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Isotopic and chemical constraints on mantle–crust evolution*

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Abstract—A formalism for the general treatment of three-layer mantle–crust evolution models is presented and various published models are shown to be special cases of this more general model. The Sm-Nd, Lu-Hf, and Rb-Sr isotopic present day mass balance for the continental crust–depleted mantle system is consistent with ~30% of the mantle being depleted. A growth curve for the continental crust is calculated on the basis of total inversion of the Sm-Nd isotopic data for all of Earth history. The curve suggests that by about 3.8 Ga ago, ~40% of the present continental volume was present. Both the estimated continental recycling and addition rates show maxima around 3.0 Ga. The resulting continental addition rates are also very high 4.5–4.0 Ga ago and during the Phanerozoic. The Sm-Nd data are not compatible with a steady state model for the crust over the past 2–3 Ga. The major uncertainty in evaluating crust–mantle evolution models is the extent of exchange between the upper and lower mantle.

INTRODUCTION

RADIOGENIC ISOTOPE VARIATIONS together with trace element patterns can be used to establish the chemical structure, dynamics, and evolution of the Earth's major reservoirs. Over the last decade, many researchers have investigated systematic coherencies between various isotopic tracers (Nd, Sr, Hf, Pb,) and trace element ratios. A number of models utilizing these data have been proposed for the structure and evolution of the continental crust-upper mantle system (JACOBSEN and WASSERBURG, 1979a, 1980a, 1981; O'NIONS *et al.*, 1979; DEPAOLO, 1980, 1983; and ALLÈGRE *et al.* 1983a,b).

The problem of understanding crust–mantle evolution from isotope and trace element data can be divided into three basic approaches. First, initial Nd, Sr, Hf, and Pb variation in recent volcanic rocks and estimates of average crustal abundances have been used to establish a present day isotope and trace element balance for these systems. This has led to constraints on the present chemical structure of the Earth's mantle. Using this approach, JACOBSEN and WASSERBURG (1979a) and DEPAOLO (1980) suggested that the depleted mantle from which the Earth's crust was extracted is only about 30% of the Earth's mantle, corresponding closely to the mass of the mantle above the 670 km discontinuity. O'NIONS *et al.* (1979) suggested that the depleted mantle comprises about 50% of the mantle. ALLÈGRE *et al.* (1983a,b) carried out this mass balance using the total inversion technique of TARANTOLA and VALETTE (1982), and concluded that the fraction of the mantle that is depleted is in the range 30–90%.

Secondly, the results of the present-day mass balance can be used to calculate the mean age of the continental crust (JACOBSEN and WASSERBURG, 1979a). The mean age of the crust is the only age information that can be extracted from the present-day balance of long lived (half life $\gg$ 4.5 Ga) isotopes such as ^{147}Sm, ^{87}Rb, ^{176}Lu, and ^{232}Th. However, the present day balance of isotopes with half life < 4.5 Ga such as ^{40}K, ^{238}U, and ^{235}U can, in principle, give more detailed information on the time evolution of the continental crust.

Finally, the evolution of the Earth's crust can be established from average curves for crust and mantle Nd, Sr, Hf, and Pb evolution. Such curves are based on initial Nd, Sr, Hf, and Pb values obtained on various ancient terrestrial rocks of known age.

For the Sm-Nd system we have a relatively well-defined set of model parameters for most of the Earth's history in addition to well-constrained bulk Earth evolution parameters. This paper presents a mantle–crust evolution model that is constrained primarily by isotopic and chemical mass balance between three reservoirs; the continental crust, the depleted mantle, and an undepleted deep mantle.

NOTATION

In this paper, the notation of JACOBSEN and WASSERBURG (1979a) will be used. The three-reservoir model is shown schematically in Fig. 1 (see figure caption for further explanation of the model). The continental crust and the depleted upper mantle are called reservoirs 3 and 2, respectively, to be consistent with previous usage. The lower mantle is called reservoir 4 and the bulk Earth is called reservoir 1; note that the bulk Earth as defined here, refers to the silicate portion of the Earth only, and excludes the core.

The number of atoms of species i in reservoir j is N_{ij} and the total mass of reservoir j is M_j. The concentration of species i in j is $C_{ij} = N_{ij}/M_j$. The species under consideration are: s, a stable nuclide with no radioactive parent; r, a radioactive nuclide with decay constant λ_r; and d, a stable nuclide (of the same chemical species as s) which is the decay product of r.

The time τ runs forward from the initial state at the formation of the Earth. The time measured backward from today (*i.e.*, the age) will be called T, such that $T = T_0 - \tau$ where T_0 is the age of the Earth today. Fractional deviations of the isotopic ratios N_{dj}/N_{sj} from the bulk Earth values are given by

$$\epsilon_{dj}(\tau) \equiv [(N_{dj}/N_{sj})/(N_{d1}/N_{s1}) - 1] \cdot 10^4. \quad (1)$$

The chemical fractionation factors between the radioactive parent isotope and the stable reference isotope are given by

$$f_j^{r/s} \equiv (N_{rj}/N_{sj})/(N_{r1}/N_{s1}) - 1. \quad (2)$$

The chemical enrichment of a species s in reservoir j relative to the bulk Earth concentration C_{s1} is called $E_{sj} \equiv C_{sj}/C_{s1}$. Initial concentrations in reservoirs 2 and 4 are assumed to be bulk Earth values such that

$$C_{i2}(0) = N_{i2}(0)/M_2(0) = N_{i1}(0)/M_1(0) = N_{i4}(0)/M_4(0) = C_{i4}(0). \quad (3)$$

The crust is assumed to grow from an initial zero mass ($M_3(0) = 0$). At any time subsequent to $\tau = 0$ the mass of the crust, $M_3(\tau)$, is >0 due to the crustal growth. In general, the concentration of an element i in new crust c_{i3} is different from that in the depleted mantle source region [$C_{i2}(\tau)$].

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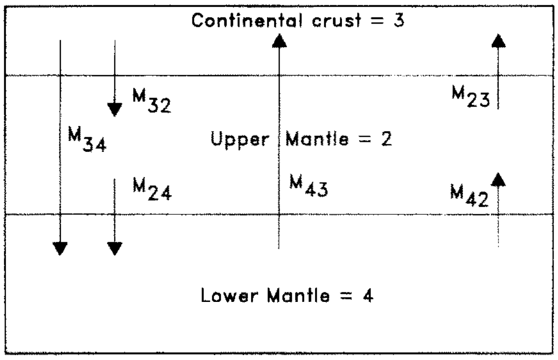

FIG. 1. Schematic three-box model for mantle–crust evolution. The various possible mass fluxes $\dot{M}_{ij}$ between reservoirs i and j are shown. The continental crust (reservoir 3; initial mass = 0) grows from an initially undifferentiated upper mantle reservoir, which becomes depleted (reservoir 2) at all times subsequent to the initial chemical differentiation as a result of continuing crustal extraction ($\dot{M}_{23}$). The mass flux $\dot{M}_{23}$ of partial melts from the upper mantle (reservoir 2) to the crust adds mass to the crust through time while a mass flux $\dot{M}_{32}$ of continental materials (sediments, etc.) is refluxed back into the depleted mantle. The crust can also grow by direct additions of partial melts ($\dot{M}_{43}$) from the lower mantle (plume flux) and leaving a residue behind in the upper mantle ($\dot{M}_{42}$). The depleted mantle is taken to include the basaltic part of the oceanic crust, since the oceanic crust is derived from the depleted mantle and is subducted back at a short time scale. If this subducted oceanic crust is recycled to the lower mantle, we have a mass flux $\dot{M}_{24}$ that would also make the lower mantle become depleted with time. The lower mantle (4) can remain undifferentiated through geologic time if $\dot{M}_{24} = \dot{M}_{34} = 0$ and only bulk material leaves the lower mantle through $\dot{M}_{42}$ and $\dot{M}_{43}$. Finally material from the lower mantle can also enter the upper mantle by bulk entrainment at their interface ($\dot{M}_{42}$) with a resulting downgoing flux of upper mantle material to the lower mantle ($\dot{M}_{24}$).

