

$$A = \frac{\mu_0 I_0 \sin(\omega(t - r/c))}{4\pi r} \rightarrow$$

$$\left\{ \begin{aligned} A_r &= \frac{\mu_0 \cos\theta I_0 \sin(\omega(t - r/c))}{4\pi r} \\ A_\theta &= -\frac{\mu_0 \sin\theta I_0 \sin(\omega(t - r/c))}{4\pi r} \\ A_\phi &= 0 \end{aligned} \right.$$

$$\phi = \frac{\mu_0 I_0 \cos\theta}{4\pi \epsilon_0 r^2} \left(\frac{q_0 \cos(\omega(t - r/c))}{r} + \frac{I_0 \sin(\omega(t - r/c))}{c} \right)$$

$$\vec{B} = \nabla \times \vec{A} = \frac{1}{r \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix} \rightarrow \left\{ \begin{aligned} B_r &= 0 \quad \checkmark \\ B_\theta &= 0 \quad \checkmark \\ B_\phi &\neq 0 \end{aligned} \right.$$

$$\Rightarrow B_\phi = \frac{1}{r \sin\theta} r \sin\theta \left(\frac{\partial}{\partial r} (rA_\theta) - \frac{\partial}{\partial \theta} A_r \right) = \frac{1}{r} \left(\frac{\partial}{\partial r} (rA_\theta) - \frac{\partial}{\partial \theta} A_r \right)$$

$$\frac{1}{r} \left\{ -\frac{\mu_0 I_0}{4\pi} (-\omega/c) I_0 \cos(\omega(t - r/c)) \sin\theta - \left(-\frac{\mu_0 I_0}{4\pi r}\right) \sin\theta I_0 \sin(\omega(t - r/c)) \right\}$$

$$= \frac{\mu_0 I_0 \sin\theta}{4\pi r} \left\{ \omega/c \cos(\omega(t - r/c)) + \frac{\sin(\omega(t - r/c))}{r} \right\} \quad \checkmark$$

$$E_\theta = -\frac{1}{r} \frac{\partial \phi}{\partial \theta} - \frac{\partial A_\theta}{\partial t}$$

$$\frac{1}{r} \frac{\partial \phi}{\partial \theta} = \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{\mu_0 I_0 \cos\theta}{4\pi \epsilon_0 r^2} \left(\frac{q_0 \cos(\omega(t - r/c))}{r} + \frac{I_0 \sin(\omega(t - r/c))}{c} \right) \right)$$

$$= -\frac{\mu_0 I_0 \sin\theta}{4\pi \epsilon_0 r^2} \left(\frac{(-I_0/c) \cos(\omega(t - r/c))}{r} + \frac{I_0 \sin(\omega(t - r/c))}{c} \right)$$

$$= \frac{\mu_0 I_0 \sin\theta}{4\pi \epsilon_0 r^2} \left(\frac{\cos(\omega(t - r/c))}{r\omega} - \frac{\sin(\omega(t - r/c))}{c} \right)$$

$$\frac{\partial A_\theta}{\partial t} = -\frac{l I_0 \mu_0}{4\pi r} \sin\theta (\omega) \cos(\omega(t-r/c))$$

$$\begin{aligned} \rightarrow E_\theta &= \frac{l I_0 \sin\theta}{4\pi \epsilon_0 r} \left\{ \mu_0 \epsilon_0 \omega \cos(\omega(t-r/c)) - \frac{\cos(\omega(t-r/c))}{r^2 \omega} + \frac{\sin(\omega(t-r/c))}{rc} \right\} \\ &= -\frac{l I_0 \sin\theta}{4\pi \epsilon_0} \left\{ -\frac{\omega}{rc^2} \cos(\omega(t-r/c)) + \frac{\cos(\omega(t-r/c))}{r^3 \omega} - \frac{\sin(\omega(t-r/c))}{r^2 c} \right\} \end{aligned}$$

$$E_r = -\frac{\partial \phi}{\partial r} - \frac{\partial A_r}{\partial t}$$

$$-\frac{\partial \phi}{\partial r} = -\frac{\partial}{\partial r} \left\{ \frac{l \cos\theta}{4\pi \epsilon_0 r} \left(\frac{q_0 \cos(\omega(t-r/c))}{r} + \frac{I_0 \sin(\omega(t-r/c))}{c} \right) \right\}$$

$$= \frac{2 \cos\theta l}{r^3 4\pi \epsilon_0} \left(q_0 \cos(\omega(t-r/c)) - \frac{l \cos\theta q_0}{4\pi \epsilon_0 r^2} (-\omega/c) (-\sin(\omega(t-r/c))) \right)$$

$$+ \frac{l \cos\theta}{4\pi \epsilon_0 r^2 c} I_0 \sin(\omega(t-r/c)) - \frac{l \cos\theta I_0}{4\pi \epsilon_0 r} (-\omega/c^2) \cos(\omega(t-r/c))$$

$$-\frac{\partial A_r}{\partial t} = -\frac{I_0 l \mu_0}{4\pi r} \cos\theta (\omega) \cos(\omega(t-r/c))$$

$$\begin{aligned} \rightarrow E_r &= \cos(\omega(t-r/c)) \left\{ -\frac{2 I_0 l \cos\theta}{r^3 4\pi \epsilon_0 \omega} + \frac{l I_0 \omega \cos\theta}{4\pi \epsilon_0 r c^2} - \frac{I_0 l \mu_0 \omega \cos\theta}{4\pi r} \right\} \\ &+ \sin(\omega(t-r/c)) \left\{ \frac{l I_0 \cos\theta}{4\pi \epsilon_0 r^2 c} + \frac{l I_0 \cos\theta}{4\pi \epsilon_0 r^2 c} \right\} \end{aligned}$$

$$\rightarrow E_r = \frac{2 l I_0 \cos\theta}{4\pi \epsilon_0} \left\{ \frac{\sin(\omega(t-r/c))}{cr^2} - \frac{\cos(\omega(t-r/c))}{\omega r^3} \right\}$$

$$E_\phi = -\frac{1}{r \sin\theta} \frac{\partial \phi}{\partial \theta} - \frac{\partial A_\phi}{\partial t} = 0 \quad \checkmark$$