## PROBLEMS

5.1 through 5.6 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the equations of the shear and bending-moment curves.


Fig. P5.1


Fig. P5.3


Fig. P5.5


Fig. P5.2


Fig. P5.4


Fig. P5.6
5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value $(a)$ of the shear, $(b)$ of the bending moment.


Fig. P5.7
Fig. P5.8
5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value $(a)$ of the shear, $(b)$ of the bending moment.


Fig. P5.9


Fig. P5.10
5.11 and 5.12 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value $(a)$ of the shear, $(b)$ of the bending moment.


Fig. P5.11
5.13 and 5.14 Assuming that the reaction of the ground is uniformly distributed, draw the shear and bending-moment diagrams for the beam $A B$ and determine the maximum absolute value $(a)$ of the shear, $(b)$ of the bending moment.


Fig. P5.13


Fig. P5.14
5.15 and 5.16 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at $C$.


Fig. P5.15


Fig. P5.12


Fig. P5.16

326 Analysis and Design of Beams for Bending
5.17 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at $C$.


Fig. P5. 17
5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section $a-a$.


Fig. P5. 18
5.19 and 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at $C$.


Fig. P5.20
5.21 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.


Fig. P5.21
5.22 and 5.23 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.


Fig. P5.22
5.24 and 5.25 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.


Fig. P5.24
5.26 Knowing that $W=12 \mathrm{kN}$, draw the shear and bending-moment diagrams for beam $A B$ and determine the maximum normal stress due to bending.
5.27 Determine (a) the magnitude of the counterweight $W$ for which the maximum absolute value of the bending moment in the beam is as small as possible, ( $b$ ) the corresponding maximum normal stress due to bending. (Hint: Draw the bending-moment diagram and equate the absolute values of the largest positive and negative bending moments obtained.)
5.28 Determine (a) the distance $a$ for which the absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)


Fig. P5.25


Figs. P5.26 and P5.27


Fig. P5.28
5.29 Determine ( $a$ ) the distance $a$ for which the absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)


Fig. P5.29
5.30 Knowing that $P=Q=480 \mathrm{~N}$, determine ( $a$ ) the distance $a$ for which the absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)


Fig. P5.30
5.31 Solve Prob. 5.30, assuming that $P=480 \mathrm{~N}$ and $Q=320 \mathrm{~N}$.
5.32 A solid steel bar has a square cross section of side $b$ and is supported as shown. Knowing that for steel $\rho=7860 \mathrm{~kg} / \mathrm{m}^{3}$, determine the dimension $b$ for which the maximum normal stress due to bending is (a) 10 MPa , (b) 50 MPa .


Fig. P5.32
5.33 A solid steel rod of diameter $d$ is supported as shown. Knowing that for steel $\gamma=490 \mathrm{lb} / \mathrm{ft}^{3}$, determine the smallest diameter $d$ that can be used if the normal stress due to bending is not to exceed 4 ksi.


Fig. P5.33

## PROBLEMS

5.34 Using the method of Sec. 5.3, solve Prob. 5.1a.
5.35 Using the method of Sec. 5.3, solve Prob. 5.2a.
5.36 Using the method of Sec. 5.3, solve Prob. 5.3a.
5.37 Using the method of Sec. 5.3, solve Prob. 5.4a.
5.38 Using the method of Sec. 5.3, solve Prob. 5.5a.
5.39 Using the method of Sec. 5.3, solve Prob. 5.6a.
5.40 Using the method of Sec. 5.3, solve Prob. 5.7.
5.41 Using the method of Sec. 5.3, solve Prob. 5.8.
5.42 Using the method of Sec. 5.3, solve Prob. 5.9.
5.43 Using the method of Sec. 5.3, solve Prob. 5.10.
5.44 and 5.45 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value $(a)$ of the shear, $(b)$ of the bending moment.


Fig. P5.44


Fig. P5.45


Fig. P5.50
5.46 Using the method of Sec. 5.3, solve Prob. 5.15.
5.47 Using the method of Sec. 5.3, solve Prob. 5.16.
5.48 Using the method of Sec. 5.3, solve Prob. 5.18.
5.49 Using the method of Sec. 5.3, solve Prob. 5.19.
5.50 For the beam and loading shown, determine the equations of the shear and bending-moment curves and the maximum absolute value of the bending moment in the beam, knowing that (a) $k=1$, (b) $k=0.5$.
5.51 and 5.52 Determine (a) the equations of the shear and bending-moment curves for the beam and loading shown, (b) the maximum absolute value of the bending moment in the beam.