MASS BALANCE EQUATIONS FOR THREE-RESERVOIR MODELS

General mass conservation equations

For any three-reservoir model, conservation of mass and species requires

$$M_1(0) = M_2(\tau) + M_3(\tau) + M_4(\tau) \quad (4)$$

$$N_{s1}(0) = N_{s2}(\tau) + N_{s3}(\tau) + N_{s4}(\tau) \quad (5)$$

$$N_{r1}(0)\exp[-\lambda_r\tau] = N_{r2}(\tau) + N_{r3}(\tau) + N_{r4}(\tau) \quad (6)$$

$$N_{d1}(0) + N_{r1}(0)[1 - \exp(-\lambda_r\tau)] = N_{d2}(\tau) + N_{d3}(\tau) + N_{d4}(\tau). \quad (7)$$

The single stage model age of a reservoir j relative to the bulk Earth reservoir is defined by:

$$T^d_{1j} = \frac{1}{\lambda_r}\ln\left[1 + \frac{(N_{dj}/N_{sj}) - (N_{d1}/N_{s1})}{(N_{rj}/N_{sj}) - (N_{r1}/N_{s1})}\right] \quad (8)$$

and represents the time in the past when the ratio $N_{dj}/N_{sj} = N_{d1}/N_{s1}$, *i.e.* the time span required to generate the observed N_{dj}/N_{sj} value in a single stage with the observed N_{rj}/N_{sj} ratio. While (8) is not strictly part of the mass balance problem, *a priori* knowledge of the mean age of a reservoir provides an additional relationship between the parameters in equations (5)–(7) as the model age T^d_{1j} would be expected to be of similar magnitude.

The equations above are written mostly in terms of extensive quantities. This problem would be more elegantly handled if the equations were written as much as possible in terms of the intensive quantities ϵ, f, and E as defined earlier. However, since we want to use this treatment to infer values for the relative sizes of the reservoirs, we also need some extensive quantities. It is therefore convenient to introduce the mass fractions $X_{sj} = N_{sj}/N_{s1}$ and $X_{Mj} = M_j/M_1$. It follows that $X_{s2} + X_{s3} + X_{s4} = 1$, $X_{M2} + X_{M3} + X_{M4} = 1$ and we have the following set of mass balance equations for ϵ, f, and E-values:

$$X_{M2}E_{s2} + X_{M3}E_{s3} + X_{M4}E_{s4} = 1 \quad (9)$$

$$X_{s2}f_2^{r/s} + X_{s3}f_3^{r/s} + X_{s4}f_4^{r/s} = 0 \quad (10)$$

$$X_{s2}\epsilon_{d2} + X_{s3}\epsilon_{d3} + X_{s4}\epsilon_{d4} = 0. \quad (11)$$

Mass balance with an undifferentiated lower mantle

For the particular case of an undifferentiated lower mantle we have that $C_{s4}(\tau) = C_{s4}(0)$ throughout Earth's history and also that $\epsilon_{d4} = f_4^{r/s} = E_{s4} = 0$ for all τ. For models with an undifferentiated lower mantle, the crust + depleted mantle system has bulk Earth composition and for this type of model we define the following mass fractions $Y_{sj} = N_{sj}/(N_{s2} + N_{s3})$ and $Y_{Mj} = M_j/(M_2 + M_3)$. The relationships between the two mass fractions Y and X are: $Y_{sj} = X_{sj}/(X_{s2} + X_{s3})$ and $Y_{Mj} = X_{Mj}/(X_{M2} + X_{M3})$.

In the case where reservoir 4 is undifferentiated, we have the following set of mass balance equations for ϵ, f, and E-values:

$$g^1 = Y_{M3}E_{s3} + (1 - Y_{M3})E_{s2} - 1 = 0 \quad (12)$$

$$g^2 = Y_{s3}f_3^{r/s} + (1 - Y_{s3})f_2^{r/s} = 0 \quad (13)$$

$$g^3 = Y_{s3}\epsilon_{d3} + (1 - Y_{s3})\epsilon_{d2} = 0 \quad (14)$$

$$g^4 = Y_{s3} - Y_{M3}E_{s3} = 0. \quad (15)$$

Finally, the ϵ and f values are related through an equation of the type

$$g^5 = \epsilon_{d2} - Q_d f_2^{r/s}\tilde{t}_{r/s} = 0 \quad (16)$$

where $\tilde{t}_{r/s} \approx T^d_{1j}$ is a parameter closely related to the mean age of the crust ($\tilde{t}_{M3}$) and $Q_d \equiv 10^4\lambda_r(N_{r1}/N_{d1})$ is a bulk Earth parameter. This bulk Earth parameter and other bulk Earth values for the Sm-Nd, Lu-Hf, and Rb-Sr systems are given in Table 1. Expressions for the parameter $\tilde{t}_{r/s}$ for several

TABLE 1. Present day bulk earth parameters for the Sm-Nd, Lu-Hf, and Rb-Sr decay systems

	Sm-Nd	Lu-Hf	Rb-Sr
N_{r1}/N_{s1}	0.1967	0.0334	0.0827
N_{d1}/N_{s1}	0.511847	0.28286	0.7045
Q_d	25.13 Ga^{-1}	22.9 Ga^{-1}	16.7 Ga^{-1}
C_{s1}	1.26 ppm	0.28 ppm	22 ppm
r	^{147}Sm	^{176}Lu	^{87}Rb
d	^{143}Nd	^{176}Hf	^{87}Sr
s	^{144}Nd	^{177}Hf	^{86}Sr
λ_r	0.00654 Ga^{-1}	0.0194 Ga^{-1}	0.0142 Ga^{-1}

Sources: DEPAOLO and WASSERBURG (1976), JACOBSEN and WASSERBURG (1979a, 1980a,b, 1984), PATCHETT *et al.* (1981), PATCHETT and CHAUVEL (1984), DEPAOLO (1983).

transport models are given by JACOBSEN and WASSERBURG (1979a, 1980a). For the simple case of unidirectional transport of matter from an undifferentiated mantle to form crust and depleted mantle, JACOBSEN and WASSERBURG (1979a) concluded that this parameter is identical to the mean age of the crust.

Thus, at any point in time, the mass balance for a single decay system is described by five equations $\mathbf{g} = (g^1, \ldots, g^5)$ and nine parameters $\mathbf{Z} = (z^1, \ldots, z^9)$. Since we have reasonable estimates of most of these parameters with uncertainties, this problem reduces to finding the least squares solution to the system of equations

$$\mathbf{g}(\mathbf{Z}) = 0 \tag{17}$$

where $\mathbf{Z} = (\epsilon_{d2}, \epsilon_{d3}, f_2^{r/s}, f_3^{r/s}, \tilde{t}_{r/s}, Y_{s3}, Y_{M3}, E_{s2}, E_{s3})$.

A set of approximate (*a priori*) values is called $\mathbf{Z}_0$ and the covariance matrix is C_0. Assuming that the errors are Gaussian, it is possible to calculate an a posteriori estimate $\hat{\mathbf{Z}}$ (adjusted values) that satisfies (17) and minimizes

$$S^2 = (\hat{\mathbf{Z}} - \mathbf{Z}_0)^T C_0^{-1} (\hat{\mathbf{Z}} - \mathbf{Z}_0) \tag{18}$$

over the set of all possible solutions of (17). TARANTOLA and VALETTE (1982) have shown that the solution to this problem is

$$\hat{\mathbf{Z}} = \mathbf{Z}_0 + C_0 \cdot G^T \cdot (G \cdot C_0 \cdot G^T)^{-1} \{G \cdot (\hat{\mathbf{Z}} - \mathbf{Z}_0) - g(\hat{\mathbf{Z}})\} \tag{19}$$

where G is the matrix of partial derivatives $G^{ik} = \partial g^i / \partial z^k$ taken at the point $\hat{\mathbf{Z}}$. They also described a fixed point method for solving (19) and obtained the covariance matrix of the adjusted parameters.

