$$
\begin{gathered}
\int_{10}^{v} \frac{d v}{v}=-\frac{1}{2} \int_{0}^{t} d t \\
\left.\ln v\right|_{10} ^{v}=-\left.\frac{1}{2} t\right|_{0} ^{t} \\
\ln \left(\frac{v}{10}\right)=-\frac{1}{2} t \\
v=\left(10 e^{-t / 2}\right) \mathrm{m} / \mathrm{s}
\end{gathered}
$$

## Problems

13.30 A bead moves along a taut wire as shown in Figure P13.30 such that its acceleration in $\mathrm{m} / \mathrm{s}^{2}$ is given as a function of time in $s$ by $a=6 t-14$. Initially , when $t=0 \mathrm{~s}$, the particle is 8 m to the left of the origin and moving $14 \mathrm{~m} / \mathrm{s}$
to the right. Determine (a) the velocity as a function of time, (b) the position as a function of time, and (c) the displacement during the time interval from $t=$ 4 s to $t=8 \mathrm{~s}$.


Figure P13.30.
13.31 The velocity function $v=t^{2}-25 t+150$ is specified for the rectilinear motion of a particle where $v$ is expressed in $\mathrm{ft} / \mathrm{s}$ and $t$ is expressed in s . The particle is at the origin when the clock is started. Determine (a) the acceleration as a function of time, (b) the position as a function of time, (c) the times for which the velocity equals zero, (d) the displacement from $t=0$ to $t=5 \mathrm{~s}$, and (e) the distance traveled during the time interval from $t=0$ to $t=8 \mathrm{~s}$.
13.32 A vehicle starts from rest at the origin at
$t=0$ and moves such that $a=10-5 t$ where $a$ is measured in $\mathrm{m} / \mathrm{s}^{2}$ and $t$ in s . Find the acceleration, velocity, and position of the particle when $t=2 \mathrm{~s}$.
13.33 The straight line motion of a small block shown in Figure P13.33 is such that $a=$ $A k^{2} \sinh k t$ where $a$ is expressed in $\mathrm{m} / \mathrm{s}^{2}$ and $t$ in s . The block is at the origin when the clock is started. When $t=0$, $v=10 \mathrm{~m} / \mathrm{s}$ and $t=1 \mathrm{~s}, v=20 \mathrm{~m} / \mathrm{s}$. Determine (a) the velocity as a function of time and (b) the position as a function of time.


Figure P13.33.
13.34 Rectilinear particle motion occurs such that the acceleration is given by $a=$ $A \sinh k t$ where $a$ is expressed in $\mathrm{ft} / \mathrm{s}^{2}$ and $t$ in s . When $t=0, v=100 \mathrm{ft} / \mathrm{s}$ and when $t=5 \mathrm{~s}, v=200 \mathrm{ft} / \mathrm{s}$. The ratio of $A$ to $k$ is 50 , and $s=0$ at $t=0$. Determine (a) the numerical values for $A$ and $k$, (b) the position, velocity, and acceleration of the particle when $t=2 \mathrm{~s}$ and (c) the units of $A$ and $k$.
13.35 A particle starts from rest at the origin when $t=0$ and moves according to $a=$ $t^{3}-6 t^{2}+11 t-6$ where $a$ is expressed in $\mathrm{m} / \mathrm{s}^{2}$ and $t$ in s . Find (a) the velocity as a function of time, (b) the position as a function of time, and (c) sketch the
motion curves (i.e., $a-t, v-t$ and $s-t$ curves) for the time interval from $t=0$ to $t=$ 4 s .
13.36 The acceleration of a small block, which oscillates along the inclined plane shown in Figure P13.36, is given by $a=$ $-\pi^{2} A \sin \pi t$ where $a$ is in inch $/ \mathrm{s}^{2}$ and $t$ is in s. When $t=0$, then, $v=\pi A$ in./s and $s=0$. The constant $A$ is the amplitude of the vibration. Determine (a) the velocity as a function of time, (b) the position as a function of time, and (c) sketch the motion curves (i.e., $a-t, v-t$ and $s-t$ curves) for the time interval from $t=0$ to $t=2 \mathrm{~s}$.


