$$f(x)=a_0+\sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{2\pi n}{T}x\right)+b_n\sin\left(\frac{2\pi n}{T}x\right)\right] \quad , \quad \begin{cases} a_0=\dfrac{1}{T}\displaystyle\int_c^{c+T} f(x)\,dx \\ a_n=\dfrac{2}{T}\displaystyle\int_c^{c+T} f(x)\cos\left(\frac{2\pi n}{T}x\right)dx \\ b_n=\dfrac{2}{T}\displaystyle\int_c^{c+T} f(x)\sin\left(\frac{2\pi n}{T}x\right)dx \end{cases}$$

$$E^{*}=\int_{-\pi}^{\pi} f^2\,dx-\pi\left[2a_0^2+\sum_{n=1}^{N}\left(a_n^2+b_n^2\right)\right]$$

$$\lim_{n\to\infty}\int_{-\pi}^{\pi}\left[f(x)-S_n(x)\right]^2dx=0$$

$$2a_0^2+\sum_{r=1}^{\infty}\left(a_r^2+b_r^2\right)=\frac{1}{\pi}\int_{-\pi}^{\pi}\left[f(x)\right]^2dx$$

$$f(x)=\int_0^{\infty}\left[A(w)\cos wx+B(w)\sin wx\right]dw.$$

$$A(w)=\frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\cos wv\,dv, \qquad B(w)=\frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\sin wv\,dv$$

$$\begin{cases} \mathcal{F}\{f(x)\}=\dfrac{k}{2\pi}\displaystyle\int_{-\infty}^{\infty} f(x)\exp(\pm i\omega x)\,dx \\ \mathcal{F}^{-1}\{F(\omega)\}=\dfrac{1}{k}\displaystyle\int_{-\infty}^{\infty} F(\omega)\exp(\mp i\omega x)\,d\omega \end{cases}$$

$$\mathcal{F}\left\{f^{(n)}(x)\right\}=(i\omega)^n F(\omega)$$

$$\mathcal{F}\left\{x^n f(x)\right\}=i^n\frac{d^n}{d\omega^n}\left[F(\omega)\right]$$

$$\mathcal{F}\left\{x^m f^{(n)}(x)\right\}=(i)^{m+n}\frac{d^m}{d\omega^m}\left[\omega^n F(\omega)\right]$$

$$\mathcal{F}\{f(ax)\}=\frac{1}{a}F(\omega/a) \quad , \quad (a>0)$$

$$\mathcal{F}\{f(x-a)\}=\exp(-i\omega a)F(\omega)$$

$$\mathcal{F}\{\exp(i\lambda x)f(x)\}=F(\omega-\lambda)$$

$${}_x\mathcal{F}\{f(x,t)\}=F(\omega,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x,t)\exp(-i\omega x)\,dx$$

$$f(x,t)={}_x\mathcal{F}^{-1}\{F(\omega,t)\}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(\omega,t)\exp(i\omega x)\,d\omega$$

$${}_x\mathcal{F}\left\{\frac{\partial^n}{\partial x^n}\left[f(x,t)\right]\right\}=(i\omega)^n F(\omega,t)$$

$${}_x\mathcal{F}\{x^n f(x,t)\}=i^n\frac{\partial^n}{\partial\omega^n}\left[F(\omega,t)\right]$$

$${}_x\mathcal{F}\left\{x^m\frac{\partial^n}{\partial x^n}\left[f(x,t)\right]\right\}=i^{m+n}\frac{\partial^m}{\partial\omega^m}\left[\omega^n F(\omega,t)\right]$$

$$F_C(\omega)=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\cos\omega x\,dx$$

$$f(x)=\sqrt{\frac{2}{\pi}}\int_0^{\infty} F_C(\omega)\cos\omega x\,d\omega$$

$$F_S(\omega)=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin\omega x\,dx$$