MANTLE–CRUST TRANSPORT MODEL

The fluxes of mass and species i from reservoir j to k are $\dot{M}_{jk}$ and J_{ijk}, respectively. The mass transport equations are:

$$\frac{dM_2}{d\tau} = \dot{M}_{32} + \dot{M}_{42} - \dot{M}_{23} - \dot{M}_{24} \tag{20}$$

$$\frac{dM_3}{d\tau} = \dot{M}_{23} + \dot{M}_{43} - \dot{M}_{32} - \dot{M}_{34} \tag{21}$$

$$\frac{dM_4}{d\tau} = \dot{M}_{24} + \dot{M}_{34} - \dot{M}_{42} - \dot{M}_{43}. \tag{22}$$

For species in the depleted upper mantle (reservoir 2), we have the following transport equations:

$$\frac{dN_{s2}}{d\tau} = J_{s32} + J_{s42} - J_{s23} - J_{s24} \tag{23}$$

$$\frac{dN_{r2}}{d\tau} = J_{r32} + J_{r42} - J_{r23} - J_{r24} - \lambda_r N_{r2} \tag{24}$$

$$\frac{dN_{d2}}{d\tau} = J_{d32} + J_{d42} - J_{d23} - J_{d24} + \lambda_r N_{r2}. \tag{25}$$

Similar equations hold for the continental crust and lower mantle (reservoirs 3 and 4).

These equations can be written with the $\epsilon - f$ notation as a system of first order differential equations

$$\frac{d\epsilon_d}{d\tau} = Q_d \mathbf{f}^{r/s} + K_s \cdot \epsilon_d - 10^{-4} Q_d \epsilon_d \tag{26}$$

where $\epsilon_d = (\epsilon_{d2}, \epsilon_{d3}, \epsilon_{d4})$ and $\mathbf{f}^{r/s} = (f_2^{r/s}, f_3^{r/s}, f_4^{r/s})$ and

$$K_s = \begin{bmatrix} -(k_{s32} + k_{s42}) & k_{s32} & k_{s42} \\ k_{s23} & -(k_{s23} + k_{s43}) & k_{s43} \\ k_{s24} & k_{s34} & -(k_{s24} + k_{s34}) \end{bmatrix} \tag{27}$$

and

$$k_{sij} = \left[\frac{J_{sij}}{N_{sj}}\right] = d_{sij}\psi_{ij}\left[\frac{N_{si}}{N_{sj}}\right] \tag{28}$$

with $\psi_{ij} = \dot{M}_{ij}/M_i$ and $d_{sij} = c_{sj}/C_{si}$. Here c_{sj} is the concentration in a new parcel of mass added to j. For the Sm-Nd, Rb-Sr, and Lu-Hf systems, Q_d is in the range $\sim$ 17–25, so the term $10^{-4} Q_d \epsilon_d$ is clearly negligible. Several of the models in the literature are special cases of the general three box model equations given above.

Constant mass of the crust-depleted mantle system and undifferentiated lower mantle

For this case the lower mantle is isolated from the depleted mantle crust system so $k_{s34} = k_{s24} = k_{s42} = k_{s43} = 0$. This is Model II of JACOBSEN and WASSERBURG (1979a, 1980a), and the $K_s \cdot \epsilon_d$ term in Eqn. (26) for this case reduces to:

$$K_s \cdot \epsilon_d = \begin{bmatrix} k_{s32}(\epsilon_{d3} - \epsilon_{d2}) \\ k_{s23}(\epsilon_{d2} - \epsilon_{d3}) \\ 0 \end{bmatrix} = \begin{bmatrix} -d_{s32}\psi_{32}(\epsilon_{d2}/Y_{s2}) \\ -d_{s23}\psi_{23}(\epsilon_{d3}/Y_{s3}) \\ 0 \end{bmatrix}. \tag{29}$$

Undifferentiated lower mantle

In Model I of JACOBSEN and WASSERBURG (1979a) the crust grows by additions from the undifferentiated lower mantle only, with the depleted upper mantle growing in size and the lower undifferentiated mantle decreasing in size with time. In this case $k_{s34} = k_{s24} = k_{s23} = 0$ and:

$$K_s \cdot \epsilon_d = \begin{bmatrix} k_{s32}(\epsilon_{d3} - \epsilon_{d2}) - k_{s42}\epsilon_{d2} \\ -k_{s43}\epsilon_{d3} \\ 0 \end{bmatrix} = \begin{bmatrix} -[d_{s32}\psi_{32} + d_{s42}\psi_{42}Y_{s4}](\epsilon_{d2}/Y_{s2}) \\ -d_{s43}\psi_{43}(Y_{s4}/Y_{s3})\epsilon_{d3} \\ 0 \end{bmatrix}. \tag{30}$$

A model similar to Model II of JACOBSEN and WASSERBURG (1979a, 1980a) but with a plume flux from the lower mantle (with $d_{s42} = 1$) was discussed by DEPAOLO (1983). In this case $k_{s34} = k_{s24} = k_{s43} = 0$ and:

$$K_s \cdot \epsilon_d = \begin{bmatrix} k_{s32}(\epsilon_{d3} - \epsilon_{d2}) - k_{s42}\epsilon_{d2} \\ k_{s23}(\epsilon_{d2} - \epsilon_{d3}) \\ 0 \end{bmatrix} = \begin{bmatrix} -[d_{s32}\psi_{32} + \psi_{42}Y_{s4}](\epsilon_{d2}/Y_{s2}) \\ -d_{s23}\psi_{23}(\epsilon_{d3}/Y_{s3}) \\ 0 \end{bmatrix}. \tag{31}$$

Differentiated lower mantle

GALER and O'NIONS (1985) discussed a model involving bulk entrainment at the interface between the upper and lower mantle, but with a constant mass of the lower mantle (*i.e.* $\dot{M}_{24} = \dot{M}_{42}$). In this model $k_{s43} = k_{s32} = k_{s34} = 0$ and $d_{sij} = 1$ except for d_{s23} which in general is $\neq 1$ and using Eqn. (11) the $K_s \cdot \epsilon_d$ term of Eqn. (26) reduces to:

$$K_s \cdot \epsilon_d = \begin{bmatrix} k_{s42}(\epsilon_{d4} - \epsilon_{d2}) \\ k_{s23}(\epsilon_{d2} - \epsilon_{d3}) \\ k_{s24}(\epsilon_{d2} - \epsilon_{d4}) \end{bmatrix}$$

$$= \begin{bmatrix} \psi_{42}[(X_{s3}/X_{s2})(\epsilon_{d3} - \epsilon_{d2}) - (\epsilon_{d2}/X_{s2})] \\ d_{s23}\psi_{23}(X_{s2}/X_{s3})(\epsilon_{d2} - \epsilon_{d3}) \\ \psi_{24}(X_{s2}/X_{s4})(\epsilon_{d2} - \epsilon_{d4}) \end{bmatrix}. \quad (32)$$

The condition $\dot{M}_{24} = \dot{M}_{42}$ implies the following relationship $\psi_{24} = \psi_{42}(M_4/M_2)$ between the mass transfer coefficients.

More general cases than the GALER and O'NIONS (1985) model including recycling of continental material can also be treated with Eqn. (26). In the case where only k_{s43} and k_{s34} are equal to zero and all d_{sij} except for d_{s23} are equal to one the equation for the depleted mantle must be modified to include a term for crustal recycling:

$$\frac{d\epsilon_{d2}}{d\tau} = Q_d f_2^{r/s} + \psi_{42}[(X_{s3}/X_{s2})(\epsilon_{d3} - \epsilon_{d2}) - (\epsilon_{d2}/X_{s2})] + \psi_{32}(X_{s3}/X_{s2})(\epsilon_{d3} - \epsilon_{d2}). \quad (33)$$

Thus from *a priori* ϵ, f, and X-values one can obtain a relationship between the rate of crustal recycling (ψ_{32}) and the rate of exchange between the upper and lower mantle (ψ_{42}).

