{12}}$: Newton 3^{rd} Law

Vector triple product

$$\frac{d\hat{l}_1\times(d\hat{l}_2\times\widehat{a_{\Re 21}})}{\Re_{21}{}^2} = \frac{d\hat{l}_2(d\hat{l}_1\bullet\widehat{a_{\Re 21}})}{\Re_{21}{}^2} - \frac{\widehat{a_{\Re 21}}(d\hat{l}_1\bullet d\hat{l}_2)}{\Re_{21}{}^2}$$

1^{st} term

$$\oint_{c1}\oint_{c2}\frac{d\hat{l}_2(d\hat{l}_1\bullet\widehat{a_{\Re 21}})}{\Re_{21}{}^2} = \oint_{c2}d\overline{l}_2\oint_{c1}\frac{d\hat{l}_1\bullet\widehat{a_{\Re 21}}}{\Re_{21}{}^2} = \oint_{c2}d\overline{l}_2\oint_{c1}d\hat{l}_1(-\overline{\nabla}_1\frac{1}{\Re_{21}})$$

$$= -\oint_{c2}d\overline{l}_2\oint_{c1}d(\frac{1}{\Re_{21}}) = 0$$

代回 $\overrightarrow{F_{21}} = -\frac{\mu_0}{4\pi}I_1 I_2\oint_{c1}\oint_{c2}\frac{\widehat{a_{\Re 21}}(d\hat{l}_1\bullet d\hat{l}_2)}{\Re_{21}{}^2} = -\overrightarrow{F_{12}}$

$\widehat{a_{\Re 21}} = -\widehat{a_{\Re 12}}$, Newton 3^{rd} Law Hold

$Ex6-21$

$\overrightarrow{F_{12}}'$ force on wire 2

$\overrightarrow{F_{12}}' = I_2(\widehat{a_z} \times \overrightarrow{B_{12}})$

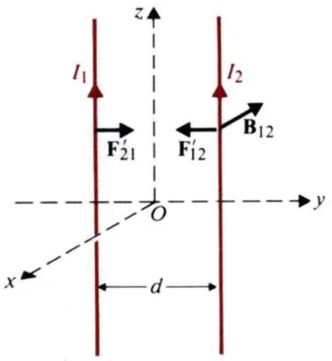
$\overrightarrow{B_{12}}$ source at wire1(I_1)

$\overrightarrow{B_{12}} = -\widehat{a_x}\dfrac{\mu_0 I_1}{2\pi d}$

$\overrightarrow{F_{12}}' = -\widehat{a_y}\dfrac{\mu_0 I_1 I_2}{2\pi d}$

Attraction

[Same polarity of current I_1 & I_2]



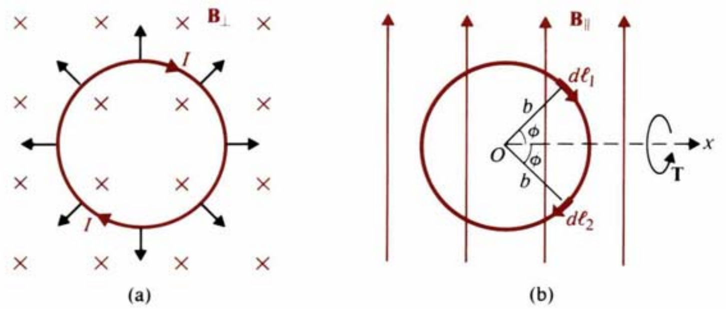


FIGURE 6–30
A circular loop in a uniform magnetic field $\mathbf{B} = \mathbf{B}_\perp + \mathbf{B}_{||}$.

$\vec{B} = \overrightarrow{B_\perp} + \overrightarrow{B_{//}}$

$\overline{B_\perp}$: expand loop 6-30(a)
no net force to more loop

$\overline{B_{//}}$: produce on upward force $d\overline{F_1}$ on $d\overline{l_1}$
downward force $d\overline{F_2}$ on $d\overline{l_2}$

$d\overline{F_1} = -d\overline{F_2}$

$$d\vec{T} = \widehat{a_x}(dF)2b\sin\phi$$
$$= \widehat{a_I}(IdlB_{//}\sin\phi)2b\sin\phi$$
$$= \widehat{a_I}\,2Ib^2B_{//}\sin^2\phi d\phi$$
$$dF = |dF_1| = |dF_2| \;\; ; \;\; dl = |dl_1| = |dl_2| = bd\phi$$
$$\vec{T} = \int d\vec{T} = \widehat{a_x}\,2Ib^2B_{//}\int_0^{\pi}\sin^2\phi d\phi$$
$$= \widehat{a_x}I(\pi b^2)B_{//}$$