Figure P13.36.
13.37 In Figure P13.37 the vibratory motion of a small body along a vertical straight line is defined by $a=-A \pi^{2} \cos \pi t$ where $a$ is in $\mathrm{m} / \mathrm{s}^{2}$ and $t$ is in s . When $t=0$, $s=1 \mathrm{~m}$, and $v=0$, determine (a) the velocity as a function of time, (b) the position as a function of time, and (c) let $A=1.00$, and sketch the motion curves (i.e., $a-t, v-t$ and $s-t$ curves) for the time interval from $t=0$ to $t=2 \mathrm{~s}$.


Figure P13.37.
13.38 A particle is at rest at the origin when $t=0$. It moves such that $a=m t+b$ where $m$ and $b$ are constants and $a$ is measured in inch $/ \mathrm{s}^{2}$ and $t$ in s. Find (a) the velocity as a function of time, (b) the position as a function of time, and (c) what special case arises for $m=0$ ?
13.39 The motion of a particle along a straight line is defined by $a=A+B \sin \pi t$ where $A$ and $B$ are constants. The acceleration is measured in $\mathrm{m} / \mathrm{s}^{2}$ and the time in s . When $t=0, s=0$ and $v=-1 / \pi$, determine (a) the velocity as a function of time and specialize it for $A=2$ and $B=1$, and (b) the position as a function of time and specialize it for $A=2$ and $B=1$.
13.40 The velocity function $v=A t^{2}+B t+C$
is specified for the rectilinear motion of a particle where $v$ is expressed in $\mathrm{m} / \mathrm{s}$ and $t$ in . When $t=0, s=0, v=1 \mathrm{~m} / \mathrm{s}$; $t=0, a=2 \mathrm{~m} / \mathrm{s}^{2}$ and at $t=1, v=6$ $\mathrm{m} / \mathrm{s}$. Determine (a) the constants $A, B$, $C$. Be sure to state their units, (b) the acceleration-time function, and (c) the position-time function.
13.41 Refer to the acceleration-time plot shown in Figure P13.41 for a vehicle which moves along a straight-line path. It has an initial velocity of $10 \mathrm{ft} / \mathrm{s}$. Plot the velocity-time curve for this vehicle for the 10 s interval shown. What is the velocity of the vehicle when $t=3 \mathrm{~s}$ and when $t=10 \mathrm{~s}$ ?
13.42 Refer to the velocity-time plot shown in Figure P13.42 for a vehicle which moves


Figure P13.41.


Figure P13.42.
13.73 The velocity of a vehicle is reduced from $50 \mathrm{ft} / \mathrm{s}$ to $30 \mathrm{ft} / \mathrm{s}$ as it moves 100 ft along a straight-line path. Determine the constant deceleration of the vehicle.
13.74 The velocity of a vehicle is increased from $30 \mathrm{~m} / \mathrm{s}$ to $40 \mathrm{~m} / \mathrm{s}$ as it moves 45 m along a straight-line path. Determine the constant acceleration of the vehicle.
13.75 A high speed train is traveling along a straight, level roadbed at a speed of 240 $\mathrm{km} / \mathrm{hr}$ as shown in Figure P13.75. Determine its stopping distance if the deceleration is constant and equal to 7.0 $\mathrm{m} / \mathrm{s}^{2}$. How much time elapsed during which the brakes were applied to stop this train?