RESULTS

Present-day mass balance

The mass balance Eqns. (12)–(16) contain seven intensive quantities (ϵ_{d2}, ϵ_{d3}, $f_2^{r/s}$, $f_3^{r/s}$, E_{s2}, E_{s3} and $\tilde{t}_{r/s}$) and two extensive quantities (Y_{s3} and Y_{M3}). *A priori* values (with maximum uncertainties treated as 2σ errors) for all of these parameters are given in Tables 2 and 3. The ϵ, f, and E values (except E in the depleted mantle) are rather well constrained. The mean ages $\tilde{t}_{Sm/Nd}$, $\tilde{t}_{Lu/Hf}$, and $\tilde{t}_{Rb/Sr}$ have been given large uncertainties because they are similar to, but in general, not equal to the mean age of the crust (JACOBSEN and WASSERBURG, 1979a, 1980a). The $\tilde{t}_{Rb/Sr}$ value in particular could be substantially older than the mean age of the crust. The mean age of the crust, $\tilde{t}_{M3}$ has been estimated at 1.8 ± 0.4 Ga from Nd model age distributions in crustal rocks and sediments (JACOBSEN, 1988). Somewhat broader limits are therefore used in Table 3 for *a priori* mean ages of the individual isotopic systems. The *a priori* values of the mass fractions Y_{s3} and Y_{M3}, and also the E-values for the depleted mantle have been given large errors as only broad limits can be placed on the values of these extensive quantities. In general, not more than 80% of Nd, Hf, and Sr in the crust-upper mantle system is believed to reside in the crust. The lower limit is 0%. The *a priori* range of the crust/depleted mantle mass ratio, Y_{M3}, is 0.005 to 0.3. The corresponding limits for the mass fraction of the mantle that is depleted are 15% to 100%.

Thus, as shown in Table 3, the adjusted values resulting from the inversion provide rather large improvements in the uncertainties and average values of the mean ages, the mass ratio Y_{M3}, and the degree of Nd, Sr, and Hf depletion in the depleted mantle (Table 2). The other ϵ, f, and E-values show only small adjustments in both errors and average values (Table 2). Table 1 also gives the bulk earth concentrations used. These are rather well established at 2.6 times $C1$ chondritic values for refractory lithophile elements like Nd, Sr, and Hf (*cf.* JOCHUM *et al.* 1986). The value used here for the bulk earth Nd concentration is 1.26 ± 0.09 ppm. This value is within the range of the HART and ZINDLER (1986) bulk earth Nd concentration of 1.17 ± 0.02 ppm.

TABLE 2. A priori and adjusted present day ϵ, f and E – values for a three–reservoir crust–mantle model with undifferentiated lower mantle†.

	Depleted Mantle	Continental Crust	Comment‡
I. Isotopic parameters (ϵ_{dj})			
Nd	10 ± 2	−15 ± 4	ap
	9.8 ± 1.6	*−16.7 ± 2.8*	ad
Hf	18 ± 2	−26 ± 8	ap
	17.9 ± 1.8	*−26.3 ± 1.7*	ad
Sr	−27 ± 4	70 ± 20	ap
	−27.5 ± 3.8	*66.6 ± 16.4*	ad
II. Chemical fractionation factors ($f_j^{r/s}$)			
Sm/Nd	0.25 ± 0.04	−0.44 ± 0.02	ap
	0.257 ± 0.02	*−0.438 ± 0.020*	ad
Lu/Hf	0.42 ± 0.09	−0.65 ± 0.10	ap
	0.434 ± 0.064	*−0.638 ± 0.082*	ad
Rb/Sr	−0.9 ± 0.1	2.0 ± 1.0	ap
	−0.89 ± 0.09	*2.2 ± 0.6*	ad
III. Chemical enrichment factors (E_{sj})			
Nd	0.6 ± 0.4	20.6 ± 2.4	ap
	0.64 ± 0.02	*20.6 ± 2.4*	ad
Hf	0.6 ± 0.4	20.6 ± 2.4	ap
	0.61 ± 0.04	*20.6 ± 2.4*	ad
Sr	0.68 ± 0.40	15 ± 2	ap
	0.72 ± 0.05	*15 ± 2*	ad

† Sources of a priori estimates: JACOBSEN and WASSERBURG (1979a, 1980a), DEPAOLO (1983), PATCHETT *et al.* (1981), GOLDSTEIN and JACOBSEN (1988), HART and ZINDLER (1986), ALLEGRE *et al.* (1983b).
‡ ap = a priori value, ad = adjusted value.

From the crust/depleted mantle mass ratio Y_{M3} we may estimate the mass fraction of depleted mantle X_{M2}:

$$X_{M2} = X_{M3}((1/Y_{M3}) - 1). \quad (34)$$

The mass fraction of continental crust X_{M3} was estimated by JACOBSEN and WASSERBURG (1979a) from various sources to be ≈0.0056. Most values used by other researchers range from 0.005 to 0.006, thus we estimate $X_{M3} = 0.0056 \pm 0.0005$. As shown in Table 4, the mass fraction of depleted mantle (X_{M2}) estimated from the Sm-Nd, Lu-Hf, and Rb-Sr systems is 0.305, 0.278, and 0.281, respectively. The mean ages range from 1.5 Ga to 1.8 Ga, consistent with previously published results by JACOBSEN and WASSERBURG (1979a, 1980a).

The inversions for Nd, Sr and Hf were conducted separately to investigate whether relatively consistent results could be obtained from these three decay systems. Rb-Sr and Sm-Nd can be inverted together as done by ALLÈGRE *et al.* (1983b); then with the values of Tables 2 and 3 we obtain $X_{M2} = 0.291 \pm 0.061$. The difference of this result from that of ALLÈGRE *et al.* (1983b) is entirely due to the difference in choice of *a priori* input values and not the numerical method used. Using the values of ALLÈGRE *et al.* (1983b) in the computer program used for the present calculations yields identical results to those presented in Table 1 of ALLÈGRE *et al.* (1983b). One

TABLE 3. Mean ages and mass fractions

	Sm–Nd	Lu–Hf	Rb–Sr	Comment†
$\bar{t}_{r/s}$	1.8 ± 0.6	1.8 ± 0.6	2.0 ± 1.0	ap
	1.51 ± 0.25	*1.80 ± 0.26*	*1.84 ± 0.30*	ad
Y_{s3}	0.4 ± 0.4	0.6 ± 0.4	0.4 ± 0.6	ap
	0.37 ± 0.04	*0.41 ± 0.04*	*0.29 ± 0.05*	ad
Y_{M3}	0.0200 ± 0.0150	0.0200 ± 0.0150	0.0200 ± 0.0150	ap
	0.0180 ± 0.0026	*0.0197 ± 0.0028*	*0.0195 ± 0.0046*	ad

† ap = a priori value, ad = adjusted value.

should note that the error estimates given in Tables 2 and 3 do not include uncertainties in the bulk Earth parameters. For Sm-Nd the uncertainties in the chondritic reference values used to represent the bulk Earth values are trivial in comparison to errors in the isotopic composition of the various reservoirs. Insofar as the earth follows a truly chondritic Sm-Nd evolution this should be valid. In contrast the "bulk Earth" values for Hf and Sr have large uncertainties; the uncertainties in Hf evolution are due to the lack of precise data on chondritic meteorites. The uncertainty in the Rb-Sr parameters is due to the method of obtaining the bulk Earth values from the mantle array, an approach that may not be entirely valid in light of more recent studies of Pb-Sr-Nd variations in oceanic basalts (HART *et al.*, 1986). These uncertainties do not appear in the final mass balance estimates as they are poorly defined, however, the apparent agreement between the three systems is encouraging. However, this may in part be caused by the fact that many of the values used were established from correlations with the Sm-Nd system.

