

## MID-TERM EXAM – Sample

### 1. QUESTION 1 (25 points)

A single phase 600 V, 60 Hz feeder supplies 2 loads in parallel: (a) a heating load with a  $10 \Omega$  resistance; (b) a 70 kVA motor load, with a power factor of 0.70 (lagging). Compute the total current, power, reactive power and power factor. Draw the V-I vector diagram. A capacitor is connected in parallel with the loads to increase the power factor to 0.95. Compute the reactive power supplied by the capacitor and the value of the capacitor bank in  $\mu\text{F}$ .

#### QUESTION 1

$$\begin{aligned}
 V_l &:= 600 & R &:= 10 & I_r &:= \frac{600}{10} & P_r &:= V_l \cdot I_r & I_r &= 60 & P_r &= 3.6 \times 10^4 \\
 S &:= 70000 & \text{pf} &:= 0.70 & P_m &:= S \cdot \text{pf} & \phi &:= \arccos(\text{pf}) & I_m &:= \frac{S}{V_l} \\
 \phi &\left(\frac{180}{\pi}\right) = 45.573 & P_m &= 4.9 \times 10^4 & I_m &= 116.667 & Q_m &:= S \cdot \sin(\phi) & Q_m &= 4.999 \times 10^4 \\
 I_{rc} &:= I_r & I_{mc} &:= I_m \cdot \cos(\phi) + I_m \sin(\phi) \cdot i & I_{mc} &= 81.667 + 83.317i & a &:= \frac{180}{\pi} \\
 I_{tc} &:= I_{rc} + I_{mc} & I_{tc} &= 141.667 + 83.317i & \left(\frac{180}{\pi}\right) \cdot \arg(I_{tc}) &= 30.461 \\
 |I_{tc}| &= 164.351 & \cos(\arg(I_{tc})) &= 0.862 \\
 P_t &:= P_r + P_m & P_t &= 8.5 \times 10^4 \\
 Q_t &:= Q_m & Q_t &= 4.999 \times 10^4 \\
 S_{tc} &:= P_t + Q_t \cdot i & S_{tc} &= 8.5 \times 10^4 + 4.999i \times 10^4 & |S_{tc}| &= 9.861 \times 10^4 \\
 \text{pft} &:= \frac{P_t}{|S_{tc}|} & \text{pft} &= 0.862 & a \cdot \arg(S_{tc}) &= 30.461 \\
 & & \text{pfc} &:= 0.95 & a \cdot \arccos(\text{pfc}) &= 18.195 \\
 S_{tcor} &:= \frac{P_t}{\text{pfc}} & Q_{tcor} &:= S_{tcor} \cdot \sin(\arccos(\text{pfc})) \\
 Q_{tcor} &= 2.794 \times 10^4 & Q_{cap} &:= Q_t - Q_{tcor} & Q_{cap} &= 2.205 \times 10^4 \\
 X_c &:= V_l \cdot \frac{V_l}{Q_{cap}} & X_c &= 16.325 \\
 I_{cc} &:= \frac{V_l}{X_c \cdot i} & I_{cc} &= -36.753i \\
 I_{tcc} &:= I_{tc} + I_{cc} & I_{tcc} &= 141.667 + 46.564i & a \cdot \arg(I_{tcc}) &= 18.195 \\
 C &:= \frac{1}{(60 \cdot 2 \cdot \pi \cdot X_c)} & C &= 1.625 \times 10^{-4}
 \end{aligned}$$

## MID-TERM EXAM – Sample

### 2. QUESTION 3 (25 points)

Three identical single-phase 69 kV/6.9 kV, 60 Hz, 33 MVA distribution transformers, with a leakage reactance of 0.10 on the transformer base, are fed from the 69 kV bus. They supply two three phase 12 kV loads connected in parallel: a 60 MW, 0.8 power factor (lagging) load (assume a Y connection) and a capacitor bank of three elements ( $-j15 \Omega$ ) connected in  $\Delta$  (hint: convert into Y). Compute: (a) total line current; (b) the total power and total reactive power drawn from the feeder; (c) the resulting power factor. Draw the single line pu circuit, using a 100 MVA, 12 kV base.