$$f(x)=\sqrt{\frac{2}{\pi}}\int_0^{\infty} F_S(\omega)\sin\omega x\,d\omega$$

$$\mathcal{F}_C\{f'(x)\}=\omega\mathcal{F}_S\{f(x)\}-\sqrt{\frac{2}{\pi}}f(0)$$

$$\mathcal{F}_C\{f''(x)\}=-\omega^2\mathcal{F}_C\{f(x)\}-\sqrt{\frac{2}{\pi}}f'(0)$$

$$\mathcal{F}_S\{f'(x)\}=-\omega\mathcal{F}_C\{f(x)\}$$

$$\mathcal{F}_S\{f''(x)\}=-\omega^2\mathcal{F}_S\{f(x)\}+\sqrt{\frac{2}{\pi}}\omega f(0)$$

$\mathcal{F}_C\{\cos(ax)\, f(x)\} = \frac{1}{2}\{F_C(\omega+a)+F_C(\omega-a)\}$	$\mathcal{F}_C\{\sin(ax)\, f(x)\} = \frac{1}{2}\{F_S(\omega+a)+F_S(\omega-a)\}$
$\mathcal{F}_S\{\cos(ax)\, f(x)\} = \frac{1}{2}\{F_S(\omega+a)+F_S(\omega-a)\}$	$\mathcal{F}_S\{\sin(ax)\, f(x)\} = \frac{1}{2}\{F_C(\omega-a)-F_C(\omega+a)\}$
$\mathcal{F}_C\{f(ax)\} = \frac{1}{a}F_C(\omega/a) \quad , \quad (a>0)$	$\mathcal{F}_S\{f(ax)\} = \frac{1}{a}F_S(\omega/a) \quad , \quad (a>0)$
${}_x\mathcal{F}_C\{f(x,y)\} = F_C(\omega,y) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x,y)\cos\omega x\,dx$	${}_y\mathcal{F}_S\{f(x,y)\} = F_S(x,\omega) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x,y)\sin\omega y\,dy$
${}_x\mathcal{F}_C\{f'(x,t)\} = \omega F_S(\omega,t) - \sqrt{\frac{2}{\pi}}f(0,t)$	${}_x\mathcal{F}_S\{f'(x,t)\} = -\omega F_C(\omega,t)$
${}_x\mathcal{F}_C\{f''(x,t)\} = -\omega^2 F_S(\omega,t) - \sqrt{\frac{2}{\pi}}f'(0,t)$	${}_x\mathcal{F}_S\{f''(x,t)\} = -\omega^2 F_S(\omega,t) + \sqrt{\frac{2}{\pi}}\omega f'(0,t)$
${}_x\mathcal{F}_C\left\{\frac{\partial^n f(x,y)}{\partial y^n}\right\} = \frac{d^n F_C(\omega,y)}{dy^n}$	$Au_{xx} + 2Bu_{xy} + Cu_{yy} = F(x, y, u, u_x, u_y)$ $Ay'^2 - 2By' + C = 0$
$F'' - \mu^2 F = 0 \quad \Rightarrow \quad F_{(x)} = C\cosh(\mu x) + D\sinh(\mu x)$ or $F_{(x)} = Ae^{\mu x} + Be^{-\mu x}$	$F'' + \mu^2 F = 0 \quad \Rightarrow \quad F_{(x)} = A\cos(\mu x) + B\sin(\mu x)$