Fig. P5.51


Fig. P5.52
5.53 Determine (a) the equations of the shear and bending-moment curves for the beam and loading shown, (b) the maximum absolute value of the bending moment in the beam.


Fig. P5.53
5.54 and 5.55 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.


Fig. P5.54
5.56 and 5.57 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.


Fig. P5.56


Fig. P5.55


Fig. P5.57
5.58 and 5.59 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.


Fig. P5.58
5.60 Beam $A B$, of length $L$ and square cross section of side $a$, is supported by a pivot at $C$ and loaded as shown. (a) Check that the beam is in equilibrium. (b) Show that the maximum stress due to bending occurs at $C$ and is equal to $w_{0} L^{2} /(1.5 a)^{3}$.


Fig. P5.60


Fig. P5.61
5.61 Knowing that beam $A B$ is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.
*5.62 The beam $A B$ supports a uniformly distributed load of $480 \mathrm{lb} / \mathrm{ft}$ and two concentrated loads $\mathbf{P}$ and $\mathbf{Q}$. The normal stress due to bending on the bottom edge of the lower flange is +14.85 ksi at $D$ and +10.65 ksi at $E$. (a) Draw the shear and bending-moment diagrams for the beam. (b) Determine the maximum normal stress due to bending that occurs in the beam.


Fig. P5.62

## PROBLEMS



Fig. P5.65


Fig. P5.67


Fig. P5.69
5.65 and 5.66 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa .


Fig. P5.66
5.67 and 5.68 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi .


Fig. P5.68
5.69 and 5.70 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa .


Fig. P5.70
5.71 and 5.72 Knowing that the allowable stress for the steel used is 24 ksi , select the most economical wide-flange beam to support the loading shown.


Fig. P5.71


Fig. P5.72
5.73 and 5.74 Knowing that the allowable stress for the steel used is 160 MPa , select the most economical wide-flange beam to support the loading shown.


Fig. P5.73


Fig. P5.74
5.75 and 5.76 Knowing that the allowable stress for the steel used is 160 MPa , select the most economical S-shape beam to support the loading shown.


Fig. P5.75


Fig. P5.76
5.77 and 5.78 Knowing that the allowable stress for the steel used is 24 ksi , select the most economical S-shape beam to support the loading shown.


Fig. P5.77


Fig. P5.78


Fig. P5.80


Fig. P5.82


Fig. P5.84
5.79 Two L102 $\times 76$ rolled-steel angles are bolted together and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 140 MPa , determine the minimum angle thickness that can be used.


Fig. P5.79
5.80 Two rolled-steel channels are to be welded back to back and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 30 ksi , determine the most economical channels that can be used.
5.81 Three steel plates are welded together to form the beam shown. Knowing that the allowable normal stress for the steel used is 22 ksi, determine the minimum flange width $b$ that can be used.


Fig. P5.81
5.82 A steel pipe of $100-\mathrm{mm}$ diameter is to support the loading shown. Knowing that the stock of pipes available has thicknesses varying from 6 mm to 24 mm in $3-\mathrm{mm}$ increments, and that the allowable normal stress for the steel used is 150 MPa , determine the minimum wall thickness $t$ that can be used.
5.83 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 24 ksi , select the most economical wide-flange beam to support the loading shown.


Fig. P5.83
5.84 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa , select the most economical wide-flange beam to support the loading shown.
5.85 and 5.86 Determine the largest permissible value of $\mathbf{P}$ for the beam and loading shown, knowing that the allowable normal stress is +6 ksi in tension and -18 ksi in compression.


Fig. P5.85
5.87 Determine the largest permissible distributed load $w$ for the beam shown, knowing that the allowable normal stress is +80 MPa in tension and -130 MPa in compression.


Fig. P5.87
5.88 Solve Prob. 5.87, assuming that the cross section of the beam is reversed, with the flange of the beam resting on the supports at $B$ and $C$.
5.89 A 54-kip load is to be supported at the center of the 16 -ft span shown. Knowing that the allowable normal stress for the steel used is 24 ksi , determine ( $a$ ) the smallest allowable length $l$ of beam $C D$ if the $\mathrm{W} 12 \times 50$ beam $A B$ is not to be overstressed, $(b)$ the most economical W shape that can be used for beam $C D$. Neglect the weight of both beams.
5.90 A uniformly distributed load of $66 \mathrm{kN} / \mathrm{m}$ is to be supported over the $6-\mathrm{m}$ span shown. Knowing that the allowable normal stress for the steel used is 140 MPa , determine $(a)$ the smallest allowable length $l$ of beam $C D$ if the $W 460 \times 74$ beam $A B$ is not to be overstressed, ( $b$ ) the most economical $W$ shape that can be used for beam $C D$. Neglect the weight of both beams.