#### QUESTION 3 - Three-phase circuits

$$V_{line} := 12000$$

$$P_{phase} := 60 \cdot \frac{10^6}{3} \quad pf := 0.8 \quad V_{phase} := \frac{V_{line}}{\sqrt{3}} \quad V_{phase} = 6.928 \times 10^3$$

$$\phi := -\arccos(pf) \quad a \cdot \phi = -36.87 \quad S_{phase} := \frac{P_{phase}}{pf} \quad S_{phase} = 2.5 \times 10^7$$

$$I_{line} := \left( \frac{S_{phase}}{V_{phase}} \right) \cdot (\cos(\phi) + i \cdot \sin(\phi)) \quad I_{line} = 2.887 \times 10^3 - 2.165i \times 10^3$$

$$|I_{line}| = 3.608 \times 10^3$$

$$I_{cphase} := \frac{V_{phase}}{\left( -i \cdot \frac{15}{3} \right)} \quad I_{cphase} = 1.386i \times 10^3$$

$$I_{cline} := I_{cphase}$$

$$I_{line} := I_{line} + I_{cline} \quad I_{line} = 2.887 \times 10^3 - 779.423i$$

$$|I_{line}| = 2.99 \times 10^3 \quad a \cdot \arg(I_{line}) = -15.11$$

$$\cos(\arg(I_{line})) = 0.965$$

$$P_t := 3 \cdot S_{phase} \cdot \cos(\phi) \quad P_t = 6 \times 10^7$$

$$Q_t := 3 \cdot S_{phase} \cdot \sin(-\phi) - 3 \cdot |I_{cline}| \cdot V_{phase} \quad Q_t = 1.62 \times 10^7 \quad a \cdot \operatorname{atan}\left(\frac{Q_t}{P_t}\right) = 15.11$$

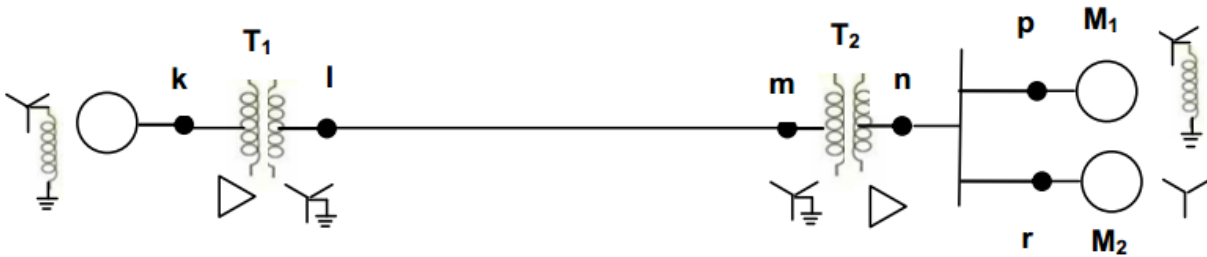
$$S_{base} := 100 \cdot \frac{10^6}{3} \quad V_{base} := \frac{12000}{\sqrt{3}} \quad I_{base} := \frac{S_{base}}{V_{base}} \quad Z_{base} := \frac{V_{base}}{I_{base}}$$

$$\text{Load} \quad p := 20 \cdot \frac{10^6}{S_{base}} \quad p = 0.6 \quad pf = 0.8 \quad Z_{base} = 1.44$$

$$\text{Capacitor} \quad x_c := \frac{15}{3 \cdot Z_{base}} \quad x_c = 3.472$$

## MID-TERM EXAM – Sample

3- A 300 MVA, 20 kV three-phase generator has a subtransient reactance of 20%. The generator supplies a number of synchronous motors over 64-km transmission line having transformers at both ends, as shown in Fig. 1.11. The motors, all rated 13.2 kV, are represented by just two equivalent motors. Rated inputs to the motors are 200 MVA and 100 MVA for M1 and M2, respectively. For both motors  $X'' = 20\%$ . The three phase transformer T1 is rated 350 MVA, 230/20 kV with leakage reactance of 10%. Transformer T2 is composed of three single-phase transformers each rated 127/13.2 kV, 100 MVA with leakage reactance of 10%. Series reactance of the transmission line is  $0.5 \Omega/\text{km}$ . Draw the impedance diagram, with all impedances marked in per-unit. Select the generator rating as base in the generator circuit.