Magnetic dipole moment

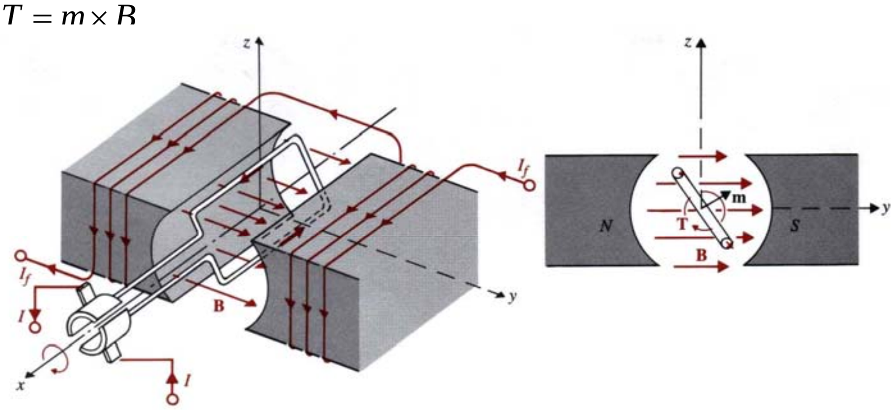
$$\vec{m} = \widehat{a_I}I(\pi b^2) = \widehat{a_N}IS$$

Hence,

$$\vec{T} = \vec{m}\times\vec{B}$$

∘DC-motor

∘Torque rotates at clockwise + X-dir



(a) Perspective view. (b) Schematic view from $+x$ direction.

$Ex6-22$

Electromagnetic

$\overline{B_\perp} = \widehat{a_Z} B_Z$; $\overline{B_{//}} = \widehat{a_X} B_x + \widehat{a_y} B_y$

$\overline{B_{//}}$ produes the following forces

$\overline{T} = \overline{T_{13}} + \overline{T_{24}} = Ib_1 b_2 (\widehat{a_x} B_y - \widehat{a_y} B_x)$

$\overline{m} = -\widehat{a_z} I b_1 b_2$

$\overline{T} = \overline{m} \times \overline{B} = \overline{m} \times (\widehat{a_x} B_y - \widehat{a_y} B_x)$

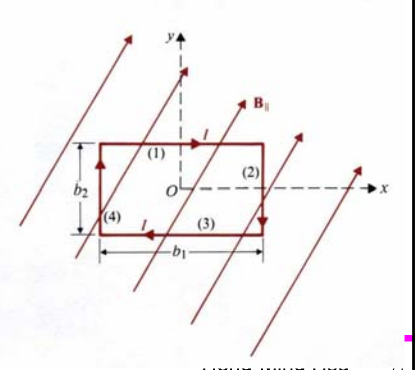
$\overline{F_1} = Ib_1 \widehat{a_X} \times (\widehat{a_x} B_x + \widehat{a_y} B_y) = \widehat{a_Z} I b_1 B_y = -\overline{F_3}$

$\overline{F_2} = Ib_2 (-\widehat{a_y}) \times (\widehat{a_X} B_x + \widehat{a_y} B_y) = \widehat{a_Z} I b_2 B_x = -\overline{F_4}$

$\overline{F_{NET}} = \sum_{i=1}^{4} \overline{F_i} = 0$

$\overline{T_{13}} = \widehat{a_x} I b_1 b_2 B_y$;$[\overline{F_1} \,\&\, \overline{F_3}]$

$\overline{T_{24}} = -\widehat{a_y} I b_1 b_2 B_x$;$[\overline{F_2} \,\&\, \overline{F_4}]$



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$6-13.3$ Forces and Torques in terms of Wm

Electromagnetic

∘ Constant Flux Linkages

[Source Spply No energy]

$\overline{F_\Phi} \bullet d\overline{l} = -dW_m = -(\overline{\nabla} W_m) \bullet d\overline{l}$

$\overline{F_\Phi} = -\overline{\nabla} W_m$

rotate about z-axis

$(T_\Phi)_Z = -\frac{\partial W_m}{\partial \phi}$

∘ Constant Currents

[*Current* sorce $\rightarrow$ increase Wm]

$dW_s = dW + dW_m$

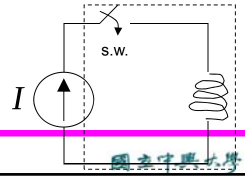
$dW_m = \frac{1}{2} dW_s$

$dW = \overline{F_I} \bullet d\overline{l} = dW_m = (\overline{\nabla} W_m) \bullet d\overline{l}$

$\overline{F_I} = \overline{\nabla} W_m$

$(T_I)_Z = \frac{\partial W_m}{\partial \phi}$

S.W.(OFF) S.W.(ON)



system

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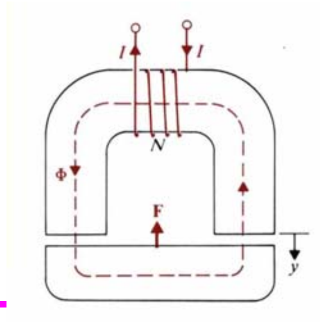
$Ex6-23$