Mass balance through time and the growth curve of the continental crust

For the Sm-Nd system, we can estimate the ϵ and f curves for depleted mantle and crust reasonably well throughout most of the Earth's history. For the Lu-Hf system, a much smaller data base is available and for Rb-Sr very few reliable initial values are available because of the effects of alteration. Also, the continental crust shows a strong layering of Rb-Sr that makes it difficult to define average crustal parameters for the Rb-Sr system. The inversion of the mass balance problem back through time is therefore here limited to the Sm-Nd isotopic system.

The data base used for the Sm-Nd system is presented in Figs. 2 and 3. Fig. 2 shows estimates of the $f^{Sm/Nd}$ values for

TABLE 4. Mass fraction estimates for the silicate portion of the earth

	Sm–Nd	Lu–Hf	Rb–Sr
Continental Crust†			
X_{M3}	0.0056 ± 0.0005	0.0056 ± 0.0005	0.0056 ± 0.0005
Depleted mantle‡			
X_{M2}	*0.305 ± 0.044*	*0.278 ± 0.040*	*0.281 ± 0.066*
X_{M2}	*0.305 ± 0.052*	*0.278 ± 0.047*	*0.281 ± 0.071*
Undepleted mantle			
X_{M4}	*0.689 ± 0.052*	*0.716 ± 0.047*	*0.713 ± 0.071*

† Value used by JACOBSEN and WASSERBURG (1979a, 1980a) with a reasonable error estimate.
‡ Obtained from estimates of the ratio of the mass of the crust to depleted mantle in Table 3 using equation (34). The uncertainties in the first row do not include uncertainty in the mass of the crust, while in the second row these are included.

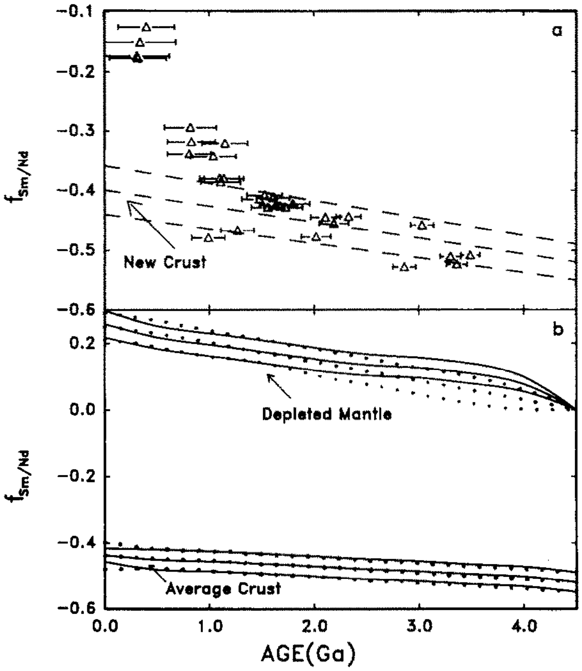

FIG. 2. $f^{Sm/Nd}$ *vs.* age for continental crust and the depleted (upper) mantle. a) The data points are river water suspended loads from GOLDSTEIN and JACOBSEN (1988). The age used for these suspended loads is a T^{Nd}_{DM} model age that should give a good estimate of the mean age of the crustal source materials for these suspended loads. The error bars correspond to using $\epsilon^{DM}_{Nd} = +6$ (lower limit) and $\epsilon^{DM}_{Nd} = +10$ (upper limit) for the present value of depleted mantle (DM). The dashed lines correspond to our estimate of average $f^{Sm/Nd}$ in new crust and the 2σ error band at various ages from these data. b) From the curve for new crust in Fig. 2a an average curve (dotted) with a 2σ error band has been estimated for the continental crust. Note that all the $f^{Sm/Nd}$ data except the data points from young island arcs with $T^{Nd}_{DM} < 1$ Ga (Japan and Philippines) fit these trends. These island arc samples appear to have a large fraction of their REE derived from weathering of tholeiitic volcanic rocks including ophiolites. The dotted curves for the depleted mantle are our estimate of the depleted mantle $f^{Sm/Nd}$ curve with a 2σ error band. The least squares inversion discussed in the text yields the solid curves (with 2σ error bands) consistent with Sm-Nd isotopic mass balance for the crust-depleted mantle system for all of Earth's history.

the crust and depleted mantle contrained through the present day values of $f_3^{Sm/Nd} = -0.438 \pm 0.020$ and $f_2^{Sm/Nd} = +0.257 \pm 0.020$. Data from suspended loads of rivers are plotted in Fig. 2a and should represent the average $f^{Sm/Nd}$ values in crust of varying age.

The average time of addition of the source material of these suspended loads to the continents is estimated with single stage depleted mantle (DM) Nd model ages; T^{Nd}_{DM} (*cf.* NELSON and DEPAOLO, 1984). For a linear evolution of ϵ_{Nd} in the depleted mantle the T^{Nd}_{DM} model ages are given by:

$$T^{Nd}_{DM} \approx \frac{\epsilon^{DM}_{Nd} - \epsilon^{meas}_{Nd}}{Q_{Nd}(f^{Sm/Nd}_{DM} - f^{Sm/Nd}_{meas})}. \quad (35)$$

This model age gives an estimate of the average crustal residence time of the continental sources of the suspended loads. The data with $T^{Nd}_{DM} > 1$ Ga plot in a band with a slightly negative slope in the $f^{Sm/Nd}$ *vs.* age diagram. Samples from young arcs with T^{Nd}_{DM} in the range of 0 to 1 Ga plot along a

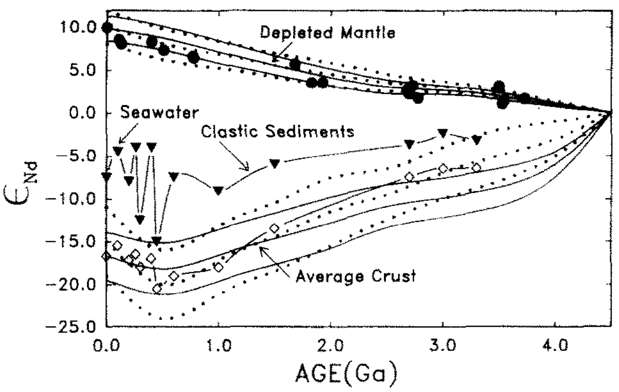

FIG. 3. ϵ_{Nd} *vs.* age for continental crust and the depleted (upper) mantle. The solid circles are data for mafic igneous rocks through time (AITKEN and ECHEVERRIA, 1984; CATTEL *et al.*, 1984; CHAUVEL *et al.*, 1985; CLAESSON *et al.*, 1984; DUPRÉ *et al.*, 1984; EDWARDS and WASSERBURG, 1985; FLETCHER *et al.*, 1984; HAMILTON *et al.*, 1983; HAWKESWORTH *et al.*, 1981; JACOBSEN and WASSERBURG, 1979b; JACOBSEN *et al.*, 1984; JAHN *et al.*, 1980; MACHADO *et al.*, 1986; MCCULLOCH and COMPSTON, 1981; MCCULLOCH *et al.*, 1981; NELSON and DEPAOLO, 1985; XUAN *et al.*, 1986; ZINDLER, 1982). The solid triangles represent our best estimates of ϵ_{Nd} in average erosion products from the continents. Through the Phanerozoic the oceans are dominated by continental input so the seawater curve of KETO and JACOBSEN (1988) has been used for this time period. For earlier times, clastic sediment data were used (see JACOBSEN, 1988, for references). It is well established that such a curve does not yield the average ϵ_{Nd} curves for the continental crust directly. The curve with the open diamonds represents the best estimate of the average crust based on seawater-clastic sediment data (see text). The dotted curves represent the best estimates of the depleted mantle and average crust curves (with 2σ error bands). The least squares inversion of the Sm-Nd mass balance problem yields the solid curves (with 2σ error bands) for the evolution of ϵ_{Nd} in the continental crust and depleted mantle for all of Earth's history.

much steeper trend that is probably caused by a substantial contribution from tholeiitic volcanics with DM-like f-values. In general, such terranes are not very typical in continental crust, so these data have been discarded in estimating $f^{Sm/Nd}$ *vs.* age for continental crust evolution.