## MID-TERM EXAM – Sample

### Solution

Base MVA = 300

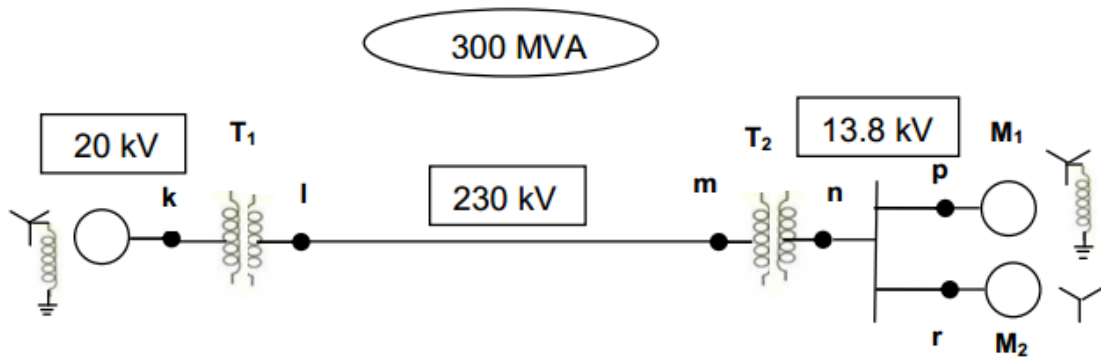
Base voltage at generator side = 20 kV

Base voltage in transmission line = 230 kV

Line to line voltages of transformer  $T_2$ :  $\sqrt{3} \times 127 / 13.2 = 220 / 13.2$  kV

Base voltage at motor side =  $230 \times \frac{13.2}{220} = 13.8$  kV

Base MVA and base voltages at different sections are marked.



MID-TERM EXAM – Sample

**Per-unit reactance of generator = 0.2**

**Per-unit reactance of transformer  $T_1 = 0.1 \times \frac{300}{350} = 0.0857$**

**Per-unit reactance of transmission line =  $0.5 \times 64 \times \frac{300}{230^2} = 0.1825$**

**Per-unit reactance of transformer  $T_2 = 0.1 \times \left(\frac{220}{230}\right)^2 = 0.0915$**

**Per-unit reactance of motor  $M_1 = 0.2 \times \frac{300}{200} \times \left(\frac{13.2}{13.8}\right)^2 = 0.2745$**

**Per-unit reactance of motor  $M_2 = 0.2 \times \frac{300}{100} \times \left(\frac{13.2}{13.8}\right)^2 = 0.549$**

MID-TERM EXAM – Sample

Per-unit impedance diagram is shown in Fig. 1.12

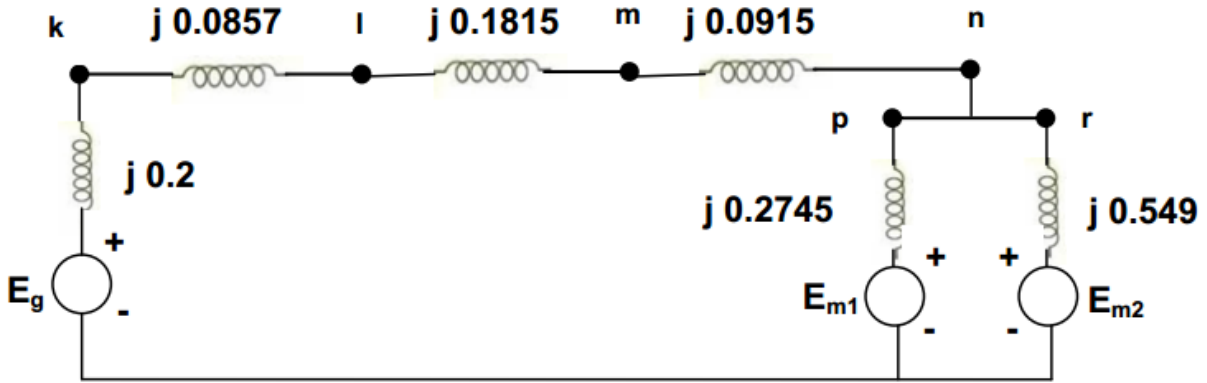
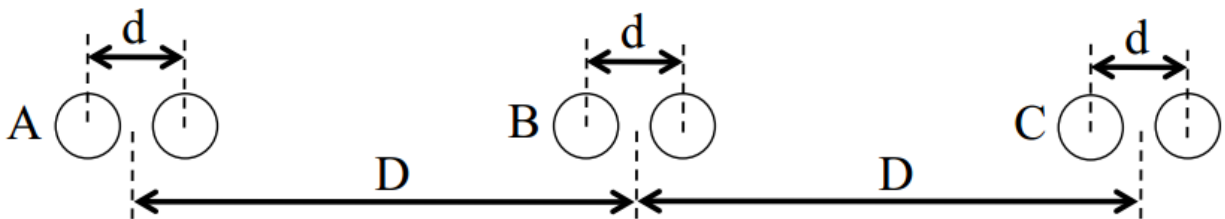


Fig. 1.12 Per-unit impedance diagram

4- A 120 MVA, 19.5 kV generator has  $X_s = 0.15$  per unit and is connected to a transmission line by a transformer rated 150 MVA, 230 Y/18Δ kV with  $X = 0.1$  per unit. If the base to be used in the calculation is 100 MVA, 230 kV for the transmission line, find the per unit values to be used for the transformer and the generator reactances.