Figure P13.75.
13.76 A supersonic jet aircraft, shown in Figure P13.76, must reduce its velocity from 1000 mph to 600 mph in 5 seconds. Determine the constant deceleration required. Express your answer in $\mathrm{ft} / \mathrm{s}^{2}$. What distance did this jet plane travel in straight, level flight during this deceleration phase?
13.77 An automobile's velocity is increased from $45 \mathrm{ft} / \mathrm{s}$ to $90 \mathrm{ft} / \mathrm{s}$ as it moves 120 ft over a straight, level highway. Determine its constant acceleration. How long did this accelerating phase last?
13.78 A toy car moves with constant acceleration of $2 \mathrm{~m} / \mathrm{s}^{2}$ along a straight-line path. The positive sense is directed to the right for this motion. Initial conditions are $t=0, s=-10 \mathrm{~m} ; t=0, v=-4 \mathrm{~m} / \mathrm{s}$. Express $s$ and $v$ as functions of time. Determine the position and velocity of the toy car when $t=2 \mathrm{~s}$.
13.79 A racing car is moving along a straight stretch of race track shown in Figure P13.79. Its velocity is increased from 120 mph to 125 mph as it moves 60 ft . Determine its constant acceleration.

Figure P13.76.

$$
v_{0}=1000 \mathrm{mph}
$$

sured in $\mathrm{rad} / \mathrm{s}$, and $t$ is the time measured in $s$. Initial conditions of the motion are $t=0, x=0, y=a, \dot{x}=a \omega$, and $\dot{y}=0$. Determine the scalar components of velocity, the $x$ and $y$ coordinates of the particle at any time $t$, and the equation of the path.
A particle moves on a path in the $x-y$ plane such that $\ddot{x}=-a \omega^{2} \sin \omega t$ and $\ddot{y}=-a \omega^{2} \cos \omega t$. Initial conditions of the motion are $t=0, x=c, y=d+a$, $\dot{x}=a \omega$, and $\dot{y}=0$. Constants $a, c$, and $d$ are measured in m , the constant $\omega$ is measured in $\mathrm{rad} / \mathrm{s}$, and the time $t$ is given in s. Determine the velocity vector and the position vector at any time $t$, and find the equation of the path.
13.100 Planar motion of a particle takes place such that the scalar acceleration components are given by $\ddot{x}=2$ and $\ddot{y}=$ $12 t^{2}$ where the units are in. and s. The initial conditions are $t=0, x=c, y=$ $d, \dot{x}=0, \dot{y}=0$. Constants $c$ and $d$ are measured in inches. Determine the velocity and position vectors as functions of time. Find the equation of the path by eliminating the parameter $t$. Sketch the path and the position and velocity vectors.
13.101 Curvilinear motion takes place in the $x-y$ plane such that the scalar acceleration components are given by $\ddot{x}=12 t^{2}$ and $\ddot{y}=56 t^{6}$. Units are m and s . Initial conditions of the motion are $t=0, x=$ $c, y=d, \dot{x}=0, \dot{y}=0$. Constants $c$ and $d$ are expressed in m . Determine the velocity and position vectors of the particle when $t=1 \mathrm{~s}$ if $c=-0.5 \mathrm{~m}$ and $d=2 \mathrm{~m}$. Sketch the path and the required vectors.
13.102 The position vector from a fixed origin to a particle moving along a curvilinear path is given by $\mathbf{r}=\left(c+t^{2}\right) \mathbf{i}+$ $\left(d+t^{6}\right) \mathrm{j}$. Units are ft and s , and the constants $c$ and $d$ are given in ft. Deter-
mine (a) the path and its equation, (b) the velocity and acceleration vectors, and (c), for $t=1 \mathrm{~s}, c=1 \mathrm{ft}, d=-2 \mathrm{ft}$, sketch the path, and show the above vectors on your sketch.
13.103 A particle moving along a curvilinear path in the $x-y$ plane is positioned by the vector $\mathbf{r}=a \sin \omega t \mathbf{i}+b \cos \omega t \mathbf{j}$, where the constants $a$ and $b$ are given in m , the constant $\omega$ is given in rad/s, and time is given in s. Determine (a) the path and its equation, (b) the velocity vector, and (c) the acceleration vector.
13.104 Acceleration components for the motion of a particle in the $x-y$ plane are given by $\ddot{x}=-a \omega^{2} \sin \omega t$ and $\ddot{y}=$ $-b \omega^{2} \cos \omega t$. Initial conditions of the motion are $t=0, x=0, y=b, \dot{x}=a \omega$, $\dot{y}=0$. Constants $a$ and $b$ are measured in ft ., the constant $\omega$ is measured in $\mathrm{rad} / \mathrm{s}$, and time is measured in s . Determine the velocity vector and the position vector at any time $t$, and find the equation of the path. Sketch the path and the velocity and position vectors. Why is this a periodic motion?
13.105 The straight-line path of a toy train is given by $y=6+4 x$ where $x=4 t$. Units are in. and s. Find $y$ as a function of $t$ and the vectors $\mathbf{r}, \mathbf{v}$ and $\mathbf{a}$. Is the train accelerating?
13.106 The position of a cannon ball shown in Figure P13.106 may be determined