The dashed band shown in Fig. 2a is our estimate of the average $f^{Sm/Nd}$ trend (with a 2σ error band) in new additions to the continental crust. This value, $f^{Sm/Nd}_{new}$, is directly related to the $f^{Sm/Nd}$ value in the depleted mantle through

$$f^{Sm/Nd}_{new} = \frac{d_{Sm}}{d_{Nd}}(f^{Sm/Nd}_2 + 1) - 1 \qquad (36)$$

where d_{Sm} and d_{Nd} are the enrichment factors in new crust relative to the depleted mantle at the same time for Sm and Nd, respectively.

In this way, the dotted band for the depleted mantle was constructed as an *a priori* evolution of $f^{Sm/Nd}$ in the depleted mantle through time. The *a priori* $f^{Sm/Nd}$ curve for average crust as a function of time was constructed by integrating the $f^{Sm/Nd}_{new}$ curve and constraining the average curve to go through the present-day average value of -0.438 ± 0.020. The result is shown as the dotted curve in the negative region of the $f^{Sm/Nd}$ *vs.* age diagram.

Data for ϵ_{Nd} in samples that may reflect average depleted mantle and continental crust evolution are shown in Fig. 3. The present value of the depleted mantle is $\epsilon_{Nd} = 10 \pm 2$ from measurements of mid-ocean ridge basalts. The solid circles represent initial ϵ_{Nd} values obtained from mafic igneous rocks. Each point represents initial values estimated from a number of samples of a given suite of rocks. In particular samples with $f^{Sm/Nd} \approx 0$ were used such that the initial values would not be very sensitive to errors in the age. On this basis, the dotted curves in the positive region of Fig. 3 were chosen to give an *a priori* estimate of the depleted mantle ϵ_{Nd}-evolution. To estimate the continental ϵ_{Nd} evolution curve, various sedimentary rocks were used (solid triangles). Through the Phanerozoic, the oceans were dominated by continental input and so the average seawater curve of KETO and JACOBSEN (1988) for this time period was used. For earlier times, clastic sediments (see JACOBSEN, 1988, for references) are used to estimate ϵ_{Nd} in average erosion products from the continents. It is well established that such a curve representing ϵ_{Nd} in average erosion products does not directly yield the average continental ϵ_{Nd} curve (ALLÈGRE and ROUSSEAU, 1984; GOLDSTEIN and JACOBSEN, 1988). As shown, the present value of ϵ_{Nd} in erosion products is ≈ -7 to -8, whereas the average crust is estimated at -15. The curve with the open diamonds represents the best estimates of average crust based on the seawater-clastic sediment data using the method discussed by ALLÈGRE and ROUSSEAU (1984). The dotted curves in the negative portion of Fig. 3 represent the *a priori* evolution curve (with 2σ error bands) for the evolution of ϵ_{Nd} in the continental crust.

In addition to the curves for $f^{Sm/Nd}$ and ϵ_{Nd} through time, rough estimates of the E_{Nd}, $\tilde{t}_{Sm/Nd}$, Y_{Nd3}, and Y_{M3} through time are necessary. The inversion was conducted in 0.1 Ga time-steps from the present to the origin of the Earth 4.5 Ga ago. Beginning with the present day, E_{Nd}, $\tilde{t}_{Sm/Nd}$, Y_{Nd3}, and Y_{M3} values were given very large errors, and for each time step of the inversion, their average value was selected from the previous time-step. (This was justified because none of these values can change drastically over 0.1 Ga). In this manner, the inversion was conducted for all of Earth's history resulting in the adjusted f and ϵ-values represented as solid curves (with 2σ error bands) in Figs. 2 and 3. The curves closely correspond to the *a priori* (dotted) curve in most cases except for the early part of the depleted mantle $f^{Sm/Nd}$ curve and the early part of the continental crust ϵ_{Nd} curve. However, for this time period there were essentially no data constraining the *a priori* curves, so it is not surprising to find the largest changes here. The inversion also yielded estimates of the mean ages and the mass fractions Y_{Nd3} and Y_{M3} in the crust. The Y_{M3} *vs.* age is the growth curve for the continental crust if $M_2 + M_3$ is constant for all τ. The resultant crustal growth curve is presented in Fig. 4. It suggests that by 3.8 Ga ago (the age of the oldest terrestrial rock) about 40% of the present crustal volume was present as continental crust. After this early period of rapid growth, the growth rate was much lower and may have increased again over the last 0.5 Ga.

Rates of crustal additions and recycling

The rate of growth of the crust ($dM_3/d\tau$) in this model for which the total inversion was carried out is simplified to

$$\frac{dM_3}{d\tau} = \dot{M}_{23} - \dot{M}_{32} \qquad (37)$$

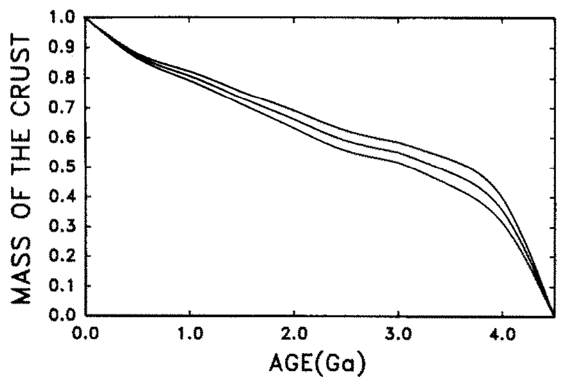

FIG. 4. Plot of the mass of the continental crust (with 2σ error band) calculated as a function of time resulting from the least squares inversion of the Sm-Nd mass balance for the depleted-mantle continental crust system. The mass of crust + depleted mantle is assumed constant for all of Earth's history in calculating this curve. The curves are normalized to the present mass of the continental crust ($M_3(T_0)$).

where $\dot{M}_{23}$ is the total rate of crustal additions and $\dot{M}_{32}$ is the rate of recycling of crust. In terms of the fractional rates, this yields

$$\frac{dM_3}{d\tau} = \psi_{23}[M_2^0 - M_3] - \psi_{32}M_3. \quad (38)$$

Since the crustal growth rate, $dM_3/d\tau$, can be derived from Fig. 4 and ψ_{32} can be evaluated from Eqn. (29) using the $\epsilon_{Nd} - f$ curves from the inversion, we can obtain the rate of crustal addition ψ_{23} from Eqn. (38). The fractional rate of recycling ψ_{32} calculated from Eqn. (29) is shown in Fig. 5a. The rate was initially low and reached a maximum of about 0.65 Ga^{-1} and then declined to a value of 0.2 Ga^{-1} from 2 to 0.5 Ga ago, reaching a value of approximately 0.33 Ga^{-1} today. This present value is essentially the same as that of DEPAOLO (1983).

The rate of crustal addition, ψ_{23}, obtained by using the results in Figs. 4 and 5a is shown in Fig. 5b. As illustrated, this results in a very high initial growth rate of $\psi_{23} \approx 0.018$ Ga^{-1} declining rapidly then reaching a maximum again of 0.008 Ga^{-1} at 3 Ga, and finally declining slowly to $\approx$0.005 Ga^{-1} before its growth increases rapidly to a value of $\approx$0.013 Ga^{-1} at the present.