Fig. P5.90


Fig. P5.89

348
Analysis and Design of Beams for Bending
5.91 Each of the three rolled-steel beams shown (numbered 1, 2, and 3 ) is to carry a 64 -kip load uniformly distributed over the beam. Each of these beams has a $12-\mathrm{ft}$ span and is to be supported by the two 24 -ft rolled-steel girders $A C$ and $B D$. Knowing that the allowable normal stress for the steel used is 24 ksi, select ( $a$ ) the most economical S shape for the three beams, (b) the most economical W shape for the two girders.


Fig. P5.91
5.92 Beams $A B, B C$, and $C D$ have the cross section shown and are pinconnected at $B$ and $C$. Knowing that the allowable normal stress is +110 MPa in tension and -150 MPa in compression, determine (a) the largest permissible value of $w$ if beam $B C$ is not to be overstressed, (b) the corresponding maximum distance $a$ for which the cantilever beams $A B$ and $C D$ are not overstressed.


Fig. P5.92
5.93 Beams $A B, B C$, and $C D$ have the cross section shown and are pinconnected at $B$ and $C$. Knowing that the allowable normal stress is +110 MPa in tension and -150 MPa in compression, determine (a) the largest permissible value of $\mathbf{P}$ if beam $B C$ is not to be overstressed, $(b)$ the corresponding maximum distance $a$ for which the cantilever beams $A B$ and $C D$ are not overstressed.


Fig. P5.93
*5.94 A bridge of length $L=48 \mathrm{ft}$ is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_{U}=60 \mathrm{ksi}$. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w=0.75 \mathrm{kips} / \mathrm{ft}$ on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a=14 \mathrm{ft}$ from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_{1}$ and $\mathbf{P}_{2}$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using LRFD with the load factors $\gamma_{D}=1.25, \gamma_{L}=1.75$ and the resistance factor $\phi=0.9$. [Hint: It can be shown that the maximum value of $\left|M_{L}\right|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $a P_{2} / 2\left(P_{1}+P_{2}\right)$.]
*5.95 Assuming that the front and rear axle loads remain in the same ratio as for the truck of Prob. 5.94, determine how much heavier a truck could safely cross the bridge designed in that problem.
*5.96 A roof structure consists of plywood and roofing material supported by several timber beams of length $L=16 \mathrm{~m}$. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_{D}=$ $350 \mathrm{~N} / \mathrm{m}$. The live load consists of a snow load, represented by a uniformly distributed load $w_{L}=600 \mathrm{~N} / \mathrm{m}$, and a $6-\mathrm{kN}$ concentrated load $\mathbf{P}$ applied at the midpoint $C$ of each beam. Knowing that the ultimate strength for the timber used is $\sigma_{U}=50 \mathrm{MPa}$ and that the width of each beam is $b=75 \mathrm{~mm}$, determine the minimum allowable depth $h$ of the beams, using LRFD with the load factors $\gamma_{D}=1.2, \gamma_{L}=1.6$ and the resistance factor $\phi=0.9$.
*5.97 Solve Prob. 5.96, assuming that the $6-\mathrm{kN}$ concentrated load $\mathbf{P}$ applied to each beam is replaced by $3-\mathrm{kN}$ concentrated loads $\mathbf{P}_{1}$ and $\mathbf{P}_{2}$ applied at a distance of 4 m from each end of the beams.


Fig. P5.94


Fig. P5.96

## PROBLEMS



Fig. P5.99


Fig. P5. 100
5.101 through 5.103 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and load-
ing shown. (b) Use the equation obtained for $M$ to determine the tions defining the shear and bending moment for the beam and load-
ing shown. (b) Use the equation obtained for $M$ to determine the bending moment at point $E$ and check your answer by drawing the
free-body diagram of the portion of the beam to the right of $E$. bending moment at point $E$ and check your answer by drawing
free-body diagram of the portion of the beam to the right of $E$.


Fig. P5.101
5.98 through 5.100 (a) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (b) Use the equation obtained for $M$ to determine the bending moment at point $C$ and check your answer by drawing the free-body diagram of the entire beam.

Fig. P5.98



Fig. P5. 102


Fig. P5. 103
5.104 (a) Using singularity functions, write the equations for the shear and bending moment for beam $A B C$ under the loading shown. (b) Use the equation obtained for $M$ to determine the bending moment just to the right of point $B$.