• Contant Flux

$$dW_m = d(W_m)_{air} = 2(\frac{B^2}{2\mu_0}Sdy)$$

$$= \frac{\Phi^2}{\mu_0 S}dy$$

$$\overline{F_\Phi} = \hat{a}_y(-\frac{dW_m}{dy}) = -\hat{a}_y\frac{\Phi^2}{\mu_0 S}$$



• Constant Current

$$W_m = \frac{1}{2}LI^2 \quad \text{Core}: \Re c$$

$$\Phi = \frac{NI}{\Re c + 2(\frac{y}{\mu_0 S})} \quad 2\text{Gap}: 2\frac{y}{\mu_0 S}$$

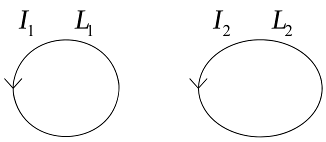
$$L = \frac{N\Phi}{I} = \frac{N^2}{\Re c + 2(\frac{y}{\mu_0 S})}$$

$$\overline{F_I} = \hat{a}_y\frac{I^2}{2}\frac{dL}{dy} = -\hat{a}_y\frac{1}{\mu_0 S}[\frac{N^2}{\Re c + 2(\frac{y}{\mu_0 S})}]^2$$

$$= -\hat{a}_y\frac{\Phi^2}{\mu_0 S}$$

(6−13.4)force and toraues in terms of mutual inductance

Two coils



$$W_m = \frac{1}{2}L_1 I_1^2 + L_{12}I_1 I_2 + \frac{1}{2}L_2 I_2^2$$

Conotant currents

$$\overline{F_I} = I_1 I_2(\overline{\nabla} L_{12})$$

$$(T_I)_Z = I_1 I_2\frac{\partial L_{12}}{\partial \phi}$$

$Ex6-24 \qquad I_1 : source$ [Ex6-7,p239]

$$\overline{A_{12}} = \hat{a}_{\varphi} \frac{\mu_0 N_1 I_1 b_1^2}{4R^2} \sin\theta = \hat{a}_{\varphi} \frac{\mu_0 N_1 I_1 b_1^2 b_2}{4[Z^2 + b_2^2]^{3/2}}$$

$$\Phi_{12} = \oint_{c2} \overline{A_{12}} \bullet d\overline{l_2} = \int_0^{2\pi} A_{12} b_2 d\phi = \frac{\mu_0 N_1 I_1 b_1^2 b_2^2 \pi}{2[Z^2 + b_2^2]^{3/2}}$$

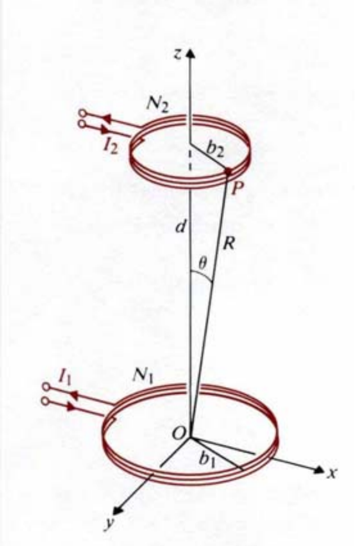
$$A_{12} = \frac{N_2 \Phi_{12}}{I_1} = \frac{\mu_0 N_1 N_2 \pi b_1^2 b_2^2}{2[Z^2 + b_2^2]^{3/2}}$$

$$\overline{F_{12}} = \hat{a}_Z I_1 I_2 \frac{dL_{12}}{dZ}\Big|_{Z=d}$$

$$\overline{F_{12}} = -\hat{a}_Z I_1 I_2 \frac{3\mu_0 N_1 N_2 \pi b_1^2 b_2^2 d}{2(d^2 + b_2^2)^{5/2}}$$

$$d \gg b_2 \;;\; m_1 = N_1 I_1 \pi b_1^2 \;;\; m_2 = N_2 I_2 \pi b_2^2$$

$$\overline{F_{12}} \simeq -\hat{a}_Z \frac{3\mu_0 m_1 m_2}{2\pi d^4} \qquad attraction$$



Home Work #6

David Cheng: Chapter6

P6-2, P6-4, P6-5, P6-6, P6-10,P6-11,P6-12
P6-13,P6-15,P6-18,P6-19,P6-22,P6-26,

P6-27,P6-29,P6-32,P6-37,P6-38,P6-39,
P6-40,P6-41,P6-42,P6-43,P6-44,P6-46,
P6-50,P6-53
Due: 2 weeks