DISCUSSION

Comparison with previous estimates of the present day Nd mass balance

The results of the present day mass balance inversion for Nd and Sr yield data that are in general agreement with previous estimates made by JACOBSEN and WASSERBURG (1979a, 1980a) and DEPAOLO (1980) using a forward calculation. The JACOBSEN and WASSERBURG (1979a) mass balance yielded a mass fraction of depleted mantle of $X_{M2} \approx 0.3$. Somewhat different results were obtained by ALLÈGRE *et al.* (1983b) in their inversion of the Nd-Sr mass balance. They used an inversion approach to obtain error estimates and obtained $X_{M2} \approx 0.3$ to 0.9. However, this disparity is not caused by differences in the technique or assumptions, but by the choice of some of the *a priori* input values of the problem.

Comparison with the results of ALLÈGRE *et al.* (1983b) is done most directly with the mass fraction of Nd in the crust, Y_{Nd3}, through the following equation:

$$Y_{s3} = \frac{\epsilon_{d2}}{(\epsilon_{d2} - \epsilon_{d3})}. \quad (39)$$

For Sm-Nd ALLÈGRE *et al.* (1983b) obtained $Y_{Nd3} = 0.289 \pm 0.06$; the input values used here yield $Y_{Nd3} = 0.370 \pm 0.018$. Whereas essentially identical depleted mantle ϵ_{Nd} values were used, the average continental ϵ_{Nd} value used here is substantially higher ($\epsilon_{Nd} \approx -16 \pm 4$) and also has a much smaller uncertainty than that obtained by ALLÈGRE *et al.* (1983b) ($\epsilon_{Nd} = -26 \pm 8$). The value used in this paper is based on the river suspended load data of GOLDSTEIN and JACOBSEN (1988). Both the very low continental ϵ_{Nd} and the very high continental ϵ_{Sr} value used by ALLÈGRE *et al.* (1983b) are inconsistent with the crustal averages based on the river water suspended load data of GOLDSTEIN and JACOBSEN (1988).

The mass fraction of depleted mantle (X_{M2}) can be obtained from Eqn. (39) since $Y_{M3} = Y_{s3}/E_{s3}$ together with Eqn. (34) yield:

$$X_{M2} = X_{M3}((E_{s3}/Y_{s3}) - 1). \quad (40)$$

Using $X_{M3} = 0.0056$ and $E_{s3} = 20.6 \pm 2.6$ we obtain $X_{M2} = 0.30 \pm 0.005$. The enrichment of Nd in the crust (E_{s3}) depends on estimates of two parameters. ALLÈGRE *et al.* (1983b) used a Nd concentration for the crust of 28.9 ± 2.6 ppm and for the bulk Earth of 1.00 ± 0.21 ppm which yields $E_{Nd3} = 28.9 \pm 6.3$. In this paper 26 ± 4 ppm is used for the

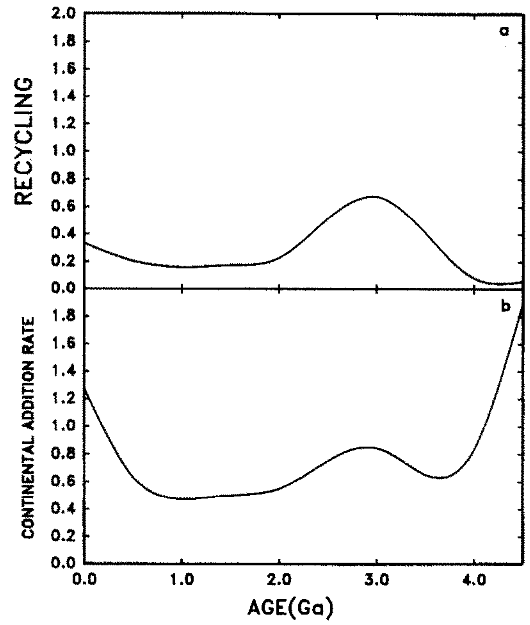

FIG. 5. a) The rate of recycling of continental crust to the depleted mantle (ψ_{32} in Ga^{-1}) as a function of time. The volumetric rate of recycling at time τ is $\dot{V}_{32}(\tau)$ (in km^3/year) $\approx 8\psi_{32}$ (in Ga^{-1})$\cdot(M_3(\tau)/M_3(T_0))$ where $M_3(T_0)$ is the present mass of the crust. b) The rate of crustal addition (ψ_{23} in 10^{-2} Ga^{-1}) as a function of time. The volumetric rate of crustal addition is $\dot{V}_{23}$ (in km^3/year) $\approx 400\psi_{23}$ (in Ga^{-1}). ψ and φ curves that also include constraints from the age distribution of crustal rocks show even stronger episodicity than those presented here (JACOBSEN, 1988).

Nd concentration in the crust (GOLDSTEIN and JACOBSEN, 1988) and it follows that $E_{Nd3} = 20.6 \pm 2.4$. A much lower crustal Nd concentration of 15 ppm has been proposed by TAYLOR and MCLENNAN (1985) which yields $E_{s3} = 12$ and $X_{M2} = 0.18$.

Another useful form of Eqn. (39) for the forward problem is:

$$X_{M2} = X_{M3}(E_{s3} - 1) - X_{M3}E_{s3}\left[\frac{Q_d f_3^{r/s}}{\epsilon_{d2}}\right]\tilde{t}_{r/s} \quad (41)$$

which shows the dependence on the mean age of the crust. Using the *a priori* Sm-Nd parameters of Tables 2 and 3 we obtain:

$$X_{M2} = 0.11(\pm 0.01) + 0.128(\pm 0.028)\tilde{t}_{Sm/Nd}. \quad (42)$$

Thus for $\tilde{t}_{Sm/Nd} = 1.5$ Ga we obtain $X_{M2} = 0.30$ while for $\tilde{t}_{Sm/Nd} = 2.5$ Ga we obtain $X_{M2} = 0.43$. This suggests that the X_{M2} value is rather well determined by the Sm-Nd mass balance alone. In contrast to ALLÈGRE *et al.* (1983b), the inversion here was conducted separately for each isotopic system to demonstrate that relatively consistent results could be obtained from each of the Sm/Nd, Rb-Sr, and Lu-Hf systems.

It is important to note that the fraction of 0.30 ± 0.05 of the mantle that has differentiated to form the crust is meaningful only in terms of the model. However, it does preclude the existence of a depleted mantle reservoir with the characteristics chosen here to extend through the whole mantle.

The Nd mass balance through time and the continental growth curve

The inversion of the Sm-Nd mass balance was carried out here for all of Earth's history yielding a set of self-consistent or adjusted curves for the ϵ_{Nd} and $f^{Sm/Nd}$ evolution of the crust and depleted mantle together with a growth curve for the crust. It is important to note that the growth curve is independent of assumptions about the transport of matter between the crust and depleted mantle through time. It depends solely on the Sm-Nd mass balance and the assumption that the mass of the crust + depleted mantle system has remained constant throughout the Earth's history.

The inversion of the mass balance through time is not very sensitive to the exact shape of the ϵ and f curves. The most notable feature of the crustal growth curve obtained is the high rate of early crustal growth. This is caused by the high ϵ_{Nd} values of about +2 in the depleted mantle at 3.8 Ga ago (HAMILTON *et al.*, 1983; JACOBSEN and DYMEK, 1988). For this to occur the $f_{Sm/Nd}$ value of the depleted mantle must grow very rapidly during this time. This can only be accomplished by a very high continental addition rate, since in this model the depleted mantle is initially fixed at its final volume. Such a rapid early crust building is not necessary in a model like the JACOBSEN and WASSERBURG (1979a) Model I, because in that model the depleted mantle and the crust grow with time from an undifferentiated source. However, there is substantial evidence that the crust grows mainly from a depleted mantle throughout most of Earth history rather than from undepleted mantle, since most new crustal additions show depleted mantle signatures (DEPAOLO, 1983). This could, of course, have been different from 4.5 to 3.8 Ga ago, since for this period we have no record. However, the oldest rocks on earth at Isua appear to be derived mostly from depleted mantle sources (JACOBSEN and DYMEK, 1988).