Answer: **0.06667 p.u.;** **0.1467 p.u.**

5- A three-phase transmission line is mounted on the tower as shown below. The radius of each conductor is  $r$  meters. Assume that  $d$  is much smaller than  $D$ . The three line currents are defined as positive into the paper and they sum to zero.



- Find the distributed flux (per meter) linking conductor A in terms of  $I_A$ ,  $I_C$ ,  $d$ ,  $D$ , and  $r$ .
- Find the distributed flux (per meter) linking conductor B in terms of  $I_B$ ,  $d$ ,  $D$ , and  $r$ .
- Find the distributed flux (per meter) linking conductor C in terms of  $I_A$ ,  $I_C$ ,  $d$ ,  $D$ , and  $r$ .

## MID-TERM EXAM – Sample

2.11 In Fig. P-2.11 which depicts two three-phase circuits on a steel tower there is symmetry about both the horizontal and vertical centre lines. Let each three-phase circuit be transposed by replacing  $a$  by  $b$  and then by  $c$ , so that the reactances of the three-phases are equal and the GMD method of reactance calculations can be used. Each circuit remains on its own side of the tower. Let the self GMD of a single conductor be 1 cm. Conductors  $a$  and  $a'$  and other corresponding phase conductors are connected in parallel. Find the reactance per phase of the system.

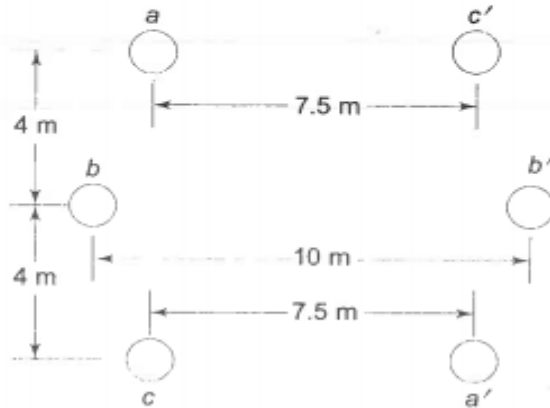


Fig. P-2.11

MID-TERM EXAM – Sample

2.11

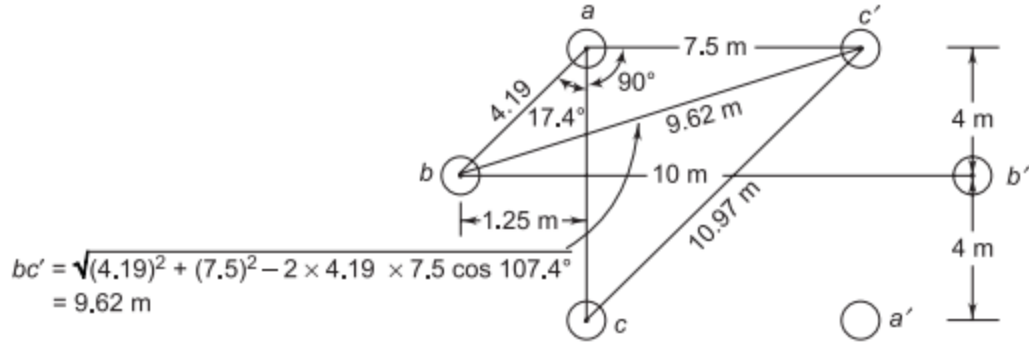


Fig. S-2.11

In section 1 of transposition cycle

$$D_{ab} = \sqrt{1.19 \times 9.62} = 6.35; \quad D_{bc} = \sqrt{4.19 \times 9.62} = 6.35$$

$$D_{ca} = \sqrt{7.5 \times 8} = 7.746$$

$$D_{eq} = \sqrt[3]{6.35 \times 6.35 \times 7.746} = 6.78$$

$$D_{sa} = \sqrt{0.01 \times 10.97} = 0.3312 = D_{sc}$$

$$D_{sb} = \sqrt{0.01 \times 10} = 0.3162$$

$$D_s = \sqrt[3]{0.3312 \times 0.3312 \times 0.3162} = 0.326 \text{ m}$$

$$X = 0.314 \times 0.461 \log \frac{6.78}{0.326} = \mathbf{0.191 \text{ } \Omega/\text{km/phase}}$$

2.12

$$r' = 0.7788 \times 1.5 \times 10^{-2} = 0.0117 \text{ m}$$