معادله‌های دیفرانسیل پاره‌ای زیر را حل کنید.

۱-

$$\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}\quad,\quad \begin{cases} u(x,0)=x+1 & , \quad 0\le x\le 2 \\ u_t(x,0)=\sin x & , \quad 0\le x\le 2 \\ u(0,t)=1 & , \quad t>0 \\ u(2,t)=3 & , \quad t>0 \end{cases}$$

Ans. $u(x,t)=x+1+\sum_{n=1}^{\infty}\frac{1}{2\pi n}\left[\frac{1}{1-n\pi/2}\sin(2-n\pi)-\frac{1}{1+n\pi/2}\sin(2+n\pi)\right]\sin(n\pi t)\sin\left(\frac{n\pi}{2}x\right)$

۲-

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}\quad,\quad 0<x<1\quad,\quad t>0$$

$$\left\{u_x(0,t)=0,\quad u_x(1,t)=0\quad,\quad u(x,0)=f(x)=\begin{cases}1 & , \quad 0<x<0.5\\ 0 & , \quad 0.5<x<1\end{cases}\right.$$

Ans. $u(x,t)=\frac{1}{2}+\sum_{n=1}^{\infty}\left(\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)\right)\exp\left(-n^2\pi^2 t\right)\cos(n\pi x)$

۳-

$$2\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}\quad,\quad -1<x<1\quad,\quad t>0$$

$$\left\{u(-1,t)=u(1,t),\quad u_x(-1,t)=u_x(1,t)\quad,\quad u(x,0)=|x|\right\}$$

Ans. $u(x,t)=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{2}{n^2\pi^2}\left[(-1)^n-1\right]\exp\left(-2n^2\pi^2 t\right)\cos(n\pi x)$

۴-

$$\frac{\partial u}{\partial t}=3\frac{\partial^2 u}{\partial x^2}\quad,\quad t>0,\quad 0<x<2$$

$$\left\{u(0,t)=2,\quad u(2,t)=6\quad,\quad u(x,0)=x\right\}$$

Ans. $u(x,t)=2(x+1)+\sum_{n=1}^{\infty}\frac{4}{n\pi}\left[2\cos(n\pi)-1\right]\exp\left(-3/4\,n^2\pi^2 t\right)\sin\left(\frac{n\pi}{2}x\right)$

۵-

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}+u\quad,\quad u(0,t)=u(1,t)=0\quad,\quad u(x,0)=\begin{cases}1 & , \quad 0<x<1/2\\ 0 & , \quad 1/2<x<1\end{cases}$$

Ans. $u(x,t)=\sum_{n=1}^{\infty}\frac{2}{n\pi}\left[1-\cos\left(\frac{n\pi}{2}\right)\right]e^{-(1+n^2\pi^2)t}\sin(n\pi x)$