$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$	$\tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1\mp\tan\alpha\tan\beta}$
$\cot(\alpha\pm\beta) = \frac{\cot\alpha\cot\beta\mp 1}{\cot\beta\pm\cot\alpha}$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha,$ $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha,$ $\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$	$\sin 2\alpha = 2\sin\alpha\cos\alpha,$ $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha,$ $\sin 4\alpha = 8\cos^3\alpha\sin\alpha - 4\cos\alpha\sin\alpha.$
$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2},$ $\cos\alpha + \cos\beta = 2\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha-\beta}{2}$ $\tan\alpha\pm\tan\beta = \frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta},$ $\tan\alpha + \cot\beta = \frac{\cos(\alpha-\beta)}{\cos\alpha\sin\beta},$	$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2},$ $\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2},$ $\cot\alpha\pm\cot\beta = \pm\frac{\sin(\alpha\pm\beta)}{\sin\alpha\sin\beta},$ $\cot\alpha - \tan\beta = \frac{\cos(\alpha+\beta)}{\sin\alpha\cos\beta}.$	$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)],$ $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$ $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)],$ $\sin^2\alpha = \frac{1}{2}(1-\cos 2\alpha),$ $\cos^2\alpha = \frac{1}{2}(1+\cos 2\alpha)$
$\sin^3\alpha = \frac{1}{4}(3\sin\alpha - \sin 3\alpha)$	$\cos^3\alpha = \frac{1}{4}(\cos 3\alpha + 3\cos\alpha)$	

$\sqrt[n]{z}=\sqrt[n]{r}\left(\cos\frac{\theta+2k\pi}{n}+i\sin\frac{\theta+2k\pi}{n}\right)$	$\lvert z-a\rvert=\rho$
$w=f(z)=u(x,y)+iv(x,y).$ $u_x=v_y, \qquad u_y=-v_x$ $f'(z)=u_x+iv_x.$ $f'(z)=-iu_y+v_y.$	$f'(z_0)=\lim\limits_{z\to z_0}\frac{f(z)-f(z_0)}{z-z_0}$ ---------------------------------- $\cos z=\frac{1}{2}(e^{iz}+e^{-iz}), \qquad \sin z=\frac{1}{2i}(e^{iz}-e^{-iz}).$
$\cos(z_1\pm z_2)=\cos z_1\cos z_2\mp\sin z_1\sin z_2$ $\sin(z_1\pm z_2)=\sin z_1\cos z_2\pm\sin z_2\cos z_1$	$\cosh z=\frac{1}{2}(e^z+e^{-z}), \qquad \sinh z=\frac{1}{2}(e^z-e^{-z}).$
$\cosh iz=\cos z, \qquad \sinh iz=i\sin z.$ $\cos iz=\cosh z, \qquad \sin iz=i\sinh z.$	$\ln z=\ln\lvert z\rvert+i\arg z=\ln\lvert z\rvert+i\,\mathrm{Arg}\,z\pm 2n\pi i$ --------------------- $-\pi<\mathrm{Arg}\,z\leqq\pi.$
$z^c=e^{c\ln z} \qquad (c\text{ complex}, z\neq 0).$	$z(t)=x(t)+iy(t) \qquad (a\leqq t\leqq b).$ $\dot{z}(t)=\frac{dz}{dt}=\dot{x}(t)+i\dot{y}(t)$
$\int_C f(z)\,dz=\int_a^b f[z(t)]\dot{z}(t)\,dt$	$\oint_C\frac{f(z)}{z-z_0}dz=2\pi i f(z_0)$ **(Cauchy's integral formula)**
$f^{(n)}(z_0)=\frac{n!}{2\pi i}\oint_C\frac{f(z)}{(z-z_0)^{n+1}}dz \qquad (n=1,2,\cdots);$	$\sum_{n=0}^{\infty}a_n(z-z_0)^n=a_0+a_1(z-z_0)+a_2(z-z_0)^2+\cdots$
سری تیلور $f(z)=\sum_{n=0}^{\infty}a_n(z-z_0)^n$ $a_n=\frac{1}{n!}f^{(n)}(z_0)$ $a_n=\frac{1}{2\pi i}\oint_C\frac{f(z^*)}{(z^*-z_0)^{n+1}}dz^*.$ $R_n(z)=\frac{(z-z_0)^{n+1}}{2\pi i}\oint_C\frac{f(z^*)}{(z^*-z_0)^{n+1}(z^*-z)}dz^*$	$\frac{1}{1-z}=\sum_{n=0}^{\infty}z^n=1+z+z^2+\cdots \qquad (\lvert z\rvert<1).$ $e^z=\sum_{n=0}^{\infty}\frac{z^n}{n!}=1+z+\frac{z^2}{2!}+\cdots$ $\cos z=\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n}}{(2n)!}=1-\frac{z^2}{2!}+\frac{z^4}{4!}-+\cdots$ $\sin z=\sum_{n=0}^{\infty}(-1)^n\frac{z^{2n+1}}{(2n+1)!}=z-\frac{z^3}{3!}+\frac{z^5}{5!}-+\cdots$