Figure P13.106.
from the equations $x=x_{0}+\left(v_{0} \sin \theta\right) t$ and $y=y_{0}+\left(v_{0} \cos \theta\right) t-\frac{1}{2} g t^{2}$, where $\left(x_{0}, y_{0}\right)$ denotes the initial position of the projectile when it is fired at an angle $\theta$ to the horizontal and $v_{0}$ is its initial velocity. The ball's range and its altitude during its motion are such that the acceleration due to gravity $g$ may be assumed constant. Atmospheric resistance is neglected. (a) Differentiate these equations for $x$ and $y$ with respect to time to write equations for the velocity components in the $x$ and $y$ directions. (b) Differentiate the velocity components equations to obtain the acceleration components in the $x$ and $y$ directions.
13.107 A baseball is hit as indicated in Figure P13.107. You are to decide which trajectory is the correct one. If trajectory 1 is correct, where does the ball hit the ground, assuming a fielder is unable to catch it? If trajectory 2 is correct, where will the ball hit the wall? If trajectory 3 is correct, by what distance will the ball clear the wall? The origin is selected at the point where the ball is hit. Break the motion into $x$ and $y$ components. The velocity in the $x$ direction remains constant if we neglect air resistance. The constant acceleration in the $y$ direction is $-g=-32.2$ $\mathrm{ft} / \mathrm{s}^{2}$.


Figure P13.107.
13.108 A projectile is fired from the origin with an initial velocity of $400 \mathrm{ft} / \mathrm{s}$ at an angle of $30^{\circ}$ to the horizontal. (a) Write the equation of its path, (b) Determine the range of this projectile, and (c) Find the maximum height above the horizontal reached by the projectile.
13.109 A projectile is fired from the point whose coordinates are $x_{0}=200 \mathrm{ft}$, $y_{0}=400 \mathrm{ft}$. It has an initial velocity of $500 \mathrm{ft} / \mathrm{s}$ and makes an initial angle of $40^{\circ}$ with the positive $x$ axis. (a) Express $x$ and $y$ as functions of time. (b) Deter-
mine the $x$ coordinate on the path corresponding to $y=-100 \mathrm{ft}$.
13.110 A projectile is launched from the origin with an initial velocity of $300 \mathrm{~m} / \mathrm{s}$ at an angle of $\theta$ to the horizontal. It strikes a point 2000 m to the right of the origin on a horizontal plane. Determine the two possible values for $\theta$, and sketch the trajectories.
13.111 A person is shot from a cannon at a circus as shown in Figure P13.111. For trajectory 1 , find $\theta_{1}$, and, for trajectory 2 , find $\theta_{2}$. Assume $v_{1}=v_{2}=200 \mathrm{ft} / \mathrm{s}$,

