We observe that all three answers are negative which means that the system is being slowed down or decelerated by these working forces. If the system were to move in a horizontal plane, rather than in a vertical plane, then, the work of gravity would vanish because point $G$ would not change elevation.

## Problems

Assume that the bodies have enough kinetic energy in their initial positions to assure that they will move to their final positions regardless of the total work done on them.
15.1 A block weighing 200 lb moves along a horizontal plane as shown in Figure P15.1. The constant force $\mathbf{P}$ has a magnitude of 50 lb and the coefficient of kinetic friction between the block and plane is 0.2 . Determine the total work
done on the block as it moves through a displacement of 10 ft directed to the right.
15.2 A block weighing 600 N moves along a horizontal plane as shown in Figure P15.2. The force $P$ has a magnitude of 100 N . If the coefficient of kinetic friction between the block and plane is 0.1 , determine the total work done on the block as it moves through a displacement of 4 m to the right.


Figure P15.1.


Figure P15.2.
15.3 A block weighing 500 lb moves 30 ft down an inclined plane as shown in Figure P15.3. If the coefficient of kinetic friction between the block and plane is 0.1 , determine the total work done on
the block as it moves 30 ft down the plane.
15.4 Determine the total work done on the block as it moves 6 m up the inclined plane shown in Figure P15.4. The block


Figure P15.3.


Figure P15.4.
has a mass of 120 kg , the constant force $P$ has a magnitude of 600 N , and the coefficient of kinetic friction for the block and plane is 0.1 .
15.5 A block with a mass of 10 slugs moves horizontally in frictionless guides, as shown in Figure P15.5. A spring ( $k=50$ $\mathrm{lb} / \mathrm{in}$ ) is attached to the block as indicated. The unstretched length of the spring is 1.00 ft . Determine the total work done on the block as it moves 4 ft horizontally to the left.
15.6 Determine the total work done on the block shown in Figure P15.6 as it moves 6 m up the inclined plane. The spring attached to the block has a constant $k=10,000 \mathrm{~N} / \mathrm{m}$ and an unstretched length of 2 m . The block has a mass of 120 kg and the coefficient of kinetic friction between the block and plane is 0.1 .


Figure P15.5.
15.7 Determine the total work done on bar OA shown in Figure P15.7 as it rotates from vertical position 1 to horizontal position 2 through a $90^{\circ} \mathrm{ccw}$ angle. The bar weighs 150 lb and the spring constant $k=600 \mathrm{lb} / \mathrm{ft}$. The unstretched length of the spring is 1.5 ft and end B of the spring remains fixed as the bar rotates.
15.8 Rod AB weighs 800 N and is subjected to a constant couple $C$ as indicated in Figure P15.8. Determine the couple $C$ such that the total work done on the rod is $+1500 \mathrm{~N} \cdot \mathrm{~m}$ as it rotates from position 1 to position 2. Assume the hinge at A to be frictionless.
15.9 Two weights $W$ are connected by a spring which is unstretched as they lie


Figure P15.6.


Figure P15.7.


Figure P15.8.
initially at rest on a horizontal table, as shown in Figure P15.9. The spring is carefully draped over a fixed peg $P$ which is indicated as position 2. Develop an equation for the work done on the spring in terms of $W$ and $k$. Ignore the size of peg $P$ in your analysis.
15.10 In Figure P15.10, two very light rods, $A B$ and DE, whose weights may be ignored, are attached to a spring which is unstretched in position 1 . Couples $C$ are applied to the rods and the rods are rotated to position 2 as indicated. Determine an expression for the total work done on the two rods. All connecting pins are frictionless.


Figure P15.9.


Figure P15.10.


Figure P15.11.
15.11 As shown in Figure P15.11, two very light rods, AB and DE , whose weights may be ignored, are attached to a spring which is unstretched in position 1 . Couples $C$ are applied to the rods and the rods are rotated to position 2 as indicated. Determine an expression for the total work done on the two rods.
15.12 Solve Problem 15.10 if each of the rods is uniform and has a weight equal to $W$. All other information given is unchanged.
15.13 Solve Problem 15.11 if each of the rods
is uniform and has a weight equal to $W$. All other information given is unchanged.
A very light rod AB supports a weight $W$ at B and is attached to a spring which is fixed at D as shown in Figure P15.14. A constant couple $C$ is applied to the rod, and it rotates through an angle $\theta$ to a new position such that $B$ travels a circular arc to $B^{\prime}$. If the total work done on the rod vanishes, determine $C$ as a function of $\theta, b, W$, and $k$. The spring has an unstretched length of $b$.

## Problems

15.15 The variable force shown acting on the block in Figure P15.15 is given by $F(s)=$ $4 s^{2}+2 s+9$ where $s$ is measured in ft and $F$ is given in lb . If the block weights 150 lb and starts from rest, determine its velocity after it has moved 10 ft to the right. What is the initial acceleration of the block? Neglect friction and air resistance. Assume that $s=0$ at $t=0$.


Figure P15.15.
15.16 A variable force acts parallel to the inclined plane shown in Figure P15.16 and is given by $F(s)=30 s+200$ where $s$ is measured in $m$ and $F$ is given in N . If the block weighs 200 N and starts from rest, determine its velocity after it has moved 5 m up the plane. What is the initial acceleration of the block? Neglect friction and air resistance. Assume that $s=0$ at $t=0$.


