[EXERCISES]

1.1 Show that the following sets satisfy the law of contradiction and law of excluded middle.

$X = \{a, b, c, d, e, f, g\}$

$A = \{a, b, c, d\}$

1.2 $A = \sum_{i=1,n} \mu_A(x_i)/x_i$ is an another form of representation of fuzzy set.

Represent the following fuzzy sets by this form.

a) A = {(2, 1.0), (3, 0.4), (4, 0.5)}

b) $B = \{(a, \mu_B(a)), (b, \mu_B(b)), (c, \mu_B(c)), (d, \mu_B(d))\}$

1.3 Consider the fuzzy sets : short, middle, tall

cm	short	middle	tall
140	1	0	0
150	1	0	0
160	0.9	0.1	0
170	0.7	1	0
180	0.3	0.8	0.3
190	0	0	1

a) Compare the support of each set.

b) What is the normalized fuzzy set?

c) Find the level set of each set.

d) Compare α-cut set of each set where α=0.5 and α=0.3.

1.4 Determine whether the following fuzzy sets are convex or not.

a) $A = \int \mu_A(x)/x$ where $\mu_A(x) = 1/(1+x^2)$

b) $B = \int \mu_B(x)/x$ where $\mu_B(x) = 1/(1+10x)^{1/2}$

1.5 Prove that all the α–cuts of any fuzzy set A defined on R^n are convex if and only if

$$\mu_A(\lambda r + (1-\lambda)s) \geq Min[\mu_A(r), \mu_A(s)]$$

such that $r, s \in \Re^n$, $\lambda \in [0,1]$

1.6 Compute the scalar cardinality and the fuzzy cardinality for each of the following fuzzy set.

a) A = {(x, 0.4), (y, 0.5), (z, 0.9), (w, 1)}
b) B = {0.5/u + 0.8/v + 0.9/w + 0.1/x}
c) C= $\sum \mu_C(x)/x$ where $\mu_C(x) = (x/(x+1))^2 \quad x \in \{0,1,2,\cdots 10\}$

1.7 Show the following set is convex.

$$\mu_A(x) = \begin{cases} 0 & x \le 10 \\ (1+(x-10)^{-2})^{-1} & x > 10 \end{cases}$$

1.8 Determine α-cut sets of the above set for α=0.5, 0.8 and 0.9.

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2.1 Let sets A, B, and C be fuzzy sets defined on real numbers by the membership functions

$$\mu_A(x) = \frac{x}{x+1},\ \mu_B(x) = \frac{1}{x^2+10},\ \mu_C(x) = \frac{1}{10^x}$$

Determine mathematical membership functions and graphs of each of the followings :

a) $A \cup B,\quad B \cap C,$

b) $A \cup B \cup C,\quad A \cap B \cap C$

c) $A \cap \overline{C},\quad \overline{B} \cup C$

d) $\overline{A \cap B},\quad \overline{A} \cup \overline{B}$

2.2 Show the two fuzzy sets satisfy the De Morgan's Law.

$$\mu_A(x) = \frac{1}{1+(x-10)}$$

$$\mu_B(x) = \frac{1}{1+x^2}$$

2.3 Show that the following sets don't satisfy the law of contradiction and the law of excluded middle.

a) $\mu_A(x) = \frac{1}{1+x}$

b) $A = \{(a, 0.4), (b, 0.5), (c, 0.9), (d, 1)\}$

2.4 Determine complements, unions, and intersections of the following sets by using Yager's operators for $\omega = 1, 2$

a) $A = \{(a, 0.5), (b, 0.9), (c, 0.1), (d, 0.5)\}$

b) $\mu_A(x) = \frac{1}{1+x}$

2.5 Compute the complements of the following sets by Yager's complements $w = 1, 2$.

a) $A = \{(a, 0.5), (b, 0.9), (c, 0.3)\}$

b) $\mu_A(x) = \frac{1}{1+(x-1)}$

2.6 Compute the complements of the following sets by using Probabilistic, Bounded, Drastic, and Hamacher product.

a) $\mu_A(x) = \dfrac{1}{1+x^2}$

b) $\mu_A(x) = 2^{-x}$

c) $A = \{(a, 0.4), (b, 0.5), (c, 0.9)\}$

2.7 Compute the simple disjunctive sum, disjoint sum, simple difference, and bounded difference of the sets

$$A = \{(x, 0.5), (y, 0.4), (z, 0.9), (w, 0.1)\}$$
$$B = \{(x, 0.4), (y, 0.8), (z, 0.1), (w, 1)\}$$

2.8 Determine the distances (Hamming, Euclidean and Minkowski for $\omega = 2$) between the following sets

$$A = \{(x, 0.5), (y, 0.4), (z, 0.9), (w, 0.1)\}$$
$$B = \{(x, 0.1), (y, 0.9), (z, 0.1), (w, 0.9)\}$$

2.9 Prove the following properties.

a) Let function Ta t-norm operation.
The following T'is a t-conorm operation.

$$T'(x, y) = 1 - T'(1-x, 1-y)$$

b) $\overline{x} \perp \overline{y} = \overline{x \top y}$ and $\overline{x} \top \overline{y} = \overline{x \perp y}$

where $\overline{x} = 1 - x$, $\overline{y} = 1 - y$ and $\overline{x \top y} = 1 \mp (x, y)$

where $\perp$ represents t-conorm operator.