Fig. P5. 104
5.105 (a) Using singularity functions, write the equations for the shear and bending moment for beam $A B C$ under the loading shown. (b) Use the equation obtained for $M$ to determine the bending moment just to the right of point $D$. tions for the shear and bending moment for the beam and loading shown. (b) Determine the maximum value of the bending moment in the beam.


Fig. P5. 106


Fig. P5. 108


Fig. P5. 107


Fig. P5. 109
5.110 and 5.111 (a) Using singularity functions, write the equations for the shear and bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.


Fig. P5. 110
Fig. P5.111
5.112 and 5.113 (a) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.


Fig. P5. 112
5.114 and 5.115 A beam is being designed to be supported and loaded as shown. (a) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the allowable normal stress for the steel to be used is 24 ksi , find the most economical wide-flange shape that can be used.


Fig. P5.114


Fig. P5.115
5.116 and 5.117 A timber beam is being designed with supports and loads as shown. (a) Using singularity functions, find the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the available stock consists of beams with an allowable stress of 12 MPa and a rectangular cross section of $30-\mathrm{mm}$ width and depth $h$ varying from 80 mm to 160 mm in $10-\mathrm{mm}$ increments, determine the most economical cross section that can be used.


Fig. P5. 116


Fig. P5.117
5.118 through 5.121 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment $\Delta L$, starting at point $A$ and ending at the right-hand support.


Fig. P5.118


Fig. P5.120


Fig. P5.119


Fig. P5.121
5.122 and 5.123 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x=0$ to $x=L$, using the increments $\Delta L$ indicated, (b) using smaller increments if necessary, determine with a $2 \%$ accuracy the maximum normal stress in the beam. Place the origin of the $x$ axis at end $A$ of the beam.


Fig. P5. 122


Fig. P5.123
5.124 and 5.125 For the beam and loading shown, and using a computer and step functions, (a) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x=0$ to $x=L$, using the increments $\Delta L$ indicated, (b) using smaller increments if necessary, determine with a $2 \%$ accuracy the maximum normal stress in the beam. Place the origin of the $x$ axis at end $A$ of the beam.


Fig. P5.124


Fig. P5.125

## *5.6 NONPRISMATIC BEAMS

Our analysis has been limited so far to prismatic beams, i.e., to beams of uniform cross section. As we saw in Sec. 5.4, prismatic beams are designed so that the normal stresses in their critical sections are at most equal to the allowable value of the normal stress for the material being used. It follows that, in all other sections, the normal stresses will be smaller, possibly much smaller, than their allowable value. A prismatic beam, therefore, is almost always overdesigned, and considerable savings of material can be realized by using nonprismatic beams, i.e., beams of variable cross section. The cantilever beams shown in the bridge during construction in Photo 5.2 are examples of nonprismatic beams.

Since the maximum normal stresses $\sigma_{m}$ usually control the design of a beam, the design of a nonprismatic beam will be optimum if the


Photo 5.2 Nonprismatic cantilever beams of bridge during construction.

## REVIEW PROBLEMS

5.152 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.


Fig. P5. 152
5.153 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.


Fig. P5. 153
5.154 Determine (a) the distance $a$ for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27.)


Fig. P5. 154
5.155 Determine (a) the equations of the shear and bending-moment curves for the beam and loading shown, (b) the maximum absolute value of the bending moment in the beam.


Fig. P5. 155
5.156 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.


Fig. P5. 156
5.157 Knowing that beam $A B$ is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.


Fig. P5. 157
5.158 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.


Fig. P5. 158
5.159 Knowing that the allowable stress for the steel used is 160 MPa , select the most economical wide-flange beam to support the loading shown.


Fig. P5.159
5.160 Determine the largest permissible value of $\mathbf{P}$ for the beam and loading shown, knowing that the allowable normal stress is +8 ksi in tension and -18 ksi in compression.


Fig. P5. 160
5.161 (a) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (b) Determine the maximum normal stress due to bending.


Fig. P5.161
5.162 The beam $A B$, consisting of an aluminum plate of uniform thickness $b$ and length $L$, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express $h$ in terms of $x, L$, and $h_{0}$ for portion AC of the beam. (b) Determine the maximum allowable load if $L=800 \mathrm{~mm}, h_{0}=200 \mathrm{~mm}, b=25 \mathrm{~mm}$, and $\sigma_{\mathrm{all}}=72 \mathrm{MPa}$.


Fig. P5. 162
5.163 A transverse force $\mathbf{P}$ is applied as shown at end $A$ of the conical taper $A B$. Denoting by $d_{0}$ the diameter of the taper at $A$, show that the maximum normal stress occurs at point $H$, which is contained in a transverse section of diameter $d=1.5 d_{0}$.


Fig. P5. 163