Rates of continental recycling and additions

For a particular version of the transport model shown in Fig. 1 with $k_{s42} = k_{s24} = k_{s43} = k_{s34} = 0$, we can easily evaluate the recycling rate (ψ_{32}) of continental crust into the depleted mantle. Whereas the present-day value (0.33) obtained here is within error of the value obtained by DEPAOLO (1983), Figs. 4 and 5 show that this does not necessarily indicate that the mass of the crust has been at steady state for the past 3 Ga as suggested by DEPAOLO (1983).

The rate of crustal addition shown in Fig. 5 suggests a strong episodicity in this parameter. Episodicity in crustal growth has long been suggested on the basis of peaks in histograms of ages on crustal rocks. Early data were mostly mineral ages that could be easily reset by thermal events much later than the time of formation of the rocks. However, now that we have Nd model age histograms of basement rocks in N. America (NELSON and DEPAOLO, 1984) illustrating similar episodicity, it is clear that episodicity is a basic feature of continental growth and not an artifact of the pattern of resetting. Nd model ages on suspended loads of rivers in N. America also show this episodicity (GOLDSTEIN and JACOBSEN, 1988).

The calculated recycling (φ) and addition (ψ) rates *versus* age curves, are in contrast to the continental mass growth curve, very dependent on the exact shape chosen for the f and ϵ curves. The time evolution data for ϵ_{Nd} are poorly constrained with regard to the exact shape of the growth curves. The fitted curve in Fig. 3 lies mainly within the range of older data points, but somewhat above the data points for the young samples. It is the slope of this curve that determines the recycling rate, and it could appear that it has been made arbitrarily larger over the past 2 Ga. This is because it was constrained to have a mid-ocean ridge basalts ϵ_{Nd} value of +10 at present. While the Phanerozoic data used in Fig. 3 are mostly obtained from ophiolites and may represent directly the depleted mantle source, such samples are not available for earlier times where the data are mostly from greenstone belts. DEPAOLO (1983) drew a slightly different depleted mantle ϵ_{Nd} curves based on island arc rocks and various continental felsic rocks and greenstone belts. The difference between the results of DEPAOLO (1983) and the current results is fully explained by the difference in approach. In contrast to previous models an average crustal ϵ_{Nd} curve is used here that constrains the crustal addition rate ψ_{23} (see Eqn. 29). The high addition rate for the past 0.5 Ga (Fig. 5) is entirely constrained by the crustal ϵ_{Nd} curve.

The rates of recycling obtained here are model dependent and it is possible that a more general three-layer model might yield quite different recycling rates. PATCHETT *et al.* (1984) argued that Lu/Hf ratios in sediments preclude the existence of large-scale return of crustal material to the mantle. A separate paper (JACOBSEN, 1988) presents an improved model that directly addresses the Nd model age distribution observed in crustal rocks. The main difference in the results is that there is a much stronger episodicity imposed on the crustal addition and recycling rates shown in Fig. 5, otherwise it is in general agreement with the results presented here.

The rate of exchange between upper and lower mantle

GALER and O'NIONS (1985) suggested that the $^{207}Pb/^{206}Pb$ difference between crust and depleted mantle as well as the low Th/U ratios in the depleted mantle preclude large-scale exchange between crust and upper mantle. They suggested instead, that large-scale exchange occurs between the upper and lower mantle. The main problem in such a model is to quantify the chemical fractionations and mass transfer between upper and lower mantle. While we cannot prove uniqueness of the recycling models favored here, the approach taken is to test whether models largely involving crust-upper mantle exchange are sufficient to explain the observed isotope and trace element patterns. As shown, the Sm-Nd, Rb-Sr and Lu-Hf isotopic systems can be well explained with such a model. ZARTMAN and HAINES (1988) have shown that the Pb-isotope arguments of GALER and O'NIONS (1985) are not definitive and that the U-Th-Pb-isotope evolution of the crust and the mantle is consistent with the recycling model suggested here.

Using Eqn. (33) with the *a priori* Sm-Nd values of Tables 2 and 3 and with $d\epsilon_{d2}/d\tau \approx 2.2$ Ga^{-1} for the present yields the following relationship between the rate of exchange between the upper and lower mantle (ψ_{42}) and the rate of continental recycling (ψ_{32}) into the mantle; ψ_{42} (in Ga^{-1}) ≈ 0.13–$0.5\ \psi_{32}$. For the case of no recycling ($\psi_{32} = 0$) this yields $\psi_{42} \approx 0.13$ Ga^{-1} and $\psi_{24} \approx 0.3$ Ga^{-1}. This suggests a residence time for the upper mantle of ≈ 3.3 Ga. However, using the range of ψ_{32} values (0.08–0.02 Ga^{-1}) favored by JACOBSEN (1988) on the basis of clastic and chemical sediment Nd isotopic data we obtain a range of ψ_{42} from 0.09 to 0.12 Ga^{-1}. This yields $\psi_{24} \approx 0.21$ to 0.28 Ga^{-1} and a residence time of the upper mantle of 3.6 to 4.8 Ga. Most of the *a priori* parameters used in this calculation are, however, poorly constrained. If a better set of adjusted parameters could be obtained through inversion of the general three layer mass balance Eqns. (8)–(11) then this could be improved. The main problem with the general three layer mass balance problem is the lack of adequate *a priori* estimates of the various relevant parameters for the lower mantle. In the GALER and O'NIONS (1985) model it is very hard to obtain $\epsilon_{Nd} \approx +2$ by 3.8 Ga because of the diluting effect of having a large undepleted mantle reservoir initially. Only for a modified version of this model where the depleted mantle grows from an initially small reservoir is it possible to produce such high ϵ_{Nd} values early and this is then essentially the JACOBSEN and WASSERBURG (1979a) Model I.

CONCLUSIONS

1) Total inversion of the present-day data for the Sm-Nd, Lu-Hf, and Rb-Sr isotopic systems are all individually consistent with ~30% of the mantle being depleted, which roughly corresponds to the volume of the Earth's upper mantle (above the 670 km discontinuity).

2) The isotopic evolution of any three layer model can be modeled by the following system of differential equations:

$$\frac{d\epsilon_d}{d\tau} = Q_d f^{r/s} + K_s \cdot \epsilon_d$$

for the Sm-Nd, Lu-Hf, and Rb-Sr as long as the material transferred from one reservoir to another reservoir has the average isotopic composition of its source reservoir. The matrix K_s (see Eqn. 27) is in general time dependent and contains all of the time dependent mass transport coefficients (ψ_{ij}; fractional mass transfer rates). Inspection of Eqns. (26)–(28) shows that even with complete knowledge of the ϵ and f curves for all three reservoirs of a single decay system, it is not possible to obtain a unique solution to the complete set of fractional mass transfer rates ψ_{ij}. To obtain all ψ_{ij}, it is necessary to use information from several decay systems showing different behavior. Various assumptions can be made to simplify the matrix K_s, and Eqns. (29)–(32) demonstrate that many of the models in the literature can be considered as special cases of the general three-layer model.

3) Total inversion of the Sm-Nd mass balance through time yields an early high growth rate with ~40% of the present crust produced by 3.8 Ga. This calculation assumes, however, that the mass of the crust + depleted mantle is constant for all of the Earth's history.

4) The rates of recycling (ψ_{32}) and crustal addition (ψ_{23}) are also evaluated for the model with a constant mass crust + depleted mantle. They both exhibit distinct peaks around 3.0 Ga ago. The addition rate is also high between 4.0 and 4.5 Ga ago and during the Phanerozoic. The results obtained here are not compatible with a constant (steady state) volume of the crust over the past 2–3 Ga.

5) The main uncertainty in evaluating the three-layer model presented here arises from inadequate constraints on the chemical and isotopic composition of the lower mantle. In the model favored here there is no exchange between the upper and lower mantle, and Nd, Sr, Hf and Pb isotope data all appear compatible with such a model, if it includes recycling of continental crust into the upper mantle. However, a range of other models involving exchange between the upper and lower mantle also appears to be compatible with available data.

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