2.10 Determine the closet pair of sets among the following sets

$A = \{(x_1, 0.4), (x_2, 0.4), (x_3, 0.9), (x_4, 0.5)\}$
$B = \{(x_1, 0.1), (x_2, 0.0), (x_3, 0.9)\}$
$C = \{(x_1, 0.5), (x_2, 0.5), (x_3, 0.9)\}$

2.11 Show the Max operator satisfies the properties boundary condition, commutativity, associativity, continuity, and idempotency.

2.12 Prove the following equation.

$0 \le \mu_{A \oplus B}(x) \le 0.5$

where $A \oplus B = (A \cap \overline{B}) \cup (\overline{A} \cap B)$

that is, $\oplus$ is simple disjunctive sum operator.

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3.1 Find the transitive closure for $A = \{a, b, c, d\}$ and $R = \{(a, b), (b, c), (c, d), (d, b)\}$.

3.2 Obtain a partition of the set $A = \{a, b, c, d, e\}$ by the equivalence relation R.

R	a	b	c	d	e
a	1	1			1
b	1	1			1
c			1	1	
d			1	1	
e	1	1			1

3.3 Compute the complements, intersection and union of the following fuzzy relations R and S.

R	a	b	c	d
a	1.0	0.2	0.4	0.0
b	0.0	0.1	0.0	0.9
c	0.1	0.0	1.0	0.0
d	0.0	0.4	0.0	1.0

S	a	b	c	d
a	1.0	0.0	0.0	0.4
b	0.0	0.0	0.4	0.9
c	0.4	0.0	0.1	0.0
d	0.5	1.0	0.0	0.0

3.4 Determine the composition relation $S \bullet R \subseteq A \times C$ where $R \subseteq A \times B$ and $S \subseteq B \times C$ are defined as follows

R	a	b	c	d
1	0.4	0.0	0.0	1.0
2	0.5	0.4	0.9	0.0
3	0.2	0.1	1.0	0.4
4	0.0	0.2	0.0	1.0

S	a	b	c
a	0.4	0.1	0.0
b	0.2	0.0	0.9
c	0.2	0.0	0.5
d	0.1	0.0	0.9

3.5 Determine the α-cut relation for the following fuzzy relation where $\alpha =$ 0.4 and 0.8.

R	1	2	3	4
a	0.4	0.0	0.5	0.8
b	0.4	0.0	0.9	0.1
c	0.0	0.4	0.0	0.2
d	0.0	0.8	0.0	1.0

3.6 Consider a fuzzy set A and a crisp set B

$A = \{(x, 0.4), (y, 0.9), (z, 1.0), (w, 0.1)\}$

$B = \{a, b, c\}$

Determine a fuzzy set $B' \subseteq B$ induced by A and the relation $R \subseteq A \times B$.

R	a	b	c
x	0.0	0.4	0.8
y	0.9	0.9	0.7
z	1.0	0.0	0.5
w	0.0	0.1	0.8

3.7 Consider a fuzzy relation $R \subseteq A \times A$ where $A = \{a, b, c\}$.

R	a	b	c	d	e
a	1	1	0	0	1
b	0	1	1	0	0
c	0	0	1	0	0
d	0	0	0	1	0
e	0	0	1	1	1

a) Determine the characteristic of this relation.

b) Show an ordinal function of the relation if it is an order relation.

3.8 Determine the fuzzy set B induced by A and $f(x) = x^2$

$A = \{(-2, 0.8), (-5, 0.5), (0, 0.8), (1, 1.0), (2, 0.4), (3, 0.1)\}$

$$B = \left\{ y \mid y = f(x), \mu_B(y) = \underset{x: y = f(x)}{Max} \mu_A(x) \right\}$$

3.9 There are fuzzy set A, crisp set B and fuzzy relation R

$A = \{(x, \mu_A(x)) \mid 0 \le x \le 1, \mu_A(x) = x^2\}$
$B = \{(y, \mu_B(y)) \mid 0 \le y \le 1\}$
$R = \{((x, y), \mu_R(x, y)) \mid 0 \le x + y = 1, x \in A, y \in B\}$
where $\mu_R(x, y) = \min[x^2, y^2]$

Determine the fuzzy set $B' \subseteq B$ induced by A, B and R

3.10 There is a relation R_{123}. $R_{123} \subset X_1 \times X_2 \times X_3 = 0.9 / (x, a, \alpha) + 0.4 / (x, b, \alpha) + 1.0 / (y, a, \alpha) + 0.7 / (y, a, \beta)$

$R_{123} \subset X_1 \times X_2 \times X_3$ where $X_1 = \{x, y\}, X_2 = \{a, b\}, X_3 = \{\alpha, \beta\}$

a) Determine $R_{12} \subset X_1 \times X_2$ and $R_{23} \subset X_2 \times X_3$ by projection.
b) Obtain R_{123} by cylindrical extension of R_{12} and R_{23}.
c) Obtain $R_{1234} \subseteq X_1 \times X_2 \times X_3 \times X_4$ by cylindrical extension where $X_4 = \{p,q\}$