Figure P15.16.
15.17 A mass $m$ is free to slide on a horizontal frictionless plane and is attached to a spring which is fixed at point O as shown in Figure P15.17. If the mass is released from rest in position 1, determine its velocity when it reached position 2 in terms of $k, m$, and $b$. In position 3 , the spring is unstretched.
15.18 In Figure P15.18, the spring is horizontal in position 1 and perpendicular to the inclined plane in position 2. The mass $m$ has a hole drilled in it which enables it to move along the rod without friction. If this mass is released from rest in position 1, determine its velocity when it reaches position 2 as a function


Figure P15.17.


Figure P15.18.
of $m, k, \theta$, and $b$. The unstretched length of the spring is $b$.
15.19 Refer to Problem 15.18. Let $\theta=30^{\circ}$, and determine the acceleration of the mass when it is in position 1 in terms of $k, b$, and $g$. Also, determine its acceleration when it is in position 2.
15.20 A block of mass $m$ is placed on an inclined plane as shown in Figure P15.20 with an initial velocity $v_{1}$. If the kinetic


Figure P15.20.
coefficient of friction is $\mu$ determine, $v_{2}$ as a function of $v_{1}, \theta, m, \mu, d$, and $g$. What condition must be satisfied among these quantities which will assure that $v_{2} \geq 0$ ?
15.21 Rod ABC shown in Figure P15.21 has negligible weight. When the system is in position 1 , the mass $m$ has a velocity $v_{1}=15 \mathrm{ft} / \mathrm{s}$ directed horizontally to the left. The spring is attached to the rod at $B$ and fixed point $D$. Let $m=0.31$ slug, $b=2 \mathrm{ft}$, and $k=5 \mathrm{lb} / \mathrm{ft}$. Then, determine the velocity $v_{2}$ for $\theta=45^{\circ}$.


Figure P15.21.
15.22 The two masses shown in Figure P15.22 are constrained to move horizontally in frictionless guides. They are released from rest in initial positions 1. Determine


Figure P15.22.
their relocities in final positions denoted by ' 2 ' as a function of $k, b$, and $m$. The unstretched length of the spring is $b$.
15.23 Mass $m$ is attached to a very light rod which rotates about A as shown in Figure P15.23. It is released from rest in position 1. Determine the speed of $m$ in position 2 as a function of $r, \theta$, and $g$. What is the corresponding speed of $m$ when it arrives at position 3? Specialize

your results for $\theta=90^{\circ}$. Ignore friction at the pin A and air resistance.
15.24 Determine the speed $v_{1}$ of mass $m$, shown in Figure P15.24, such that, when the rod has rotated cw through $90^{\circ}$, the final speed of mass $m$ will be zero. The unstretched length of the spring is $3 b$. Express $v_{1}$ as a function of $m, k, g$ and $b$. What relationship must hold among these quantities to assure a solution? Neglect the rod mass, air resistance and pin friction at A .
15.25 Mass $m$ slides along a frictionless vertical rod as shown in Figure P15.25. It is released from rest in position 1 and the spring has an unstretched length of $b$. Determine the speed of the mass when it arrives at position 2. Express your answer in terms of $k, m, g$, and $y$.
15.26 As shown in Figure P15.26, the mass $m$ is held against a spring which has a deformation $\delta$ and then released in position 1. The spring is not attached to the mass. Determine the velocity of the mass when it arrives at position 2 as a function of $k, \delta, m, g, L$, and $\theta$. Find a numerical value for this velocity corresponding to $k=600 \mathrm{lb} . / \mathrm{ft}$., $\delta=1 \mathrm{ft}$., $m=0.5$ slug, $g=32.2 \mathrm{ft} / \mathrm{s}^{2}, L=6 \mathrm{ft}$. and $\theta=30^{\circ}$. Next, the particle becomes a projectile launched from position 2,


Figure P15.24.


Figure P15.56.
15.57 Use the conservation of energy principle for the system depicted in Figure P15.57. The system is released from rest when the spring is unstretched, and mass $m_{1}$ moves a distance $s=1 \mathrm{ft}$ along the frictionless inclined plane. Determine the corresponding final velocities of these masses, given that $m_{1}=1$ slug, $m_{2}=10$
15.58 The system shown in Figure P15.58 is slug, and $k_{1}=400 \mathrm{lb} / \mathrm{ft}$. frictionless. It is released from rest, and


Figure P15.57.
mass $m_{2}=4$ slug moves downward a distance of 2 ft . Determine the velocities of the masses after this downward movement, given that $m_{1}=2$ slug and $k_{1}=$ $40 \mathrm{lb} / \mathrm{ft}$. The cable is inextensible and the springs are unstretched initially.
15.59 Refer to Figure P15.59 which depicts a frictionless system. Initially the spring is unstretched and the system is at rest. Determine the final velocity of mass $m_{3}$ after it moves a distance $s=1 \mathrm{ft}$ down the inclined plane. Given $m_{1}=2$ slug, $m_{2}=3$ slug, $m_{3}=6$ slug, and $k_{1}=20$ $\mathrm{lb} / \mathrm{ft}$.


Figure P15.58.