$$f(z) = f(z_0) + \frac{z - z_0}{1!} f'(z_0) + \frac{(z - z_0)^2}{2!} f''(z_0) + \cdots + \frac{(z - z_0)^n}{n!} f^{(n)}(z_0) + R_n(z).$$

$$f(z) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(z_0)(z - z_0)^n \qquad (|z - z_0| < R),$$

$$\cosh z = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \cdots$$

$$\sinh z = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} = z + \frac{z^3}{3!} + \frac{z^5}{5!} \cdots$$

$$\frac{1}{(1+z)^m} = (1+z)^{-m} = \sum_{n=0}^{\infty} \binom{-m}{n} z^n$$

$$= 1 - mz + \frac{m(m+1)}{2!} z^2 - \frac{m(m+1)(m+2)}{3!} z^3 + \cdots$$

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$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z - z_0)^n}$$

$$= a_0 + a_1(z - z_0) + a_2(z - z_0)^2 + \cdots$$

$$\cdots + \frac{b_1}{z - z_0} + \frac{b_2}{(z - z_0)^2} + \cdots$$

$$0 < |z - z_0| < R.$$

$$a_n = \frac{1}{2\pi i} \oint_C \frac{f(z^*)}{(z^* - z_0)^{n+1}} dz^*,$$

$$b_n = \frac{1}{2\pi i} \oint_C (z^* - z_0)^{n-1} f(z^*) \, dz^*,$$

$$\oint_C f(z)\, dz = 2\pi i b_1.$$

$$b_1 = \operatorname*{Res}_{z=z_0} f(z)$$

$$\operatorname*{Res}_{z=z_0} f(z) = b_1 = \lim_{z \to z_0} (z - z_0) f(z)$$

$$\operatorname*{Res}_{z=z_0} f(z) = \operatorname*{Res}_{z=z_0} \frac{p(z)}{q(z)} = \frac{p(z_0)}{q'(z_0)}$$

$$\oint_C f(z)\, dz = 2\pi i \sum_{j=1}^{k} \operatorname*{Res}_{z=z_j} f(z).$$

$$J = \int_0^{2\pi} F(\cos\theta, \sin\theta)\, d\theta$$

$$J = \oint_C f(z) \frac{dz}{iz}$$

$$\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta}) = \frac{1}{2}\left(z + \frac{1}{z}\right)$$

$$\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta}) = \frac{1}{2i}\left(z - \frac{1}{z}\right)$$

$$\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{+\infty} \frac{P(x)}{Q(x)}\,dx = 2\pi i \sum_{y>0} \operatorname{Res} \frac{P(z)}{Q(z)}$$

$$\int_{-\infty}^{+\infty} f(x)\cos(ax)\,dx = (-2\pi)\operatorname{Im}\left\{\sum_{y>0} \operatorname{Res}\left[f(z)e^{iaz}\right]\right\}$$

$$\int_{-\infty}^{+\infty} f(x)\sin(ax)\,dx = (2\pi)\operatorname{Re}\left\{\sum_{y>0} \operatorname{Res}\left[f(z)e^{iaz}\right]\right\}$$

$$\operatorname{Res}\left[f(z)\right]_{z=z_0} = \lim_{z\to z_0} \frac{1}{(N-1)!} \frac{d^{N-1}}{dz^{N-1}}\left[(z - z_0)^N f(z)\right]$$

$$\text{pr. v.} \int_{-\infty}^{\infty} f(x)\, dx = 2\pi i \sum \operatorname{Res} f(z) + \pi i \sum \operatorname{Res} f(z)$$